1

Reliability, Robustness and Structural Control

1.1 PRELIMINARY CONCEPTS

The third author of this book began his PhD thesis (Marazzi 2003) with these words:

Since ancient times the designers of civil engineering structures have been responsible for building collapse: in the Code of Hammurabi, King of Mesopotamia during the XVIII century B.C., builders were punished with the death penalty in case the building failed. Article 229 says: “The builder has built a house for a man and his work is not strong: if the house he has built falls in and kills a householder, that builder shall be slain.” It was implicit in the law that collapse could happen for ordinary loads, let’s say dead loads (caused for example by a large number of people in the same room) or common dynamic loads (for example, a strong but common wind). On the contrary, collapse caused by extraordinary loads, such as, for example, earthquakes and hurricanes, was not taken into consideration, because it was considered beyond the control of the designer and the builder and was retained as a manifestation of a “supernatural” event sent by the gods as a punishment or simply for playing a joke on human beings. Later on, as the
belief that these events were of supernatural origin progressively disappeared, the idea that the designer could not take into account the effects of such loads still persisted. On the one hand earthquakes and hurricanes were considered completely unpredictable, in both occurrence and intensity, while, on the other hand, there were no suitable techniques to reduce the risk of collapse.

The Code of Hammurabi is a collection of laws written in 51 columns on a stele discovered at the beginning of the twentieth century and now held at the Louvre Museum in Paris. It consists of a prologue and an epilogue celebrative of the king and of 282 articles regarding various aspects of the civil, penal and commercial law. It is one of the first examples of written laws. The question is: how far has human society moved after 37 centuries? A quick, nearly blasphemous, answer might be: we are slowly coming back to the starting point! Indeed, after one century of prescriptive rules (which in some countries were made laws), as well as after 30 years of attempts at unifying them across Europe (Eurocodes) and around the world, the so-called “performance-based design” is rediscovering the fascinating Hammurabi idea of performance. This prevailing framework is generally promoted by those designers who want to preserve their identity, role and responsibilities, rather than being replaced by sophisticated software able to navigate across prescriptions better than any human being.

But the reader must have noted that the border between predictable and unpredictable events in the previous scheme is rather fuzzy. Are we still there after 37 centuries? It must be said that a rational effort was made and today the results are summarized in the probabilistic model code prepared by the Joint Committee of Structural Safety and which has been available on the Internet for 10 years [http://www.jcss.ethz.ch/] as follows.
Structures and structural elements shall be designed, constructed and maintained in such a way that they are suited for their use during the design working life and in an economic way. In particular they shall, with appropriate levels of reliability, fulfil the following requirements:

- They shall remain fit for the use for which they are required (serviceability, limit state requirement)
- They shall withstand extreme and/or frequently repeated actions occurring during their construction and anticipated use (ultimate limit state requirement)
- They shall not be damaged by accidental events like fire, explosions, impact or consequences of human errors, to an extent disproportionate to the triggering event (robustness requirement)

There are three main innovations:

1. Success and failure are situations characterized by their probability of occurrence; it is the society which decides the target to be pursued for ultimate limit states.

2. Serviceability limit states are ruled by the desiderata of the contractor, as well as the relevant probability of failure.

3. In any case the designer cannot conceive structural architectures which result in weakness to accidental events. They can be introduced as likely events, without the chance of associating a probability of occurrence to them. In other words, they can be conceived, but not assessed on a statistical basis.

The usual carrying capacity design, originally developed for static loading conditions, evolved through the assignment of a proper level of ductility reserve to the structural
members. The structure remains elastic for the major part of its life (under ordinary loads), but it will enter the plastic state under exceptional lateral loads; in this way the input energy should be dissipated. Even if there might be permanent damage to the structural members, the building is designed in such a way that it should not collapse, so no loss of life should result. On the other hand, however, the structure should be retrofitted after each strong event in which damage occurs in the structure, and this can be very expensive and time consuming.

This capacity design approach also has another drawback: it is not able to mitigate vibrations that do not induce damage in the structure. This means that comfort aspects cannot be considered with this technique. So the problem of swaying of tall buildings caused by not very intense winds, for example, is not resolvable. It must be noted that, for very flexible structures such as long bridges or tall buildings, the comfort requirements can be more stringent than the ones related to the resistance.

For all these reasons the engineering community has moved towards the concept of “structural control”. This means that the structure is regarded as a dynamic system in which some properties, typically the stiffness or the damping, can be adjusted in such a way that the dynamic effect of the load on the building decreases to an acceptable level. The natural frequency of the structure, its natural shape and the corresponding damping values are changed in such a way that the dynamic forces from the environmental loads are reduced. This can be done using a large variety of techniques that can be collected in four classes: passive, active, hybrid and semiactive. From an historical point of view, the first class of control techniques (passive) was extensively studied from both the theoretical and the experimental sides, and many practical realizations have already been implemented, especially in the USA, Japan,
China and Italy. A large number of researchers are still working in this field on a second generation of passive devices, while the first generation (for example, rubber bearings) needs only the approval of official designing and application rules for its certification.

Active control techniques have been studied extensively from a theoretical, numerical and, more recently, an experimental point of view. They are surely the most effective ones, but they also show some disadvantages (for example, the need for a lot of power to operate) that have lead to a low number of implementations. Hybrid techniques are a combination of the first two techniques, so their development directly follows that of the first two. Their application is still limited, even if some tuned mass dampers equipped with a little active mass driver were patented and installed in some buildings in Japan. Semiactive techniques are, at present, the most studied solution, from theoretical, numerical and experimental points of view, because of their excellent characteristics intermediate between those of active and passive techniques.

1.2 DEFINITIONS

Before entering into the core of the discussion, a clarification must be made about the terminology used in this book. The following definitions of some key terms are provided:

**Definition 1 (Active Control)** An active control system is a system in which an external source powers one or many control actuators that apply forces to the structure in a prescribed manner. These forces can be used either to add or dissipate energy in the structure.

In an active feedback control system, the signals sent to the control actuators are functions of the response of
the system measured by physical sensors. Active control makes use of a wide variety of actuators, including active mass dampers, hybrid mass dampers and active tendons, which may employ hydraulic, pneumatic, electromagnetic or motor-driven ball–screw actuation.

An essential feature of the active control system is that external power is used to effect the control action. This makes such systems vulnerable to power failure, which is always very likely during a strong environmental event.

**Definition 2 (Passive Control)** A passive control system consists of an appended or embedded device that modifies the stiffness or the damping of the structure in an appropriate manner without requiring an external power source to operate and feeding energy to the system.

Passive control devices impart forces that are generated by the mutual displacement of the two connection points of the device inside the protected structure. Passive control may depend on the initial design of the structure, on the addition of viscoelastic material to the structure, on the use of impact dampers, or on the use of tuned mass dampers. The initial design consists of tapered distributions of mass and stiffness, or uses techniques of base isolation, where the lowest floor is deliberately made very flexible, thereby reducing the transmission of forces into the upper storeys. The energy of a passively controlled structural system cannot be increased by the passive controller devices.

Though seldom as effective as active control, passive control has three main advantages:

1. It is usually relatively inexpensive.
2. It consumes no external energy.
3. It is inherently stable.
Definition 3 (Hybrid Control) The common meaning of the term “hybrid control” implies the combined use of active and passive control systems.

A hybrid control system may use active control to supplement and improve the performance of a passive control scheme. Alternatively, passive control may be added to an active control scheme to decrease its energy requirements. For example, as a structure equipped with distributed viscoelastic damping supplemented with an active mass damper on the top of the structure, or a base-isolated structure with actuators actively controlled to enhance performance.

It should be noted that the only essential difference between an active and a hybrid control scheme is, in many cases, the amount of external energy used to implement control. Hybrid control schemes alleviate some of the limitations that exist for either a passive or an active control acting alone, thus leading to an improved solution.

A side benefit of hybrid control is that, in the case of a power failure, the passive component of the control still offers some degree of protection, unlike an active control system.

Definition 4 (Semiactive Control) Semiactive control systems are a class of systems for which energy is used to change the mechanical properties of the device.

For this reason, usually the semiactive control system energy requirements are orders of magnitude smaller than typical active control systems. Typically, battery power is sufficient to make them operational. Semiactive control devices do not add mechanical energy to the structural system, therefore bounded-input bounded-output stability is guaranteed, in the sense that no instability can
occur, except for the considerations summarized in the last chapter of this book. Indeed, the system overall energy can be increased by adding a passive system! This depends on the external excitation. The passively controlled system may have resonances that coincide with the main excitation frequencies differently from those that happened before “appending” the passive device. In this case much more energy than in the unprotected case will go into the system. This means that bad performances can be obtained if passive or even semiactive systems are improperly tuned. Passive or semiactive devices can even be dangerous because they allow the external excitation to feed a system that would be (partially) isolated otherwise.

Semiactive control devices are often viewed as controllable passive devices. Preliminary studies indicate that appropriately implemented semiactive systems perform significantly better than passive devices and have the potential to achieve the performance of fully active systems, thus allowing for the possibility of effective response reduction during a wide array of dynamic loading conditions. Examples of such devices include variable-orifice fluid dampers, controllable friction devices, variable-stiffness devices, semiactive impact dampers, adjustable tuned liquid dampers, and controllable fluid dampers (with electrorheological and magnetorheological fluids) (Casciati 2004; Spencer and Sain 1997). Details are provided in Chapter 3.

### 1.3 SYSTEM REPRESENTATION

Dynamic system representation increases in complexity due mainly to the requirements of high accuracy and reliability to analyse complex functionalities and to represent device characteristics. Complex systems may have multiple
inputs and outputs, may be linear or non-linear and may be
time invariant or time varying. A powerful technique for
analysing such systems is the state-space approach, based
on the concept of state. An essential benefit of this method
is the combination of the concept of state and the capability
of high-speed solution of differential equations by use of
a digital processor. Furthermore the state-space approach
is very general and can be applied to both linear and non-
linear systems and to both time-invariant and time-variant
systems.

Let us consider a linear time-invariant system formulated
in the state space by the following equations:

\[
\dot{x}(t) = Ax(t) + Bu(t) \quad (1.1a)
\]

\[
y(t) = Cx(t) + Du(t) \quad (1.1b)
\]

where

1. \(x\) is the \(2n\)-dimensional vector of the state variables,
2. \(y\) is the \(p\)-dimensional vector of the measurable
   variables,
3. \(u\) is the \(m\)-dimensional vector of the controllable and
   forcing variables,
4. \(A \in \mathbb{R}^{2n \times 2n}, B \in \mathbb{R}^{2n \times m},
5. C \in \mathbb{R}^{p \times 2n}, D \in \mathbb{R}^{p \times m}.

This \textit{state-space representation} is commonly used in struc-
tural control engineering.

The equation of motion of a generic system with \(n\)
degrees of freedom can be considered:

\[
M\ddot{z} + C\dot{z} + Kz = F(t) \quad (1.2)
\]

where \(M, C\) and \(K\) are square matrices of dimension \(n\), \(z\)
are the inter-mass displacements (in the case of a building
structure, the inter-storey displacements) and $F(t)$ are the external forces. These forces can be generated by external excitations such as seismic disturbance or wind load. In controlled structures, these forces also include the actuators’ load. Note that in the following, to retain agreement with the control engineering community, we will substitute the notation $F(t)$ by $u$.

By calling the displacement vector $x_1$ and the velocity vector $x_2$, the system represented by (1.2) can be rewritten in the following form:

$$
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -M^{-1}Kx_1 - M^{-1}Cx_2 + M^{-1}u
\end{align*}
$$

(1.3)

or in state-space representation:

$$
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} =
\begin{bmatrix}
0 & I \\
-M^{-1}K & -M^{-1}C
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} +
\begin{bmatrix}
0 \\
M^{-1}
\end{bmatrix} u
$$

(1.4)

where the state matrix $\mathcal{A}$ and the control matrix $\mathcal{B}$ are given by:

$$
\mathcal{A} =
\begin{bmatrix}
0 & I \\
-M^{-1}K & -M^{-1}C
\end{bmatrix}
$$

(1.5)

and

$$
\mathcal{B} =
\begin{bmatrix}
0 \\
M^{-1}
\end{bmatrix}
$$

(1.6)

The output vector (1.1b) (or measurable variables) is a weighted sum of the state and control variables. Matrix $\mathcal{C}$ is a weighting matrix that combines the states of the system (1.2), matrix $\mathcal{D}$ is a weighting matrix that combines the control input, and the sum of these two quantities gives the output $y(t)$. Because the direct action of control variable $u$ on the output vector $y(t)$ is often negligible, the matrix $\mathcal{D}$ is null in most practical applications.
The transfer function $G(s)$ related to system (1.1) can be very easily obtained with the following expression:

$$G(s) = \mathcal{C}(sI - \mathcal{A})^{-1}\mathcal{B} + \mathcal{D}$$

(1.7)

with $s$ indicating the Laplace variable. It expresses the link between output and input Laplace transforms. It must be recalled that this representation is not applicable to non-linear systems.

Just for the sake of exemplification, a shear-type $n$-storey building is a good example of the use of the state-space representation in civil engineering. This type of building can be idealized as lumped masses at floors connected by springs and dashpots describing the columns’ behaviour under horizontal loadings (Figure 1.1). These assumptions are true if the mass of the columns is one order of magnitude lower than the storey mass and if the floors are very rigid (and so are not subject to significant deformation). In practice the mass of the columns is incorporated in the mass of the floors. The stiffness and the damping are completely due to columns, walls, non-structural vertical elements.

In this case, the three matrices $\mathcal{M}$, $\mathcal{C}$ and $\mathcal{K}$ can be obtained as follows. The forces acting on each storey are depicted in Figure 1.2. The inertial force due to the acceleration of the storey is proportional to the acceleration itself times the floor mass.

On both the lower and the upper side of the mass there are two forces: the viscous and the elastic forces. These forces are related to the inter-storey displacement and the inter-storey velocity.

Balancing all the forces acting on each storey, the generic equation for the $i^{th}$ storey can be obtained:

$$m_i \ddot{z}_i + (c_i + k_i) \dot{z}_i + (k_i + c_i) z_i - c_{i+1} \dot{z}_{i+1} - c_i \dot{z}_{i-1} - k_{i+1} z_{i+1} - k_i z_{i-1} = u_i$$

(1.8)

where $u_i$ is the generic control force acting on the $i^{th}$ floor.
Figure 1.1  Simplified model of a multi-storey building
After some calculations, $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ can be obtained from (1.8) as follows (the omitted terms are all zero):

$$
\mathbf{M} = \begin{bmatrix}
m_1 \\
m_2 \\
\vdots \\
m_i \\
\vdots \\
m_n
\end{bmatrix}
$$

$$
\mathbf{C} = \begin{bmatrix}
c_1 + c_2 & -c_2 & \cdots & \cdots & \cdots \\
-c_2 & c_2 + c_3 & -c_3 & \cdots & \cdots \\
\vdots & \vdots & \ddots & \ddots & \ddots \\
\vdots & \vdots & \ddots & -c_i & c_i + c_{i+1} & -c_{i+i} & \cdots \\
\vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots \\
-c_{n-1} & c_{n-1} + c_n & -c_n & \cdots & \ddots & \ddots & \ddots \\
-c_n & \cdots & \cdots & \cdots & \cdots & \ddots & c_n
\end{bmatrix}
$$
It can be observed that $M$ is a diagonal matrix, while $C$ and $K$ are tri-diagonal. Matrix $A$ can be obtained from Equation (1.5). Matrix $B$ usually has more than one column: some of them are usually related to the forcing actions (such as the forces induced by the ground acceleration caused by an earthquake or given by the wind acting on the façade of the building) and can be obtained by Equation (1.6). The columns of $B$ related to the action of the control actuators must be evaluated following the idea that matrix $B$ will be a kind of topological operator indicating where the actuators are acting. For example, if this multi-storey building is equipped with only one actuator on the top floor, matrix $B$ is zero everywhere except for that degree of freedom. Matrix $D$, as already said, is usually set to zero.

As regards matrix $C$, it must be constructed according to the aims of the particular problem. One favourable choice is to use the identity matrix in order to have direct access to the states of the system: in this case each output is directly related with a state variable.

### 1.4 A COMPARISON OF PASSIVE, ACTIVE AND SEMIACTIVE CONTROL STRATEGIES

A brief comparison among passive, active and semiactive control strategies is given hereafter considering a single degree-of-freedom (SDOF) system. This system consists of
a rigid body representing a mass (that could be a machine) connected to a foundation by an isolator that consists of a spring and a damper. In this simple case the function of the vibration isolator is to reduce the amplitude of force transmitted from the vibratory mass to its foundation or to reduce the magnitude of motion transmitted from a vibratory foundation to the mass.

The transmissibility of this system is a measure of the reduction of transmitted force or motion provided by the isolator. If the source of vibration (excitation force) is attached to the mass, transmissibility is the ratio of the force amplitude transmitted to the foundation to the amplitude of the exciting force. If the source of vibration is a vibratory motion of the foundation (excitation motion), transmissibility is the ratio of the vibration amplitude of the mass to the vibration amplitude of the foundation.

Figure 1.3 shows the transmissibility of the system in terms of damping ratio $\xi$ and frequency ratio $\beta$ (where $\beta$ is the ratio of the excitation frequency to the natural frequency). From this graphic representation it appears clearly that increased damping decreases the transmissibility for frequencies lower than the system’s natural frequency multiplied by $\sqrt{2}$, but increases the transmissibility at higher frequencies. This means that the insertion of passive dissipation devices into a structure can sometimes lead to unwanted effects, especially during transient response (Pinkaew and Fujino 2001). Usually an augmented level of damping also induces a higher level of forces at some structural connection. With a semiactive device it is possible to adjust the damping in the most proper manner, for example using an on–off control law that switches the damping value from a high value to a low one. In the example of Figure 1.3 a switching frequency ratio $\beta$ of 1.414 can be chosen. Using more sophisticated control laws (for example, skyhook control or clipping control, see
Figure 1.3 Transmissibility of a SDOF system for several values of supplemental damping

Chapter 4) performances comparable with active devices can be achieved.

Figure 1.4 summarizes this basic concept. The hatched region between the active and passive response curves is the theoretically possible working area of a semiactive system (for more details on the transfer function of semiactive systems see also (Pinkaew and Fujino 2001)). This figure has the advantage of showing in a very simple manner how the semiactive control is better than a passive one and how it can approach the performance of the active systems. However, in real applications, the damping is not the only critical parameter that can be modified. The forces that are present at the various storeys of the structure, the inter-storey drifts and the accelerations must also be attentively considered. It is usually necessary to obtain a balance among all these constrains.
Figure 1.4 Transmissibility of a SDOF system with active, semiactive and passive control systems

To assume the role of structural designers again, one understands that safety against ultimate limit states can be matched by passive control ideas, provided quality control and maintenance plans are introduced in the management of the structure in use, due to the limited lifetime of such devices. To state a similar affirmation for active control systems would require the full availability of the corresponding devices. Nevertheless, such a high availability level is not required if the target is no longer safety against ultimate limit states, but serviceability or robustness. Such important features, however, do not justify the high costs of active control realizations. Semiactive solutions emerge from this discussion as the approach which could better fit a cost–benefit analysis in structural design.