Draw a Diagram

Diagrams are often the key to getting started on a problem. They can clarify relationships that appear complicated when written. Electrical engineers draw diagrams of circuit boards to help them visualize the relationships among a computer’s electrical components.
You’ve probably heard the old saying “One picture is worth a thousand words.” Most people nod in agreement when this statement is made, without realizing just how powerful a picture, or a diagram, can be. (Note that words in bold type are terms that are defined in this book’s glossary.) A diagram has many advantages over verbal communication. For example, a diagram can show positional relationships far more easily and clearly than a verbal description can. To attempt to clarify ideas in their own minds, some people talk to themselves or to others about those ideas. Similarly, a diagram can help clarify ideas and solve problems that lend themselves to visual representations.

One of the best examples of a diagram in the professional world is a blueprint. An architect’s blueprint expresses ideas concisely in a visual form that leaves little to interpretation. Words are added only to indicate details that are not visually evident. A blueprint illustrates one of the strengths of diagrams: the ability to present the “whole picture” immediately.

Problem solving often revolves around how information is organized. When you draw a diagram, you organize information spatially, which then allows the visual part of your brain to become more involved in the problem-solving process. In this chapter, you will learn how you can use diagrams to clarify ideas and solve a variety of problems. You’ll improve your diagramming abilities, and you’ll discover that a diagram can help you understand and correctly interpret the information contained in a problem. You’ll also see the value of using diagrams as a problem-solving strategy.

Solve this problem by drawing a diagram.

VIRTUAL BASKETBALL LEAGUE

Andrew and his friends have formed a fantasy basketball league in which each team will play three games against each of the other teams. There are seven teams: the (Texas A&M) Aggies, the (Purdue) Boilermakers, the (Alabama) Crimson Tide, the (Oregon) Ducks, the (Boston College) Eagles, the (Air Force) Falcons, and the (Florida) Gators. How many games will be played in all? Do this problem before reading on.
As you read in the Introduction, you’ll see many different problems as you work through this book. The problems are indicated by an icon of an attentive dog. To get the maximum benefit from the book, solve each of the problems before reading on. You gain a lot by solving problems, even if your answers are incorrect. The process you use to solve each problem is what you should concentrate on.

You could use many different diagrams to solve the Virtual Basketball League problem, but you could also solve this problem in ways that do not involve diagrams. As you also read in the Introduction, throughout this book you will see some of the same problems in different chapters and solve them with different strategies. You will become a better problem solver in two ways: by solving many different problems and by solving the same problem in many different ways. In this chapter, the solutions involve diagrams. If you solved the Virtual Basketball League problem without using a diagram, try solving it again with a diagram before reading on.

What comes next is a solution process that is attributed to a student. The people mentioned in this book are real students who took a problem-solving class at either Sierra College in Rocklin, California, or at Luther Burbank High School in Sacramento, California. In those classes, the students presented their solutions on the board to their classmates. Ted Herr and Ken Johnson, two of the authors of this book, taught these classes. Our students presented their solutions because we felt that the other students in class would benefit greatly from seeing many different approaches to the same problem. We didn’t judge each student’s solution in any way. Rather, we asked each member of the class to examine each solution that was presented and decide which approach or approaches were valid or, perhaps, better. The purpose behind shifting this responsibility from the instructor to the students is to give the students practice in evaluating problem solving.

We have tried to re-create the same learning atmosphere in this book. Sometimes you’ll see several different approaches to a problem in this book, but for the most part those approaches and the resulting solutions won’t be judged. You are encouraged to evaluate the quality of the approaches. You may have been led to believe that there is always one right way—and many wrong ways—to solve problems. This notion couldn’t be further from the truth. There are many right ways to solve problems, and you are encouraged to solve the problems in this book more than once, using different methods.
Here’s how Rita solved the Virtual Basketball League problem: She drew a diagram that showed the letters representing each team arranged in a circle.

She then drew a line from A to B to represent the games played between the Aggies and the Boilermakers. Then she drew a line from A to C to represent the games played between the Aggies and the Crimson Tide.

She finished representing the Aggies’ games by drawing lines from A to D, E, F, and G.

Next she drew the lines for the Boilermakers. She’d already drawn a line from A to B to represent the games the Boilermakers played against the Aggies, so the first line she drew for the Boilermakers was from B to C.

She continued drawing lines to represent the games that the Boilermakers played against each other team.
From C she drew lines only to D, E, F, and G because the lines from C to A and from C to B had already been drawn. She continued in this way, completing her diagram by drawing the lines needed to represent the games played by the rest of the teams in the league. Note that when she finally got to the Gators, she did not need to draw any more lines because the games the Gators played against each other team had already been represented with a line.

She then counted the lines she’d drawn. There were 21. She multiplied 21 by 3 (remember that each line represented three games) and came up with an answer of 63 games. Finally, Rita made sure that she’d answered the question asked. The question was “How many games will be played in all?” Her answer, “Sixty-three games will be played,” accurately answers the question.

Mirka solved this problem with the diagram below. She also used the letters A, B, C, D, E, F, and G to represent the teams. She arranged the letters in a row and, as Rita did, she drew lines from team to team to represent games played. She started by drawing lines from A to the other letters, then from B to the other letters, and so on. She drew 21 lines, multiplied 21 by 3, and got an answer of 63 games.

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**MODEL TRAIN**

Esther’s model train is set up on a circular track. Six telephone poles are spaced evenly around the track. The engine of Esther’s train takes 10 seconds to go from the first pole to the third pole. How long would it take the engine to go all the way around the track? Solve the problem before reading on.
If you read the problem quickly and solved it in your head, you might think the answer is 20 seconds. After all, the problem states that the engine can go from the first pole to the third pole in 10 seconds, which is three poles out of six and apparently halfway around the track. So it would take the engine 2 times 10, or 20 seconds, to go all the way around the track. But this answer is wrong. The correct answer becomes apparent when you look at a diagram.

Rena’s diagram is shown at right. Rena explained that the train goes one-third of the way around the track in 10 seconds, not halfway around the track. So the train goes around the entire track in 3 times 10 seconds, or 30 seconds.

Phong drew the same diagram, but he interpreted it differently. He explained that if it takes 10 seconds to go from the first pole to the third pole, then it takes 5 seconds to go from the first pole to the second pole. So it takes 5 seconds to go from pole to pole. There are six poles, so it takes the train 30 seconds to go all the way around the track.

Pete interpreted the problem as Phong did, but he didn’t draw a diagram. Thus, he neglected the fact that the train must return from the sixth pole to the first pole in order to travel all the way around the track. Therefore, he got the incorrect answer 25 seconds.

The diagram helped Rena and Phong solve the Model Train problem. If you used a diagram to solve the problem, you probably got the correct solution. If you were able to get the correct solution without drawing a diagram, think back on your process. You probably visualized the train track in your mind, so even though you didn’t actually draw a diagram, you could “see” a picture.
Do you get the picture? Do you see why diagrams are important? Research shows that most good problem solvers draw diagrams for almost every problem they solve. Don’t resist drawing a diagram because you think that you can’t draw, or that smart people use only equations to solve problems, or whatever. Just draw it!

**THE POOL DECK**

Curly used a shovel to dig his own swimming pool. He figured he needed a pool because digging it was hard work and he could use it to cool off after working on it all day. He also planned to build a rectangular concrete deck around the pool that would be 6 feet wide at all points. The pool is rectangular and measures 14 feet by 40 feet. What is the area of the deck? As usual, solve this problem before continuing.

Jeff drew the diagram below to show the correct dimensions of the deck and pool, which together are 12 feet longer and 12 feet wider than the pool alone.

The diagram helps show the difficult parts of the problem. However, Jeff solved the problem incorrectly by finding the outside perimeter of the pool and the deck together, then multiplying the perimeter by the width of the deck.

\[
52\text{ feet} + 26\text{ feet} + 52\text{ feet} + 26\text{ feet} = 156\text{ feet}
\]

\[
156\text{ feet} \times 6\text{ feet} = 936\text{ square feet}
\]

His approach is incorrect because it counts each corner twice.
Rajesh used the same diagram, but he solved the problem by first computing the area of the deck along the sides of the pool, then adding in the corners of the deck.

Two lengths: $40 \text{ ft} \times 6 \text{ ft} \times 2 = 480 \text{ sq ft}$
Two widths: $14 \text{ ft} \times 6 \text{ ft} \times 2 = 168 \text{ sq ft}$
Four corners: $6 \text{ ft} \times 6 \text{ ft} \times 4 = 144 \text{ sq ft}$
Total $792 \text{ sq ft}$

May’s diagram shows the corners attached to the length of the deck.

She calculated the area as follows:
$52 \text{ ft} \times 6 \text{ ft} = 312 \text{ sq ft}$
$312 \text{ sq ft} \times 2 = 624 \text{ sq ft}$ for extended lengths
$14 \text{ ft} \times 6 \text{ ft} = 84 \text{ sq ft}$
$84 \text{ sq ft} \times 2 = 168 \text{ sq ft}$ for widths
Total $624 \text{ sq ft} + 168 \text{ sq ft} = 792 \text{ sq ft}$
Herb solved this problem by first computing the area of the pool and the deck together, then subtracting the area of the pool, leaving the area of the deck.

Area of entire figure = 52 ft × 26 ft = 1,352 sq ft
Area of pool alone = 40 ft × 14 ft = 560 sq ft
Area of deck = 1352 ft – 560 ft = 792 sq ft

FARMER BEN

Farmer Ben has only ducks and cows. He can’t remember how many of each he has, but he doesn’t need to remember because he knows he has 22 animals and that 22 is also his age. He also knows that the animals have a total of 56 legs, because 56 is also his father’s age. Assuming that each animal has all legs intact and no extra limbs, how many of each animal does Farmer Ben have? Do this problem, and then read on.

Trent drew the following diagram and explained his thinking: “These 22 circles represent the 22 animals. First, I made all of the animals into ducks.” (Trent is not much of an artist, so you just have to believe that these are ducks.) “I gave each animal two legs because ducks have two legs.”
“Then I converted the ducks into cows by drawing extra legs. The ducks alone had 44 of the 56 legs initially, so I drew 12 more legs, or six pairs, on 6 ducks to turn them into cows. So there are 6 cows and 16 ducks.”

Of course, Farmer Ben might have a problem when his father turns 57 next year.

**Draw a Diagram**

Any idea that can be represented with a picture can be communicated more effectively with that picture. By making visible what a person is thinking, a diagram becomes a problem-solving strategy. A diagram clarifies ideas and communicates those ideas to anyone who looks at it. Diagrams are used in many jobs, especially those that require a planning stage. Occupational diagrams include blueprints, project flow charts, and concept maps, to name a few. Diagrams are often necessary to show position, directions, or complicated multidimensional relationships, because pictures communicate these ideas more easily and more clearly than words.

**Problem Set A**

You must draw a diagram to solve each problem.

1. **Worm Journey**

A worm is at the bottom of a 12-foot wall. Every day the worm crawls up 3 feet, but at night it slips down 2 feet. How many days does it take the worm to get to the top of the wall?
2. **UP AND DOWNS OF SHOPPING**

Roberto is shopping in a large department store with many floors. He enters the store on the middle floor from a skyway and immediately goes to the credit department. After making sure his credit is good, he goes up three floors to the housewares department. Then he goes down five floors to the children’s department. Then he goes up six floors to the TV department. Finally, he goes down ten floors to the main entrance of the store, which is on the first floor, and leaves to go to another store down the street. How many floors does the department store have?

3. **FOLLOW THE BOUNCING BALL**

A ball rebounds one-half the height from which it is dropped. The ball is dropped from a height of 160 feet and keeps on bouncing. What is the total vertical distance the ball will travel from the moment it is dropped to the moment it hits the floor for the fifth time?

4. **FLOOR TILES**

How many 9-inch-square floor tiles are needed to cover a rectangular floor that measures 12 feet by 15 feet?

5. **STONE NECKLACE**

Arvilla laid out the stones for a necklace in a big circle, with each stone spaced an equal distance from its neighbors. She then counted the stones in order around the circle. Unfortunately, before she finished counting she lost track of where she had started, but she realized that she could figure out how many stones were in the circle after she noticed that the sixth stone was directly opposite the seventeenth stone. How many stones are in the necklace?

6. **DANGEROUS MANEUVERS**

Somewhere in the Mojave Desert, the army set up training camps named Arachnid, Feline, Canine, Lupine, Bovine, and Thirty-Nine. Several camps are connected by roads:

- Arachnid is 15 miles from Canine, Bovine is 12 miles from Lupine,
- Feline is 6 miles from Thirty-Nine, Lupine is 3 miles from Canine,
- Bovine is 9 miles from Thirty-Nine, Bovine is 7 miles from Canine,
- Thirty-Nine is 1 mile from Arachnid, and Feline is 11 miles from Lupine. No other pairs of training camps are connected by roads.

*Note:* This problem continues on the next page.
Answer each of the following questions (in each answer, indicate both the mileage and the route): What is the shortest route from

- Feline to Bovine?
- Canine to Thirty-Nine?
- Lupine to Thirty-Nine?
- Lupine to Bovine?
- Canine to Feline?
- Arachnid to Feline?
- Arachnid to Lupine?

7. RACE

Becky, Ruby, Isabel, Lani, Alma, and Sabrina ran an 800-meter race. Alma beat Isabel by 7 meters. Sabrina beat Becky by 12 meters. Alma finished 5 meters ahead of Lani but 3 meters behind Sabrina. Ruby finished halfway between the first and last women. In what order did the women finish? What were the distances between them?

8. A WHOLE LOTTAT SHAKIN’ GOIN’ ON!

If six people met at a party and all shook hands with one another, how many handshakes would be exchanged?

9. HAYWIRE

A telephone system in a major manufacturing company has gone haywire. The system will complete certain calls only over certain sets of wires. So, to get a message to someone, an employee of the company first has to call another employee to start a message on a route to the person the call is for. As far as the company can determine, these are the connections:

- Cherlondia can call Al and Shirley (this means that Cherlondia can call them, but neither Al nor Shirley can call Cherlondia). Al can call Max.
- Wolfgang can call Darlene, and Darlene can call Wolfgang back.
- Sylvia can call Dalamatia and Henry. Max can get calls only from Al.
- Carla can call Sylvia and Cherlondia. Shirley can call Darlene.
- Max can call Henry. Darlene can call Sylvia. Henry can call Carla.
- Cherlondia can call Dalamatia.

How would you route a message from

- Cherlondia to Darlene?
- Shirley to Henry?
- Carla to Max?
- Max to Dalamatia?
- Sylvia to Wolfgang?
- Cherlondia to Sylvia?
- Henry to Wolfgang?
- Dalamatia to Henry?
10. **ROCK CLIMBING**

Amy is just learning how to rock climb. Her instructor takes her to a 26-foot climbing wall for her first time. She climbs 5 feet in 2 minutes but then slips back 2 feet in 10 seconds. This pattern (up 5 feet, down 2 feet) continues until she reaches the top. How long will it take her to reach the very top of the wall?

This problem was written by Jen Adorjan, a student at Sierra College in Rocklin, California.

11. **CIRCULAR TABLE**

In Amanda and Emily’s apartment, a round table is shoved into the corner of the room. The table touches the two walls at points that are 17 inches apart. How far is the center of the table from the corner?

12. **THE HUNGRY BOOKWORM**

Following is an expansion of a well-known problem:

The four volumes of *The World of Mathematics* by James R. Newman are sitting side by side on a bookshelf, in order, with volume 1 on the left. A bookworm tunnels through the front cover of volume 1 all the way through the back cover of volume 4. Each book has a front cover and a back cover that each measure $\frac{1}{16}$ inch. The pages of each book measure $1\frac{1}{8}$ inches. How far does the bookworm tunnel?

13. **BUSING TABLES**

Brian busses tables at a local café. To bus a table, he must clear the dirty dishes and reset the table for the next set of customers. One night he noticed that for every three-fifths of a table that he bused, another table of customers would get up and leave. He also noticed that right after he finished busing a table, a new table of customers would come into the restaurant. However, once every table was empty (no diners were left in the restaurant), nobody else came into the restaurant. Suppose there were six tables with customers and one unbused table. How many new tables of customers would come in before the restaurant was empty? After the last table of customers had left, how many tables were unbused?

This problem was written by Brian Strand, a student at Sierra College in Rocklin, California.
14. **WRITE YOUR OWN PROBLEM**

In each chapter, you’ll be given the opportunity to write your own problem that can be solved by using the strategy you studied in that chapter. The book will give you suggestions for how to go about writing these problems. Each time you write your own problem, solve it yourself to be sure that it’s solvable. Then give it to another student to solve and, as needed, to help you with the problem’s wording.

Create your own Draw a Diagram problem. Model it after either this chapter’s Worm Journey problem or Ups and Downs of Shopping problem.

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**CLASSIC PROBLEMS**

15. **THE WEIGHT OF A BRICK**

If a brick balances with three-quarters of a brick and three-quarters of a pound, then how much does the brick weigh?


16. **THE MOTORCYCLIST AND THE HORSEMAN**

A motorcyclist was sent by the post office to meet a plane at the airport. The plane landed ahead of schedule, and its mail was taken toward the post office by horse. After half an hour, the horseman met the motorcyclist on the road and gave him the mail. The motorcyclist returned to the post office 20 minutes before he was expected. How many minutes early did the plane land?

Adapted from *The Moscow Puzzles* by Boris Kordemsky.

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**MORE PRACTICE**

1. **APARTMENT BUILDING**

Joden just moved into a 12-story apartment building, and he is still having trouble finding which floor he lives on. He knows that he lives in the first apartment on the floor, but doesn’t know which floor. He starts by going to the first floor and knocks on the door. Mrs. Smith answers and tells him to go up 8 floors and ask Mr. Jones. Joden does that and asks Mr. Jones where he lives. Mr. Jones doesn’t know, but he says, “Go down 2 floors to Bryn’s apartment and ask him.” Bryn didn’t know either. He told Joden to go up 5 floors to see his friend Trudie, because
she knows where everyone lives. Trudie had no clue who Joden was. She said that the only one who might know where he lived was the new guy 7 floors below her. Joden goes down 7 floors and knocks on the new guy’s apartment. No one answers. He stands there thinking for a while and finally realizes that he is the new guy. He opens the door and walks into his apartment. Which floor does Joden live on?

This problem was written by Jeremy Chew, a student at Sierra College in Rocklin, California.

2. MOVIE LINE

A bunch of people were standing in line for a movie. Averi got there late and realized that she knew every person in line. She decided not to get in line until she figured out who to cut in with. She first stopped and talked to Jake, who was at the back of the line. Then she moved forward by passing 3 people and talked to Alexandra. She then moved forward again by passing 9 people and talked to Walter. Then she moved backward by passing 4 people and talked to Annie. Then she moved forward by passing 12 people and talked to Katie. Finally she moved backward by passing 2 people and joined Carli in line. Carli was originally the person in the exact middle of the line. Including Averi (who was now in line) how many people are in the line?

Note: Moving forward refers to moving toward the front of the line and moving backward refers to moving toward the back of the line.

3. BACKBOARD

Ei liked to play tennis. One day she didn’t have anyone to play with, so she took her racket and tennis ball and began to hit the ball against the backboard. She hit it at the backboard, the ball bounced off the backboard and came back to her, and she hit it again, and so on. She started out 40 feet from the backboard. But she didn’t hit the ball hard enough—the bounce came back only 90% as far, so she had to run up to hit it again. Again she didn’t hit it hard enough, and it again only came back 90% as far as she had hit it. This continued for two more hits, each time the ball coming back 90% as far. Finally on the fifth hit she hit it really hard and it came back five times as far as she had hit it, going way over her head and hitting the fence. She got frustrated and picked up her ball and went home. How far was the distance from the backboard to the fence? What was the total horizontal distance that the tennis ball traveled?
4. **WORKING OUT**

At the gym, there are 12 weight machines that Gina liked to use: 6 upper body machines and 6 lower body machines. All 12 machines were in one row, with 8 feet between each upper body machine and 8 feet between each lower body machine. The upper body machines were separated from the lower body machines by 16 feet. The lower body machines were numbered 1 through 6, and the upper body machines were numbered 7 through 12. Gina started at machine 6 (lower body) and then walked to machine 7 (upper body), then to machine 5 (lower body), then to machine 8 (upper body), and so on, alternating between lower body and upper body. After each 3 lower body machines, she would walk to the mat that was 20 feet past machine 1 and do some stretches. After each 3 upper body machines, she would walk to the wall that was 30 feet past machine 12 and do some stretching. When she finally finished lifting and stretching (the last thing she did was upper body stretching on the wall), how many feet had she walked in all?

5. **ANYA AND SOPHIA AND MASHA AND MIKE**

Anya, Sophia, Masha, and Mike went to New York City to see a few Broadway shows. They stayed on a floor in a small hotel which had four rooms along one long hallway. Anya was staying in the first room, Sophia in the second room, Masha in the third room, and Mike in the fourth room, at the opposite end of the hall from Anya. The elevator was along the hallway, exactly halfway between Sophia’s door and Masha’s door. One afternoon, Sophia and Masha got out of the elevator and walked 90 feet to Mike’s door to say hello. Then they walked back the other direction all the way down the hall to Anya’s door to leave a note. The total distance from Mike’s door to Anya’s door was 160 feet. Then they turned around and walked 30 feet to Sophia’s door. How far did Masha have to walk to return from Sophia’s door to her own room’s door?

This problem was written by Sierra College professor Jill Rafael.