Contents

List of Tables xi
List of Figures xiii
Preface xvii

CHAPTER 1
A Quick Tour 1
1.1 Market-Based Valuation 1
1.2 Structure of the Book 2
1.3 Why Python? 3
1.4 Further Reading 4

PART ONE
The Market

CHAPTER 2
What is Market-Based Valuation? 9
2.1 Options and their Value 9
2.2 Vanilla vs. Exotic Instruments 13
2.3 Risks Affecting Equity Derivatives 14
2.3.1 Market Risks 14
2.3.2 Other Risks 15
2.4 Hedging 16
2.5 Market-Based Valuation as a Process 17

CHAPTER 3
Market Stylized Facts 19
3.1 Introduction 19
3.2 Volatility, Correlation and Co. 19
3.3 Normal Returns as the Benchmark Case 21
CONTENTS

3.4 Indices and Stocks 25
  3.4.1 Stylized Facts 25
  3.4.2 DAX Index Returns 26
3.5 Option Markets 30
  3.5.1 Bid/Ask Spreads 31
  3.5.2 Implied Volatility Surface 31
3.6 Short Rates 33
3.7 Conclusions 36
3.8 Python Scripts 37
  3.8.1 GBM Analysis 37
  3.8.2 DAX Analysis 40
  3.8.3 BSM Implied Volatilities 41
  3.8.4 EURO STOXX 50 Implied Volatilities 43
  3.8.5 Euribor Analysis 45

PART TWO

Theoretical Valuation

CHAPTER 4
Risk-Neutral Valuation 49
  4.1 Introduction 49
  4.2 Discrete-Time Uncertainty 50
  4.3 Discrete Market Model 54
    4.3.1 Primitives 54
    4.3.2 Basic Definitions 55
  4.4 Central Results in Discrete Time 57
  4.5 Continuous-Time Case 61
  4.6 Conclusions 66
  4.7 Proofs 66
    4.7.1 Proof of Lemma 1 66
    4.7.2 Proof of Proposition 1 67
    4.7.3 Proof of Theorem 1 68

CHAPTER 5
Complete Market Models 71
  5.1 Introduction 71
  5.2 Black-Scholes-Merton Model 72
    5.2.1 Market Model 72
    5.2.2 The Fundamental PDE 72
    5.2.3 European Options 74
  5.3 Greeks in the BSM Model 76
  5.4 Cox-Ross-Rubinstein Model 81
  5.5 Conclusions 84
  5.6 Proofs and Python Scripts 84
    5.6.1 Itô’s Lemma 84
### Contents

5.6.2 Script for BSM Option Valuation  
5.6.3 Script for BSM Call Greeks  
5.6.4 Script for CRR Option Valuation  

#### CHAPTER 6
**Fourier-Based Option Pricing**

6.1 Introduction  
6.2 The Pricing Problem  
6.3 Fourier Transforms  
6.4 Fourier-Based Option Pricing  
6.4.1 Lewis (2001) Approach  
6.4.2 Carr-Madan (1999) Approach  
6.5 Numerical Evaluation  
6.5.1 Fourier Series  
6.5.2 Fast Fourier Transform  
6.6 Applications  
6.6.1 Black-Scholes-Merton (1973) Model  
6.6.2 Merton (1976) Model  
6.6.3 Discrete Market Model  
6.7 Conclusions  
6.8 Python Scripts  
6.8.1 BSM Call Valuation via Fourier Approach  
6.8.2 Fourier Series  
6.8.3 Roots of Unity  
6.8.4 Convolution  
6.8.5 Module with Parameters  
6.8.6 Call Value by Convolution  
6.8.7 Option Pricing by Convolution  
6.8.8 Option Pricing by DFT  
6.8.9 Speed Test of DFT  

#### CHAPTER 7
**Valuation of American Options by Simulation**

7.1 Introduction  
7.2 Financial Model  
7.3 American Option Valuation  
7.3.1 Problem Formulations  
7.3.2 Valuation Algorithms  
7.4 Numerical Results  
7.4.1 American Put Option  
7.4.2 American Short Condor Spread  
7.5 Conclusions  
7.6 Python Scripts  
7.6.1 Binomial Valuation  
7.6.2 Monte Carlo Valuation with LSM  
7.6.3 Primal and Dual LSM Algorithms
PART THREE

Market-Based Valuation

CHAPTER 8
A First Example of Market-Based Valuation 147
8.1 Introduction 147
8.2 Market Model 147
8.3 Valuation 148
8.4 Calibration 149
8.5 Simulation 149
8.6 Conclusions 155
8.7 Python Scripts 155
8.7.1 Valuation by Numerical Integration 155
8.7.2 Valuation by FFT 157
8.7.3 Calibration to Three Maturities 160
8.7.4 Calibration to Short Maturity 163
8.7.5 Valuation by MCS 165

CHAPTER 9
General Model Framework 169
9.1 Introduction 169
9.2 The Framework 169
9.3 Features of the Framework 170
9.4 Zero-Coupon Bond Valuation 172
9.5 European Option Valuation 173
9.5.1 PDE Approach 173
9.5.2 Transform Methods 175
9.5.3 Monte Carlo Simulation 176
9.6 Conclusions 177
9.7 Proofs and Python Scripts 177
9.7.1 Itô’s Lemma 177
9.7.2 Python Script for Bond Valuation 178
9.7.3 Python Script for European Call Valuation 180

CHAPTER 10
Monte Carlo Simulation 187
10.1 Introduction 187
10.2 Valuation of Zero-Coupon Bonds 188
10.3 Valuation of European Options 192
10.4 Valuation of American Options 196
10.4.1 Numerical Results 198
10.4.2 Higher Accuracy vs. Lower Speed 201
10.5 Conclusions 203
10.6 Python Scripts 204
10.6.1 General Zero-Coupon Bond Valuation 204
10.6.2 CIR85 Simulation and Valuation 205
Contents

10.6.3 Automated Valuation of European Options by Monte Carlo Simulation 209
10.6.4 Automated Valuation of American Put Options by Monte Carlo Simulation 215

CHAPTER 11
Model Calibration 223

11.1 Introduction 223
11.2 General Considerations 223
11.2.1 Why Calibration at All? 224
11.2.2 Which Role Do Different Model Components Play? 226
11.2.3 What Objective Function? 227
11.2.4 What Market Data? 228
11.2.5 What Optimization Algorithm? 229
11.3 Calibration of Short Rate Component 230
11.3.1 Theoretical Foundations 230
11.3.2 Calibration to Euribor Rates 231
11.4 Calibration of Equity Component 233
11.4.1 Valuation via Fourier Transform Method 235
11.4.2 Calibration to EURO STOXX 50 Option Quotes 236
11.4.3 Calibration of H93 Model 236
11.4.4 Calibration of Jump Component 237
11.4.5 Complete Calibration of BCC97 Model 239
11.4.6 Calibration to Implied Volatilities 240
11.5 Conclusions 243
11.6 Python Scripts for Cox-Ingersoll-Ross Model 243
11.6.1 Calibration of CIR85 243
11.6.2 Calibration of H93 Stochastic Volatility Model 248
11.6.3 Comparison of Implied Volatilities 251
11.6.4 Calibration of Jump-Diffusion Part of BCC97 252
11.6.5 Calibration of Complete Model of BCC97 256
11.6.6 Calibration of BCC97 Model to Implied Volatilities 258

CHAPTER 12
Simulation and Valuation in the General Model Framework 263

12.1 Introduction 263
12.2 Simulation of BCC97 Model 263
12.3 Valuation of Equity Options 266
12.3.1 European Options 266
12.3.2 American Options 268
12.4 Conclusions 268
12.5 Python Scripts 269
12.5.1 Simulating the BCC97 Model 269
12.5.2 Valuation of European Call Options by MCS 274
12.5.3 Valuation of American Call Options by MCS 275