Index

Above par 8
Accounting for dividends 88–90
Agreements 1, 2, 8, 34–41, 199, 321
American option 151, 155–6
    early exercise boundary 156–8, 224
    pricing 158–9
Annual bond 23
Annual compounding factor 5
Annual coupons 10
Annual equivalent yield 29
Annual rate 3
Arbitrage pricing 82, 87–8, 92–3, 144, 224
Arbitrageurs 87
Arithmetic Brownian motion 139, 141, 291
Arithmetic process 18
Asian option 208, 221–4
Asset management, factor models in 326
Asset-or-nothing option 152
ATM option 154, 155, 184, 190, 238, 239, 240, 318
At par 8
At-the-money (ATM) option 154, 155, 18, 190, 238, 308
Average price option 208, 222–4
Average strike option 208, 221–4

Bank of England forward rate curves 57–8
Banking book 1, 47
Barrier option 152, 219–21
Base rate 8
Basis 68, 95
    commodity 100–1
    fair value of 95, 99
    no arbitrage range 95–7
    risk 67
    swap 38
Basket hedging 111–12
Bearer bond 10

Bear spread 169
Below par 8
Bermudan option 151
Bermudan swaption 195
‘Best of’ option 209
Beta, volatility
    estimation 357–9
    mapping 356–7
Binary option 152, 214
Binomial option pricing model 138, 148–51
Binomial tree 148, 244
BIS Quarterly Review 73
Bivariate GARCH model 262
Black-Scholes-Merton (BSM) formula 137, 139, 173, 176, 179
Black-Scholes-Merton (BSM) model 173–85
    assumptions 174
    implied volatility 183, 231–42
    interpretation of formula 180–3
    partial differential equation (PDE) 139, 175–6
    prices adjusted for stochastic volatility 183–5
    pricing formula 178–80
    underlying contract 176–8
Black–Scholes–Merton Greeks 186–93
    delta 187–8
    gamma 189–90
    static hedges for standard European options 193–4
    theta and rho 188–9
    vega, vanna and volga 190–2
Bond convexity 24–8, 37
Bond duration 2, 10, 20–4
Bond futures 70–3
Bond holder 60
Bond market numeraire 145
Bond portfolio 124–6
Cross-currency basis swap 38
Cross-gamma effect 349
Cubes 81
Cubic splines 51
Currency forwards
  and futures 73, 91–2
  and interest rate differential 91–2

Day count convention 3
Debt financing 8, 319
Degree of homogeneity 292–3
Deliverable grades 70
Delta 139, 159, 187–8, 329
dollar 342
  hedge 103, 166, 175, 300
  minimum variance 279–303
  neutral 161
  position 163–4, 340–2
  value 322, 342–4
Delta–gamma approximation 321, 344–6, 344–7, 349–51
Delta–gamma hedged portfolio 165, 297, 319
Delta–gamma–theta approximation 351
Delta–gamma–theta–rho approximation 351
Delta–gamma–vega–theta–rho approximation 355
Delta–gamma–vega–volga–vanna approximation 354
Delta hedged portfolio 175, 297, 319
Diamond 81
Differential swap 327
Discount
  bond 5
  bond numeraire 145
  factor
    basis point sensitivity of 42, 364
    continuously compounded 89
    discretely compounded 4–5, 7
Discrete compounding 2, 3, 4–8
Discretely compounded discount factor 4
Discretely compounded yield 12
Discretization of stochastic volatility processes 274–5
Dividend payments 89
Dividend risk and interest rate risk 90–1, 114–18
Dividend yield 89
Dollar delta 342
  see also Value delta
Dollar duration 2
  see also Value duration

Dollar gamma, 343
  see also Value gamma
Dollar rho 343
  see also Value rho
Dollar theta 343
  see also Value theta
Dollar vega 355
  see also Value vega
Domestic bond 9
Double jump stochastic volatility model 288
Down and in barrier options 220
Drift 140, 145, 308
Dupire’s equation 228, 245–8, 317
Duration 20–8
dollar 2, 24
  invariance 336
  Macaulay duration 21–3
  modified duration 23–4
  value 2, 24
Duration and convexity 20–8
  approximations to bond price change 25–6
  duration and convexity of a bond portfolio 24–5
  immunizing bond portfolios 27–8

Early exercise boundary 151, 156–8
Early exercise premium 151, 158
Ederington effectiveness 112
Electronic trading system 66
Emissions allowances 84–5
Energy futures 74–9, 134
Energy futures portfolio 118–24
Equity portfolio 324–7
Equity swap 40
Euler’s homogeneous function theorem 293
Eurobonds 10
European option, definition 151
European swaption 198–9
Exchange option 209
Exchange traded fund (ETF) 80–2, 134
  futures on 80–2
Exotic option 139, 207
  price for European
    Asian 221–4
    barrier 219–21
    best/worst of two asset options 209–11
    capped 216–17
    chooser 214
    compound 216
    contingent 214–16
    currency protected 213–14
Exotic option (Continued)
exchange 209–11
ladder 216–18
look-back 208, 218–19
look-forward 218–19
pay-off 208–9
power 214
spread 211–13
Expiry date 8, 65, 134, 151
Exponentially weighted moving average (EWMA) 98, 127–33, 262, 263–4, 358–60
Face value 8
Factor model 326–7
in asset management 326
in risk management 326–7
Fair bet 65
Fair value 67, 88, 94
Financial security 1
Fixed coupon bond, definition 2, 10
Fixed-for-floating interest rate swap, definition 35
Fixed strike volatility spread 256–61
Fixed term 68
Fixing dates 35
Floater 10, 28, 33
Floating rate notes 28, 31–33, 63
Floating smile 247–9, 256–7, 292–3, 317
model 256–7, 292–3
property 247–9
Floor 139, 194, 196
caplets and 195–6
Floorlets
implied volatilities 196–8
Floored floater 33
Foreign bond 10
Forex forward 73
Forex risk 113–14
Forward interest rate 2, 3
agreement 2, 33–34
correlations 56–7
volatilities 55–6
Forward rate agreements 33–40
Forward start option 209
Forward volatility 244–5
see also Local volatility
Foundations of option pricing 140–51
Brownian motion 140–1
calibrating model parameters 147–8
market prices and model prices 146–7
numeraire 144–6
risk neutral valuation 142–4
Free boundary pricing method 158
Fundamental theorem of arbitrage 143
Futures contract 65
credit index 86–7
on emissions allowances 84–5
real estate 87
volatility futures 83–4
weather 85–6
Futures and forwards 65–7
bond futures 70–3
credit index 86–7
currency 73
on emissions allowances 84–5
energy and commodity futures 74–9
exchange traded funds and ETF futures 80–2
hedging 101–2, 164
academic literature on minimum variance 129–30
basket hedging 111–12
bond portfolio 124–6
energy futures portfolio 118–24
forex risk 113–14
‘insurance’ approach 102–3
international stock portfolio 114–18
in liquid stock markets 130–3
mean–variance approach 104–6
minimum variance hedge ratio 106–8, 126–9
performance measures 112–13
position risk 108–9
proxy hedging 110–111
interest rate and swap futures 68–9
real estate 87
stock futures and index futures 79–80
Futures markets 82–7
Futures-spot relationship
accounting for dividends 88–90
commodity forwards, carry costs and convenience yields 93–4
currency forwards and the interest rate differential 91–2
dividend risk and interest rate risk 90–1
fair values of 94
no arbitrage pricing 87–8, 92–3
Future volatility 83–4
Gamma 139, 162, 167, 189–90, 329
on portfolio risk, effect of 346–7
position 163–4
value 322, 342–4
GARCH diffusion 280–5
Generalized autoregressive conditionally heteroscedastic (GARCH) model 127–9
Geometric Brownian motion 139, 140–1, 174
Geometric process 18
Gilts 48
Greeks 139, 159, 162, 163, 178, 179, 187–204, 225, 322, 329
Growth model 140

Heating degree days (HDD) 82
Hedgers 66, 74, 133
Hedger’s beliefs 107
Hedging with futures and forwards 101–2, 164
academic literature on minimum variance 129–30
basket hedging 111–12
bond portfolio 124–6
ergy futures portfolio 118–24
forex risk 113–14
‘insurance’ approach 102–3
international stock portfolio 114–18
in liquid stock markets 130–3
mean–variance approach 104–6
minimum variance hedge ratio 106–8, 126–9
performance measures 112–13
position risk 108–9
proxy hedging 110–11
Hedging options 101–2, 138, 139
delta 159–61
delta–gamma 164–5
delta–gamma–vega 165–7
other Greeks 161–3
performance measures 112–13
position Greeks 163–4
Heston jump model 287
Heston model 270, 278–80, 299, 300–3, 318
Horizontal spread 169
Hybrid securities 1

Idiosyncratic return 325
Implicit volatility 227
Implied tree 244
Implied volatility 183, 329
at-the-money term structure of 238
dynamics 227, 230, 257–66
equity index volatility skew 233–6

index (as in volatility index) 83, 230, 302, 312–14
market 183, 225, 227, 228, 231, 266, 316
from market price 231–3
model 227
principal component analysis of 257–61
smiles and skews in other markets 236–8
sticky models 255–7
surface 178, 239–40
swaption 243
term structures of 238–9
Incomplete market 270, 297, 319
Information set 280
Instantaneous variance 274
‘Insurance’ approach to hedging 102–3
Interest rate
caplets and floorlets 195–6
caps, floors and their implied volatilities 196–8
differential 91
European swaption 198–9
futures 68–9
LIBOR model 201–3
LIBOR model calibration 203–7
sensitive portfolios 323–4, 332–7
sensitive securities 1
short rate models 199–201
swaps 2, 31–41
Internal rate of return 12
Interest rate risk 45–8
Interest rate sensitivities 41–8, 351–3
Interest rate swaps 33–34, 35–6
International Capital Markets Association (ICMA) 10, 61
International exposures 327–8
International stock portfolios 114–18
In-the-money (ITM) option 154, 184, 185
Intrinsic value of vanilla call 154, 156
Invariant cash flow maps 332–7
Investment banks 138
Investment grade bonds 9
Irreducible risk 326
I-Share 80
ITM option 154, 184, 185
Itô’s lemma 141, 174, 175, 177, 270, 298, 307
Jump intensity 287
Junior bond holders 9
Junk bond 9
Kalman filter 262
Key rates 332
Kurtosis 112, 113, 271, 283

Law of one price 87
Lévy model 268
LIBOR (London Inter Bank Offered Rate) 2, 8, 53–9, 195, 201–7
Linear portfolio 321
Liquid stock markets 130–3
Local volatility 243–4, 268
CEV model 285–7, 299, 300–3
Dupire’s equation 245–8
forward 244–5
lognormal mixture diffusion 249–50
parametric models of 248–9
surface 243, 277
dynamics of 228
floating smile 247, 249, 257, 317
static smile 247–8, 249, 317
Lock-out period 31
Lognormal distribution 174
Lognormal mixture diffusion 249–50, 317
London Inter Bank Offered Rate (LIBOR) 2, 8, 53–9, 195, 201–7
Long collar 168
Long run volatility 147, 239, 255, 278, 282
Long term bonds 10
Look-back option 208, 218–19
Look-forward option 208, 218–19

Macaulay duration 20–1
Mapping options portfolios 340–63
Mapping options portfolio to price risk factors 340–1
delta–gamma approximation 344–6, 349–51
price beta mapping 347–9
Taylor expansion 341–2
time and interest rates sensitivities 351–3
value delta and value gamma 342–4
Mapping implied volatility 353
vanna and volga 354–5
vega bucketing 355–6
vega risk in options portfolios 353–4
volatility beta mapping 356–7
Marché à Prime 137
Margin 65
Market
beta 325
crash 257
delta 266, 276
implied measure 143
implied volatility 1–2, 183, 225, 227, 228, 231, 266, 295, 316
interest rate 1, 2, 10–11
local volatility 227, 228
makers 138
model of interest rates 195, 201–3
price 11, 138
of volatility risk 270
Market interest rates, behavior of 17–19
Marking to market 340
Marking to model 340
Martingale measure 145
Martingale process 65
Maturity 3, 10, 14, 15, 151
of bond 2
effect 95
mismatch 67, 102, 103, 105, 107
Mean reversion in volatility 273–4
Mean-reverting stochastic process 17, 140, 147, 271
Mean–variance approach to hedging 67, 101–2, 104–6
Measure 142–6
Medium term bonds 10
Merton’s jump diffusion 287
Micro-econometric foundations of option pricing 280
Minimum variance hedging 102, 290, 297–9
delta 297–303, 319
gamma 297–303, 319
hedge ratios 67, 103, 106–8, 129–30, 230
hedging with futures 102, 106–8, 126–9, 300–3
Mispreding 95
Model free property 229
hedge ratios in scale invariant models 292, 294–7
Model implied volatility 228, 229, 275, 316
Model local volatility 227, 228, 230, 277
Model prices 138, 146–7, 277
Model risk of volatility mapping 360
Modified duration 20–1, 63
Money market 1
account 144
Moneyness, definition 138, 154–5
Multivariate delta–gamma approximation 349–51
Multivariate Taylor expansion 351–3
Naïve hedge 107
Naked call 168
Near futures 74
Negative skew 112, 113, 235, 238
Net carry cost 93
Net convenience yield 93, 95, 99
Net position delta 348
Net position gamma 348
Net value Greeks 349–53
Net value vega 356–63
No arbitrage 138, 139, 142
pricing 87–8, 92–3, 224
range 95–7
valuation principle 88, 144
Nominal and percentage risk factors and sensitivities 331–2
Notional bond 65, 70, 92
Numeraire 140, 144–6
Objective measure 142
One-way arbitrage 93
On-the-run swap 36
Open interest 70
Open outcry 65
Option premium 146, 167, 168
Option pricing 140, 148–51, 207–24, 227, 228, 254, 268–89
Options, characteristics of
American options 155–6
Bermudan 151, 226
early exercise boundary 156–8
elementary options 152–3
European 148, 178, 193, 233, 268, 317
exotics 207–24
moneyness 154–5
pricing 158–9
put–call parity 153–4
Options portfolios 193, 328–30, 332, 340–4, 353–4
Options trading strategies 167
bear strategies 168–9
bull strategies 167–8
P&L profiles, replication of 172–3
spread strategies 169–70
volatility strategies 170–2
Orthogonalization of Brownian motions 273
Orthogonalization of risk factors 330
Out-of-the-money (OTM) option 154, 184, 185
Partial differential equation (PDE) 174
Black–Scholes–Merton model 175–6
stochastic volatility model 269–71
Par value 10
Par yield curves 16
Path-dependent options 148, 151, 152, 207, 219–20, 221, 275–276, 341
Pay-off function 151
Percentage delta 161
Performance measures for hedged portfolios 112–13
Period discount factor 16
Physical delivery 68
Physical measure 142
Plain vanilla 152
Point value 69
Portfolio Greeks 322, 325, 341, 342, 348
Portfolio mapping 321–66
Position
delta 161
gamma 164, 165, 348
greeks 163–4, 225, 365
risk 67, 108–9, 117
theta 164
Power option 209, 214
Premium of option 146, 167, 168
Present value 3, 11, 41
Present value of a basis point (PV01) 2, 41–5, 64, 321, 323, 347
invariance condition for portfolio mapping 332–7
Present value, price and yield 11–13
Price
beta mapping 322, 347–9
discovery 82
process, scale invariant 291–4
risk 104
Price sensitivity, of volatility 262, 266
Price–volatility correlation 147, 269, 271
Price–yield curve 13, 14
Price and yield, relationship between 13–14
Primary market 8
Primitive security 1
Principal 3
Principal component analysis (PCA) 121, 202–7, 257, 330, 361–3
Principal value 8
Private placements 1
Prompt futures 74
Proxy hedging 67, 102, 110–111
Put on a call 216
Put–call parity relationship 138, 151, 153–4
Put option 137, 152, 199, 233
Put on a put 216
PV01 2, 41–8, 64, 321, 323, 332–6, 347
approximations to 44–5
invariance condition for portfolio mapping 332–7
and value duration 41–4
Quadratic covariation 269
Quanto option 209, 213
Quarterly bond 28
Rainbow option 209
Range bounded market 257
Rate of mean reversion 17, 147, 273
Ratio spread 169
Real estate futures 87
Realized volatility 304, 305, 306, 314–16, 320
Redemption value 8
Reference rate 8
Regression based minimum variance hedge
ratios 126–9
Replication portfolio 144, 172, 182
Reset of swap 35
Residual time to maturity 151
Returns sensitivity, of volatility 262
Reverse floater 33
Rho 163–4
dollar 343
position 164, 351, 352
value 343, 351
Risk 67, 74
Risk factor 119–21, 159
betas 325
identification of 121–2
portfolio sensitivity to 122–3
Risk factors and risk factor sensitivities
commodity portfolios 328
equity portfolios 324–7
interest rate sensitive portfolios 323–4
international exposures 327–8
nominal versus percentage 331–2
options portfolios 328–30
orthogonalization of risk factors 330
Risk free portfolio 142, 175
Risk horizon 47
Risk neutral investor 142
Risk neutral measure 142
Risk neutral probability 150
Risk neutral valuation 139, 142–4
principle 144, 175
Risk premium, volatility 147, 270
Rolling the hedge 74
SABR model 285–7, 299, 300–3
see also Stochastic volatility
Sato process 229
Savings account 144–145
Scale invariance 180, 290–2
Scale invariance and hedging 289–90
and change of numeraire 291
empirical results on hedging 300–3
and homogeneity 292–4
model free price hedge ratios 294–6
Secondary market 8
Self-financing portfolio 144, 182
Semi-annual bond 28
Semi-annual coupon 10
Semi-annual and floating coupons 28–9
floaters 10, 28, 33
floating rate notes 31–3
semi-annual and quarterly coupons 29–30
Semi-annual yield 29
Senior bond holders 9
Settlement price 68
Short collar 169
Short rate models 195, 199–201
Short term bonds 10
Short term hedging with futures 67, 126–33
Simple interest 2
Skewness 112, 233, 236, 283, 308–9
Smile 227, 228, 236–8, 243, 247, 249, 257,
271–2
see also Floating smile and Static smile
Sources of emissions 84
Spark spread options 211
Specific risk 117
Speculators 66, 74, 133, 138
Spider 81
Splines 51–2
Spot and forward term structures, characteristics
of 19–20
Spot-futures relationship
accounting for dividends 88–90
commodity forwards, carry costs and
canvenience yields 93–4
correlation 97–8
currency forwards and interest rate
differential 91–2
dividend risk and interest rate risk 90–1
fair values of 94
no arbitrage pricing 87–8, 92–3
Index

Spot interest rates 2–8
  continuously compounded rates 3–4
  discretely compounded rates 4–5
Spot variance and spot volatility 274
Spread option 209
Stable trending market 257
Stack and strip hedge 74
Static hedge 193–4
Static smile 249
Sticky delta 256
Sticky strike 256
Sticky tree 256
Stochastic differential equation (SDE) 141, 148
Stochastic process 17, 137, 140–1, 174, 179, 241, 269, 273
Stochastic variance versus stochastic volatility 272
Stochastic volatility 139, 140, 147, 183–5, 227, 231, 233, 268
  processes, discretization of 274–5
Stochastic volatility models 268–302
  SABR model 285–7, 299, 300–3
  GARCH diffusions 280–5
  Heston model 270, 278–80, 283–5, 299, 300–3, 318
  model implied volatility surface 275–6, 277–8
  PDE 269–71
  prices 287–9
  properties 271–5
Stock futures and index futures 79–80
Stock-specific return 325
Straddle 170
Strangle 170
Strike of option, definition 151
Strong GARCH diffusion 280–5, 318
Structured products 35
Subjective measure 142
Swap 38, 40, 304
  basis 38, 64
  futures 68–9
  interest rate 35
  rate 35
  vanilla 36, 63
  variance 304
  volatility 304
Swaption 1, 137, 139, 194, 201, 226
  Bermudan 139, 195
  European 198–9
  volatility 199, 202, 243
Systematic risk 117, 326
Tax arbitrage 80
Taylor approximation 25, 321, 329, 332
Taylor expansion 2, 184, 330, 341–2, 355
Tenor 3, 88
Theoretical value 88
Theta 163–4
  dollar 343
  mapping 351–3
  and rho 188–9
  value 343
Time spread 169
Tracking error risk 117
Trading book 1
Trading options and volatility
  bear strategies 168–9
  bull strategies 167–8
  construction of a volatility index 307–8
  forecasts to trade volatility 314–16
  P&L profiles, replication of 172–3
  skew, effect of 308–9
  spread strategies 169–70
  trading forward volatility 305–6
  variance risk premium 306–7
  variance swaps and volatility swaps 304–5
  vix 310–311
volatility indices
  futures 312–13
  options on 313–14
  term structures of 309–10
volatility strategies 170–2
Transition probability 149
Two-way arbitrage 100
Underlying contract 176–8
Undiversifiable risk 326
Up and in barrier option 220
Up and out call 153, 219–21
US Treasury bills 10
Value convexity 2, 24
Value delta 322, 342–4
  vector 350
Value duration 2, 24
Value gamma 322, 342–4
  matrix 350
Value rho 343
Value theta 343
Value vega 343, 355–9
Vanilla options 138, 159
  American 155–6, 158–9
  early exercise boundary 156–8
Vanilla options (Continued)
  elementary 152–3
  moneyness 154–5
  put–call parity 153–4
  swap 36
  cash flows on 36
Vanna 159, 163–4
Vanna-volga mapping 354–5
Varb-X benchmark index 313, 316
Variance risk premium 147, 270, 306
Variance swap 230, 304
  rate 304
Vasicek process 200
Vega 139, 150, 162
  bucketing 355–6
  dollar 343, 355
  risk in options portfolios 353–4
  value 343, 353–6
  weighted volatility iteration 241–3
Vega bucketing 355
Vega, vanna and volga 190–2
Vftse index 309, 357–64
Volatility 227–320
  beta 356–9
  construction of volatility index 307–8
  forward volatility 305–6
  futures 83–4
  implied
    cap and caplet 240–3
    equity index volatility skew 233–6
    from market price 231–3
    smiles and skews in other markets 236–8
    surfaces 239–40
    swaption 243
    term structures of 238–9
  implied, modeling the dynamics of
    case studies 257–65
    hedging options, applications 265–7
    sticky models 255–7
  index 83, 230, 302, 3312–14
  index futures 312–13
  index, options on 313–14
  index, term structures of 309–10
  invariant maps 334–6
  local 243–4
    Dupire’s equation 245–8
    forward 244–5
    lognormal mixture diffusion 249–50
    parametric models of 248–9
    mean reversion in 147, 273–4
price sensitivity 262, 266
risk premium 147, 270
scale invariance and hedging 289–90
  and change of numeraire 291
  empirical results 300–3
  and homogeneity 292–4
  model free price hedge ratios 294–6
  skew, effect of 308–9
  smile 227, 228, 236–8, 243, 247, 249, 257, 271–2
stochastic volatility model 268
  CEV and SABR 285–7
  GARCH diffusions 280–5
  Heston model 278–80, 299, 300–3
  model implied volatility surface 275–6
  277–8
  PDE 269–71
  properties 271–5
  swap 304
  trade volatility 314–16
  trading 304–15
  variance risk premium 306–7
  variance swaps and volatility swaps 304–5
  vix 310–311
Volatility risk
  mapping to term structures of volatility
  indices 361
  model risk of volatility mapping 360
  volatility betas 357–9, 361–3
Volga 159, 162–3
Weak GARCH diffusion 281–5, 318
Weather futures 85–6, 100
Wiener process 140, 174
Yankee 10
Yield curves 14–16, 48–59
  fitting 48–59
  model 16, 52, 53, 58, 59
  bootstrapping 49–51
  calibration instruments 48–9
  LIBOR rates 53–9
  parametric models 52–3
  splines 51–2
Yield to redemption 12
Zero coupon
  bond 5, 16
  interest rate 43
  yield curve 1, 2, 10, 15, 25–26