Contents

List of Contributors xiii
Foreword xv
Preface xxv

1 Introduction to Lattice Materials 1
A. Srikantha Phani and Mahmoud I. Hussein
1.1 Introduction 1
1.2 Lattice Materials and Structures 2
1.2.1 Material versus Structure 3
1.2.2 Motivation 3
1.2.3 Classification of Lattices and Maxwell’s Rule 4
1.2.4 Manufacturing Methods 6
1.2.5 Applications 7
1.3 Overview of Chapters 8
Acknowledgment 10
References 10

2 Elastostatics of Lattice Materials 19
D. Pasini and S. Arabnejad
2.1 Introduction 19
2.2 The RVE 21
2.3 Surface Average Approach 22
2.4 Volume Average Approach 25
2.5 Force-based Approach 25
2.6 Asymptotic Homogenization Method 26
2.7 Generalized Continuum Theory 29
2.8 Homogenization via Bloch Wave Analysis and the Cauchy–Born Hypothesis 32
2.9 Multiscale Matrix-based Computational Technique 34
2.10 Homogenization based on the Equation of Motion 36
2.11 Case Study: Property Predictions for a Hexagonal Lattice 38
2.12 Conclusions 42
References 43
3 Elastodynamics of Lattice Materials 53
A. Srikantha Phani
3.1 Introduction 53
3.2 One-dimensional Lattices 55
3.2.1 Bloch's Theorem 57
3.2.2 Application of Bloch's Theorem 59
3.2.3 Dispersion Curves and Unit-cell Resonances 59
3.2.4 Continuous Lattices: Local Resonance and sub-Bragg Band Gaps 61
3.2.5 Dispersion Curves of a Beam Lattice 62
3.2.6 Receptance Method 64
3.2.7 Synopsis of 1D Lattices 67
3.3 Two-dimensional Lattice Materials 67
3.3.1 Application of Bloch's Theorem to 2D Lattices 67
3.3.2 Discrete Square Lattice 70
3.4 Lattice Materials 72
3.4.1 Finite Element Modelling of the Unit Cell 75
3.4.2 Band Structure of Lattice Topologies 77
3.4.3 Directionality of Wave Propagation 84
3.5 Tunneling and Evanescent Waves 85
3.6 Concluding Remarks 87
3.7 Acknowledgments 87
References 87

4 Wave Propagation in Damped Lattice Materials 93
Dimitri Krattiger, A. Srikantha Phani and Mahmoud I. Hussein
4.1 Introduction 93
4.2 One-dimensional Mass–Spring–Damper Model 95
4.2.1 1D Model Description 95
4.2.2 Free-wave Solution 96
State-space Wave Calculation 97
Bloch–Rayleigh Perturbation Method 97
4.2.3 Driven-wave Solution 98
4.2.4 1D Damped Band Structures 98
4.3 Two-dimensional Plate–Plate Lattice Model 99
4.3.1 2D Model Description 99
4.3.2 Extension of Driven-wave Calculations to 2D Domains 100
4.3.3 2D Damped Band Structures 101
References 104

5 Wave Propagation in Nonlinear Lattice Materials 107
Kevin L. Manktelow, Massimo Ruzzene and Michael J. Leamy
5.1 Overview 107
5.2 Weakly Nonlinear Dispersion Analysis 108
5.3 Application to a 1D Monoatomic Chain 114
5.3.1 Overview 114
5.3.2 Model Description and Nonlinear Governing Equation 114
10  **Topology Optimization of Lattice Materials**  217  
*Osama R. Bilal and Mahmoud I. Hussein*

10.1  Introduction  217

10.2  Unit-cell Optimization  218

10.2.1  Parametric, Shape, and Topology Optimization  218

10.2.2  Selection of Studies from the Literature  218

10.2.3  Design Search Space  219

10.3  Plate-based Lattice Material Unit Cell  220

10.3.1  Equation of Motion and FE Model  221

10.3.2  Mathematical Formulation  222

10.4  Genetic Algorithm  223

10.4.1  Objective Function  223

10.4.2  Fitness Function  224

10.4.3  Selection  224

10.4.4  Reproduction  224

10.4.5  Initialization and Termination  225

10.4.6  Implementation  225

10.5  Appendix  226

References  228

11  **Dynamics of Locally Resonant and Inertially Amplified Lattice Materials**  233  
*Cetin Yilmaz and Gregory M. Hulbert*

11.1  Introduction  233

11.2  Locally Resonant Lattice Materials  234

11.2.1  1D Locally Resonant Lattices  234

11.2.2  2D Locally Resonant Lattices  241

11.2.3  3D Locally Resonant Lattices  243

11.3  Inertially Amplified Lattice Materials  246

11.3.1  1D Inertially Amplified Lattices  246

11.3.2  2D Inertially Amplified Lattices  248

11.3.3  3D Inertially Amplified Lattices  253

11.4  Conclusions  255

References  256

12  **Dynamics of Nanolattices: Polymer-Nanometal Lattices**  259  
*Craig A. Steeves, Glenn D. Hibbard, Manan Arya, and Ante T. Lausic*

12.1  Introduction  259

12.2  Fabrication  259

12.2.1  Case Study  262

12.3  Lattice Dynamics  263

12.3.1  Lattice Properties  264

Geometries of 3D Lattices  264

Effective Material Properties of Nanometal-coated Polymer Lattices  265

12.3.2  Finite-element Model  266

Displacement Field  266

Kinetic Energy  268
Contents

Strain Potential Energy  269
Collected Equation of Motion  270
12.3.3 Floquet–Bloch Principles  271
Generalized Forces in Bloch Analysis  272
Reduced Equation of Motion  274
12.3.4 Dispersion Curves for the Octet Lattice  275
12.3.5 Lattice Tuning  277
Bandgap Placement  277
Lattice Optimization  277
12.4 Conclusions  278
12.5 Appendix: Shape Functions for a Timoshenko Beam with Six Nodal Degrees of Freedom  279
References  280

Index  283