1

Introduction to Power Transmission

The term power transmission refers to a collection of devices assembled to transmit power from one physical point to another. In this chapter, we describe the most common types of power transmissions and introduce the subject of hydrostatic transmissions and actuators. This chapter is divided into six parts:

- Mechanical transmissions
- Hydrodynamic transmissions
- Hydrostatic transmissions
- Hydromechanical transmissions
- Mechanical actuators
- Hydrostatic actuators

It is important to mention that there are other types of power transmissions. For example, an electricity gridline is a type of power transmission – from the generator to the final user. However, when mechanical energy is involved (kinetic and potential), the aforementioned types are the most representative.

The majority of this chapter deals with the topic ‘transmissions’, with a smaller portion dedicated to ‘actuators’, as actuators can be seen as a special type of hydrostatic transmission where the motor is replaced by a hydraulic cylinder. We start with a basic concept common to both mechanical and hydrostatic transmissions: the transmission ratio.

1.1 Transmission Ratio

1.1.1 Generalities

Figure 1.1 illustrates a typical situation where a power transmission can be applied. The input shaft is rotating with an angular speed $\omega_i$ and is connected to a prime mover (such as
an electric motor or an engine) whose output power is $P_i$. We connect the input shaft to an output (driven) shaft that must rotate at an angular speed $\omega_o$. The angular speed of the driven shaft may be greater or lesser than the angular speed of the input shaft or even have an opposite direction in relation to the input shaft’s angular speed.

When mechanical transmissions, such as gearboxes, belts and chains, are considered, the spatial arrangement of the driving and driven shafts is of paramount importance because it dictates the technology to be used. For example, in the case of parallel shafts, a gear transmission may be used if their distance from each other is not too great. However, the farther the shafts are from each other, the heavier the gearbox becomes, leading to more demanding requirements with respect to alignment and lubrication. Belts and pulleys may be used for transmissions between shafts that are separated by a considerable length, but the problem of spatial arrangement remains. Moreover, the power to be transmitted becomes considerably limited due to the belt-to-pulley friction coefficient. Chain transmissions are noisy and require the shafts to be perfectly parallel with constant lubrication. Additionally, these types of transmissions – with the exception of some special arrangements of chains and belts – do not allow for continuous transmission ratios, as will be explained shortly.

Mechanical transmissions, in general, have the following limitations:

1. They require the driving and driven shafts to be relatively near one another.
2. They are usually not flexible with regard to the spatial arrangement of the components.
3. Typically, they do not provide a continuous transmission ratio, that is, the ratio between the angular speed of the driven shaft and the angular speed of the driving shaft assumes discrete values.

Hydrostatic transmissions transmit power through a hydraulic fluid that travels inside flexible hoses or other types of conduits. As a result, there is great spatial flexibility, and the input and output shafts can be placed almost anywhere in relation to one another. Continuous transmission ratios are an inherent property of hydrostatic transmissions, whereas in
mechanical transmissions, continuous transmission ratios are usually attained through more complex mechanisms. Energy losses are typically higher in hydrostatic transmissions when compared to mechanical transmissions. The pros and cons of hydrostatic transmissions will become more clear throughout this book.

1.1.2 Definition

The transmission ratio, $R_T$, is commonly defined as the quotient between the angular speed of the input shaft and the angular speed of the output shaft:

$$R_T = \frac{\omega_i}{\omega_o}$$

However, the definition of transmission ratio given above is frequently inconvenient. For example, in cases where the angular speed of the output shaft ($\omega_o$) becomes zero, the transmission ratio becomes infinite. In this case, it is better to define the transmission ratio as the quotient between the output and the input speeds \[1\]. We adopt this convention in this book and use the following expression for $R_T$:

$$R_T = \frac{\omega_o}{\omega_i} \quad (1.1)$$

It is important to note that the concept of transmission ratio only makes sense when the power transmission occurs between two rotating shafts, as shown in Figure 1.1. When there is a conversion from a rotary to a linear motion, as is the case for mechanical or hydrostatic actuators, it makes no sense to use the transmission ratio as a design parameter.

The term *transmission* implies that the power transfer occurs between two rotating shafts. If, in the process of power transmission, there is a conversion from a rotary motion to a linear or a limited angular motion, the term *actuator* will be applied instead.

1.1.3 Classification

In what follows, let $c_1, c_2, \ldots, c_N$ be nonzero real constants. Concerning the transmission ratio, we can have the following types of power transmission:

- Fixed-ratio transmission ($R_T = c_1$): There is only one possible value for the transmission ratio.
- Discretely variable transmission ($R_T = [c_1, c_2, c_3, \ldots, c_N]$): There is a set of finite values for the transmission ratio.
Fixed-ratio transmission

- Gear trains
- Belt
- Chains

Discretely variable transmission

- Sequential gearboxes
- Planetary gearboxes

Continuously variable transmission

- Toroidal
- Conic pulley

Infinitely variable transmission

- Combined: toroidal + planetary gearbox
- Hydro-mechanical transmission
- Hydrostatic transmission

Figure 1.2 Classification of power transmissions according to the transmission ratio

- Continuously variable transmission (CVT) \((c_1 \leq R_T \leq c_2 \text{ and } c_1 c_2 > 0)\): The transmission ratio varies continuously between two positive or negative\(^1\) limits.
- Infinitely variable transmission (IVT) \(0 \leq |R_T| \leq |c_1|\): The absolute value of the transmission ratio varies continuously between zero and a positive value \([2]\). Strictly speaking, an IVT ratio would be such that \(-\infty \leq R_T \leq +\infty\). However, due to engineering restrictions, this condition is usually relaxed\(^2\) \([1, 2]\).

Figure 1.2 shows a few selected types of power transmissions, organized according to their transmission ratio. The first two categories – fixed-ratio and discretely variable transmissions – cover most of the existing mechanical transmissions today. Continuously and infinitely variable mechanical transmissions are restricted to some special designs.

In the following sections, we briefly review each mechanical transmission type presented in Figure 1.2.

1.2 Mechanical Transmissions

There are many different types of mechanical transmissions, and it is not our goal to address all of them. Rather, we aim to give a succinct introduction to the theme in order to present an overview of the main transmission types. Therefore, only a small subset of the existing technologies will be covered here – enough to exemplify each type of mechanical transmission and give brief explanation about their characteristics.

1.2.1 Gear Trains

Gear trains are a typical example of fixed-ratio transmissions. They can be better understood if we first consider a disc train (Figure 1.3), where we assume that every disc is perfectly rigid.

\(^1\) A negative transmission ratio indicates a reversal of direction. Observe that the definition of a continuously variable transmission implies that the rotation of the output shaft is never reversed within the transmission range.

\(^2\) Some authors have defined the term ‘Infinitely Variable Transmission’ based on the definition of the transmission ratio as being the ratio between the input and output speeds, \(\omega_i/\omega_o\). In this aspect, a zero output speed would correspond to an infinite transmission ratio \([2]\). Using this argumentation, an infinitely variable transmission would be simply defined as a Continuously Variable Transmission for which \(0 \leq |R_T| \leq |c_1|\).
This kind of transmission is not practical, but the equations obtained here are representative of the most general scenario. By assuming that no slip occurs between the discs, we can say that the tangential velocities on the surface of the two contacting discs are equal. This allows us to obtain the rotating speed of the shaft between discs 2 and 3 (represented by the subscript 23) through the following expression:

$$\omega_{23} = -\left(\frac{r_1}{r_2}\right)\omega_i$$  \hspace{1cm} (1.2)

where $r$ is the radius of the corresponding disc (disc 1 or disc 2).

If we proceed in this manner for every pair of discs, we obtain the following general relation for a train with $N/2$ pair of discs ($N$ is an even number of discs, as shown in Figure 1.3):

$$\omega_o = (-1)^{\frac{N}{2}}\left(\frac{r_1r_3r_5\cdots r_{N-1}}{r_2r_4r_6\cdots r_N}\right)\omega_i$$  \hspace{1cm} (1.3)

The transmission ratio, in this case, is given by:

$$R_T = \frac{\omega_o}{\omega_i} = (-1)^{\frac{N}{2}}\left(\frac{r_1r_3r_5\cdots r_{N-1}}{r_2r_4r_6\cdots r_N}\right)$$  \hspace{1cm} (1.4)

We observe that friction discs lose their non-slip characteristic as soon as the resisting torque reaches a certain value. That is why gear trains are used instead of disc trains. Nevertheless, Eqs. (1.3) and (1.4) are still valid for gear trains if we replace $r_i$ by the pitch radii of the gears, or, as it is usually the case, the corresponding number of teeth. An almost identical transmission ratio can be obtained for a chain of belts and pulleys under the non-slipping assumption.$^3$ It is also possible to show that chain drives behave similar to belts and pulleys.

$^3$ In fact, the only difference would be the absence of the reversal term, $-1$, in Eq. (1.4), given that the direction of rotation between each pair of pulleys is the same.
1.2.2 Gearboxes

Gearboxes are typical examples of discretely variable transmissions. We describe two types of gearboxes here: sequential and planetary. Within the automotive industry framework, sequential gearboxes are used in manual transmission cars, whereas planetary gearboxes are usually found in automatic transmission vehicles.

1.2.2.1 Sequential Gearboxes

The train shown in Figure 1.3 is limited because the whole mechanism gives only one constant transmission ratio, and that cannot be easily changed. In many applications, however, it is important for the transmission to offer the choice of more than one transmission ratio. The ideal case would be the one in which the transmission could be able to produce an infinite number of transmission ratios. Gearboxes allow for multiple transmission ratios and, in this sense, represent a step forward in relation to the fixed train represented by Figure 1.3.

Figure 1.4 shows a simple two-speed sequential gearbox. The collar C is mechanically engaged to the splined shaft B and is allowed to slide horizontally on the shaft ridges. Gears 2 and 4, on the other hand, are mounted on bearings, being mechanically decoupled from shaft B. Teeth are placed on the inner sides of gears 2 and 4 to allow for the coupling with the sliding collar and, as a result, with shaft B. Gears 1 and 3 are rigidly coupled to shaft A and rotate with it. The collar C may engage with either gear 2 or gear 4, with the aid of the fork F directly connected to a shifting lever, providing a choice of two transmission ratios as only one pair of gears is actually transferring power at a time: gears 1 and 2 or gears 3 and 4. We could proceed in this manner and add more gear couplings to the gearbox, creating a multiratio gearbox. Note, however, that still only a few discrete values for the transmission ratio would be available in the end, and the shifting between each ratio would

![Figure 1.4 Schematic representation of a two-speed sequential gearbox](image)
require decoupling the driving shaft A from its prime mover by means of a clutch to avoid damaging the collar mechanism.

Figure 1.5 shows a typical sequential gearbox.

1.2.2.2 Planetary Gearboxes

In the gearboxes shown in Figures 1.4 and 1.5, the gears’ teeth engage externally with one another. Figure 1.6 shows another arrangement called a planetary gearbox in which the engagements occur both externally and internally. Gears S and P (sun and planet) are held against each other by a rotating bar C (planet carrier) that is pivoted on each gear in a way that gear P can rotate around its centre and orbit around gear S simultaneously. Both gears P and S are placed inside an outer gear (ring R).

Figure 1.5  Sequential gearbox

Figure 1.6  Schematic representation of a planetary gearbox
Planetary gearboxes can have different gear ratios depending on the way in which power is being transferred. For instance, consider the pitch radii of the sun and ring gears: \( r_S \) and \( r_R \). It can be easily shown that the angular speeds of the ring, sun and carrier, \( \omega_R \), \( \omega_S \) and \( \omega_C \), respectively, are related through the following equation\(^4\):

\[
\frac{r_S}{r_R} = -\left( \frac{\omega_R - \omega_C}{\omega_S - \omega_C} \right)
\] (1.5)

Suppose, for example, that the input power source is connected to the carrier \( C \). If we connect the output to the ring \( R \) and fix the sun \( S \) (\( \omega_S = 0 \)), we get, from Eq. (1.5),

\[
\frac{r_S}{r_R} = \frac{\omega_R - \omega_C}{\omega_C}
\] (1.6)

From Eqs. (1.6) and (1.1), we obtain the transmission ratio, in this case:

\[
R_T = \frac{\omega_R}{\omega_C} = \frac{r_S}{r_R} + 1
\] (1.7)

We may proceed in the same manner for other possible configurations. Note that when the carrier \( C \) is fixed (\( \omega_C = 0 \)), a reversion of the rotation occurs (see Eq. (1.5)). As expected, Eqs. (1.5)–(1.7) can be written as a function of the gear teeth instead of the pitch radii by substituting \( r_S \) and \( r_R \) by the corresponding number of teeth \( z_S \) and \( z_R \).

Planetary gearboxes are more complex to manufacture when compared to sequential gearboxes. However, changing gears only requires a set of simple devices to keep the right element stationary. For example, we may use a belt around the ring gear (brake band) to keep it stationary while connecting the output shaft to the sun gear with the aid of a clutch. A similar procedure can be applied to the other gears. In planetary gearboxes, the transmission ratio can be changed without the need to disengage the gears simply by holding the right elements; this makes this type of gearbox attractive in automatic automotive transmissions. Another important feature of planetary gearboxes is that they can be used as either mechanical power dividers or combiners. For example, by connecting the ring and the sun to different power sources, the total power input can be combined into an output shaft connected to the carrier. This feature will be better explained later on in this chapter when we introduce the theme of power-split transmissions. A typical planetary gearbox is illustrated in Figure 1.7.

### 1.2.3 Efficiency

The transmission ratio given by Eq. (1.4) was determined under the premise that the velocities on the surface of two contacting discs were equal. That is true as long as there is no slippage between the discs and no deformation in the point of contact so that the discs remain perfectly round. However, by transitioning to the usage of gears, the picture of two friction discs is replaced by two surfaces pressing against one another. For this reason we cannot use

\(^4\)We leave this demonstration as an exercise for the student at the end of this chapter.
either the external or the internal radii of the gears in relation (1.4); instead, we must use a virtual dimension called pitch radius. Fortunately, this is equivalent to using the number of gear teeth instead [3], which makes the determination of the transmission ratio much easier.

Figure 1.8 shows the moment when two gears are engaging. The instantaneous speed of a point in gear A, located at the intersection of the two contacting surfaces, is $v_A = r_A \omega_A$. Similarly, the instantaneous speed of a point in gear B, ‘in touch’ with point A, is $v_B = r_B \omega_B$. The projection of these two speeds onto line $t$, which is tangent to the contacting surfaces, will be different most of the time. For example, in the figure, we see that the projection of
\( v_A \) onto \( t \) is bigger than the projection of \( v_B \) onto \( t \). Such difference causes the two gear teeth to slide against each other.

By multiplying the magnitude of the difference between the velocity projections by the friction force between the teeth, we obtain the instantaneous friction dissipative power that is lost in the form of heat. Friction dissipation will be greater when either the normal force or the friction coefficient between the teeth increases. The relative velocity also increases with the angular velocities \( \omega_A \) and \( \omega_B \). Therefore, higher angular speeds favour energy losses.

From what has been examined so far, we observe that not all the input power is transferred to the output shaft; a certain amount of that power is lost. Moreover, the amount of energy dissipation is not constant, given that the losses depend on factors such as torque, angular speed, lubricant, surface roughness of the gear teeth and temperature. In the particular case of mechanical transmissions, the energy loss is translated into a smaller torque available at the output shaft.

It has been reported that gear transmissions are quite efficient, each pair being capable of transmitting as much as 99.1–99.9% of the input power [4]. However, it must be pointed out that depending on the size of the gearbox, its design and the choice of the lubricant, this efficiency could drop significantly. For example, miniature planetary gearboxes have been reported to have a maximum efficiency near 73% [5].

We finish this discussion about efficiency by giving it a formal definition that will be used throughout this book:

\[
\eta = \frac{P_o}{P_i}
\]  

(1.8)

If we recall that power is the product of the torque and the angular speed \( (P = T\omega) \), we may write Eq. (1.8) as,

\[
\eta = \frac{T_o\omega_o}{T_i\omega_i}
\]

(1.9)

Using the transmission ratio definition (Eq. (1.1)), we obtain,

\[
T_o = \frac{\eta T_i}{R_T}
\]

(1.10)

From Eq. (1.10), we see that, for a fixed transmission ratio, the relation between the input torque and the output torque in gearboxes is constant, as long as the efficiency remains constant. As said earlier, lower efficiencies reflect directly on the magnitude of the output torque.

In closing this section, it is very important to mention another aspect of gear transmissions: teeth geometry. Depending on the gear diameters and the number of teeth, the tooth of one
of the gears may carve into the other’s causing a serious problem known as interference. The way to avoid this is to limit the minimum number of teeth of the smallest gear; therefore, the size of the transmission must be inferiorly limited (the interested reader may consult [3] for a more detailed discussion on this topic). As the dimensions of the gear teeth must grow for higher torques, the size of the gearbox is also affected by the power to be transferred. Gearboxes, therefore, can become bulky and heavy, and this characteristic can be seen as a negative feature for some particular applications (such as in wind turbines [6]).

1.2.4 Continuously and Infinitely Variable Transmissions

1.2.4.1 Basic Design

The idea of a CVT is illustrated with the aid of three friction discs, as shown in Figure 1.9. The three discs A, B and C have radii \( r_A, r_B \) and \( r_C \), respectively. In this figure, discs A and B rotate in opposite directions with vector angular speeds \( \vec{\omega}_A \) and \( \vec{\omega}_B \). The angular speed of the intermediate disc, C, is \( \vec{\omega}_C \). Disc C touches both discs A and B in two points whose distance to the centre lines are given by \( r_1 \) and \( r_2 \), respectively. Although it is not shown in the figure, both \( r_1 \) and \( r_2 \) can have negative values. If \( r_1 \) and \( r_2 \) have opposite signs, the discs A and B rotate in the same direction. Observe that the distance between discs A and B varies with both \( r_1 \) and \( r_2 \), reaching a maximum for \( r_1 = r_2 \).

In order to transmit high powers, the device illustrated in Figure 1.9 needs a high friction between the discs. As a result, the contact pressure must be high in order to create sufficient friction torques and, since the contact between the discs occurs practically along a line segment, the superficial deformation becomes significant. This is the main reason why this kind of device is not used in practice. However, the toroidal transmission, which will be introduced next, constitutes a natural development of this basic scheme and has been commercially used.

From Figure 1.9, we obtain the transmission ratio between discs A and B:

\[
R_T = -\frac{\omega_B}{\omega_A} = \frac{r_1}{r_2}
\]  

(1.11)
Equation (1.11) shows that the transmission illustrated in Figure 1.9 can have an infinite transmission ratio since the values of $r_1$ and $r_2$ can range from a negative minimum to a positive maximum. Therefore, if we disregard any physical limitation, what we have is an IVT in the strictest sense of the expression. Figure 1.10 shows the transmission ratio in a graph format as given by Eq. (1.11). Two situations are shown: one in which we fix the radius $r_1$ at its maximum value, $r_{\text{max}}$, and another in which the radius $r_2$ is fixed at $r_{\text{max}}$. In both cases, we plot the variation of the transmission ratio for the other radius changing from $-r_{\text{max}}$ to $+r_{\text{max}}$. In the first case, we obtain a speed amplification at the output shaft as the transmission ratio grows infinitely when $r_2$ approaches zero. In the second case, the output speed is reduced in relation to the input speed, becoming zero at $r_1 = 0$ and smoothly reversing as disc C is gradually inclined.

1.2.4.2 Toroidal Continuously Variable Transmissions

Toroidal CVTs are based on the IVT model shown in Figure 1.9. However, because of the way in which they are built, they are not IVTs.

Figure 1.11 shows the working principle of a toroidal transmission. The input and output shafts are connected to the toroidal discs A and B, respectively. Discs C_1 and C_2 work in a synchronized way, moving symmetrically and playing the same role of disc C in Figure 1.9.\footnote{Although only one disc would be necessary in this case, two discs are used to keep the vertical shaft mechanically balanced.} Discs A and B are free to slide over the vertical shaft that passes through them so that they may move towards or apart from one another as needed.

As can be seen in Figure 1.11, a reversal of the rotation is not possible in this configuration, and the transmission ratio is limited within a range that is determined by the geometry of the discs. Toroidal CVTs can be found in automobile transmissions, being regarded as ‘smooth’ and efficient [7].

Even though the toroidal design does not allow for the construction of an IVT or reverse the rotation of the driven shaft in relation to the driving shaft, it is possible to combine it with a planetary gearbox in a way that a reversal of the output speed can be continuously
attained [1]. The idea here is to ‘split’ the input power so that part of it goes through a CVT, also known as a \textit{toroidal variator},\footnote{In power split transmissions, it is common to refer to the CVT component as \textit{variator}.} and the other part goes through a discretely variable transmission (gearbox), as illustrated in Figure 1.12.\footnote{Torque converters will be introduced in Section 1.3.}

The power-split concept shown in Figure 1.12 can be generalized to include other types of CVTs. For instance, as will be seen in Section 1.6, the same idea can be applied to a combined hydrostatic transmission and gearbox.

In order to understand the operation of a power-split transmission, consider the arrangement\footnote{Not necessarily the same arrangement be used in the transmission shown in Figure 1.12.} shown in Figure 1.13. In this figure, we see that the prime mover shaft is connected to both the input shaft of the variator, A, and the carrier of a planetary gearbox, C. Furthermore, the output shaft of the variator, B, is connected to the sun gear of the gearbox, S. Finally, the output shaft of the combined transmission is connected to the planetary ring, R. Note that we have

\[
\begin{cases} 
\omega_C = \omega_A = \omega_i \\
\omega_B = \omega_S = -\omega_A R_{CVT}^T = -\omega_i R_{CVT}^T
\end{cases}
\]

where $R_{CVT}^T$ is the CVT ratio of the toroidal transmission.
From Eqs. (1.5) and (1.12), we have

\[
\frac{r_S}{r_R} = - \left( \frac{\omega_o - \omega_i}{-R_{CVT}^T \omega_i - \omega_i} \right) \tag{1.13}
\]

We can rewrite Eq. (1.13) in a more meaningful way:

\[
\omega_o = \omega_i \left[ 1 + \frac{r_S}{r_R} \left( R_{CVT}^T + 1 \right) \right] \tag{1.14}
\]

The transmission ratio of the combined transmission, \( R_T \), can now be obtained:

\[
R_T = \frac{\omega_o}{\omega_i} = 1 + \frac{r_S}{r_R} \left( R_{CVT}^T + 1 \right) \tag{1.15}
\]

Table 1.1 shows the behaviour of the transmission ratio, \( R_T \), in terms of the number of teeth of the sun and the ring gears, \( z_S \) and \( z_R \), for different values of the toroidal transmission ratio, \( R_{CVT}^T \). We observe that through the use of the combined toroidal and gearbox transmissions, we can obtain a transmission ratio that continuously ranges from a negative value to a positive value, that is, an IVT.

### 1.2.4.3 Variable Diameter Pulleys

Figure 1.14 shows a continuously variable mechanical transmission with conic pulleys and a steel belt [7] or a chain [8]. The pulley cones can slide over their shafts towards or apart from one another. In order to keep the belt tightly adjusted if one pair of pulley cones moves towards one another, the other must be set apart by the same length. For example, in Case 1, the pulley cones in the top are close to each other, whereas the pulley cones at the bottom are separated. When we consider pulley A as the input pulley, in this case, \( R_T > 1 \) (\( \omega_A < \omega_B \)),

![Combination of a toroidal CVT and a gearbox](Image.png)
Table 1.1  Transmission ratio range for the power-split transmission

<table>
<thead>
<tr>
<th>CVT transmission ratio</th>
<th>Combined CVT and gearbox transmission ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_{CVT}^T &lt; - \left( \frac{z_R}{z_S} + 1 \right) )</td>
<td>( R_T &lt; 0 )</td>
</tr>
<tr>
<td>( R_{CVT}^T = - \left( \frac{z_R}{z_S} + 1 \right) )</td>
<td>( R_T = 0 )</td>
</tr>
<tr>
<td>( R_{CVT}^T &gt; - \left( \frac{z_R}{z_S} + 1 \right) )</td>
<td>( R_T &gt; 0 )</td>
</tr>
</tbody>
</table>

Figure 1.14  Continuously variable transmission with conic pulleys: (a) Case 1 \( \omega_A < \omega_B \) and (b) Case 2 \( \omega_A > \omega_B \)

we have a speed amplification. In Case 2, the top pulley cones are separated, whereas the bottom pulley cones are brought together. The transmission ratio then changes to \( R_T < 1 \) \((\omega_A > \omega_B)\) and a reduction occurs. Note that it is not possible to reverse the direction of rotation in this transmission design.

Variable diameter transmissions have been widely used in the automobile industry. Figure 1.15 shows a typical CVT with a steel belt.

1.3  Hydraulic Transmissions

In hydraulic transmissions, power is transmitted by means of a fluid connecting a hydraulic pump to a hydraulic motor. The pump receives mechanical energy from a rotating shaft connected to the prime mover and transfers it to the fluid in the form of flow and pressure (hydraulic energy). The fluid then carries the hydraulic energy into the motor where it is transformed back into mechanical power at the output shaft connected to a mechanical
device (Figure 1.16). The basic elements in a hydraulic transmission are, therefore, the pump, the fluid and the motor, whose individual roles can be summarized as follows:

The pump converts mechanical energy into hydraulic energy.
The fluid transports hydraulic energy from the pump into the motor.
The motor converts hydraulic energy into mechanical energy.

Depending on the type of pump and motor used in the hydraulic transmission of Figure 1.16, the result can be either a hydrodynamic transmission or a hydrostatic transmission. Despite this general definition, it is not unusual to find the terms ‘hydraulic transmission’ and ‘hydrostatic transmission’ being used interchangeably, especially in older references [9, 10]. In the automotive industry, the term ‘hydrodynamic transmission’ itself is not very popular, being usually substituted by the expression ‘torque converter’, as will be described shortly.

Hydrodynamic transmissions require the connection of a hydrodynamic pump to a hydrodynamic motor. In a hydrodynamic motor, torque is created through a change in the fluid velocity as it passes through the internal blades and channels. Similarly, the torque input at a hydrodynamic pump causes the fluid velocity to change in intensity and direction, producing a flow. This is better visualized in Figure 1.17, which shows a typical
Figure 1.16 Schematic representation of a hydraulic transmission

Figure 1.17 Torque converter: (a) schematic diagram and (b) exploded view (courtesy of BD Diesel Performance)

hydrodynamic transmission used in automatic cars, best known as a ‘torque converter’ as mentioned earlier.

In the schematic representation given in Figure 1.17(a), we identify the pump (impeller), the energy-carrying fluid and the motor (turbine). The pump receives power from the engine and rotates at an angular speed $\omega_i$. As the rotor connected to the pump shaft revolves, the fluid is radially accelerated towards the case. Due to the case curvature, the flow changes
direction and leaves the pump parallel and opposite to the incoming flow (see view A–A’). Likewise, as soon as the fluid reaches the turbine (motor), the case redirects the stream so that it now flows radially towards the centre. Due to the curvature of the turbine blades, the velocity of the fluid changes direction, as shown by their projections $\vec{v}_1$ and $\vec{v}_2$. The change in the velocity vector as the fluid travels from the turbine case towards the centre produces a tangential force and, as a consequence, an output torque, $T_o$, that depends on the magnitude of the incoming speed and the aerodynamic design of the blades.

Due to the geometry of the turbine blades, the flow returning from the turbine into the impeller is no longer horizontally oriented. If this flow, coming from the turbine, hit the impeller blades just as it were, that is, inclined in relation to the transmission axis, it would end up contributing to a deceleration of the impeller as an opposite torque would be created. Therefore, in order to redirect the flow from the turbine into the pump, a stator (a helical fluid redirector) is placed in between the impeller and the turbine [11]. Note that, even if the turbine stops, torque at the output shaft will still exist as long as fluid is pumped through the turbine blades. At this stage, the input power would be totally converted into torque and heat (due to the viscosity of the fluid). Because of this particular feature, it seems appropriate to use the term ‘torque converter’ instead of ‘hydrodynamic transmission’ when referring to the device illustrated in Figure 1.17, given that power is not always transmitted between the input and output shafts.\(^9\)

When both pump and motor in a hydraulic transmission are ‘hydrostatic’, we have a hydrostatic transmission. In general terms, we may say that in hydrostatic pumps, fluid is literally ‘pushed’ into the circuit, while, on the motor side, it is the fluid pressure that causes the motor to move. A simple illustration of the operational principle of hydrostatic pumps and motors is given in Figure 1.18, where the crank and shaft mechanism on the left (pump) pushes the fluid with a force $F_i$, creating pressure $p$ inside the hydraulic circuit. On the other hand, the piston on the right (motor) turns the crank by the action of this same pressure ($p$). Therefore, the input power is transmitted to the output shaft.

Depending on the relative diameters of the pump and motor pistons in Figure 1.18, we can obtain a speed amplification or reduction. Note, however, that differently from the torque converter shown in Figure 1.17, it is not possible to stop the motor shaft and still have the pump shaft rotating. In other words, if power (torque and speed) is input at the pump, we must have a power output at the motor\(^{10}\) – the basic characteristic of any power

\[\text{Figure 1.18} \quad \text{Operational principle of hydrostatic pumps and motors (left: pump and right: motor)}\]

\(^9\)Remember that power requires two components: torque and angular speed. Therefore, when the turbine shaft is stationary, although torque is being generated, no power is being transmitted.

\(^{10}\)Here, we assume that there is no fluid leakage in the circuit.
transmission. Figure 1.19 shows a typical hydrostatic pump that can be used in hydrostatic transmissions.\footnote{There is no crank mechanism in the pump shown in Figure 1.19 (keep in mind that Figure 1.18 is merely illustrative). Details about hydrostatic pumps and motors, including the one shown in Figure 1.19, will be presented in Chapter 3.}

In what follows, we explore hydrostatic transmissions in more detail.

### 1.4 Hydrostatic Transmissions

Having introduced the main differences between hydrodynamic and hydrostatic transmissions, we now focus on hydrostatic transmissions, one of the theme subjects of this book. We begin by exploring the way in which hydrostatic transmissions operate, with emphasis on the circuit components. We then move on to give a formal definition of ‘hydrostatic transmissions’, followed by some considerations about efficiency and classification. Before we move on, however, we need to say something about the way in which hydraulic circuits are represented in this book.

As a general rule, the ISO 1219-1 hydraulic symbols (see Appendix A) are frequently used in this book to describe hydraulic circuits. However, a few non-standard symbols are also employed to enhance text comprehension or to represent a particular situation for which a standard symbol is not available.\footnote{It must be noted here that the use of non-standard ISO symbols is a common practice in hydraulics, as can be confirmed by a number of references quoted in this book.} Table 1.2 shows two non-standard symbols used throughout this book, representing a generic prime mover, such as an internal combustion engine or an electric motor.

#### 1.4.1 Operational Principles

Figure 1.20 illustrates a simple situation where pump and motor are connected as a hydrostatic transmission. In Figure 1.20, the prime mover, PM, is connected to a reversible hydrostatic pump\footnote{A reversible hydrostatic pump is a pump that can output flow in both directions.} whose input and output ports 1 and 2 link to the motor input and output.

![External gear-type hydrostatic pump (courtesy of Parker-Hannifin Corp.)](image)
Table 1.2  Non-standard prime mover hydraulic symbols

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prime mover</td>
<td>PM</td>
</tr>
<tr>
<td>Variable speed prime mover</td>
<td>PM, PM</td>
</tr>
</tbody>
</table>

Figure 1.20  Pressure rise in the pump–motor conduit

ports 3 and 4. Imagine that the motor shaft is slowed down by a resistive load while the pump continues sending its flow, \( q_p \), into port 3. As a consequence, we observe that the pressure in lines 2–3 will rise, as illustrated in the figure.\(^{14}\)

One possible solution for lowering the pressure in the conduit 2–3 of the circuit shown in Figure 1.20 is to reduce the pump output flow. This can be done in two different ways in a hydrostatic pump:

1. By varying the prime mover speed.
2. By varying the pump displacement, that is, by adjusting the pump flow without changing its speed.\(^{15}\) It is important to mention that, in this particular case, only variable-displacement pumps can alter their output flows for a constant speed of the shaft. For fixed-displacement pumps, this is not possible.

We will give a formal definition of displacement shortly. In the meantime, note that the adoption of the first or the second solution changes the hydraulic symbols of Figure 1.20 accordingly. Figure 1.21 shows the representation of a variable-speed prime mover and a variable-displacement pump, respectively.

\(^{14}\) As seen in Section 1.3, this is one of the basic differences between a hydrostatic and a hydrodynamic transmission. In hydrodynamic transmissions, stopping the motor shaft will also elevate the internal pressure of the circuit, whereas in hydrostatic transmissions, the situation is much more complicated because stopping the motor would imply stopping the pump as well. Thus, the rise in pressure can be extremely high in hydrostatic transmissions, depending on the input power. A means of alleviating the pressure in the circuit must, therefore, be provided.

\(^{15}\) Displacement is a concept unique to hydrostatic pumps and motors.
1.4.1.1 Pump Displacement and Flow

We have mentioned that a hydrostatic pump can have a fixed or a variable displacement. A general definition for displacement in a pump is as follows:

Displacement of a hydrostatic pump is a technical term given to the maximum theoretical fluid volume displaced by the pump when the rotor performs a complete revolution, considering the hypothetically perfect situation where no volumetric losses are present.

It is easy to see that a fixed-displacement pump, connected to a constant-speed prime mover, will output a constant flow. For instance, if the pump displacement is 100 cm³/rev and the prime mover rotates at 1800 rpm, the pump will output a flow of 180,000 cm³/min, or, in more common units, 180 l/min. We can, therefore, express the average pump output flow, \( q_p \), as a function of its displacement, \( D_p \), and the angular speed of its shaft, \( \omega_p \):

\[
q_p = D_p \omega_p
\]  

(1.16)

In a typical variable-displacement pump, \( D_p \) changes continuously between two limiting values. As a result, the output flow, \( q_p \), can be altered even if the angular speed, \( \omega_p \),

---

16 It will be shown in Chapter 3 that the pump flow is not, actually, constant in time. Therefore, \( q_p \) as defined by Eq. (1.16) must be seen as an average value [12], obtained for the hypothetical case where no volumetric losses are present. The same observation is valid for hydrostatic motors.
does not change. Moreover, in a variable-displacement pump, whenever the displacement changes sign, the pump flow changes its direction.\footnote{Note that in Eq. (1.16), the pump displacement, \( D_p \), can be negative, positive or zero.} This is equivalent to say that the output port of the pump becomes the input port, and vice versa. Figure 1.22 illustrates the three possibilities.

From Eq. (1.16), we observe that we can also alter the output flow by varying the rotation of the pump shaft, \( \omega_p \). In the particular case, when \( \omega_p \) is reversed, the pump flow changes its direction as illustrated in Figure 1.23.

\subsection{Motor Displacement and Speed}

We saw that the pump output flow can be controlled by changing the displacement, \( D_p \). Similarly, the angular speed of hydrostatic motors, \( \omega_m \), is related to the displacement, \( D_m \), and the input flow, \( q_m \), through the following expression:

\[ \omega_m = \frac{q_m}{D_m} \]  

(1.17)

The concept of displacement in a hydraulic motor\footnote{It is a common practice to use the term ‘hydraulic motor’ instead of ‘hydrostatic motor’. Likewise, the term ‘hydrodynamic motor’ is usually replaced by some more specific term, such as ‘turbine’. In this book, to keep with the usual convention, whenever we refer to a hydraulic motor, we are, in fact, referring to a hydrostatic motor.} is a little bit different from the concept given for the pump. Note that, here, the fluid that flows into the motor causes its shaft to...
rotate, whereas in the pump, it was the rotation of the pump shaft that caused the fluid to flow. We can, therefore, say that:

Displacement of a hydraulic motor is a technical term given to the theoretical fluid volume that, when flowing through the motor, causes the shaft to perform a complete revolution, considering a perfect situation where no volumetric losses are present.

It is easy to see that the units of the motor displacement are the same as the units used for the pump displacement. Also, similar to pumps, motors can be of either fixed-displacement or variable-displacement types.

In Eq. (1.17), if the motor displacement approaches zero, the angular speed, \( \omega_m \), approaches infinity. Mathematically speaking, \( D_m = 0 \) is a singular point in Eq. (1.17). We know, by experience, that infinite angular speeds are not possible. Moreover, due to dynamic considerations, every rotary machine must have a maximum speed beyond which operation is not recommended. Thus, variable-displacement hydraulic motors should never operate near zero displacement when there is flow coming from the pump. In fact, the displacement of variable hydraulic motors is usually set between two limiting values so that there is no risk of reaching the maximum angular speed. Figure 1.24 illustrates what has been said so far. Notice that an inversion of the flow direction automatically results in a reversal of the motor rotation.

### 1.4.1.3 Motor Displacement and Pressure

Suppose that the motor input flow comes from a fixed-displacement pump, as shown in Figure 1.25. If we assume that there are no energy losses between the pump and the motor, we can write the following relation between the torque applied to the pump shaft and the torque output by the motor (see Eq. (1.10)):

\[
T_m = \frac{T_p}{R_T}
\]  

(1.18)
Figure 1.25 Pressure in a hydrostatic transmission with a variable-displacement motor

We observe from Eq. (1.18) that a high transmission ratio produces a low torque at the transmission output, and vice versa.

From Eqs. (1.1) and (1.18), the torque at the pump, \( T_p \), necessary to produce the torque \( T_m \), at the motor, is given by

\[
T_p = T_m R_T = T_m \left( \frac{\omega_m}{\omega_p} \right)
\]  

(1.19)

In the hypothesis that the motor displacement becomes zero, we have seen that the angular speed of the motor shaft becomes infinite and, therefore, the torque at the pump also goes to infinity (Eq. (1.19)). We shall see, in Section 3.4.4, that the input torque for an ideal and completely efficient pump can be written as

\[
T_p = D_p (p_2 - p_1)
\]  

(1.20)

where \( p_2 \) is the pressure at the pump output port (high-pressure line) and \( p_1 \) is the pressure at the pump input port (low-pressure line). Observe that in a 100% efficient transmission, \( p_2 = p_3 \) and \( p_4 = p_1 \), as shown in Figure 1.25.

By writing \( q_p = q_m \) and using Eqs. (1.16) and (1.17), the transmission ratio in the absence of transmission losses is

\[
R_T = \frac{\omega_m}{\omega_p} = \frac{D_p}{D_m}
\]  

(1.21)

After substituting \( T_p \) and \( R_T \), given by Eqs. (1.20) and (1.21), into Eq. (1.19), we arrive at the following expression for the pressure at the pump output, \( p_2 \):

\[
p_2 = \frac{T_m}{D_m} + p_1
\]  

(1.22)

Equation (1.22) shows that whenever the motor displacement becomes small, the pressure in the pump output, \( p_2 \), becomes high. Moreover, it tends to infinity when the motor displacement, \( D_m \), tends to zero. Therefore, we have another limitation for the minimum displacement of the motor. This limitation now describes the whole circuit, not just the motor...
itself (the high pressure in line 2–3 will affect everything that is connected to the output port of the pump).

1.4.1.4 Pressure Overshoot Attenuation

We have seen that the solution for the problem of the high pressures in the pump–motor line 2–3 (Figure 1.25) is to reduce the pump output flow. However, there may be momentary pressure elevations for which a faster relief is necessary (e.g. during unexpected high loads at the motor shaft). In those situations, there is usually no time to reduce the pump output flow so that the pressure is quickly attenuated. The usual solution is to use a pressure relief valve (Figure 1.26), which opens a passage between the two lines, 2–3 and 3–4, as soon as the pressure rises in one of the conduits.

Figure 1.27 illustrates the use of pressure relief valves in hydrostatic transmissions. In this figure, a variable-displacement pump is coupled with a fixed-displacement motor. Note that two valves are employed, and each one of them can become operational for a corresponding flow direction.

![Pressure relief valve](image1.png)

**Figure 1.26** Pressure relief valve: (a) picture, (b) operational principle and (c) ISO representation

![Pressure overshoot attenuation using relief valves](image2.png)

**Figure 1.27** Pressure overshoot attenuation using relief valves
In a pressure relief valve, the pressure acts against the force of a spring. As the pressure rises beyond a certain value, the corresponding valve spring is compressed, opening a passage from the high-pressure conduit to the low-pressure conduit, as shown in the figure for the valve $R_1$. The pressure in the circuit can then be relieved, preventing an eventual damage of the parts connected to the high-pressure line.

The ISO representation of the circuit in Figure 1.27 is shown in Figure 1.28.

1.4.1.5 Leakages and Fluid Replenishment

Up until this point, we have been considering that all the flow that comes out of the pump makes its way through the conduits, passes through the motor and returns to the pump. However, this is not what happens in real life. Hydraulic components usually leak to some degree. To understand where the leakages happen in a hydrostatic transmission, consider the flow at point 1 immediately before the pump (Figure 1.27), and let us follow it in a clockwise direction. As we travel from point 1 to point 2, part of the flow leaks through the internal clearances of the pump into the pump case. From point 2 to point 3, there may be some flow through the relief valve $R_1$, as indicated in the figure, but this flow remains in the circuit, being diverted into line 4–1. From point 3 to 4, fluid leaks from the high-pressure chambers of the motor into the motor case in the same way it leaked in the pump. In the return line 4–1, no leakages are likely to happen. When we reach point 1 again, the circuit will be depleted of hydraulic fluid because of the external leakages at the pump and motor.

In a hydrostatic transmission, there are two kinds of leakages: internal and external. Internal leakages remain in the main circuit (pump–motor), whereas external leakages happen between the main circuit and the outside. By ‘the outside’, we mean to describe the elements through which the main flow does not circulate, such as the pump and the motor cases.

If we consider the external leaking flows we have seen so far (between points 1 and 2 and points 3 and 4), it is clear that the flow reaching the pump input, $q_1$, will be smaller than
the flow leaving the pump, \( q_2 \). Because of that, a charge circuit is provided to replenish the fluid that has been lost through external leakages. Usually, this circuit consists of a small pump (driven by the same prime mover, PM, that is connected to the main pump), a pressure relief valve (to keep the charge pressure limited), an oil reservoir (tank) and a check valve (which allows for the passage of the fluid in only one direction). The check valve is important because we want the replenishing fluid to flow into the main circuit while preventing the fluid in the main circuit from flowing back to the reservoir. In many cases, the charge pump also powers the pump servo-control cylinder that is responsible for changing the pump displacement. Figure 1.29 illustrates how the charge circuit operates in a simple unidirectional transmission. Relief valves between lines 2–3 and 4–1 are not shown in the figure to keep things simplified.

In Figure 1.29, we see that the check valve, C, is simply represented by a sphere that is loosely placed inside a case. If the flow comes from the indicated direction, the sphere is pushed downwards until it stays held in the mid-position so that fluid can flow into the conduit 4–1. If the flow tries to go the other way around, the sphere will close the passage between the conduit 4–1 and the charge pump. The relief valve, R, keeps the pressure in the line 4–1 at a pre-determined level,\(^{19}\) given by the degree of compression of its spring. As shown in the figure, the relief valve R usually possesses an external adjustment to the spring compression, which allows for the control of the pressure at the charge circuit. Figure 1.30 shows a typical check valve used in hydraulic circuits.\(^{20}\)

Figure 1.31 shows the ISO representation of the complete circuit, with all the elements represented in Figures 1.27 and 1.29. Note that there are two independent check valves, \( C_1 \) and \( C_2 \), each one connecting to a different side of the circuit.

### 1.4.1.6 Energy Storage and Hydraulic Accumulators

One of the great advantages of using a fluid as an energy transportation medium is the capacity to store energy for later reuse. The way in which this is usually done is by either

\(^{19}\) Typical values for the pressure in the charge circuit lie between 7.5 and 26 bar [13].

\(^{20}\) Despite the ISO representations shown in Figure 1.30(c), commercially available check valves are usually built with an internal spring. The same observation is valid for the circuit shown in Figure 1.29. In this book, we adopt a more relaxed approach and use both symbols in Figure 1.30(c) to represent check valves in general.
Figure 1.30  Check valve (spring loaded and standard): (a) picture, (b) operational principles and (c) ISO representations

Figure 1.31  ISO representation of a hydrostatic transmission with charge circuit

having the hydraulic fluid press an elastic element (a spring or a gas encapsulated in a chamber) or lift a weight. In any case, the energy storage element is called *hydraulic accumulator*.

There are, basically, three types of hydraulic accumulators, namely, the spring-loaded accumulator, the gas-loaded accumulator and the weight-loaded (or gravity) accumulator. Figure 1.32 illustrates the operational principle of each type. In weight-loaded accumulators, a volume of fluid, $V$, is kept at a constant pressure, $p$, by the weight, $W$, acting on the piston. Spring-loaded accumulators, on the other hand, store elastic energy in the spring. Given that for the usual spring type, the elastic spring force, $F_s$, is proportional to the spring compression, the pressure inside the accumulator changes linearly between zero and a maximum value as the volume of stored hydraulic fluid increases. Gas-loaded accumulators also store energy elastically, though the pressure does not vary linearly with the stored fluid volume as in spring-loaded accumulators. The elastic medium is a gas held in an internal chamber by a flexible membrane, for example\(^{21}\) (Figure 1.32(c)). The initial volume and pressure\(^ {22}\)

---

\(^{21}\) Other configurations are possible that are not shown in Figure 1.32. For instance, the gas could have been kept inside a bladder or separated from the oil by a piston. Under restrictive conditions, it is even possible to have the gas mixed-up with the oil itself [12].

\(^{22}\) The initial pressure at the gas chamber, $p_{g0}$, is called ‘pre-charge pressure’ and is usually 70–90% of the minimum operating pressure at the accumulator [12]. The volume $V_{g0}$, on the other hand, is a design parameter that defines the accumulator size.
inside the gas chamber are $V_{g0}$ and $p_{g0}$, respectively. As the accumulator discharges, the gas expands causing the pressure inside the gas chamber, $p_g$, to decrease.\textsuperscript{23}

If we compare the three types of accumulators illustrated in Figure 1.32, we observe that the weight-loaded accumulator is the only one that supplies a constant pressure while discharging. However, some limitations are evident. For example, there is a need for the accumulator to be stationary and vertically placed. Another drawback, which is also shared with the spring-loaded accumulator, is the use of a piston separating the oil chamber from the energy storage element. Since there must be a clearance between the piston and the case, energy will inevitably be lost through leakages, especially at high pressures. Moreover, the fatigue of the spring, when subject to cyclic operation will, in the end, affect its elasticity and, as a result, the energy storage capacity.

Some types of gas-loaded accumulators, such as membrane and bladder accumulators, do not leak internally and are, by far, the most used in practical applications. Due to their mode of operation, these accumulators can store fluid at very high pressures, even for small gas volumes. As a result, the energy storage capacity is considerably high.\textsuperscript{24} Moreover, since there is no need for a moving piston, these accumulators are much lighter and provide a faster response, being able to immediately compensate for pressure variations in the circuit. Figure 1.33 qualitatively compares the typical behaviour of the three types of accumulators studied so far. The figure shows the oil pressure versus time when the accumulator is being discharged considering a (hypothetical) quasistatic\textsuperscript{25} process.

Membrane and bladder accumulators are pre-charged with a non-reacting gas (usually nitrogen) to prevent explosion during an eventual failure of the membrane/bladder at high

\textsuperscript{23} If an idealized reversible process is considered, the pressure of the gas inside the gas chamber, $p_g$, can be related to its volume $V_g$ through the state equation: $p_g = C/V_g^n$ where $C$ is a constant and $n$ is the polytropic index ($1 \leq n \leq 1.4$). In actual situations, however, the process of charging and discharging the accumulator is irreversible and the relation between $p_g$ and $V_g$ can be considerably complex [14].

\textsuperscript{24} It is important to note here that thermal losses due to the irreversible charge and discharge process of gas-loaded accumulators are significant and may amount to 40% of the input energy [14]. The use of elastomeric foam inside the gas chamber in bladder accumulators, for example, has proved to minimize such losses by reducing the temperature gradients gas (see [15] for a thorough discussion about the theme).

\textsuperscript{25} ‘Quasistatic’ means that the charge/discharge of the accumulator is so slow that the process can be considered as thermodynamically reversible.
Figure 1.33 Oil pressure, $p$, as a function of time during the accumulator discharge

Figure 1.34 Gas-loaded membrane accumulator: (a) picture, (b) operational principle and (c) ISO representation of the corresponding hydraulic circuit

pressures. Figure 1.34(a) shows a small membrane accumulator with the usual shut-off and relief valves required for safety. The corresponding hydraulic scheme is shown in Figure 1.34(b). The relief valve, R, limits the maximum pressure inside the accumulator. The shut-off valve, $V_2$, connects the accumulator to the main hydraulic line through port, P, and the throttle valve, $V_1$, is needed to discharge the accumulator to the tank through port T in case of an eventual emergency.

Figure 1.35 illustrates one of the ways in which energy can be stored in a hydrostatic transmission using accumulators. As the weight $W$ descends (Figure 1.35(a)), the pulley

$^{26}$ We must remember that hydraulic fluids can combust when exposed to the air oxygen at certain temperature and pressure conditions.
turns the motor shaft, which, as a result, operates as a pump. The gravitational potential energy of the weight is, therefore, stored inside the accumulator $A_2$, which acts as a braking device. Note that the pump displacement has been set to zero during the energy recovery process. At this moment, the prime mover, PM, can be disconnected from the pump or even turned off, as represented in the figure.

The energy stored in the accumulator $A_2$ can be reused at a posterior moment, as illustrated in Figure 1.35(b) where we have removed the weight, $W$, to eliminate the load at the pulley. At this time, the pressure differential between the accumulators $A_2$ and $A_1$ causes the fluid to flow back and rotate the motor shaft the other way around. The energy that had been stored in the previous moment is now recovered and turned into mechanical work again.

Before we move on to the next topic, it is important to mention another application of accumulators in hydraulic circuits. In addition to being used to store energy, accumulators can also act as shock absorbers, making operation smoother and reducing vibrations. Note, however, that the presence of accumulators might affect the response time of the transmission. By ‘response time’, we mean the time lapse the motor takes to respond to an angular rotation of the pump shaft. Accumulators reduce the ‘stiffness’ of the hydraulic medium, causing the motor to take longer to sense an input at the pump. All these things must be carefully considered when using accumulators in a hydraulic circuit.

$^{27}$ A more technical term is energy regeneration. We will discuss this theme more deeply at Chapter 6, when we study hydrostatic actuators.
1.4.2 Formal Definition of Hydrostatic Transmissions

It may seem strange that we have delayed a formal definition of hydrostatic transmissions until now. However, the information provided so far will prove to be valuable at this stage. Interestingly, in spite of the apparent simplicity of the matter, we notice that in the search for a suitable definition of the term ‘hydrostatic transmission’, we must be cautious not to be too general. For example, if our definition is too broad to cover all the circuits in which power is transferred from a prime mover into a mechanically driven rotary device, we will eventually conclude that any hydraulic circuit containing a pump on one end and a motor on the other can be categorized as a hydrostatic transmission.

In this book, we adopt the following line of thought: whenever we speak about a ‘hydrostatic transmission’, we speak of a pump, connected to a motor, with the particularity that the motor control is performed exclusively by the prime mover, the pump or the motor itself. As an example, consider the scheme shown in Figure 1.25. The speed of the motor is directly controlled by the pump flow or the motor displacement. If we had placed a flow control valve between the pump and the motor in order to control the motor speed, or else if we had made use of a directional valve to control the direction of the motor rotation, we would no longer have a hydrostatic transmission, but a conventional hydraulic circuit. We, therefore, give the following definition for hydrostatic transmissions:

The term hydrostatic transmission encompasses all the hydrostatic power transmissions between two rotating shafts whereby the motor (output) is directly controlled by the prime mover, the pump, the motor, or by any combination of the three elements.

The definition given above has not been arbitrarily created but should be viewed as an attempt to summarize the general concept found in the specialized literature. For instance, in Ref. [16] we find that ‘a hydrostatic transmission is simply a pump and motor connected in a circuit’. On the other hand, in Ref. [17], it is said that ‘hydrostatic transmissions are hydraulic systems specifically designed to have a pump drive a hydraulic motor’. From these statements, it is clear that a hydrostatic transmission should be composed of one pump and one motor, and that the pump should control the motor. Merritt [18] speaks of a hydrostatic transmission as being a type of pump controlled system, which he defines as consisting of ‘… a variable delivery pump supplying fluid to an actuation device’. If we add the possibility of a variable-displacement motor to these descriptions, we naturally arrive at our definition.

At this point, it is interesting to introduce the general classification of hydraulic circuits proposed in Ref. [19]. In this classification, the term ‘hydrostatic drive’ is used as a synonym to the hydraulic circuits where an actuator (cylinder or motor) is driven by a flow source (pump) or a pressure source (accumulator). The hydrostatic drives have been divided into displacement-controlled and resistance-controlled drives. Within each of these categories, we also have the flow-supplied circuits and pressure-supplied circuits. Figure 1.36 shows the details.
If we carefully observe Figure 1.36, it is easy to see that the circuits represented in region III, that is, flow-supplied and displacement-controlled circuits, precisely fit into our definition of hydrostatic transmissions in the particular case where the actuator is a hydraulic motor. Therefore, based on the classification of Figure 1.36, a ‘hydrostatic transmission’ could be regarded as a flow-supplied/displacement-controlled drive where the actuator is a hydraulic motor.

Similarly, as will be seen later in this chapter, hydrostatic actuators can also be identified with the circuits in region III in the case where the actuator is a hydraulic cylinder. In addition, regions II and IV define the ‘Common Pressure Rail’ systems, which will be addressed in Chapter 6, and region I can be identified with valve-controlled actuators in general.
1.4.3 Classification of Hydrostatic Transmissions

1.4.3.1 Classification According to the Transmission Ratio

We can have four possible pump–motor combinations related to the transmission ratio of a hydrostatic transmission (Eq. (1.21)):

1. Fixed-displacement pump and fixed-displacement motor.
2. Variable-displacement pump and fixed-displacement motor.
3. Fixed-displacement pump and variable-displacement motor.
4. Variable-displacement pump and variable-displacement motor.

Figure 1.37 Transmission ratio for a variable-displacement pump and fixed-displacement motor transmission

Figure 1.38 Power, speed and torque output for a variable-displacement pump and fixed-displacement motor transmission for a constant load at the motor
In what follows, we briefly explore the main characteristics of each of these transmission types:

1. **Fixed-displacement pump and fixed-displacement motor.** In this configuration, the transmission ratio, $R_T$, is constant given that both $D_p$ and $D_m$ in Eq. (1.21) have constant values. Therefore, the only means of obtaining different speeds at the motor is through altering the speed of the prime mover that is directly connected to the pump. This kind of hydrostatic transmission is the least flexible and constitutes a fixed-ratio transmission. The only advantage lies in the spatial flexibility of the whole set, which allows for both pump and motor to be placed mostly anywhere, in contrast to the strict requirements of mechanical transmissions.

2. **Variable-displacement pump and fixed-displacement motor.** This constitutes the most popular configuration and can be regarded as an IVT. By keeping the motor displacement, $D_m$, constant while changing the pump displacement from negative ($-D_{p1}$) to positive ($+D_{p2}$), the transmission ratio can have any value within the interval $[-D_{p1}/D_m, +D_{p2}/D_m]$. Note that a reversal in the motor shaft rotation happens smoothly and continuously in this configuration. Figure 1.37 shows the transmission ratio for two different values of the motor displacements $D_{m1}$ and $D_{m2}$ ($D_{m2} < D_{m1}$).

   In situations where the motor needs to drive a constant torque, the pressure differential between the input and output ports of the motor remains constant (Eq. (1.22)). On the other hand, since the motor speed can only be altered through changing the pump flow, the power output at the motor will not be constant in this situation (remember that power is the product between torque and speed). Figure 1.38 shows the speed ($\omega_m$), torque ($T_m$) and power ($P_m$) for a fixed-displacement motor driving a constant load, with the pump displacement varying between 0 and $+D_{p2}$.

3. **Fixed-displacement pump and variable-displacement motor.** This configuration is not very practical due to the limitations associated with small motor displacements (see comments after Eq. (1.22)). Figure 1.39 shows a typical transmission ratio curve for a fixed value of the pump displacement and the motor displacement changing between $-D_{m1}$ and $+D_{m2}$.
and $+D_{m2}$. We observe that the magnitude of the motor displacement should not become smaller than a minimum established value, $D_{\text{min}}$, to keep the circuit pressure and angular speed within acceptable limits (Eqs. (1.21) and (1.22)).

Typically, the fixed-displacement pump and variable-displacement motor configuration produces a constant power transmission, as can be seen in Figure 1.40, where the torque, speed and power output at the motor are shown for $D_m$ varying between $D_{\text{min}}$ and $D_{\text{max}}$.

4. **Variable-displacement pump and variable-displacement motor.** This configuration is the most flexible one. In theory, we may say that this is an example of an IVT in the strictest sense of the term, that is, $-\infty < R_T < +\infty$. The graphical representation of a transmission of this kind is similar to the mechanical IVT shown in Figure 1.10, and can be seen in Figure 1.41 for symmetrical values of the pump and motor displacements (both changing between $-D$ and $+D$). The graph on (a) shows the situation where the pump displacement is kept constant while the motor displacement varies between $-D_{\text{max}}$ and $+D_{\text{max}}$. In (b), we invert the situation and keep the motor displacement constant while changing the pump displacement. We see that, by varying $D_p$ and $D_m$, the transmission ratio can, in theory, have any value within the interval $]-\infty, +\infty[. However, that when this configuration is used, we are not free from the high pressures that occur when the motor displacement approaches zero. This demands a limiting value for $R_T$ in such a way that, in the real case scenario, the transmission ratio should stay within the finite interval $]-R_{T_{\text{min}}}, +R_{T_{\text{max}}}[.$

### 1.4.3.2 Classification According to the Spatial Arrangement

It is common to classify compact (or integral) hydrostatic transmission units according to the geometrical placement of pump and motor. It is possible to find expressions such as ‘in-line’,...
\[ R_T = D_m^{\text{max}} + D_m^{\text{max}} - 1 \]

\[ R_T = D_m^{\text{max}} + D_m^{\text{max}} - 1 \]

**Figure 1.41** Transmission ratio for a variable-displacement motor and variable-displacement pump transmission

'U-shaped', 'S-shaped' (also called 'Z-shaped') and 'split' associated with each particular hydrostatic transmission type [16]. Figure 1.42 shows a picture of an actual Z-shaped hydrostatic transmission (bottom) and the schematic representation of the three configurations mentioned earlier. The 'split' configuration corresponds to hydrostatic transmissions where pump and motor are placed separately, as in the generic scheme shown in Figure 1.16.
1.4.3.3 Classification According to the Circuit Construction

Hydrostatic transmissions can also be divided according to the way their circuits are designed. In this context, we can have either a closed-circuit or an open-circuit transmission. Technically speaking, a closed-circuit transmission is one in which the flow that leaves the motor is immediately directed to the pump input, whereas in an open-circuit transmission, the flow that comes out of the motor returns to a reservoir from where it is pumped again to the circuit. An example of a closed-circuit transmission is given in Figure 1.43, which is similar to the circuit shown in Figure 1.31 with the addition of an in-line oil filter F, a heat exchanger H, and the pump and motor drain lines connecting the cases to the tank.

With reference to Figure 1.43, a filter F is placed between the charge pump and the tank to filter out any eventual fluid impurity. Note that only the replenishing fluid is filtered. The heat exchanger H may or may not be present, depending on the amount of heat generated by the viscous flows inside the transmission. Sometimes, the fluid temperature drops down naturally as heat is exchanged with the environment through conduits, pump and motor surfaces. However, there are occasions when a heat exchanger is necessary to bring the oil back to the temperature recommended by the manufacturer. In those circumstances, the usual procedure is to overcool the replenishment fluid so that, as it mixes with

![Diagram of Closed-circuit transmission](image)

Figure 1.43 Closed-circuit transmission

---

29 Hydrostatic transmissions can also be subdivided into ‘open-loop’ and ‘closed-loop’ transmissions. Open-loop transmissions are those in which the pump displacement is independently adjusted in relation to the hydraulic circuit [20]. This nomenclature follows the reasoning used for open-loop and closed-loop control systems and expresses the ability of the hydrostatic transmission input (e.g. pump displacement) to be constantly adjusted to match the output requirements (e.g. torque and angular speed). In that sense, the words ‘closed’ and ‘open’ relate to the way the transmission is controlled, rather than the physical configuration [21].

30 Some have argued about the effectiveness of placing the filter in the charge circuit and not in the main line [22]. However, it is important to observe that placing the filter in the main line can reduce the efficiency of the transmission by adding a resistive load between the pump and the motor.
the main flow, the temperature inside the transmission can be brought down to an acceptable level. An additional technique consists of sporadically flushing fluid from the lower pressure conduit into the tank so that a larger fraction of oil can be filtered and cooled, as will be seen later on in Section 4.6.2, when we address the theme of loop-flushing in hydrostatic transmissions.

Figure 1.44 shows an open-circuit configuration. Note that this time, the hydraulic fluid flows from the oil reservoir into the pump and then from the pump into the motor. The fluid that leaves the motor returns to the tank instead of going back to the pump. We observe that, in this particular circuit, there is no possibility of reversing the motor rotation by changing the direction of the pump flow unless the gauge pressure in the line between the pump output and the motor input becomes negative (an undesirable situation). There is also no need for a charge circuit, as the tank now plays the role of the low-pressure conduit, and the relief valve only exists to alleviate any pressure spikes in the line connecting the pump to the motor.

The open-circuit configuration illustrated in Figure 1.44 is relatively simple and, in typical situations, some new elements should be added for a sound operation. For example, consider the case where the load accelerates the motor, creating a low-pressure zone between the pump output and the motor input. Given that the motor output connects to the tank, we must add a counterbalance valve to the circuit, as illustrated in Figure 1.45, to prevent the risk of fluid evaporation in the pump–motor line.
1.4.4 Efficiency Considerations

Figure 1.46 shows the energy balance in a hydrostatic transmission. The input and the output powers are represented by $P_i$ and $P_o$. Losses in the transmission are divided into heat losses, $P_q$, and fluid leakage losses, $P_L$. Here, we see one of the reasons why mechanical transmissions are, in general, superior to hydrostatic transmissions in terms of efficiency. There is no fluidic system to transfer the power from the input to the output, and therefore, losses due to fluid leakages do not exist in mechanical transmissions. Therefore, one of the challenges of hydrostatic transmission technology is to reduce fluid leakages as much as possible.

External leakages are not the only factors detrimental to transmission efficiency; internal leakages also play an important part since the fluid that backflows from the high-pressure conduit into the low-pressure conduit will not transfer its energy to the motor in the end, but will return the energy to the pump input instead.

Considering the definition of efficiency given by Eq. (1.8), the efficiency of a hydrostatic transmission is given as

$$\eta = \frac{P_i - (P_q + P_L)}{P_i} \quad (1.23)$$

We will come back to Eq. (1.23) later on, in Chapter 4, when we study the steady-state operation of hydrostatic transmissions.

1.5 Hydromechanical Power-Split Transmissions

In order to take advantage of the high efficiency of gearboxes and the flexible transmission ratios of hydrostatic transmissions, a power-split scheme, similar to the one shown in Figure 1.13, can be used. The resulting scheme is denominated ‘hydromechanical (power-split) transmission’ (HMT) [23]. Technically, an HMT can be seen as ‘an energy translation device in which mechanical energy at the input is converted into mechanical and hydrostatic energy and then is reconverted into mechanical energy before leaving at the output’ [24]. The idea is illustrated in Figure 1.47, where we can visualize the power flow.

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31 Heat loss is a direct result of the friction and viscous forces and, therefore, can also be thought of as mechanical losses.
Figure 1.47  Power flow in a typical HMT

in a typical HMT. The input and output powers are represented by $P_i$ and $P_o$, respectively, and $x$ represents the fraction of the input power diverted into the hydraulic leg of the circuit. For simplicity, transmission losses have not been represented in the figure.

1.5.1 General Classification

HMTs are usually classified as ‘input-coupled’, ‘output-coupled’ and ‘compound’ [24]. This classification has to do with the way in which power is input to the hydrostatic transmission (or ‘variator’). In an input-coupled HMT, there is a direct connection between the input shaft and the variator, whereas in an output-coupled HMT, the variator is connected to the output shaft. In the compound configuration neither the input nor the output shafts are directly connected to the variator. Figure 1.48 illustrates these three types [23–25].

As an example, consider the particular case of the input-coupled architecture, shown in Figure 1.48(a) and presented in more details in Figure 1.49. In this configuration, the hydraulic unit M at the output side usually has a fixed displacement [23]. From the figure, we see that the input power $T_1\omega_1$ is split between the shaft connected to the sun gear S of the planetary train and the variable-displacement pump $P$ through gears 1 and 2. Gear 3, on the other hand, receives the power $T_3\omega_3$ coming from the hydrostatic transmission output. Finally, the powers entering the sun and the ring gears of the planetary gear train, S and R, are added into the output shaft, connected to the carrier C.

In what follows, we study the input-coupled HMT shown in Figure 1.49 in more detail.

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32 We are not going to study the three configurations shown in Figure 1.48 in detail in this book. Rather, we focus only on the input-coupled type. The interested student can consult Ref. [24] for further information on double- and compound-type HMTs.

33 Observe that it is the input torque $T_1$ and not the input speed $\omega_1$, which is responsible for splitting the input power between the hydrostatic transmission and the planetary train. In fact, given that $\omega_S = \omega_1$, the power entering the sun S can only be smaller than the input power $T_1\omega_1$ because of the torque division at gear 1. Therefore, it is also common to refer to input-coupled HMTs as ‘split-torque’ HMTs [24].
1.5.2 Transmission Ratio

The relation between the output speed \( \omega_C \) and the ring and sun speeds \( \omega_R \) and \( \omega_S \) in the planetary gear train can be obtained through substituting the pitch radii \( r_S \) and \( r_R \) by the corresponding number of teeth \( z_S \) and \( z_R \) in Eq. (1.5):

\[
\frac{z_S}{z_R} = - \left( \frac{\omega_R - \omega_C}{\omega_S - \omega_C} \right)
\]  
(1.24)
Considering the hypothetical situation of a 100% efficient transmission, we have (see Eq. (1.21)):

\[
\frac{\omega_3}{\omega_2} = \frac{D_p}{D_m}
\]  
(1.25)

Observing Figure 1.49, the following additional relations can be written

\[
\frac{\omega_2}{\omega_1} = \frac{\omega_2}{\omega_S} = -\frac{z_1}{z_2} \quad \text{and} \quad \frac{\omega_R}{\omega_3} = -\frac{z_3}{z_R}
\]  
(1.26)

From Eqs. (1.25) and (1.26), we obtain

\[
\omega_R = -\frac{z_3}{z_R} \omega_3 = -\frac{z_3 D_p}{z_R D_m} \omega_2 = \frac{D_p z_1 z_3}{D_m z_2 z_R} \omega_S
\]  
(1.27)

Substituting \(\omega_R\), given by Eq. (1.27), into Eq. (1.24), we arrive at the following expression:

\[
\frac{z_S}{z_R} = \frac{\omega_C - \frac{D_p z_1 z_3}{D_m z_2 z_R} \omega_S}{\omega_S - \omega_C} = \frac{\omega_C - \frac{D_p z_1 z_3}{D_m z_2 z_R}}{1 - \frac{\omega_C}{\omega_S}} = \frac{R_T - \frac{D_p z_1 z_3}{D_m z_2 z_R}}{1 - R_T}
\]  
(1.28)

where \(R_T\) is the transmission ratio \((R_T = \omega_C/\omega_1 = \omega_C/\omega_S)\).

Equation (1.28) can be rearranged to give the transmission ratio explicitly as follows:

\[
R_T = \frac{\frac{z_S}{z_R} + \frac{D_p}{D_m} \left( \frac{z_1 z_3}{z_2 z_R} \right)}{1 + \frac{z_S}{z_R}}
\]  
(1.29)
Observing Eq. (1.29), we see that it is possible to have the transmission ratio changing from $-R_{T_{\text{min}}}$ to $+R_{T_{\text{max}}}$, by varying the pump displacement from $-D_{p_{\text{min}}}$ to $+D_{p_{\text{max}}}$. In practice, as will be seen in Section 1.5.4, input-coupled HMTs are not efficient at negative transmission ratios [24], and a reversal of rotation at the output shaft is usually carried out mechanically [23].

### 1.5.3 Lockup Point

If we use a variable-displacement pump/fixed-displacement motor configuration in the hydrostatic transmission, as shown in Figure 1.49, the energy flow will be dictated by the pump displacement, $D_p$. For instance, it is easy to see that the choice of $D_p = 0$ corresponds to $x = 0$ in Figure 1.47. In this case, the ring of the planetary gearbox stops moving, and all the power is mechanically transmitted from the input to the output shaft. At this stage, the transmission reaches the point of highest efficiency, also known as ‘lockup point’ [24]. The corresponding transmission ratio $R_{TL}$ can be easily obtained by making $D_p = 0$ in Eq. (1.29):

$$R_{TL} = \frac{\frac{z_S}{z_R}}{1 + \frac{z_S}{z_R}}$$

(1.30)

As will be seen in the following section, the lockup point is very significant to HMTs.

### 1.5.4 Power Relations

It is possible to obtain a mathematical expression for the proportion, $x$, of the input power that goes into the hydraulic branch of the input-coupled HMT, as shown in Figure 1.49.

Consider the forces acting on the shaft connecting gear 1 to the sun gear, $S$, of the planetary train, as shown in Figure 1.50. The nomenclature used in the figure is the following: $F_{XY}$ indicates the tangential component of the force $F$, coming from element X onto element Y.

![Figure 1.50 Forces acting on the sun gear shaft and the planet gear](image-url)
where X and Y are generic representations of the transmission parts. Likewise, \( r_X \) is the radius corresponding to element X. For instance, \( F_{21} \) is the tangential component\(^{34}\) of the force that gear 2 exerts on gear 1 at their point of contact, \( r_p \) is the pitch radius of the planet gear and so on. A momentum balance over the sun gear shaft and the planet gear\(^{35}\) gives us the following equations:

\[
\begin{align*}
T_1 - F_{21}r_1 - F_{PS}r_S &= 0 \\
F_{3R}r_p + F_{PS}r_p &= 0
\end{align*}
\]

(1.31)

Similar to what was assumed in Figure 1.50, let \( r_2 \) and \( r_3 \) be the pitch radii of gears 2 and 3 in Figure 1.49. The magnitude of the contact forces between gears 2 and 1 and between gear 3 and the ring of the planetary train can then be written as

\[
F_{21} = \frac{T_2}{r_2} \quad \text{and} \quad F_{3R} = \frac{T_3}{r_3}
\]

(1.32)

Using Eqs. (1.31) and (1.32), it is possible to obtain the following expression for the input torque \( T_1 \):

\[
T_1 = T_2 \left( \frac{r_1}{r_2} \right) + T_3 \left( \frac{r_S}{r_3} \right)
\]

(1.33)

The torques \( T_2 \) and \( T_3 \), on the other hand, can be related to the pump and motor displacements, \( D_p \) and \( D_m \), and the pressure differential between the conduits, \( \Delta p \) (see Eq. (1.20)). If we also observe that \( \frac{r_1}{r_2} = \frac{\omega_2}{\omega_1} \), Eq. (1.33) can be rewritten as

\[
T_1 = D_p \Delta p \left( \frac{\omega_2}{\omega_1} \right) + D_m \Delta p \left( \frac{r_S}{r_3} \right)
\]

(1.34)

Equation (1.34) can be further modified into the following expression:

\[
\frac{D_p \Delta p \omega_2}{T_1 \omega_1} = 1 - \frac{D_m \omega_1 \Delta p}{T_1 \omega_1} \left( \frac{r_S}{r_3} \right)
\]

(1.35)

Note that

\[
\frac{D_p \Delta p \omega_2}{T_1 \omega_1} = \frac{T_2 \omega_2}{T_1 \omega_1} = x
\]

(1.36)

We can then write Eq. (1.35) as

\[
x = 1 - \frac{D_m \Delta p}{T_1} \left( \frac{r_S}{r_3} \right)
\]

(1.37)

---

\(^{34}\) Tangential component, that is, the component of the contact force between the two gears that is tangent to the pitch circle (see Ref. [3] for a more complete discussion).

\(^{35}\) Without loss of generality, we are treating the planetary gear train as if it had only one planet when we know that the torques exerted by the ring and the sun are equally divided between all the planets in the train. In our representation, all these forces are summed up in \( F_{3R} \) and \( F_{PS} \).
Since there are no energy losses in the transmission, $T_1 \omega_1 = T_C \omega_C$ in Figure 1.49. We can also perform another momentum balance over the planet gear having the pitch point of contact between the planet and the sun as a reference (Figure 1.50):

$$ F_C = \frac{2F_{3R}r_p}{r_p} = 2F_{3R} \quad (1.38) $$

If we make $T_C = F_C r_C$, the following chain of relations can be developed:

$$ T_1 = \frac{T_C \omega_C}{\omega_1} = \frac{F_C r_C \omega_C}{\omega_1} = \frac{2F_{3R}r_C \omega_C}{r_3 \omega_1} = \frac{2T_{3R}r_C \omega_C}{r_3 \omega_1} = \frac{2D_m \Delta p r_C \omega_C}{r_3 \omega_1} \quad (1.39) $$

Substituting $T_1$, given by Eq. (1.39) into Eq. (1.37), it is possible to arrive at the following expression for $x$ (we leave the demonstration as an exercise for the student):

$$ x = 1 - \frac{r_S}{2r_C} \left( \frac{\omega_1}{\omega_C} \right) = 1 - \frac{r_S}{2r_C} \left( \frac{1}{r_T} \right) \quad (1.40) $$

Equation (1.40) can be written in a more interesting way. First, we note that we can write the lockup transmission ratio, $R_{TL}$, in Eq. (1.30) as a function of the pitch radii of the gears, $r_S$ and $r_R$, instead of the number of teeth, $z_S$ and $z_R$:

$$ R_{TL} = \frac{r_S}{1 + \frac{r_S}{r_R}} = \frac{r_S}{r_R + \frac{r_S}{r_R}} = \frac{r_S}{r_R + r_S} \quad (1.41) $$

From Figure 1.50, we obtain the following geometric relations:

$$ \begin{cases} r_C + r_p = r_R \\ r_C - r_p = r_S \end{cases} \quad (1.42) $$

Adding each side of the two equations in (1.42), we get

$$ r_R + r_S = 2r_C \quad (1.43) $$

Finally, using Eqs. (1.40), (1.41) and (1.43), we arrive at the following expression for $x$:

$$ x = 1 - \frac{R_{TL}}{R_T} \quad (1.44) $$
Remarks

1. Given that the value of the lockup transmission ratio, $R_{TL}$, is constant, the proportion of hydraulic power, $x$, as a function of the transmission ratio, $R_T$, has the form of a hyperbole.

2. For $R_T > R_{TL}$, $0 < x < 1$, meaning that a fraction of the input power, $P_i$, is diverted through the hydrostatic variator to be later added to the input power through the ring gear of the planetary set. If $R_T = R_{TL}$, $x = 0$ and the HMT behaves like a mechanical transmission. On the other hand, if $R_T < R_{TL}$, $x$ becomes negative, meaning that power coming from the variator is joined to the input power, $P_i$, before entering the planetary gear train through the sun gear. In the limit, when $R_T \to 0$, there will be infinitely more ‘hydraulic power’ than the input power, $P_i$, itself. This may be confusing or even ambiguous now, but we will make it clear shortly.

Figure 1.51 shows a graphical representation of Eq. (1.44). A brief analysis of the curves will be given in the sequence.

To better understand the curves in Figure 1.51, we have divided the picture into four regions, A through D, read from right to left. Observe that $\omega_C > \omega_1$ at region A and $\omega_C < \omega_1$ at regions B and C (remember that $\omega_C = R_T \omega_1$). In region D, a reversal of the output speed occurs as the transmission ratio becomes negative.

![Figure 1.51](image-url)  
*Figure 1.51  Power relation in a 100% efficient input-coupled HMT*
1.5.4.1 Operation at Regions A and B

Observing the curve at the right of Figure 1.51, we see that for both regions A and B, $R_T > R_{TL}$, and the proportion of input power going into the hydraulic leg of the circuit, $x$, increases from zero ($R_T = R_{TL}$), converging asymptotically to 1 as $R_T \to \infty$. Clearly this is merely theoretical because the pump displacement in Figure 1.49 is limited to a maximum value, $D_{p\text{max}}$, while the motor displacement is constant. Therefore, in practice, there is a maximum positive value of $x$ that can be obtained in this HMT.

Given that the proportion of input power going into the hydrostatic transmission increases with $x$, the input-coupled HMT becomes less efficient as we move further to the right of the lockup point. As mentioned earlier, the highest efficiency is achieved when $R_T = R_{TL}$. At this point, all the input energy is mechanically transmitted to the output shaft. Figure 1.52 shows the HMT operation in regions A and B. Note that the output torque, $T_C$, becomes smaller than the input torque, $T_1$, in region A, when the output speed, $\omega_C$, is higher than the input speed, $\omega_1$ ($\omega_1 = \omega_3$). As expected, the opposite happens in region B, when $\omega_C < \omega_S$.

1.5.4.2 Operation at Region C

We have seen that the transmission ratio, $R_T$, can be reduced by reducing the pump displacement, $D_p$ (see Eq. (1.29)). Therefore, if we want to decrease the transmission ratio down to values below the lockup ratio, $R_{TL}$, we need to make the pump displacement negative. This is what happens in region C, where we have $0 < R_T < R_{TL}$. However, if we make the pump displacement negative, the oil flow in the hydrostatic transmission also changes direction. As a result, gear 3, which is connected to the output shaft of the motor M, rotates the other way around, as does the ring gear of the planetary set, as shown in Figure 1.53(a). Note that the motor M now operates as a pump, recirculating the unused power back into the transmission again. In this case, the HMT is said to be operating in regenerative mode [24].

As $R_T \to 0^+$ (i.e. when $R_T$ tends to zero from the positive side of the abscissa axis), more and more power recirculates through the hydrostatic transmission into gear 1. In addition, given that $\omega_C \to 0^+$ when $R_T \to 0^+$, the output power, $T_C\omega_C$, is also reduced. We know that in a 100% efficient transmission, the power input equals the power output. Therefore, the input power, $T_1\omega_1$, and, consequently, the input torque, $T_1$, also tend to zero as $R_T \to 0^+$, which results in $x \to -\infty$, as shown in Figure 1.51.

Figure 1.53(b) graphically illustrates the situation in an actual HMT, where the efficiency is smaller than 100%. Because of the losses in the hydrostatic transmission, the power entering gear 2 is smaller than the power leaving gear 3. Therefore, a minimal input power is required to cover the energy losses. In other words, in real-life situations, $x$ will have a finite negative value when the transmission ratio, $R_T$, becomes zero.

1.5.4.3 Operation at Region D

We have seen that by changing the pump displacement to negative values, the ring gear rotates in the opposite direction of the sun gear. As a result, the output speed $\omega_C$ is reduced,
and eventually becomes zero. It is, therefore, natural to infer that if we keep on increasing the magnitude of the pump displacement to the negative side, the ring gear will, eventually, drive the carrier to the opposite direction and the transmission ratio will become negative. This is precisely what happens, and is illustrated in Figure 1.54, where we represent the particular case when $-1 < R_T < 0$.

Figure 1.52  Energy flow (regions A and B in Figure 1.51): (a) region A, (b) region B and (c) lockup point

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36 We must observe here that it is usually difficult to precisely match the angular speeds of the ring and the sun gears in the planetary train in such a way that the neutral gear ($\omega_C = 0$) is obtained [26].
Figure 1.53  Energy flow (region C in Figure 1.51): (a) $0 < R_T < R_{TL}$ and (b) $R_T \to 0^+$

Figure 1.54  Energy flow (region D in Figure 1.51): (a) $R_T \to 0^-$ and (b) $R_T < 0$
Figures 1.54(a) and (b) can be understood as follows. The ring gear rotates in the opposite direction in relation to the sun gear, dragging the carrier counter-clockwise at the same time that it creates a positive torque at the sun gear, adding to the torque $T_1$. The power flow must, therefore, be towards gear 1. The difference between the power entering and leaving the planetary train is then output to the planetary carrier. In the particular situation shown in Figure 1.54(a), the carrier speed approaches zero from the negative side (i.e. $\omega_C \rightarrow 0^-$). Therefore, almost all the power circulates back to the sun gear, and the output torque approaches zero. Because of the energy conservation, the input power is also very small, which causes the ratio between the hydraulic power and the input power, $x$, to go to plus infinity (see Figure 1.51). As the ring gear rotates faster and faster, more power is transmitted to the carrier and, consequently, the value of $x$ is reduced. Note that $x$ will always be higher than 1, meaning that in the negative range of transmission ratios, the power transfer mechanism will be mostly hydraulic. As a result, the input-coupled HMT becomes considerably less efficient when operating at negative transmission ratios [24].

The subject of hydromechanical power-split transmissions is vast and will not be addressed in more details in this book. What we have done here, however, can be used as a basis for the study of other configurations. For instance, it is possible to show that the ratio $x$, in an output-coupled configuration (Figure 1.48(b)), is given by [24]:

$$x = 1 - \frac{R_T}{R_{TL}} \quad (1.45)$$

If we compare Eq. (1.44) with Eq. (1.45), we observe that the transmission ratio now appears in the numerator of the second term. As a result, the curve $x(R_T)$ will be a straight line and not a hyperbole as in Figure 1.51. We leave the demonstration of Eq. (1.45), and the subsequent analysis of the power flows as an exercise to the student.

### 1.6 Mechanical and Hydrostatic Actuators

In mechanical and hydrostatic transmissions, we saw that power was transferred between two rotating shafts. The term actuator is commonly used in the context where the input is a rotary device (e.g. an electric motor), and the output is either a semi-rotary device or a linear actuator. The way in which the power is transferred between the input and the output defines the type of actuator.

#### 1.6.1 Mechanical Actuators

Consider the rack and pinion mechanism shown in Figure 1.55. As the pinion performs a rotary movement, clockwise and counter-clockwise, the rack moves to the left and to the right, respectively. This is an example of a mechanical actuator, where the prime mover, PM, connects to the gear (pinion) and the rack A is the linear actuator.

---

37 Note that the planets reverse the direction of the torque coming from the ring onto the sun. Therefore, when the ring is being pushed in the opposite direction, it actually acts towards accelerating the sun gear.
The mechanical actuator shown in Figure 1.55 has the spatial rigidity of mechanical transmissions. In addition, the prime mover must be near the actuator and is responsible for the actuator control. These characteristics are common to every mechanical actuator and make them a poor option for situations where the actuator must be placed away from the prime mover. However, it must be said that in many applications, such as the opening and closing of a gate, mechanical actuators like the one shown in Figure 1.55 can be the best choice.

Figure 1.56 shows another type of mechanical actuator, where a platform, P, is driven by an endless screw, S, mechanically connected to a stepper-motor. As the motor shaft is turned to a certain angle, the platform moves accordingly. A linear potentiometer, R, is attached to the platform to detect its position. This type of device provides an excellent positioning and is widely used in machine tools.

1.6.2 Hydrostatic Actuators

Hydrostatic actuators are composed of a prime mover, a pump and a hydraulic cylinder. Figure 1.57 shows the basic configuration. The cylinder is connected to the pump through a hydraulic circuit. Depending on which element is responsible for the cylinder control – the variable-displacement pump or the variable-speed electric motor – the hydrostatic actuator is called displacement-controlled actuator or electrohydrostatic actuator. In both cases, the control of the hydraulic cylinder is entirely carried out by either the variable-displacement pump or the variable-speed electric motor.

---

38 A stepper-motor is an electric motor whose shaft can rotate at discrete angles. This type of motor is very common in positioning devices.
Figure 1.57 Displacement-controlled and electrohydrostatic actuators: (a) displacement-controlled actuator and (b) electrohydrostatic actuator

pump (displacement-controlled actuator) or by the electric motor (electrohydrostatic actuator).\textsuperscript{39} We may therefore say that

The term \textit{hydrostatic actuator} encompasses all the hydrostatic power transmissions between a rotating shaft and a linear or rotary hydraulic actuator,\textsuperscript{40} where the actuator control is carried out solely by the pump or by the prime mover.

1.6.3 Hydrostatic Actuation Versus Valve Control

As an example of a valve-controlled actuator, consider the situation illustrated in Figure 1.58, where the cylinder is controlled through a four-way, three-position directional valve. The number of ways (4) represents the number of external connections and the positions (3) correspond to the possible spool stops (we only see one stop in the figure, where the spool is totally displaced to the left; other stops are the spool at the centre and at the right position). A relief valve limits the pressure inside the cylinder. With the valve positioned as it is, the cylinder moves to the right until it reaches the end of its stroke. When that happens, the flow into the left chamber is blocked and the pressure rises in the conduit connected to the pump output. In the simplest case scenario, the pump flow is diverted into the tank through the relief valve.\textsuperscript{41} In order to bring the cylinder back to the left position, we must move the spool of the directional valve to the right. If the spool is centred, the cylinder is held in place.

Figure 1.59 shows the ISO representation of the circuit illustrated in Figure 1.58.

\textsuperscript{39} Note the change from the generic label PM into the more specific label M (electric motor) in the electrohydrostatic actuator scheme (Figure 1.57).
\textsuperscript{40} We make the distinction here between a rotary actuator and a motor, in that a rotary actuator moves within a limited angle and a motor rotates continuously.
\textsuperscript{41} Here we are assuming that the pump has a fixed displacement and that the prime mover has a constant speed. A better solution is to use a variable-displacement pump so that the flow can be reduced when the output line is blocked (pressure-compensated circuit).
An example of a hydrostatic actuator is given in Figure 1.60. The circuit is identical to the circuit of the hydrostatic transmission shown in Figure 1.31 and is only repeated here for convenience. Again, the charge pump exists only to replenish the external leakage\(^{42}\) and to keep the low-pressure side of the cylinder at a minimal level, regulated by the relief valve, \(R\). The connection between the charge circuit and the main lines is made by two symmetrically disposed check valves, \(C_1\) and \(C_2\), in the same way it was done in

\(^{42}\)It must be observed that the use of a charge pump is not the only way to replenish the external volumetric losses of the actuator. Alternative pressure sources like accumulators or even direct connections to the oil reservoir can be used as well, given that the need for fluid replenishment is usually much smaller when compared to hydrostatic transmissions.
the hydrostatic transmission of Figure 1.31. Since pressure overshoots can also exist in hydrostatic actuators, a couple of relief valves, $R_1$ and $R_2$, must be present in the circuit for safety reasons.

1.6.4 Multiple Cylinder Actuators

The hydrostatic actuator concept can be easily extended to applications where multiple cylinders are involved. Figure 1.61 illustrates how this can be done for two displacement-controlled, double-rod actuators sharing one single-charge circuit and the same prime mover, PM (pressure overshoot relief valves are not shown for simplicity reasons). The two cylinders $A_1$ and $A_2$ are independently controlled by the variable-displacement pumps $P_1$ and $P_2$, which are driven by the same prime mover. The prime mover also connects to the charge pump, which together with the check valves $C_1$–$C_4$ and the relief valve $R$ is responsible for replenishing the external volumetric losses in the circuit. A considerable drawback lies in the fact that multiple cylinder actuators require multiple pumps, which increases the complexity and the weight of the system.
Exercises

(1) For two gears to engage perfectly, their module, $m$, defined as the ratio between the pitch diameter and the number of teeth ($m = D_p/z$), must be the same. Use this fact to prove that Eq. (1.3) can be written as

$$\omega_i = (-1)^{N} \left( \frac{z_2 z_4 z_6 \cdots z_N}{z_1 z_3 z_5 \cdots z_{N-1}} \right) \omega_o$$

(2) Based on the planetary gearbox illustration (Figure 1.6), give a formal demonstration of Eq. (1.5).

(3) Suppose that the diameter of the ring gear of the planetary gearbox represented by Figure 1.6 is 300 mm. Consider that the minimum number of teeth allowed for the smallest gear in the gearbox is 18 (to avoid interference), and that the module of the gears is 2 mm. Assuming that one of the elements – sun, ring or carrier – remains fixed while the others are allowed to turn, obtain the number of teeth of the sun and the planet gear that gives the highest transmission ratio possible. What is the element that needs to be fixed in that case?

(4) Show that the slippage between the two gear teeth shown in Figure 1.8, given by the difference between the linear speed projections onto the line, $t$, increases with the angular speeds of the gears.

(5) The gear train shown in Figure 1.62 must be placed inside a metal case, and is therefore vertically limited by the dimension $L_m = 300$ mm. To avoid interference, the number of teeth of each gear must be equal to or greater than 18. Considering that the distance between the pitch circle and the external circle in each gear is equal to the module $m = 2$ mm (see problem 1 for the definition of module), what is the maximum transmission ratio that can be obtained and what is the corresponding number of teeth of each gear?

![Figure 1.62 Two-speed gearbox](image)

(6) Consider the dual-motor transmission shown in Figure 1.63 and prove that the following relation holds (disregard the circuit losses):

$$\frac{D_p}{D_{m2}} = R_{T1} \left( \frac{D_{m1}}{D_{m2}} \right) + R_{T2}$$
Consider the hydrostatic actuator shown in Figure 1.60 and show that the (constant) linear speed of the piston rod, in the absence of leakages, is given by

\[ v = \frac{4\omega_p D_p}{\pi(d_p - d_r)^2} \]

where \(\omega_p\) is the pump speed, \(D_p\) is the pump displacement and \(d_p\) and \(d_r\) are the diameters of the piston and the rod of the cylinder, respectively.

References


