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Marx Generators and Marx-Like Circuits

The simplest and most widely used high-voltage impulse generator is the device Erwin Marx introduced in 1925 for testing high-voltage components and equipment for the emerging power industry. The basic operation of a Marx generator is simple: Capacitors are charged in parallel through high impedances and discharged in series, multiplying the voltage. This simplicity, however, is somewhat misleading: The design of a Marx generator, when stray reactance is included and reliability and precise timing are needed, can be incredibly complex.

This chapter discusses the principles of operation and overall performance of Marx generators. For instruction, the design formulas for simple Marx generators based on their equivalent circuits are given in considerable detail. Some aspects are highlighted in the discussion of modified Marx configurations. The importance of overvoltages to Marx operation, as well as advanced triggering techniques, are reviewed. Various aspects of Marx generators such as electrical insulation, delay time and jitter, and the selection of components are discussed. A rigorous analysis is performed for pulse shaping using resistors, which is common in impulse generators.

1.1 Operational Principles of Simple Marxes

A Marx generator is a voltage-multiplying circuit that charges a number of capacitors in parallel and discharges them in series. The process of transforming from a parallel circuit to a series one is known as “erecting the Marx.” In the common parlance, a “stage” is comprised of energy storage and switch. The energy storage elements are usually one or more capacitors, but pulse forming networks or transmission lines may also be used. The switches are almost always gas-insulated spark gaps, with varying sophistication, but other types of switches with low leakage current may be used. A careful evaluation of the role of overvoltage on the switches, however, is recommended.
Figure 1.1 shows two simple ladder-type Marx generators, where a number $N$ of capacitors with a capacitance value $C_0$ are charged in parallel through charging resistors $R$ to a voltage $V_0$ and discharged in series through the spark gaps, producing an open-circuit voltage $V_{OC}$. The resistors play a dual role: During the charge cycle, the capacitors charge through the resistors on one side while the other completes the circuit to ground. During the discharge cycle, the resistors provide a high-impedance path, forcing the current through the spark gap. The resistance values are chosen sufficiently high to limit the current through the resistors and $R \sim$ few k$\Omega$ to a few $\sim$ M$\Omega$ is sufficient. The charge and discharge cycles and the erection of the Marx are treated separately. Inductors may also be used as isolation impedances.

The circuit of Figure 1.1a is an implementation that produces an output with the same polarity as the charge voltage. The advantage of this circuit is the elimination of a switch if the load can tolerate a modest DC voltage during the charge cycle. However, if the first switch is a triggered spark gap, as the Marx erects, a high-voltage transient is introduced into the trigger circuit. This issue is resolved in the circuit of Figure 1.1b. The first spark gap is at ground potential, making the choice of a trigatron as the triggered switch particularly attractive as

![Figure 1.1](image)

**Figure 1.1** Ladder-type Marx generators may either (a) preserve the polarity of the charging voltage or (b) invert it.
the trigger pin may be embedded directly into the grounded electrode. The polarity of the output voltage is inverted from the charging polarity.

### 1.1.1 Marx Charge Cycle

During the charge cycle, the Marx charges a number of stages, $N$, each with capacitance $C_0$ to a voltage $V_0$, through a chain of charging resistors $R$, as shown in Figure 1.2.

The capacitors do not charge instantaneously, but do charge at different rates and sequentially. The time to charge the $n$th stage with a DC source is given approximately by Fitch [1] and validated by a rigorous analysis by Swift [2], as

$$\tau_{ch} = n^2 R C_0, \quad \text{where} \quad 1 \leq n \leq N$$

(1.1)

It is often advantageous to minimize the charge time $T_M$ of Marx, which may be done by using a constant current-charging source. The $N$th stage is the last to reach the full charge voltage, and the minimum charge time may be determined by the acceptable difference in charge voltage between the first and $N$th (last) stage, illustrated in Figure 1.2, and given by [3]

$$\frac{V_{C,N} - V_{C,1}}{V_C} = \frac{N^2 R C_0}{T_M}$$

(1.2)

Given sufficient time, the last stage will charge to the full charge voltage $V_0$. However, the charge time determines the time of electrical stress on the insulation of the Marx, increasing the probability of unintentional insulation failure. Thus, minimizing the time to charge the Marx increases the reliability in highly stressed designs (Figure 1.3).

Moreover, during Marx erection, as the each stage is switched, the stored energy in that stage begins to discharge through the resistors $R$ on each side with a time constant:

$$\tau_{disch} = \frac{R C_0}{2}$$

(1.3)

Figure 1.2 The Marx circuit during the charge cycle. Larger stage capacitances require longer charge times.
Energy dissipated in the resistor chain is energy lost to the load and contributes to inefficiency. The discharge time of the Marx is load dependent and should be kept short compared to \( \tau_{\text{disch}} \) for maximum extraction of energy and generator efficiency. The maximum energy stored in the Marx is

\[
E_{\text{stored}} = \frac{1}{2} (N C_0) V_0^2 \quad (1.4)
\]

where \((N C_0)\) is the parallel combination of the stage capacitance. The actual value is reduced from this maximum because of the reduced charge voltage on the upper stages and the energy lost to the resistors during the charge process.

### 1.1.2 Marx Erection

The Marx erection is the process of sequentially closing the switches to reconfigure the capacitors from the parallel charging circuit to the series discharge circuit. Marx erection is initiated when a spark gap fires resulting in an increased voltage across the remaining stages. The spark gap is said to be overvolted when the voltage exceeds its DC self-breakdown value. Sufficient spark gap overvoltages are critically important to reliable operation of the Marx.

Any switch in the Marx generator can initiate the erection process, but the maximum output voltage is ensured when the first spark gap initiates the discharge and fires each successive stage. Nonsimultaneity in the firing of spark gaps reduces the amplitude of the output voltage and distorts the waveform, as shown in Figure 1.4.

Each switch contributes its delay time and jitter to the Marx erection time and its overall jitter. Fast, reliable Marx erection requires that large overvoltages appear on each stage during discharge. The overvoltage as the Marx erects aids
in minimizing the switch jitter, but triggering methods may also be used. As Marx generators were used as primary energy storage for very high peak power applications, it was found the process of erecting a Marx is not necessarily straightforward as stray capacitances may limit the achievable stage overvoltage, the details of which are discussed in Section 1.3. Here, all stray impedances are neglected, and the erection of an ideal Marx is examined.

1.1.2.1 Switch Preionization by Ultraviolet Radiation
A convenient method of ensuring low jitter in simple Marx generators is to use the ultraviolet light generated by the firing of the first spark gap to trigger the next stage. The firing of the second stage generates ultraviolet light that aids the erection of the next stage and the Marx erects in a cascade fashion. The circuits of Figure 1.1 may be reconfigured as shown in Figure 1.5 so that the switches are arranged in line-of-sight. The spark gaps are often inserted into a pressurized gas column with the remaining components insulated with oil. This easily fabricated Marx can produce hundreds of kilovolts and erect reliably.

1.1.2.2 Switch Overvoltages in an Ideal Marx
Switch overvoltages during erection can be investigated with the aid of Figure 1.6 where, for simplicity, an ideal Marx has infinitely large charge resistors that draw no current. The remaining lossless circuit is a string of capacitors. Each stage capacitor \( C_0 \) is charged to \( V_0 \). Note that in this model, the voltages across each spark are equal and represented by \( V_s \).

Figure 1.4 Sequential Marx erection minimizes pulse distortion and maximizes voltage.

Figure 1.5 A Marx circuit arranged to allow the ultraviolet light generated by the closing of a spark gap to preionize the other spark gaps. The solid line represents a cylindrical support for the spark gaps and the resistors may be wound around the cylinder to connect the capacitors.
The unfired Marx draws no current and $V_{OC} = 0$. Applying Kirchoff’s voltage law (KVL) to each stage, the voltage across each spark gap is equal in magnitude and opposite in sense to the stage capacitor voltage. When the first spark gap fires, its voltage goes to zero and the voltage at its upper node becomes the stage voltage $V_0$. The Marx remains unfired and the voltage is redistributed across the remaining unfired spark gap changing from $V_0$ to a higher value, determined by the number of switches that have fired. Using KVL,

$$V_{OC} = 0 = \sum_{n=1}^{N} V_0 - \sum_{n=1}^{N-1} V_g = NV_0 - (N-1)V_g$$

$$V_g = \frac{N}{N-1}V_0 \quad (1.5)$$

The firing sequence is given by the following [4]:

<table>
<thead>
<tr>
<th>Condition</th>
<th>Expression 1</th>
<th>Expression 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before firing, voltage on first gap $V_{g1}$</td>
<td>$NV_0 = NV_g$</td>
<td>$V_g = V_0$</td>
</tr>
<tr>
<td>First gap fires:</td>
<td>$NV_0 = (N-1)V_g$</td>
<td>$V_g = \frac{N}{N-1}V_0$</td>
</tr>
<tr>
<td>Second gap fires:</td>
<td>$NV_0 = (N-2)V_g$</td>
<td>$V_g = \frac{N}{N-2}V_0$</td>
</tr>
<tr>
<td>$n$th gaps fires:</td>
<td>$NV_0 = (N-2n)V_g$</td>
<td>$V_g = \frac{N}{N-n}V_0$</td>
</tr>
<tr>
<td>$(N-1)$th gap fires:</td>
<td>$NV_0 = V_g$</td>
<td>$V_g = NV_0 = V_{OC}$</td>
</tr>
</tbody>
</table>

As each spark gap fires, the voltage across each spark gap increases until the full open-circuit voltage is across the final spark gap, leading to the full erection of the Marx. Insufficient overvoltage results in late firing or nonfiring of some stages or energy following unintended paths. In practice, this model is too simplistic and implies significant overvoting that is not realized. In addition to losses in finite isolation resistors, the degree of overvoting is significantly affected by stray capacitance, which is explored further in Section 1.3.

### 1.1.3 Marx Discharge Cycle

The discharge of a Marx has two distinct phases, which will be treated separately. The first is the case where the final output gap of the Marx does not fire. The other is the case when the last gap discharges into a load.
1.1.3.1 No Fire
When the final output gap of the Marx does not fire, the circuit is represented by the equivalent circuit of Figure 1.7. The charge on the capacitors is discharged through the two charging resistors in parallel, with an effective time constant of $1/2RC_0$.

1.1.3.2 Equivalent Circuit Parameters During Discharge
Under proper Marx operation, where the last gap does fire, the Marx may be represented by the circuit in Figure 1.8. The output is essentially a capacitive discharge whose characteristics depend on the load. The Marx inductance, $L_M$, accounts for the inductance of the switches, capacitors, and connectors in the discharge circuit, and is given by

$$L_M = L_{\text{Switches}} + L_{\text{Capacitors}} + L_{\text{Connections}}$$

Marx generators may be charged in a single polarity or differentially charged. Both configurations are common and may be treated identically if the Marx parameters $C_M$, $L_M$, and the energy are properly defined.

1.1.3.2.1 Single-Polarity Charging
If the resistor values are chosen so that little discharge current is drawn, the equivalent circuit is that given in Figure 1.8. The following are the parameters of the erected Marx:

Marx equivalent erected capacitance:

$$C_M = \frac{C_0}{N} \quad (1.6)$$

Figure 1.7 The equivalent circuit of the erected Marx. If the last gap in the Marx does not fire, the charged capacitors must discharge their energy through the resistor network.

Figure 1.8 The equivalent circuit of an ideal Marx generator.
Marx erected impedance:
\[ Z_M = \sqrt{\frac{L_M}{C_M}} \]  
(1.7)

Marx intrinsic discharge time:
\[ T_M = \sqrt{L_MC_M} \]  
(1.8)

Erected Marx open-circuit voltage:
\[ V_{OC} = -NV_0 \]  
(1.9)

The Marx has an equivalent erected inductance, \( L_M \), comprised of contributions from the capacitors, switches, and connections. When fully erected, the Marx is a capacitive discharge circuit whose output characteristics depend on the load. The peak output voltage under load conditions is designated \( V_M \), to differentiate it from the open-circuit voltage \( V_{OC} \). Fast Marx generators may be characterized by their intrinsic discharge times, with 500 ns being the state of the art, but designs for significantly smaller intrinsic discharge times exist [5–7].

The discharge cycle of the Marx is strongly dependent on the load characteristics. Of particular importance is to determine the discharge time so that it can be compared with Equation 1.3. This section calculates the discharge time for the cases where the Marx discharges into two important capacitive loads \( (C_M \sim C_2 \text{ and } C_M \gg C_2) \) and a resistive load.

### 1.1.3.2.2 Dual-Polarity (Differential) Charging

A differentially charged, or bipolar, Marx, shown in Figure 1.9, uses two capacitors per stage charged to equal but opposite voltages. The differentially charged Marx has half the number of switches but at twice the stage voltage for a given output voltage. Two capacitors in series make up the stage capacitance.

In this arrangement, the Marx capacitance is given by
\[ C_M = \frac{C_0}{2N} \]  
(1.10)

The series Marx inductance, however, is lowered (compared to a single-polarity Marx with the same voltage) since \( N \) switches are used to switch \( 2N \) capacitors:
\[ L_M = NL_S + 2NL_C \]  
(1.11)

where \( L_S \) is the switch inductance and \( L_C \) is the capacitor inductance (Figure 1.9). The voltage per stage is \( 2V_0 \), giving an open-circuit voltage of
\[ V_{OC} = 2NV_0 \]  
(1.12)
The energy stored in a bipolar Marx is

\[ E_S = \frac{1}{2} (2NC_0)V_0^2 = N C_0 V_0^2 \]  

(1.13)

A high-impedance resistor to ground \( R_g \) is connected between the two capacitors of each stage. Generally, the value of \( R_g \) is chosen to be much greater than the charging resistors \( R_c \) to minimize the energy loss.

A differentially charged Marx generator is shown schematically in Figure 1.10, where the isolation resistor chain is replaced by an inductor chain to enable faster charging of the Marx. This allows higher pulse repetition rate capability and reduces the time of electric stress on the oil insulation during charging [8]. Bipolar charging, extensively used in very high energy storage generators, enables the efficient use of symmetrical three-electrode triggered

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**Figure 1.9** A differentially charged Marx generator uses a resistor to ground \( R_g \) to complete the circuit during the charge cycle. The first three switches are resistively coupled.

**Figure 1.10** A differentially charged Marx using inductors as the charging and ground isolation impedances allows for higher repetition rates.
gaps. The center electrode, held to ground potential during charging, is the triggered electrode.

1.1.4 Load Effects on the Marx Discharge

A fully erected Marx generator is essentially a capacitive discharge. Thus, the load voltage depends not only on the characteristics of the Marx but also on the characteristics of the load. This is illustrated with the important cases when the load is a capacitor and when it is a resistor.

1.1.4.1 Capacitive Loads

The case of a Marx generator charging capacitive loads is of tremendous importance in pulsed power and forms the basis of many pulse compression schemes. A charged capacitor can transfer almost all of its energy to an uncharged capacitor if connected through an inductor. It is the basis for the intermediate storage capacitor architecture used in multigigawatt pulsed power machines. A Marx generator may also be used with a peaking switch to increase its rise time on an output load.

The energy transfer from a Marx generator to a capacitive load is considered by the equivalent circuit shown in Figure 1.11. The inductance includes the internal inductance of the Marx, $L_M$, as well as any additional inductance that may be added [9]. When the spark gap switch closes, the energy stored in the Marx capacitance $C_M$ discharges through the inductor to charge the capacitor $C_2$.

Assume the erected Marx capacitance $C_M$ has an initial charge of $V_M$, which may be different from the open-circuit value $V_{OC}$. Applying Kirchhoff’s law to the circuit of Figure 1.11,

$$V_1(t) - L \left( \frac{di}{dt} \right) = V_2(t)$$  \hspace{1cm} (1.14)

where

$$V_1(t) - V_M = \frac{1}{C_M} \int i(t)dt$$  \hspace{1cm} (1.15)

**Figure 1.11** A Marx, with an equivalent capacitance $C_M$ charging a capacitive load $C_2$. 

Differentiating (1.15) once, we obtain

\[ L \left( \frac{d^2 i(t)}{dt^2} \right) + i(t) \left( \frac{1}{C_M} + \frac{1}{C_2} \right) = 0 \]  
(1.17)

With the initial conditions \( i(0) = 0 \) and \( L(di(0)/dt) = V_M \). Defining

\[ \omega = \sqrt{\frac{1}{L} \left( \frac{1}{C_M} + \frac{1}{C_2} \right)} = \sqrt{\frac{(C_M + C_2)}{LC_M C_2}} \]  
(1.18)

The solution to (1.17) for the current in the circuit is

\[ i(t) = \frac{\omega V_M \sin \omega t}{(1/C_M) + (1/C_2)} = \frac{V_M \sin \omega t}{\omega L} \]  
(1.19)

The voltages on the two capacitors are

\[ V_1(t) = V_M - \int \frac{i}{C_M} dt = V_M \left( 1 - \int \frac{\sin \omega t}{\omega L C_M} dt \right) \]  
(1.20)

which simplifies to

\[ V_1(t) = V_M - \frac{V_M C_2}{(C_M + C_2)} (1 - \cos \omega t) \]  
(1.21)

Similarly,

\[ V_2 = \int \frac{i}{C_2} dt = V_M \int \frac{\sin \omega t}{\omega L C_M} dt \]  
(1.22)

\[ V_2(t) = \frac{V_M C_M}{(C_M + C_2)} (1 - \cos \omega t) \]  
(1.23)

From Equation 1.23, the capacitive ringing gain can be defined as \( V_2/V_M \) and has a maximum value of

\[ \left. \frac{V_2}{V_M} \right|_{\text{Max}} = \frac{2C_M}{C_M + C_2} \]  
(1.24)

The ringing gain is easily measured and may be used to baseline a realized circuit against its design. When a Marx charges another capacitor through an inductor, the charging waveform has a \((1 - \cos \omega)\) waveshape.

Two cases of specific importance in pulsed power technology occur when (i) the Marx capacitance is approximately equal to the load capacitance \( C_M \sim C_2 \) and (ii) the Marx capacitance is much greater than the load capacitance, \( C_M \gg C_2 \).
1.1.4.1 Equal Marx and Load Capacitances

The case $C_M \sim C_2$ is the basis for the pulse compression schemes used in many pulsed power machines because the energy from the charged Marx generator can be transferred efficiently to the load. If the switch in Figure 1.11 is closed when the current is zero ($\omega t = \pi$), the energy is transferred from $C_M$ to $C_2$,

$$V_1 \left( t = \frac{\pi}{\omega} \right) = 0$$

and

$$V_2 \left( t = \frac{\pi}{\omega} \right) = \frac{V_M C_M}{C_M + C_2}$$

(1.26)

The voltage waveforms across the two capacitors are shown in Figure 1.12 for the case $C_M \approx C_2$. Since $C_M \sim C_2$ and $V_2 \sim V_M$, most of the energy initially stored in $C_M$ has been transferred to $C_2$, and the energy transfer can be efficient. The capacitor $C_2$ is often an intermediate storage capacitor, as discussed in Chapter 3.

![Figure 1.12](image-url)  

**Figure 1.12** For the case when $C_2 \approx C_M$, the voltage $V_1(t)$ across the capacitor $C_M$ charges the capacitor $C_2$ to a nearly identical voltage $V_2(t)$ and the energy transfer is very efficient. This is often used for the first stage of a pulse compression where the Marx charges an intermediate store capacitor in a time $t = \pi/\omega$ and the energy transfer is efficient.
1.1.4.1.2 The Peaking Circuit: $C_M \gg C_2$

The case $C_M \gg C_2$ is known as a peaking circuit. Here, energy transfer occurs when $(1 - \cos(\omega t)) = 2$, that is, when $(\omega t = \pi)$.

$$V_1\left(t = \frac{\pi}{\omega}\right) \approx V_M$$  \hspace{1cm} (1.27)

and

$$V_2\left(t = \frac{\pi}{\omega}\right) \approx 2V_M$$  \hspace{1cm} (1.28)

The voltage $V_2(t)$ across the capacitor $C_2$ is driven to nearly twice the Marx voltage, while the voltage $V_1(t)$ remains nearly the same, as shown in Figure 1.13. The energy transfer, however, is inefficient. It is usually sufficient to have a peaking capacitance such that $(C_M / C_2) \approx 10$.

1.1.4.1.3 A Peaking Circuit Driving a Resistive Load

A peaking capacitor ($C_2 = C_p$) can be used in conjunction with a Marx generator to sharpen the rise time of a Marx. The circuit is shown in Figure 1.14,

![Normalized voltage V(t)/V_M](image)

**Figure 1.13** For the case when $C_2 = 0.1C_M$, the voltage $V_1(t)$ across the capacitor $C_M$ varies little, while the voltage $V_2(t)$ on capacitor $C_2$ nearly doubles. The energy transfer is not efficient. Peaking capacitors are used to increase the pulse rise time and can be very difficult to implement.
The switch $Sw_2$ is called a peaking switch and is set to switch when the current is maximum [3]. An exponential waveform can be delivered to the resistive load if the value of the peaking capacitor $C_p$ is chosen to be [10]

$$C_p = \frac{L_M C_M}{R_L^2 C_M + L_M} \tag{1.29}$$

And the switch $Sw_2$ is closed at a time $t_p$ chosen:

$$t_p = \frac{1}{\omega} \cos^{-1} \left( \frac{-C_p}{C_M} \right) = \sqrt{\frac{L_M C_M C_p}{C_M + C_p}} \cos^{-1} \left( \frac{-C_p}{C_M} \right) \tag{1.30}$$

The frequency $\omega$ is usually quite high and care must be taken on the breakdown time and jitter of the peaking switch $Sw_2$, which manifest as variations of the load voltage. In practice, it may be difficult to make the low-value, high-voltage capacitor necessary for a peaking circuit.

### 1.1.4.2 A Marx Charging a Resistive Load

The case of a Marx generator charging a resistive load finds application where the current and voltage are in phase and proportional, such as for relativistic electron beam generation. If the Marx was comprised of a pure capacitance and charged a pure resistance, the voltage on the load resistor is

$$V_L(t) = V_M e^{-t/R_L C_M} \tag{1.31}$$

When the Marx fires at $t = 0$, the load voltage jumps instantaneously to the peak Marx voltage $V_M$ and decays with a time constant equal to $R_L C_M$. This ideal case, however, is nonphysical since a Marx generator has a significant inductance. The effect of the equivalent series inductance of the Marx on a resistive load is analyzed using the circuit of Figure 1.13a.

$$V_1(t) - L \left( \frac{di}{dt} \right) = Ri(t) \tag{1.32}$$

and

$$i(t) = C_M \frac{dV_1(t)}{dt} \tag{1.33}$$

![Figure 1.14 A peaking capacitor circuit may be used to sharpen the rise time of the Marx to a load.](image)
Differentiating (1.32), substituting (1.33), and simplifying,

\[
\frac{d^2 i(t)}{dt^2} + \frac{R}{L} \left( \frac{di}{dt} \right) - \frac{1}{LC_M} i(t) = 0 \tag{1.34}
\]

With the initial conditions \(i(0) = 0\) and \(L(di(0)/dt) = V_M\) and setting

\[
\gamma = \sqrt{\left( \frac{R}{L} \right)^2 - \frac{4}{LC_M}}
\]

the solution is

\[
i(t) = \frac{V_M}{\gamma L_M} e^{-\gamma t} \left[ 1 - e^{-\gamma t} \right] \tag{1.35}
\]

Equation 1.31 is a double exponential. The rise time is determined by the \((1 - e^{-\gamma t})\) term. As \(t\) gets large, this term is overtaken by the other decaying exponential term, as illustrated in Figure 1.15b. The series inductance of the Marx plays an important role in the performance of Marx generators because it increases the rise time of the output waveform in the case of a resistively loaded Marx generator and reduces the peak current capability.

1.2 Impulse Generators

Impulse generators are an important and long-used application of Marx generators. The shape of the pulse must be tailored for the specific testing requirements.

1.2.1 Exact Solutions

For impulse testing, the load is assumed to be capacitive. The equivalent circuit of the \(N\) stage Marx generator of Figure 1.16 is shown in Figure 1.17.
The pulse duration can be adjusted by choosing proper values for the front resistor $R_F$ and tail resistor $R_T$. The value of the tail resistor in the equivalent circuit of Figure 1.17 is

$$R_T = \frac{NR}{2}$$

(1.36)

In the Laplace domain, the impedance of the equivalent circuit is

$$Z(s) = \frac{1}{C_MS} + \frac{R_T(R_F + (1/C_LS))}{R_T + R_F + (1/C_LS)}$$

$$= \frac{R_TC_Ls + R_FC_Ls + 1 + C_MC_LR_Fs^2 + C_MR_Ts}{C_MC_LR_Fs^2 + C_MC_LR_Fs^2 + C_MS}$$

(1.37)

and the voltage on the load is

$$V_L(s) = \frac{V}{s} \frac{1}{Z(s)(R_T + R_F + (1/C_LS))} C_LS$$

(1.38)

Substituting $Z(s)$ from (1.33) into (1.34) leads to

$$V_L(s) = \frac{V}{C_LR_Fs^2 + s[(1/R_FC_M) + (1/R_TC_M) + (1/R_FC_L)] + (1/R_FR_TC_MC_L)}$$

$$= \frac{V}{C_LR_F(s + \alpha)(s + \beta)}$$

(1.39)
where \( \alpha \) and \( \beta \) are roots of the following equation:

\[
 s^2 + s \left( \frac{1}{R_F C_M} + \frac{1}{R_T C_M} + \frac{1}{R_F C_L} + \frac{1}{R_F R_T C_M C_L} \right) + \frac{1}{R_F R_T C_M C_L} = 0
\]

The \( \alpha \) and \( \beta \) are related to the circuit values by

\[
\alpha + \beta = \frac{1}{R_F C_M} + \frac{1}{R_T C_M} + \frac{1}{R_F C_L} \quad \quad (1.40)
\]

\[
\alpha \beta = \frac{1}{R_F R_T C_M C_L} \quad \quad (1.41)
\]

Converting Equation 1.39 into the time domain by taking the inverse Laplace transform, the Marx output voltage \( V_L(t) \) is

\[
V_L(t) = \frac{V_M}{R_F C_L} \left( e^{-\alpha t} - e^{-\beta t} \right) \quad \quad (1.42)
\]

A typical waveform representing Equation 1.42 is shown in Figure 1.18. The time \( t' \) when the output voltage \( V_L(t) \) reaches the maximum value can be found by taking a time derivative of (1.42) and equating to 0, yielding

\[
t' = \frac{\ln(\beta/\alpha)}{\beta - \alpha} \quad \quad (1.43)
\]

The maximum amplitude of the pulse voltage \( V_L^{\text{Max}} \) is obtained by substituting the value of \( t' \) from (1.43) into (1.42):

\[
V_L^{\text{Max}} = V_L(t') = \frac{V_M}{C_L R_F} \left( \frac{1}{\beta - \alpha} \right) \left( e^{-\alpha t'} - e^{-\beta t'} \right) \quad \quad (1.44)
\]
1.2.2 Approximate Solutions

Equations 1.36–1.44 are the design formulas for the Marx generator output waveform. In most cases, the approximate formulas may suffice and are convenient for pulse shaping. If this is the case, Figure 1.18 can be used for the calculation of rise time $t_r$ and front time $t_f$. The rise time $t_r$ is defined as the time interval required for the voltage to rise from $0.1 \ V_L^{Max}$ to $0.9 \ V_L^{Max}$. The front time $t_f$ is defined as

$$t_f = 1.25 \cdot t_r$$  \hspace{1cm} (1.45)

Figure 1.19 can be used for calculation of tail time $t_t$, the time for the falling edge of the output voltage $V_L(t)$ to fall to half its peak value. The values of $t_r$ and $t_t$ are indicated on the waveforms of Figures 1.18b and 1.19b.

The approximate solution for the output voltage $V_L(t)$ for the circuit shown in Figure 1.18 has the following form [11]:

$$V_L(t) \approx \frac{V_{OC} C_M}{C_M + C_L} (1 - e^{-t/RC_T})$$  \hspace{1cm} (1.46)

where

$$C_T = \frac{C_M C_L}{C_M + C_L}$$  \hspace{1cm} (1.47)

The values of $t_r$ and $t_f$ deduced from Equation 1.46 are as follows:

$$t_r = 2.2 C_T R_F$$  \hspace{1cm} (1.48)
$$t_f = 1.25 t_r = 2.75 C_T R_F$$  \hspace{1cm} (1.49)

The approximate solution for the output voltage $V_L(t)$ for the circuit shown in Figure 1.17 has the following form [12]:

$$V_L(t) \approx \frac{V_{OC} C_M}{C_M + C_L} e^{-t/(R_T C_L)}$$  \hspace{1cm} (1.50)

**Figure 1.19** Approximate equivalent circuit for the tail time calculation and (b) the output voltage during the tail time.
The value of \( t_t \) deduced from Equation 1.50 is as follows:

\[
t_t = (0.7) \cdot R_f (C_L + C_M) \tag{1.51}
\]

Precise waveshaping can be attained with these techniques [13]. A voltage monitor is often added in parallel with the load to obtain a direct measurement. The impulse insulation testing of high-voltage insulators is a typical case of a capacitive load and therefore the above formulas apply. Impulse testing is usually notated in the form: (pulse rise time/FWHM fall time) and typical testing waveforms are 1.2/50\( \mu \)s or 8/20\( \mu \)s and are specified by specific standards. The effective capacitance of an erected Marx should be in the range of 4–10 times the load capacitance for effective voltage gain [14].

1.2.3 Distributed Front Resistors

One such modified configuration [12] is shown in Figure 1.20, where the externally connected front resistor \( R_F \) of Figure 1.16 is uniformly distributed throughout the individual stages of the Marx generator, forming an internal part of the Marx generator. The distributed front resistors need to only withstand a fraction of the total voltage. This design, however, increases the number of components in the Marx and makes varying the resistance value difficult.

1.3 Effects of Stray Capacitance on Marx Operation

The elegant Marx concept was used for many years with little change. Marx generators can be made to operate very simply when two-electrode spark gaps are triggered by overvoltages from switching previous stages and each switch contributes its delay time and jitter to the Marx erection time and the overall jitter. In the 1960s, interest in pulsed power spread rapidly as applications emerged in high-energy density physics. The increasingly specialized demands

![Figure 1.20](image-url)
revealed its limitations and sparked a period of careful study of the Marx circuit operation and construction.

These emerging capabilities had requirements that exceeded the state of the art in voltage, energy, and energy density. As facilities became large, efforts to reduce the Marx volume included insulating the Marx with transformer oil. The generation of very high voltages resulted in physically long Marx generators with many stages. These changing requirements resulted in increased stray capacitances that could no longer be neglected. Stray capacitance may limit the degree of spark gap overvoltage by establishing transient voltage dividers, resulting in increased erection time and jitter. Fast, reliable Marx erection requires that large overvoltages appear on each stage during discharge. The reduced overvoltage may have severe and unintended consequences: In developing a physically long, 42-stage, 18 MV Marx generator, Prestwich and Johnson [15,16] observed that after 23 stages fired, the switches began firing from the top, in reverse order from the output end, with the last 7 stages firing simultaneously from overvoltages, as discussed in Section 1.5.1.

There are three significant sources of stray capacitance. The first is the stray capacitance across the spark gaps. Prior to closing, a spark gap consists of two electrodes separated by an insulator, which also describes a capacitor. Thus, during the charge cycle, the spark gap may be represented by a capacitance and denoted $C_g$. The second source is the conducting connections of each stage of the Marx that are isolated from the system ground and are represented by the stray capacitance $C_s$. The third source is stray capacitance between stages. In many cases, the electrodes of the energy storage capacitors $C_0$ have only small separations between adjacent stages, leading to the stages being coupled by a capacitance $C_c$. In some designs, the stray capacitance between adjacent stages is minimized by physically separating the stages, in which case the dominant stray stage capacitance is between alternate stages and denoted $C_{R}$. The relative magnitudes of these stray capacitances determine the performance of the Marx.

This section is organized to first illustrate how stray capacitance can establish a voltage divider in the case when the nonnegligible stray capacitance consists of $C_g$ and $C_s$ and then show how the stray capacitance can be exploited. The wave erection Marx is a case when the energy is small, the component dimensions make for small inductance and coupling impedance, and the conventional Marx becomes capable of rapid erection. The next sections show the effect of the coupling capacitances $C_c$ and $C_{R}$ and how the Marx can be designed so that the stages with weak overvoltages are reinforced by capacitive or resistive coupling.

### 1.3.1 Voltage Division by Stray Capacitance

During the charge cycle, a spark gap consists of conductors separated by insulators, and is modeled as a stray capacitance $C_g$ as shown. Each conductor
in the Marx also has capacitance to ground. The equivalent circuit of a Marx, including the stray capacitance of the gap and the conductors to ground, \( C_s \), is shown in Figure 1.21.

Assume the first gap is triggered by an external source. The first stage erects, and the voltage at point \( B \), \( V_B \), equals \( V_0 \) because one terminal of the first-stage capacitance is connected directly to ground. The voltage at point \( A \) is given by \( V_A = V_0 \). Invoking \( V_B = V_0 \) yields

\[
V_A = 2V_0
\]  

(1.52)

The voltage across gap 2 is \( V_{g2} = V_A - V_D \). Recognizing the voltage at point \( D \) is the voltage across the stray capacitance \( C_s \), \( V_D = V_s \). The circuit stray capacitance across the spark gap, \( C_g \), and the stray capacitance to ground, \( C_s \), establish a voltage divider. The voltage across the spark gap can be calculated from Figure 1.21b and the voltage divider relation.

\[
2V_0 - V_{g2} = V_s
\]  

(1.53)

\[
V_s = 2V_0 \frac{C_g}{C_s + C_g}
\]  

(1.54)
Solving for $V_{g2}$,

$$V_{g2} = \frac{2V_0}{1 + (C_g/C_s)}$$

The overvoltage on gap 2 is maximized from Equation 1.55 when $C_g \ll C_s$. The stray capacitance to ground, $C_s$, can be tailored somewhat by placing a ground plane next to the Marx, which is often accomplished by enclosing the Marx in a grounded cylinder.

Switch overvoltages can be increased by triggering several switches in a Marx. If, instead of triggering just the first gap, the first $k$ gaps are triggered simultaneously, the overvoltage on the next $(k+1)$ gap is [3]

$$V_{g,k+1} = \frac{kV_0}{1 + \sqrt{1 + (4C_g/C_s)}}$$

Spark gap overvoltages are transient events. As the potential across $C_s$ rises to $V_0$, the stray capacitance to ground of the next stage begins to charge to $V_0$, limiting the overvoltage on gap 2. Care must be taken in the design to ensure the spark gaps fire before an excessive reduction in its overvoltage happens. A fast erection time ensures that switching occurs at the maximum overvoltage leading to low jitter.

### 1.3.2 Exploiting Stray Capacitance: The Wave Erection Marx

The case where the coupling capacitance between stages, $C_C$, is small is typically one using ceramic capacitors with values of a few nanofarads. Platts [17] used this approach to develop a Marx in a compact geometry that generated a high peak voltage with low energy content. The circuit is arranged in a line so that the spark gaps are line-of-sight and the ultraviolet radiation produced in the breakdown acts as a preionization source for successive spark gaps. The Marx was encased in a grounded metal tube and insulated throughout with moderate pressure gas producing an open-circuit voltage of 200 kV. This sparked a renewed interest in Marx generators for new applications directly driving a load.

The stray capacitance to ground may be utilized to produce a Marx generator with a very fast rise time by designing the Marx to act like a cascading peak circuit [3,18]. The Marx is arranged so that $C_s \gg C_g$, and the circuit may be represented as shown in Figure 1.22a. As each stage of the Marx is switched, the peaking effect, shown in Figure 1.22b, becomes more marked, resulting in a very fast rise time.

The capacitance to ground, $C_s$, may be controlled by a suitably designed, grounded metal enclosure. The stage capacitance $C_0$ and the total number of stages $N$ are selected to satisfy

$$\frac{C_0}{n} = C_n \gg C_s \quad n < N$$

(1.57)
where \( C_n \) is the capacitance of the Marx as the \( n \)th stage has switched into the series discharge circuit. As the Marx erects, the Marx capacitance \( C_n \) and the stray capacitance to ground \( C_s \) produce a peaking circuit as shown in Figure 1.23b. Each Marx stage becomes a peaking circuit charging the next stage in an increasingly fast erection rate.

Impressive performance has been reported [19–23]. Kekez built a 600 kV, 1 ns output unit [19] and a 200 kV Petit Marx had an estimated 50 ps rise time into a 100 \( \Omega \) load [20]. Mayes and his colleagues have also reported spectacular results using a wave erection scheme [21–23].

### 1.3.3 The Effects of Interstage Coupling Capacitance

For high energy storage applications, the open geometry yields a large device with significant inductance. For applications requiring high energies and relatively fast discharges, architectures become more compact to reduce the inductance but result in a significant increase in the coupling impedances. Typical high-energy density capacitors have capacitance values of a few
microfarads and are arranged so that the metallic enclosure is one of the capacitors terminals. When stacked in a Marx generator, as shown in Figure 1.24, the capacitance established between stage capacitors may be significant [24]. The coupling capacitance slows the erection process, which can only proceed as fast as the interstage capacitances can be charged and discharged. Thus, the analysis of the erection process described in Section 1.1.2 predicts large overvoltages on the upper stages that do not materialize and, more importantly, do not predict the order in which the switches fire.

The circuit of Figure 1.1 must be modified to include the stray capacitances as shown in Figure 1.24.

The stray capacitances shown in Figure 1.24 are given as follows:

\( C_{gn} \): The capacitance of the gap of the \( n \)th switch.
\( C_C \): The coupling capacitance between adjacent capacitors.

Figure 1.24 The (a) layout and (b) equivalent circuit of a Marx generator with significant stray coupling capacitances \( C_C \) between adjacent stages, as well as stray capacitance to ground and across the spark gap. The relative magnitudes determine the Marx operation.
When the Marx is fully charged, the voltage across the second spark gap, $V_{g2}$, is the voltage on capacitor 2, $V_0$. From the simplified analysis of Section 1.1.2, the voltage across the second spark gap should increase to $2V_0$ when the first spark gap fires, but, instead, the transient overvoltage encounters the capacitive voltage divider formed from the stray capacities $C_c$ and $C_{s2}$, reducing the overvoltage on the spark gap (Figure 1.25).

Only the voltage contribution from the erection of the first stage of the Marx sees the voltage divider. The charge voltage on stage 2 must be added to the divided contribution from stage 1, yielding the voltage on spark gap 2 to be

$$V_{g2} = V_0 + \frac{V_0 C_{s3}}{C_{s3} + C_c} \tag{1.58}$$

Morrison and Smith [4] did a thorough analysis of a 10-stage generator where the voltages in the Marx, including voltage division effects, were examined as the Marx erected. Their analysis showed that unexpected (and frightening) patterns can occur for various ratios of stray capacitances. Morrison and Smith use the nonsequential switch firing described by Prestwich and Johnson [15] to validate their analysis.

The effects of large interstage capacitance became apparent as Marx generator designs moved from the large, open constructions used in impulse testing to more compact configurations producing higher voltages. The high energy storage, fast discharge capacitors used in these generators typically use their outer case as one of the capacitor electrodes. When stacked in a Marx, as shown in Figure 1.24, the large area of the capacitor case results in significant interstage capacitance. Concerns about interstage capacitance are not only for high-energy generators. Ordinarily, in conventional Marx generators, the switch capacitance $C_g$ is sufficiently low such that the stray stage to ground capacitance $C_s$ is adequate to deliver a sufficient overvoltage to the next stage. In compact generators, $C_g$ may be quite high because the switches may be set with small gaps to minimize the inductance in the discharge path. Discrete interstage coupling capacitors can be used to overcome this shortcoming [25].
1.4 Enhanced Triggering Techniques

Jitter assumes importance especially in cases where the Marx generator has to be synchronized to trigger other events in a given system or to other Marx generators. A Marx’s erection time and jitter are closely linked to the degree of overvoltage on the spark gaps that may be adversely affected during the erection sequence by stray capacitance. The effects of stray capacitance can be mitigated by coupling key stages to control transients during the erection process. Interstage coupling is typically achieved with resistors or capacitors. Interstage capacitors may be lumped elements but are often attained by careful design of the layout.

1.4.1 Capacitive Back-Coupling

In the previous section, the effects of large interstage stray capacitance were shown to potentially severely affect the erection of the Marx. The interstage coupling capacitance can be reduced by arranging the Marx into columns, with each column containing alternating stages of the Marx, as illustrated in Figure 1.26.

The column arrangement changes the equivalent circuit, so the stray coupling capacitance between alternate stages, denoted $C_R$, is greater than between adjacent stages, $C_C$. This arrangement aids the erection of the Marx generator.

![Diagram](image)

**Figure 1.26** The (a) layout of a Marx arranged in two columns to promote capacitive back-coupling and (b) its equivalent circuit.
because $C_R$ is in parallel to the gap capacitance $C_g$, increasing the overvoltage. Thus, capacitive coupling between alternate stages leads to enhanced triggering, and is sometimes referred to as “capacitive back-coupling” [1].

The resultant large overvoltages on the switches permit the Marx to be operated at a fraction of its self-breakdown voltage. By rearranging the Marx into two columns, an operating range approaching 2:1 can be achieved by triggering only 1 gap. In general, the Marx can be arranged in $p$ columns with an operating range approaching $p:1$, but the first $(p-1)$ gaps must be triggered. In practice, due to stray impedances, the number of columns is limited to three. This analysis gave rise to a naming convention, not often used anymore, where the Marx is characterized by its interstage coupling with a designation of $n = p$. Thus, a $n = 2$ Marx couples to every other stage. Another type of transient voltage appears in this type of construction. For a two-column Marx, if the conductors and switches were perfect, an overvoltage of $2V_0$ would appear between the two capacitors coupled by $C_R$. By including the inductance of the connections and switches, the actual voltage is [26]

$$V_2 = 2V_0 \cdot \left(1 - \cos \left(\frac{t}{\sqrt{LC_c}}\right)\right)$$

This high-frequency oscillation can reach a peak of $4V_0$, aiding triggering further, but is generally limited in practice to $3V_0$.

### 1.4.2 Resistive Back-Coupling

The use of one or more triggered gaps is common in many Marx generators in order to reduce jitter. The erection process can be further enhanced with strategically placed three-electrode triggered spark gaps [27–29]. This technique, pioneered by J.C. (“Charlie”) Martin, uses more than one triggered spark gap combined with back-coupling to produce a highly reliable, low-jitter system and is sometimes referred to as a Martin Marx [1].

The Martin Marx is characterized by switches with trigger electrodes and an enhanced erection process. In addition to the triggered spark gaps to initiate the erection process from an external trigger generator, the Martin Marx places triggered spark gaps in upper stages and resistively couples the trigger electrode back to the earlier stages. The coupling impedance can be any combination of capacitors, inductors, or resistors but resistors are common. A common triggering scheme is shown in Figure 1.27 where every third gap is coupled and the first three stages are externally triggered.

In this circuit, the triggered spark gaps in the upper stages are used in a fundamentally different manner than in the first three stages. In the first three stages, the potential of the trigger electrode is changed, resulting in a large over-voltage and the spark gap fires. In the upper stages, the resistive coupling impedance
holds the trigger electrode at a voltage determined by its lower stage coupling partner, while the voltage on the main electrodes responds to the erection process, resulting in large potential differences between the spark gap electrodes. The use of resistive back-coupling can make the design rather complex.

The main advantages of this Marx configuration are its control of prefires and its wide operating range, which in practice depends on the magnitude and distribution of the stray capacitance [29]. Voltages as low as 30% of the self-breakdown voltage can be achieved by varying the charging voltage with no adjustments to the spark gap spacing [15,16].

1.4.3 Capacitive and Resistively Coupled Marx

Coupling can be illustrated with the aid of Figure 1.28. The capacitors can be arranged so that the stray capacitance between alternate stages, $C_R$, is large. These stages are connected with charging resistors.
By triggering the first two spark gaps, point A is maintained at a voltage $V_0$ and point B at $3V_0$. The voltage at point D will initially be determined by the voltage division among stray capacitances. If $C_s \ll C_R$, as in the case of a long Marx generator, the voltage across the third spark gap, $V_{g3}$, approximately is

$$V_{g3} = \frac{2V_0C_R}{C_g + C_R}$$  \hspace{1cm} (1.60)

The voltage across $C_R$ decays with a time constant $\tau_{dis} = R(C_g + C_R)$ and the voltage across the spark gap approaches $(2V_0)$, making an $n = 2$ Marx. These relations can be generalized [16]. If the capacitors are coupled across every $X$ capacitors, the voltage across the third spark gap, caused by the capacitive division, would immediately be

$$V_{g3} = X \frac{V_0C_R}{C_g + C_R}$$  \hspace{1cm} (1.61)
The charge resistors, coupled across every $Y$ stages, would discharge through $C_g$ and $C_R$ to give $Y V_0$ across the spark gaps, making it an $n = Y$ Marx generator.

1.4.4 The Maxwell Marx

Very high energy storage Marx generators use physically large capacitors with large stray impedances. On the other hand, when the stored energy is small, the component dimensions have small coupling impedances and the low-energy Marx can erect rapidly. This rapid erection may be exploited to trigger a larger Marx, resulting in a precise, high-energy generator, with some added complexity. A small Marx generator, with the same basic characteristics of the large Marx, may be coupled, stage by stage, with the large Marx [1]. The small, low-energy fast Marx runs in parallel with the large Marx. The erection time is set by the small, fast Marx and can result in very precise operation of the large high-energy Marx and determines its erection time. This technique has been referred to as a Maxwell Marx [1].

Thus, for a long and large Marx system, the individual trigger generators can be arranged in ascending potential to match the potential of the erecting main Marx. An implementation of this design is shown in Figure 1.29, where the high energy storage Marx, with stage capacitance $C_{0M}$, is triggered by a small fast Marx with stage capacitance $C_{0m}$. The trigger Marx is arranged so that each of its stages acts as a trigger generator for a stage of the main Marx so that the erection rate of the large high-energy Marx is dictated by the small, low-energy trigger Marx.

Delays in erection between stages are caused by the channel formation time in the switches and the time to charge the stray capacitance associated with the next stage. Care must be taken to not to discharge the main Marx through the trigger Marx.

**Figure 1.29** The Maxwell Marx uses a low-energy Marx to dictate the erection speed of a large high-energy Marx [1].
1.5 Examples of Complex Marx Generators

1.5.1 Hermes I and II

Using a differential charging scheme, where half the capacitors are charged to a positive voltage $+V_0$ and the other half to a negative voltage $-V_0$, Prestwich and Johnson [15] studied Marx generators with many stages for the HERMES program. By using a stage voltage of $2V_0$, half the number of switches is required to yield the same output voltage.

One of their generator designs is shown schematically in Figure 1.30. It is an $n = 3$ Marx, using both resistive and capacitive coupling, with a measured inductance of 69 $\mu$H. It was demonstrated that with three gaps triggered, the Marx would fire down to 30% of its self-breakdown voltage. Triggering five gaps did not increase the firing range. The firing times of the individual spark gaps were measured and shown in Figure 1.31. It is interesting that after the first 23 stages erected, the Marx began firing from the high-voltage end down to the ground end, with the last seven gaps broke down simultaneously.

These studies yielded the design for the Marx generators for the Hermes I and Hermes II machines. Hermes I is a 100 kJ design consisting of six complete
rows and 1 partial row to provide electrical grading at the high-voltage output end. The Hermes II Marx has 93 spark gaps (stages) arranged in 31 rows and stores 1 MJ when charged to $\pm 103\text{kV}$. The parameters for these two generators are summarized in Table 1.1.

### 1.5.2 PBFA and Z

The innovation of triggered three-electrode switches allowed the development of advanced triggering for precision control of even large, high energy storage Marx generators. The triggering schemes can become quite complex. The Marx developed for the Particle Beam Fusion Accelerator programs at Sandia National Laboratories is a good example of the triggering complexities.

The PBFA II energy storage section consists of 36 bipolar Marx generators. Each Marx is composed of 60 capacitors charged to $\pm 95\text{kV}$, storing $370\text{kJ}$ of energy, and yielding an erected output voltage of $5.7\text{MV}$. The ringing gain is nearly unity and the time to peak voltage is $1.1 \mu\text{s}$. The output for each Marx is connected through a single-pole, double-throw transfer switch to the first stage in the pulse compression scheme. The transfer switch is initially set to a liquid resistor dump load until the Marx is fully charged, and then the transfer switch is rotated into the firing position using a pneumatic actuator [30]. The Marx

![Figure 1.31](image)

Figure 1.31 The firing times of individual gaps in the Marx of Figure 1.30. After erection was initiated, the generator began firing from the high-voltage end [15].

<table>
<thead>
<tr>
<th></th>
<th>$V_M$</th>
<th>$R_c$</th>
<th>$C_M$</th>
<th>$L_M$</th>
<th>$R_M$</th>
<th>$C_g$</th>
<th>$C_R$</th>
<th>$C_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hermes I</td>
<td>4 MV</td>
<td>1.2 kΩ</td>
<td>13.1 nF</td>
<td>22 μH</td>
<td>4 Ω</td>
<td>$\sim 45\text{pF}$</td>
<td>$\sim 90\text{pF}$</td>
<td>$&lt;20\text{pF}$</td>
</tr>
<tr>
<td>Hermes II</td>
<td>18 MV</td>
<td>1.5 kΩ</td>
<td>5.4 nF</td>
<td>80 μH</td>
<td>20 Ω</td>
<td>$\sim 45\text{pF}$</td>
<td>$\sim 190\text{pF}$</td>
<td>$&lt;10\text{pF}$</td>
</tr>
</tbody>
</table>
switches are three-electrode, SF6-filled spark gaps triggered through a midplane, field-enhanced electrode located at the midplane.

The reliability of low-jitter Marx generators was improved dramatically for PBFA II, and the basic design, with the exception of higher energy density capacitors, is still used on Z. Circuit modeling and design prototypes were studied extensively to determine the implications of stray charging paths. It was found that stray capacities within the Marx reduce the energy available to the spark gaps, and prevent cascade erection. Considerable effort was invested in the development of the trigger generator architecture. Trigger and ground resistors were installed to decouple detrimental strays and maintain sequential erection characteristics [31]. The resultant circuit layout is shown in Figure 1.30 [30,31].

The trigger system is arranged so that the operation of nine “MPUs” is initiated simultaneously with a 100 kV pulse with a 10 ns rise time. The MPU is also a bipolarly charged Marx generator providing a 540 kV pulse with an 80 ns rise time to the six gaps of row 1. The other gaps are sequentially triggered by the voltage pulses from the forward-feeding trigger resistors. Each MPU fires four Marx generators, resulting in all 36 modules of PBFA being triggered (Figures 1.32 and 1.33).

1.5.3 Aurora [9]

The Aurora Marx bank, with output parameters of 11 MV, 120 kA, and a stored energy of 5 MJ, was comprised of four individual Marx generators connected in parallel, as shown in Figure 1.34a. This design reduced the overall Marx inductance to an internal inductance of 12 μH. The Marx generators share a common 120 kV DC charging supply and are triggered by a common 600 kV Marx generator.

Each of the Marx generators has 95 stages with a stage capacitance of 1.85 μF charged to 120 kV. The basic schematic of a representative portion of the Aurora Marx generator [9] is shown in Figure 1.34b and uses both capacitive and resistive coupling. Weak overvoltages at particular spark gaps (S4, S6, S7, S9, S10) are resistively coupled to earlier stages, greatly increasing the reliability of the Marx erection upon command trigger, but the number of stages between the resistive coupling is not fixed. The Aurora Marx bank’s erection time is 1 μs with an erection jitter of 10 ns. The enhanced coupling scheme allows operation down below 50% of its self-breakdown voltage with a prefire probability of less than 1%.

1.6 Marx Generator Variations

As a result of the large number of applications for Marx generators, a variety of modified versions of the conventional Marx generator have evolved, each of
Figure 1.32 The circuit of the PBFA II Marx generator [30]. The six spark gaps of row 1 are triggered and the Marx generator uses both capacitive and resistive coupling to the upper stages to ensure a reliable erection process. The physical layout of the circuit is shown in Figure 1.33.

Figure 1.33 The implementation of Sandia Marx generator circuit is complex. The Marx is arranged in two columns of capacitors where each “stage” is connected by spark gaps and which constitutes a row. Five rows are assembled into the Marx with coupling connections as shown in Figure 1.32. The Marx banks are very sophisticated and have been shown to be astonishingly reliable. (Reproduced with permission of Sandia National Laboratories.)
which is able to meet a specific application in a more efficient way. The salient features of a few of these modified Marx generators are described in this section.

### 1.6.1 Marx/PFN with Resistive Load

Conventional Marx designs do not produce rectangular pulse. The principles of Marx operation may be modified, however, to produce a flat-top pulse by making the Marx “capacitors” energy storage elements that produce a pulse with a flat-top pulse. Erection of the Marx adds the voltages of each stage while preserving the pulse shape. The circuit, shown in Figure 1.35, is known as a Marx-PFN.

Pulse forming networks (PFNs) made from lumped elements are very common and coaxial cables have also been used. Any type of PFN may be used, but E-type PFNs are common.

![Figure 1.34](image)

**Figure 1.34** The Aurora Marx bank (a) is comprised of four Marx generators in parallel. The Aurora Marx generator design, shown in part (b), reinforces the weak overvoltage on various spark gaps with both capacitive and resistive back-coupling [9].

![Figure 1.35](image)

**Figure 1.35** A PFN-Marx generator driving a resistive load.
From given requirements of pulse voltage, pulse current, and pulse duration, the equations for fixing the basic parameters assuming matched load conditions are given below:

Characteristic impedance ($R'$):
\[ Z_{PFN} = \sqrt{L/C} \]  

(1.62)

Pulse duration ($T$):
\[ T = 2n\sqrt{LC} \]  

(1.63)

Matched load voltage ($V_L$):
\[ V_L = \frac{NV_0}{2} \]  

(1.64)

Load resistance ($R_L$):
\[ R_L = NZ_{PFN} \]  

(1.65)

Maximum stored energy ($E_s$):
\[ E_s = \frac{1}{2} N(nC)V_0^2 \]  

(1.66)

where $L$ and $C$ are parameters of the PFN, $n$ is the number of capacitors in the PFN, $N$ is number of Marx stages, and $V_0$ is the charging voltage. The output waveform of a Marx-PFN into a matched load is shown in Figure 1.36. The rise time ($t_r$) in this case is governed by $L'/R_L$ of the circuit and the main contribution to $L'$ is from the spark gaps.

The energy content is reduced from its maximum value by inefficiencies, including the energy lost in the spark gaps and the resistors as well as charge remaining in the capacitors. Energy dissipated in the switches is reduced by low-gap spacing but necessitating high dielectric strength mediums. Both single insulator (gas) and hybrid insulation (gas/oil) schemes are common in Marx-PFNs. When a single insulating gas is used for insulation and switching, the contribution to the inductance from the connections is minimized, but the

Figure 1.36 The output voltage of a Marx-PFN measured at a matched load.
switch inductance may increase because the gap length is set by the system pressure. At high voltages, transformer oil or epoxy may be used to insulate the Marx and pressurized gas is used in the switches. Figure 1.37 illustrates a single stage consisting of a PFN, resistors, and spark gap of a 20-stage Marx-PFN producing a 300 kV, 100 ns pulse [32,33]. The switches are aligned in a straight-line path so that the ultraviolet light generated when the first spark gap fires irradiates the rest of the spark gaps, providing initiatory electrons for the breakdown process, reducing the statistical lag time, and increasing reliability. The Marx-PFN uses pressurized N₂ gas for spark gaps and mineral oil for the rest of the components.

A Marx-PFN can be constructed using any type of PFN. Riepe [34] made the differentially charged Marx-PFN shown in Figure 1.38 with a type-C PFN to produce a 2.5 μs long pulse at 120 kA and 300 kV into a matched load with a reasonably flat top. The first spark gap is triggered.

More recently, Adler et al. [35] have developed a Marx-PFN with a novel charging supply to produce pulses with a duration greater than 1 μs with less than a 5% droop with high reliability. The Marx stages are type-C PFN network and generate 500 kV into a 50 Ω load with a rise time of less than 200 ns.

Figure 1.37 A single stage (a) circuit and (b) layout of a Marx/PFN using a Type E PFN.

Figure 1.38 A Marx-PFN made with a type-C Guilleman network. (Reproduced with permission from Ref. [34]. Copyright 2008, AIP Publishing LLC.)
1.6.2 Helical Line Marx Generator

Helical lines can be used as energy storage elements in a Marx generator [36], as shown in Figure 1.39. The helical line Marx-PFN is useful for the generation of flat-top pulses of long duration (a few microseconds). The Marx capacitors are replaced by helical lines made by replacing the braided outer conductor of a typical low-inductance coaxial cable with a helical winding. The helical winding is a conductor of diameter $d$ wound over a polyethylene insulator of diameter $D$ with $n$ turns per unit length.

The design formulas for a helical Marx are as follows:

Characteristic impedance:

\[ Z_{\text{helix}} = \sqrt{\frac{L}{C}} \]  

(1.67)

Figure 1.39 A Marx generator using helical lines as stage energy storage elements [36]. (Reprinted with permission from Ref. [36]. Copyright 2001, AIP Publishing LLC.)
Marx internal impedance:

\[ Z_M = NZ_{\text{helix}} \]  

(1.68)

Pulse duration:

\[ T = 2\ell \sqrt{L_h C_h} \]  

(1.69)

Matched load voltage:

\[ V_L = NV_0 \]  

(1.70)

Here

\[ L_h = \text{inductance per unit length of helical line} \]
\[ C_h = \text{capacitance per unit length of helical line} \]
\[ \ell = \text{length of helical conductor} = \pi nD \text{ for } D \gg d \]

The maximum load voltage can be greatly reduced by coupling between the cables of adjacent lines. A 20-stage Marx generator based on the above design using modified coaxial cables as helical line storage elements has delivered 400 kV and 20 A at a pulse duration of 1 \( \mu \)s into a matched load.

1.7 Other Design Considerations

This section deals with the salient features of Marx generators: (a) DC charging voltage and number of stages, (b) Marx capacitor selection, (c) pulse charging schemes, (d) Marx spark gap considerations, (e) Marx resistors, (f) delay time and jitter, and (g) triggering.

1.7.1 Charging Voltage and Number of Stages

The open-circuit output voltage of the Marx can be increased by increasing either the number of stages or the charging voltage per stage. Larger number of stages increases the number of switches, the inductance, and the cost. However, it also allows a common switching and insulating medium. The voltage per stage is set to be much lower than the self-breakdown voltage of the spark gaps to minimize the possibility of prefire. Prefires are often a significant risk in the operation of high-energy Marx banks and are often operated at 50–70% of their self-breakdown voltage. Sufficient overvoltages are ensured by carefully designed resistive triggering and capacitive coupling.

In advanced versions of fast Marx generators, a high DC charge voltage and fewer switches are preferred. Since the overall reliability of a Marx generator depends mainly on the precision of firing the spark gaps, any method that
reduces the number of spark gaps increases the performance of the Marx generator. In practical systems, the maximum DC voltages are limited to approximately 100 kV, and such high DC voltages necessarily involve complicated insulation systems. This insulation requirement is one reason to consider bipolar charging since the insulation requirement per stage is similar to that of a single-polarity Marx, but the required number of stages (and switches) for a given output voltage is cut in half.

A Marx generator may be pulse charged, where the capacitors are charged in a very short time and immediately discharged. This was proposed to reduce the prefire rate and extend the useful life of the energy storage capacitors. Buttram and Clark built a pulse charged 1 MV, 10 kJ, 10 Hz Marx generator by interposing a triggered spark gap between the HVDC supply and the Marx generator [37]. Pulse charging greatly increases the complexity and necessitates sophisticated synchronization and a low-impedance HVDC supply.

### 1.7.2 Insulation System

Marx generators generally used by teaching institutions and high-voltage testing laboratories are usually of open type of construction utilizing atmospheric air insulation. Because of poor dielectric strength of air, these Marx generators tend to be very large in size and usually require high ceilings and large side clearances for the installation room. An open construction has the advantage that the components are accessible, easing maintenance and fault detection, as well as modification of the waveform. This is particularly important for impulse testing, where distinctly different waveforms are required or if the Marx is being used as a teaching aid. In an open construction, the insulation strength is easily affected by humidity levels and other environmental changes. The insulation systems that can be used in a Marx generator (other than switches) are (a) high-pressure gas, (b) insulating oil, and (c) pourable solids (potting).

Gas at elevated pressures, usually N₂, SF₆, or a mixture of the two, has good dielectric strength, considerably reducing the overall size of the Marx. The Marx containment vessel, however, becomes a pressure vessel. To ensure personnel safety, the pressure vessel should be designed according to an appropriate mechanical engineering standard, such as the ASME. Depending on the specific design, a single gas pressure may be used throughout the Marx or separate gas pressures may be used for the switches and the insulation. Oil insulation is widely used to insulate Marx generators in very high-voltage operation and have the considerable advantage of not requiring a pressure vessel. Oils are also suitable for repetitive Marx operation because it has a high heat capacity. However, contaminants and repeated stressing of the oil tend to decompose the oil and degrade its dielectric strength.

The most popular insulation system is one that uses gas in the spark gaps with the remainder of the Marx immersed in oil. The Marx output is easily variable
by changing the pressure of the gas in the spark gaps, without affecting the basic insulation level of the overall Marx. By employing recirculation and filter system for this insulating oil, the Marx generator can be easily adapted for high repetitive pulsing.

1.7.3 Marx Capacitors

The Marx capacitors are generally fast-discharge, low-inductance energy storage capacitors with higher current capability. Considerable ringing may occur in Marx generators with capacitive and inductive loads and the voltage reversal capability of the capacitors should be considered. Resistive isolation resistors are preferable to inductive isolation since it damps the reversal.

For smaller Marx generators, ceramic capacitors such as those using a barium titanate dielectric can be used [38]. Experience indicates that with open Marx construction, barium titanate capacitors with epoxy enclosure are suitable but those with PVC and other plastic enclosures are susceptible to permeation of moisture on the dielectric surface, and subsequent failure by surface tracking.

The state of the art in capacitors for multimegavolt Marx generators is those made by General Atomics for Sandia National Laboratories’ ZR project [39]. The new 2.6 μF, 100 kV capacitors doubled the energy density of the older 1.3 μF capacitors by doubling the capacitance in the same volume, while maintaining the low inductance (<30 nH) and high peak current (170 kA) capabilities. Testing predicts capacitor lifetimes of about 11,000–13,000 shots at 100 kV and 8000 shots at 110 kV. Several other manufacturers have comparable products.

1.7.4 Marx Spark Gaps

The first one or two spark gaps in a Marx generator are generally externally triggered and the remainder fire from the resultant overvoltage. Common electrode materials are brass [38], copper [40], and stainless steel [41]. Brass is often used in SF₆ insulated spark gaps because the chemical reaction with the SF₆ dissociation products stabilizes the self-breakdown level, greatly reducing the probability of prefire [42]. In three-electrode Marx spark gaps, the center plate may be made from stainless steel. For small or low-energy Marx, spherical electrodes are often used. In generators with many stages or high energies, long-lived commercial spark gaps are used. Triggered three-electrode spark gaps are used in combination with resistive or capacitive coupling to other stages to control overvoltages.

For a high-power repetitively pulsed Marx generator of 1 MV, 10 kJ, 10–100 Hz, Buttram and Clark employ a water-cooled spark gap with a high-velocity gas flow to remove the ionization products from the gap medium and reduce the erosion of the switch electrodes by not allowing the electrodes to heat up [37].
1.7.5 Marx Resistors

The purpose of the Marx resistors $R$ in Figure 1.14 is to prevent the Marx capacitors from being short circuited when the Marx spark gaps fire, provide a dump for the energy stored in the Marx capacitors in case of emergency or malfunction, and shape the fall time of the output wave to specification. The maximum voltage drop across $R$ corresponds to the charging voltage $V_0$, and it should be sized to prevent flashover across its surface. The resistor $R_F$ should have a pulse insulation level for the full output voltage of the Marx. Resistors incorporated into Marx generators should be noninductive to avoid the introduction of oscillations in the erected Marx circuit.

Noninductive resistors may be fabricated by hand-winding on a fine-tooth comb, as shown in Figure 1.40 [43]. High-resistivity materials such as Nichrome, Manganin, and Kanthal can be conveniently used for this purpose. Manganin has the advantage of being easily soldered and has good stability with temperature. Potting in epoxy gives the resulting resistor mechanical rigidity and counteracts environmental influences such as humidity [44].

When designing for Marx-PFN, the value of $R$ should be such as to make the $R_C$ time constant much larger than the output pulse duration. Liquid resistors, particularly copper sulfate with copper electrodes or sodium thiosulfate with aluminum electrodes, can be used [29,45]. For copper sulfate solutions with resistivity between 60 and 1200 $\Omega$ cm and applied electric fields between 2 and 50 kV/cm, the resistance is linear [45,46]. Liquid resistors are low inductance and compact, are easily constructed, and have high energy dissipation but may be excessively capacitive in some applications.

1.7.6 Marx Initiation

A Marx generator can be self-fired, but this imposes severe limitations on its functionality: The generator must be operated at a large percentage of its self-breakdown voltage, which results in difficulty in varying the output voltage and low overvoltages which leads to poor rise time performance. These drawbacks are overcome by externally triggering the first one or two gaps as shown in Figure 1.38, and Figure 1.39.

![Figure 1.40 Hand-winding on a fine-tooth comb.](image)
A trigatron, discussed extensively in Chapter 4, has two main electrodes, the anode A and the cathode K with an insulated trigger T embedded in the cathode. The trigger pulse is formed by the discharge of capacitor $C'$ into the primary of a pulse transformer through switch $S'$, as shown in the dotted box of Figure 1.41. The pulse transformer produces a high-voltage pulse of sufficient magnitude and duration to close the trigatron, leading to a command-triggered cascade erection of the generator.

Triggering along the midplane of a spark gap switch is a popular choice for Marx generators and is illustrated in Figure 1.39. For instance, in Kukhta’s implementation [47], the spark gap consists of three electrodes: the cathode K, the anode A, and the trigger electrode T. The trigger electrode is located in the middle of the gap and held at potential $(V_0/2)$ by a voltage divider comprised of $R_1$ and $R_2$. The trigger pulse is generated by the cable assembly, charged to a voltage $V_0/2$. The cable is short circuited when the spark gap $S'$ fires, generating a pulse of magnitude $(-V_0/2)$. When this pulse reaches the trigger electrode, it inverts its potential leading to breakdown of the spark gap $S_1$ (Figure 1.42).
A Marx generator may be used as a trigger generator for another “main” Marx generator. This arrangement is used extensively on large high-energy banks to inject a high-energy, high-voltage pulse into a number of trigger points of the main Marx. Figure 1.43 shows a main Marx generator whose switches are all double-trigger mode, described in detail in Chapter 4, and fired from the trigger Marx [48]. The trigger Marx must have a fast rise time and firing reliability.

1.7.7 Repetitive Operation

The repetitive capability of Marx generators is severely limited relative to even conventional pulse transformers. For repetitive operation, the main advantage of Marx generators over other methods of generating high voltage is its versatility of packaging. The sequential erection of a Marx generator can be made to follow any number of paths because of the inherent flexibility in its architecture.

The pulse repetition rate is limited by the Marx cycle, which can be considered in three distinct parts: the time to charge the Marx, the high-voltage time, and the switch recovery time. Inductors can replace the isolation resistors for faster charging times. The Marx charging time has been investigated in both series and parallel charging using resistors, inductors, and resistive/inductor combinations as charging elements [49].

Pulse repetition rates of 600 Hz have been demonstrated with a series inductive charging system and spark gap switches insulated with 3 atm of an air/SF$_6$ mixture [50] (L. Veron, personal communication). Corona-stabilized switches, demonstrated to operate at 20 kHz, have been used in Marx

![Diagram of Marx generator](image)

**Figure 1.43** Triggering of a large Marx generator with a smaller Marx.
generators at pulse repetition frequencies of 400 Hz [51–53] and Marx generators using hydrogen-filled spark gaps have been demonstrated at 10 kHz in burst mode [54]. In practice, depending on the energy per pulse, the power supply may be the limiting factor.

1.7.8 Circuit Modeling

Circuit modeling is an important task of pulsed power design and both commercial and custom codes are used for this purpose. Circuit modeling is especially important for machines where currents are added, because even small deviations can manifest as large deviations from design when tens of modules are connected to the load. Custom codes may contain time-varying circuit elements to represent components unique to pulsed power, and commercial codes can usually be made to mimic them.

Marx banks can be quite intricate, but even a simple Marx generator would add unnecessary complexity to a circuit code. Instead, the Marx may be represented by the circuit of Figure 1.44. The capacitance $C_M$ is the erected Marx capacitance with its discharge rate limited by the resistor $R_T$. The Marx inductance $L_M$ accounts for the inductance of the switches, capacitors, and connectors in the discharge circuit. These inductances are difficult to calculate and, in practice, the Marx is charged to a reduced voltage and discharged into a short circuit. The equivalent circuit values of the Marx can be deduced from the ringing frequency and the decay time. These values are largely independent of voltage.

The equivalent Marx circuit can represent the energy storage stage quite effectively. Figure 1.45 is an overlay of a circuit code simulation and a measured current waveform from a pulse compression circuit showing excellent agreement. The dip that occurs at approximately 1700 ns is a reflection from the next capacitive energy storage stage.

1.8 Marx-Like Voltage-Multiplying Circuits

Other circuits may be used to multiply voltages that are not Marx generators. Like a Marx, these schemes feature a staged energy storage resulting in voltage multiplication. However, the fundamental principle of a Marx generator—namely, charging a circuit in parallel and discharging it in series—is missing.
These circuits, particularly the LC inversion generator, are often mistakenly referred to as a Marx generator.

1.8.1 The Spiral Generator

A spiral generator [56,57] is a voltage multiplier constructed by placing another layer of insulation on a stripline and wrapping it around itself \((n - 1)\) times in a spiral fashion, as shown in Figure 1.46. The resulting geometry, a three-electrode stripline, comprised of line 1 and line 2, is wound on an insulating form of diameter \(D\) in the form of a spiral of \(n\) turns with a width \(w\) and dielectric thickness \(h\) for each two-electrode line. In a spiral generator, voltage multiplication is achieved with a single switch—a big advantage—but the output waveshape is triangular, which limits its usefulness. The load is connected from the center of the generator to ground. The start switch \(S\) may be located as shown in Figure 1.46 or at the midpoint of the line length with the electrical lengths of the lines being equal.

The spiral generator operated on the manipulation of reflected waves, similar to the operation of a Blumlein. Line 1 is charged to \(V_0\), and when the switch \(S\) is closed at \(t = 0\), a maximum voltage of \((-2nV_0)\) appears on the output terminal \(V_{out}\) at a time \(T\) after the switch closure. The time \(T\) corresponds to the sum of the electrical lengths of line 1 and line 2, which are equal. The status of the electric field vectors on line 1 and line 2 through the radial cross section of the
spiral generator at times corresponding to \( t = 0 \) and \( t = T \) are shown in Figure 1.47. At \( t = 0 \), just before the start switch closes, the \( E \)-field vectors in alternate sections of lines are in opposite directions and hence the net output voltage at the load is zero, as shown in Figure 1.47a. At \( t = T \), the \( E \)-field vectors

Figure 1.46 An ideal spiral generator.

Figure 1.47 A view of the status of the electric field on the cut \( XX \) in Figure 1.46. The electric field vectors on lines 1 and 2 of the spiral generator are initially opposed, but align to produce an output voltage \( V_{out} \) with a maximum value of \(-2nV_0\) at the output terminals at the time \( t = T \).
across the entire line 1 are reversed by the reflected voltage waveform, as shown in Figure 1.47b, producing an open-circuit output voltage $V_{out}$ at the load.

If $D$ represents the mean diameter of the spiral, under the assumptions that $D \gg 2nh$, the important parameters for a lossless spiral generator can be derived from its equivalent circuit \[58\]:

a) Time to reach maximum output voltage:

$$T = 2\pi D n \sqrt{\varepsilon_0 \varepsilon_r \mu_0}$$  \hspace{1cm} (1.71)

b) Output capacitance:

$$C = \frac{\varepsilon_0 \varepsilon_r \pi Dw}{(2n)h}$$  \hspace{1cm} (1.72)

c) Maximum electrostatic stored energy:

$$E = \frac{\varepsilon_0 \varepsilon_r \pi D w n V_0^2}{h}$$  \hspace{1cm} (1.73)

The output voltage is triangular in shape, as shown in Figure 1.48, mainly due to the integrating effect of the inductive coupling of the spiral turns \[3\]. Fitch and Howell \[57\] provide excellent insight into losses in these types of generators. In particular, if the condition $D \gg 2nh$ is not satisfied, which is frequently the case, the voltage multiplication factor is reduced \[3\].

### 1.8.2 Time Isolation Line Voltage Multiplier

Lewis \[59,60\] conceived a high-voltage multiplying circuit, shown in Figure 1.49, that does not use switches. Instead, it uses long sections of transmission line, usually coaxial, driven by a low-voltage, low-impedance,

![Figure 1.48](image-url)  

**Figure 1.48** The output waveform of a spiral generator initially charged to a voltage $V_0$.  

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high-current source. It is termed a time isolation voltage multiplier because the long line propagation time $T_T$ permits parallel excitation at the source and voltage multiplication by a series connection at the load.

The time isolation generator consists of $N$ transmission lines connected in parallel at the input end and in series at the output end. When the input end is fed with a pulse voltage of duration, $T_p$, under the conditions of $T_p \ll T_T$, the output voltage is multiplied by a factor of $(2N)$. The performance of this voltage multiplying is similar to the $LC$ inversion generator described in the next section. Careful consideration must be shown regarding impedance matching at the input and output ends to avoid the waveform distortions from reflections. The effects of stray capacitances have been studied in considerable detail by Chodorow [61] who states a 700 kV, fast rise time Time Isolation generator was used as an EMP simulator. Voltage multiplication in coaxial Time Isolation generators was reported by Soto and Soto and Altamirano [62] and Carmel et al. [63]. The maximum theoretical output voltage is typically reduced, sometimes greatly, by coupling between the cables.

### 1.8.3 The LC Inversion Generator

The $LC$ inversion generator is a voltage-multiplying circuit that was patented by Richard Fitch in 1968 [1,64]. Although similar in appearance to a Marx generator, the operation of an $LC$ generator or inversion generator is not like a Marx generator but instead is related in its operation to the spiral generator, for which Fitch also has a patent [56], and a Blumlein generator.
The LC generator operates on the transient inversion of alternate potentials in a series system of alternately opposed potentials, as shown in Figure 1.50. Each “stage” consists of two capacitors charged to equal and opposite potentials. During charge cycle, the net stage potential is zero and then switched at a precise time so that the potentials add.

Figure 1.50 The three-stage LC inversion generator in part (a) has alternating potentials on the capacitor, as shown in part (b) for a net stage voltage of 0 when fully charged. The potentials add (c) when the spark gaps are simultaneously fired.

The LC generator operates on the transient inversion of alternate potentials in a series system of alternately opposed potentials, as shown in Figure 1.50. Each “stage” consists of two capacitors charged to equal and opposite potentials. During charge cycle, the net stage potential is zero and then switched at a precise time so that the potentials add.

Moderate power LC generators using diodes across the capacitors, often called a Fitch circuit, has found widespread use in industrial applications, such as lasers and pollution control circuits because the circuit can be made repetitive, highly reliable, and stable [65–67]. The principle of operation is most easily understood by looking at circuit where diodes or other rectifiers are used. The diodes in the circuit prevent the second half-cycle in the discharge cycle instead of relying purely precise switching times and relax requirements on the output pulse duration.
A single-stage LC generator is shown in Figure 1.51. During the charge cycle, the capacitors $C_1$ and $C_2$ are charged through the diode $D_1$ and inductor $L$. The resistor $R$ completes the charging circuit for $C_2$. $D_2$ is not involved in the charging cycle, but when more than one stage is used, it passes current through to subsequent capacitor pairs. Recognizing that when the capacitors are fully charged, no current flows in the circuit $C_1–C_2–R$, the application of Kirchoff’s voltage law shows the voltages on capacitors $C_1$ and $C_2$ must be equal in magnitude and opposite in sign.

When the spark gap $Sw_1$ fires, the capacitor $C_2$ begins to discharge and current flows in the circuit $C_2–L–D_2$. The diode $D_1$ is reverse-biased and capacitor $C_1$ does not discharge but remains fully charged to the voltage $V_0$. The current in the circuit $C_2–L–D_2$ during this time, $I_2(t)$, is given by

$$I_2(t) = V_0 \sqrt{\frac{C_2}{L}} \sin \omega t$$

(1.74)

where

$$\omega = \frac{1}{\sqrt{LC_2}}$$

(1.75)

As this resonant circuit reaches its zero crossing at a time

$$t_0 = \frac{\pi}{\omega} = \pi \sqrt{LC_2}$$

(1.76)

the sinusoidal current changes sign, the diode $D_2$ is now reverse-biased, and the spark gap extinguishes and opens. Capacitor $C_2$ is now inverted, and has a charge of $+V_0$. The capacitors $C_1$ and $C_2$ have the same polarity in the second period and the voltage at the output spark gap is $2V_0$. Switch $Sw_2$ may now be fired to discharge the circuit into the load. The switch $Sw_2$ may be omitted. The resistance $R$ determines the decay time, in a manner similar to that derived for Marx generators. Switch timing and jitter are critical to the operation of the LC generator.

In general, an LC inversion generator stacks several of these circuits for voltage multiplication, yielding an open-circuit voltage:

$$V_{OC}(t) = NV_0 \left[ 1 - e^{-\omega t} \cos (\omega t) \right]$$

(1.77)
with

\[ \alpha = \frac{R}{2L} \]  

(1.78)

where \( N \) is the number of stages and \( R \) is the effective series resistance in the \( LC \) circuit. The \( e^{-at} \) term accounts for losses in the discharge path including those introduced by component connections, and so on. The rise time of the output pulse is determined by the resonant portion of the circuit and is given by

\[ t_r = \pi \sqrt{LC_2} \]  

(1.79)

The pulse rise time can be recognized as the time of zero crossing of the \( LC_2 \) resonant circuit. For pulse generation with a fast rise time, a small value of inductance is required, but the chosen value also affects the pulse width. The realized inductor must also be designed to store the same energy as the capacitor \( C_2 \).

A high-power \( LC \) generator must achieve the same basic operation as described above without the benefit of diodes. Instead, the inversion is achieved by careful and precise firing of the switches. Figure 1.52 shows two variations of inversion generators with their charging impedances. With no diodes to prevent

![Figure 1.52](image_url)
the second half-cycle, the inversion time must be made two or three times longer than the required pulse duration.

In an inversion circuit, the switches are not part of the discharge circuit and the switch inductance and switch resistance do not contribute to inefficiency. The load currents are also independent of the switches and a low-inductance, fast current pulse can be produced. The output begins to appear as soon as the switches fire, with a rise time determined by the ringing portion of the circuit. Particularly for high-power operation, an output switch is typically included as part of the circuit.

The LC inversion generator has two very significant disadvantages: triggering and fault modes. The switches in an LC inversion generator must all be triggered simultaneously. Unlike a Marx, the discharge of the generator does not produce an overvoltage on the switches. In an inversion generator, each switch is independently fired and the additional cost of multiple trigger generators could be prohibitive. Trigger generators with multiple outputs may be used in lieu of independent triggers. Instead of independent switch triggers, nearby stages can be linked capacitively [1], as shown in Figure 1.53.

Protection against fault modes in an LC inversion generator is serious. If a switch fails to fire, the generator could develop large overvoltages or voltage reversals that can be dangerous to some capacitors. Switch prefires do not necessarily fire the remaining stages and prefire protection system is difficult to design. The complexity of protecting the circuit against its fault modes reduces the initial appeal of the LC inversion generator for some applications.

**Figure 1.53** Coupling capacitors linking switch may be used so that each switch does not require a trigger generator.
An LC inversion generator, developed by Harris and Milde [68], is shown in Figure 1.54. It is charged in dual polarity and produces an output voltage between 270 kV and 1 MV with a maximum stored energy of 15 kJ and a discharge time of 5 μs. When the capacitive voltages align, the load circuit is connected through the self-breaking transfer switch \( S'' \). Protective spark gaps \( S''' \) are incorporated into the design to prevent damage if one of the spark gaps fail to fire and puts a dangerously large overvoltage across its capacitor set.

### 1.9 Design Examples

#### Example 1.1

A four-stage Marx generator, having capacitors of 0.125 μF in each stage, discharges into a capacitive load of 200 pF. Using a simplified circuit approximation, calculate the values of stage resistors and front resistors to give (a) e-folding rise time of 50 ns, and (b) e-folding tail time of 50 μs. Calculate (c) overvoltage factor on the remaining spark gaps, when the first spark gap is triggered. Assume a charging voltage of 30 kV and self-breakdown voltage of the spark gap as 35 kV.

**Solution**

The equivalent circuit of this Marx generator is shown in Figure 1.2, with the following values:

\[
C_M = \frac{C_0}{N} = \frac{0.125}{4} \text{ µF} = 31.25 \text{ nF}
\]

\[
C_L = 200 \text{ pF}
\]
a) The approximate circuit for rise time is shown in Figure 1.4, from which

\[
e\text{-folding rise time} = 50 \text{ ns} = \left[ \frac{C_M \times C_L}{C_M + C_L} \right] \times R_F
\]

Front resistor \( R_F = 250 \Omega \)

b) The approximate circuit for tail time is shown in Figure 1.5, from which

\[
e\text{-folding tail time} = 50 \mu s = (C_M \times C_L) \times R_T
\]

Tail resistor \( R_T = \frac{2R_T}{N} = 1.59 \text{ k}\Omega \)

Stage resistor \( R = \frac{2R_T}{N} = 795 \Omega \)

c) When the first spark gap fires, additional voltage on the remaining spark gaps

\[
V_g = \frac{N}{N-1} V_0 = \frac{4}{3} (30 \text{ kV}) = 40 \text{ kV}
\]

Overvoltage factor \( \frac{V_{\text{actual}}}{V_{\text{breakdown}}} = \frac{40}{30} = 1.14 \)

Example 1.2

A 10-stage Marx generator has the following parameters: (a) DC charging voltage =100 kV, (b) capacitance per stage =0.25 \( \mu \)F, and (c) tail resistor per stage =720 \( \Omega \). Calculate the magnitude of voltage reached on the output terminal at \( t = 90 \mu s \), when the Marx generator is fired on no load.

Solution

The equivalent circuit of a Marx generator with “no load” is similar to Figure 1.2, with \( C_L \) being open circuited. The values of \( C_M \) and \( R_T \) are as follows:

\[
C_M = \frac{C_0}{N} = \frac{0.25 \mu \text{F}}{10} = 25 \text{ nF}
\]

\[
R_T = \frac{NR}{2} = \frac{10 \times 720}{2} = 3600 \Omega
\]

(continued)
(continued)

\[ C_M R_T = 90 \]

Output voltage \( V_T = NV_0 e^{-\frac{t}{C_L R_T}} \)

\[ V_T(t = 90 \mu s) = (10 \times 100)e^{-1} = 367.8 \text{ kV} \]

Example 1.3

A four-stage Marx generator discharging into an output capacitor \( (C_L) \) has the following parameters: (a) DC charging voltage = 100 kV, (b) capacitance per stage = 0.125 \( \mu \)F, (c) front resistor = 200 \( \Omega \), and (d) load capacitance = 200 pF.

Calculate the following: (a) output voltage at any time \( t \), (b) amplitude of voltage at \( t = 0.01 \mu s \), (c) time to reach maximum voltage, and (d) amplitude of peak voltage.

Solution

In Equation 1.6 and solving for the roots by Equations 1.4 and 1.5, we get the following corresponding values for the Marx equivalent circuit of Figure 1.2:

\[ C_M = \frac{C_0}{N} = \frac{0.125}{4} = 31.25 \text{ nF} \]

\[ R_T = \frac{NR}{2} = \frac{4 \times 1200}{2} = 2.4 \text{ k} \Omega \]

\[ C_L = 200 \text{ pF}, \quad R_F = 200 \text{ } \Omega \]

Substituting the above values:

\[ \alpha = -0.0135 \left( \frac{1}{\mu \text{s}} \right) \quad \text{and} \quad \beta = 25.159 \left( \frac{1}{\mu \text{s}} \right) \]

a) Output voltage (V):

\[ V_M(t) = \frac{NV_0}{C_LR_F} \times \frac{1}{(\beta - \alpha)} \times (e^{-\beta t} - e^{-\alpha t}) \]

\[ = (0.994)(4)(100) \left( e^{-0.0135x10^6t} - e^{-25.159x10^6t} \right) \]

\[ = 397.6 \left( e^{-0.0135x10^6t} - e^{-25.159x10^6t} \right) \text{ kV} \]
b) Amplitude of voltage at $t = 0.01 \mu s$:

By substituting $t = 0.01 \times 10^{-6}$ in the relation (a) above, we get

$$V = 397.6 \times 0.222 \text{ kV} = 88.27 \text{ kV}$$

c) Time ($t'$) to reach maximum voltage $V_{\text{max}}$:

By substituting the values of $\alpha$ and $\beta$ in relation (3a), we get

$$t' = \ln \left( \frac{\beta}{\alpha} \frac{\beta - \alpha}{\alpha} \right) = 0.299 \mu s$$

d) Amplitude of peak voltage ($V_{\text{max}}$):

By substituting the value of ($t'$) in relation (a), we get

$$V_{\text{max}} = (397.6)(0.995) = 395.6 \text{ kV}$$

References


28 J.C. Martin, CESDN Note #3. Available at http://www.ece.unm.edu/summa/notes.


49 C.E. Baum and J.M. Lehr, Parallel Charging of Marx Generators for High Pulse Repetition Rate, in *Ultra-Wideband, Short-Pulse Electromagnetics 5*, Kluwer Academic/Plenum Publishers, pp. 415–422, 2002. (∗Cited references such as Circuit and Electromagnetic System Design Notes, Switching Notes, Sensor and Simulation Notes and Interaction Notes, formerly edited by Dr. C.E. Baum, are available electronically at http://www.ece.unm.edu/Summa/ The Note Series is currently edited by Dr. D.V. Giri.)


