## CONTENTS

**Preface**  

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Congruency</td>
<td>3</td>
</tr>
<tr>
<td>1.1</td>
<td>Introduction</td>
<td>3</td>
</tr>
<tr>
<td>1.2</td>
<td>Congruent Figures</td>
<td>6</td>
</tr>
<tr>
<td>1.3</td>
<td>Parallel Lines</td>
<td>12</td>
</tr>
<tr>
<td>1.3.1</td>
<td>Angles in a Triangle</td>
<td>13</td>
</tr>
<tr>
<td>1.3.2</td>
<td>Thales' Theorem</td>
<td>14</td>
</tr>
<tr>
<td>1.3.3</td>
<td>Quadrilaterals</td>
<td>17</td>
</tr>
<tr>
<td>1.4</td>
<td>More About Congruency</td>
<td>21</td>
</tr>
<tr>
<td>1.5</td>
<td>Perpendiculars and Angle Bisectors</td>
<td>24</td>
</tr>
<tr>
<td>1.6</td>
<td>Construction Problems</td>
<td>28</td>
</tr>
<tr>
<td>1.6.1</td>
<td>The Method of Loci</td>
<td>31</td>
</tr>
<tr>
<td>1.7</td>
<td>Solutions to Selected Exercises</td>
<td>33</td>
</tr>
<tr>
<td>1.8</td>
<td>Problems</td>
<td>38</td>
</tr>
</tbody>
</table>
2 Concurrency
  2.1 Perpendicular Bisectors
  2.2 Angle Bisectors
  2.3 Altitudes
  2.4 Medians
  2.5 Construction Problems
  2.6 Solutions to the Exercises
  2.7 Problems

3 Similarity
  3.1 Similar Triangles
  3.2 Parallel Lines and Similarity
  3.3 Other Conditions Implying Similarity
  3.4 Examples
  3.5 Construction Problems
  3.6 The Power of a Point
  3.7 Solutions to the Exercises
  3.8 Problems

4 Theorems of Ceva and Menelaus
  4.1 Directed Distances, Directed Ratios
  4.2 The Theorems
  4.3 Applications of Ceva's Theorem
  4.4 Applications of Menelaus' Theorem
  4.5 Proofs of the Theorems
  4.6 Extended Versions of the Theorems
    4.6.1 Ceva's Theorem in the Extended Plane
    4.6.2 Menelaus' Theorem in the Extended Plane
  4.7 Problems

5 Area
  5.1 Basic Properties
    5.1.1 Areas of Polygons
    5.1.2 Finding the Area of Polygons
    5.1.3 Areas of Other Shapes
  5.2 Applications of the Basic Properties
6 Miscellaneous Topics

6.1 The Three Problems of Antiquity 159
6.2 Constructing Segments of Specific Lengths 161
6.3 Construction of Regular Polygons 166
   6.3.1 Construction of the Regular Pentagon 168
   6.3.2 Construction of Other Regular Polygons 169
6.4 Miquel's Theorem 171
6.5 Morley’s Theorem 178
6.6 The Nine-Point Circle 185
   6.6.1 Special Cases 188
6.7 The Steiner-Lehmus Theorem 193
6.8 The Circle of Apollonius 197
6.9 Solutions to the Exercises 200
6.10 Problems 201

PART II TRANSFORMATIONAL GEOMETRY

7 The Euclidean Transformations or Isometries 207

7.1 Rotations, Reflections, and Translations 207
7.2 Mappings and Transformations 211
   7.2.1 Isometries 213
7.3 Using Rotations, Reflections, and Translations 217
7.4 Problems 227

8 The Algebra of Isometries 235

8.1 Basic Algebraic Properties 235
8.2 Groups of Isometries 240
   8.2.1 Direct and Opposite Isometries 241
8.3 The Product of Reflections 245
8.4 Problems 250
9 The Product of Direct Isometries 255
  9.1 Angles 255
  9.2 Fixed Points 257
  9.3 The Product of Two Translations 258
  9.4 The Product of a Translation and a Rotation 259
  9.5 The Product of Two Rotations 262
  9.6 Problems 265

10 Symmetry and Groups 271
  10.1 More About Groups 271
    10.1.1 Cyclic and Dihedral Groups 275
  10.2 Leonardo's Theorem 279
  10.3 Problems 283

11 Homotheties 289
  11.1 The Pantograph 289
  11.2 Some Basic Properties 290
    11.2.1 Circles 293
  11.3 Construction Problems 295
  11.4 Using Homotheties in Proofs 300
  11.5 Dilatation 304
  11.6 Problems 306

12 Tessellations 313
  12.1 Tilings 313
  12.2 Monohedral Tilings 314
  12.3 Tiling with Regular Polygons 319
  12.4 Platonic and Archimedean Tilings 325
  12.5 Problems 332

PART III INVERSIVE AND PROJECTIVE GEOMETRIES

13 Introduction to Inversive Geometry 339
  13.1 Inversion in the Euclidean Plane 339
  13.2 The Effect of Inversion on Euclidean Properties 345
  13.3 Orthogonal Circles 353
  13.4 Compass-Only Constructions 362
  13.5 Problems 371
14 Reciprocation and the Extended Plane
   14.1 Harmonic Conjugates 375
   14.2 The Projective Plane and Reciprocation 385
   14.3 Conjugate Points and Lines 396
   14.4 Conics 402
   14.5 Problems 409

15 Cross Ratios 411
   15.1 Cross Ratios 411
   15.2 Applications of Cross Ratios 422
   15.3 Problems 431

16 Introduction to Projective Geometry 435
   16.1 Straightedge Constructions 435
   16.2 Perspectivities and Projectivities 445
   16.3 Line Perspectivities and Line Projectivities 450
   16.4 Projective Geometry and Fixed Points 450
   16.5 Projecting a Line to Infinity 453
   16.6 The Apollonian Definition of a Conic 457
   16.7 Problems 463

Bibliography 466

Index 471