Contents

Preface xv
Frequently Used Symbols and Abbreviations xxiii

1 Introduction: Signal Digitizing and Digital Processing 1
  1.1 Subject Matter 1
  1.2 Digitizing Dictates Processing Preconditions 4
    1.2.1 Connecting Computers to the Real-life World 5
    1.2.2 Widening of the Digital Domain 5
    1.2.3 Digital Signal Representation 7
    1.2.4 Complexity Reduction of Systems 10
  1.3 Approach to the Development of Signal Processing Systems 12
  1.4 Alias-free Sampling Option 14
    1.4.1 Anti-aliasing Irregularity of Sampling 14
    1.4.2 Sparse Nonuniform Sampling 17
    1.4.3 Nonuniform Sampling Events 21
  1.5 Remarks in Conclusion 23

Bibliography 25

Part 1 Digitizing

2 Randomization as a Tool 31
  2.1 Randomized Versus Statistical Signal Processing 32
  2.2 Accumulation of Empirical Experience 33
    2.2.1 Using Monte Carlo Methods for Signal Processing 33
    2.2.2 Polarity Coincidence Methods 36
    2.2.3 Stochastic–Ergodic Method 39
    2.2.4 Stochastic Computing 40
### Contents

2.2.5 Dithering 45  
2.2.6 Generalized Scheme of Randomized Digitizing 48  
2.3 Discovery of Alias-free Signal Processing 49  
2.3.1 Early Academic Research in Randomized Temporal Sampling 49  
2.3.2 Early Research in Randomized Spatial Signal Processing 51  
2.3.3 Engineering Experience 52  
2.4 Randomization Leading to DASP 53  
2.4.1 DASP Mission 55  
2.4.2 Demonstrator of DASP Advantages and Limitations 55  
2.5 Some of the Typically Targeted Benefits 58  
Bibliography 60  

3 Periodic Versus Randomized Sampling 63  
3.1 Periodic Sampling as a Particular Sampling Case 63  
3.1.1 Generalized Sampling Model 65  
3.2 Spectra of Sampled Signals 67  
3.2.1 Spectra of Periodically Sampled Signals 68  
3.2.2 Spectra of Randomly Sampled Signals 70  
3.3 Aliasing Induced Errors at Seemingly Correct Sampling 71  
3.4 Overlapping of Sampled Signal Components 76  
3.5 Various Approaches to Randomization of Sampling 79  
Bibliography 83  

4 Randomized Quantization 87  
4.1 Randomized Versus Deterministic Quantization 87  
4.1.1 Basics 88  
4.1.2 Input–Output Characteristics 90  
4.1.3 Rationale of Randomizing 92  
4.2 Deliberate Introduction of Randomness 93  
4.2.1 Various Models 94  
4.3 Quantization Errors 97  
4.3.1 Probability Density Function of Errors 98  
4.3.2 Variance of Randomly Quantized Signals 101  
4.4 Quantization Noise 103  
4.4.1 Covariance between the Signal and Quantization Noise 104  
4.4.2 Spectrum 105  
Bibliography 106
## 5 Pseudo-randomized Quantizing

5.1 Pseudo-randomization Approach 107  
5.2 Optimal Quantizing 109  
  5.2.1 Single-threshold Quantizing 109  
  5.2.2 Multithreshold Quantizing 112  
  5.2.3 Implementation Approaches 113  
5.3 Input–Output Relationships 115  
5.4 Quantization Errors 115  
5.5 Quantization Noise 117  
  5.5.1 Covariance between Signal and Quantization Noise 119  
  5.5.2 Spectrum of the Pseudo-randomized Quantization Noise 120  
  5.5.3 Noise Reduction by Oversampling 121  
5.6 Some Properties of Quantized Signals 122  
5.7 Benefits 125  
Bibliography 126

## 6 Direct Randomization of Sampling

6.1 Periodic Sampling with Jitter 128  
6.2 Additive Random Sampling 131  
6.3 Sampling Function 132  
6.4 Elimination of Bias Errors 136  
Bibliography 138

## 7 Threshold-crossing Sampling

7.1 Sampling at Input and Reference Signal Crossings 140  
  7.1.1 Level-crossing Sampling 141  
  7.1.2 Time-variant Threshold Crossings 142  
7.2 Representing Signals Using Timing Information 142  
7.3 Sine-Wave Crossings 144  
  7.3.1 Recovery of Signal Sample Values 144  
  7.3.2 Various Realizations 147  
7.4 Remote Sampling Based on Sine-Wave Crossings 150  
7.5 Advantages and Disadvantages 152  
Bibliography 155

## 8 Derivatives of Periodic Sampling

8.1 Phase-shifted Periodic Sampling 158
8.1 Dependence of Aliasing on the Sampling Phase 158
  8.1.1 Dependence of Aliasing on the Sampling Phase 158
  8.1.2 Reconstruction of Sampled Signals 160

8.2 Periodic Sampling with Random Skips 163
  8.2.1 General Model 163
  8.2.2 Typical Use 165

8.3 Compensation Effect 165
  8.3.1 Display of Fourier Transforms 166
  8.3.2 Observing the Aliasing Processes 167

8.4 Generation of Randomized Sampling Pulse Trains 171
  8.4.1 Basic Approach 171
  8.4.2 Practical Experience 173

Bibliography 174

9 Fuzzy Aliasing 177
  9.1 Meaning of the DFT of a Nonuniformly Sampled Signal 177
  9.2 Concept of Fuzzy Aliasing 179
    9.2.1 Generic Periodic Sampling with Random Skips 179
    9.2.2 Primary and Secondary Aliasing 181
    9.2.3 Decomposition of Sampling Point Processes 183
  9.3 Anatomy of Fuzzy Aliasing 185
    9.3.1 Tracking of Particular Contributions 185
    9.3.2 Incomplete Compensation of Aliases 187
    9.3.3 Aliasing at Multiple Frequencies 188
  9.4 Object Lesson 188

Bibliography 190

10 Hybrid Sampling 191
  10.1 Hybrids of Periodic and Random Sampling 192
    10.1.1 Basic Approach 192
    10.1.2 Arrangements for Sample Value Processing 194
  10.2 Hybrid Double Sampling 198
    10.2.1 Providing for Short Sampling Intervals 199
    10.2.2 Double Periodic Sampling with Jitter 201
    10.2.3 Double Additive Pseudo-random Sampling 203
    10.2.4 Periodic Additive Pseudo-random Sampling 204
  10.3 Mixing Hybrid Sampling with Periodic Sampling 206
  10.4 Comments in Conclusion 208

Bibliography 210
Contents ix

Part 2 Processing

11 Data Acquisition 213
  11.1 Data Acquisition from Wideband Signal Sources 214
     11.1.1 Practical Results Confirming the Theory 214
     11.1.2 Sampling with Reduced Uncontrolled Jitter 215
  11.2 Application of Hybrid Double Sampling 218
  11.3 Pseudo-randomized Multiplexing 219
  11.4 Massive Data Acquisition 221
     11.4.1 Specifics of Multichannel Data Acquisition 221
     11.4.2 Reconfigurable Distributed Structure ADC 223
  Bibliography 225

12 Quantizing-specific Signal Parameter Estimation 227
  12.1 Theoretical Limits 228
     12.1.1 Minimal Observation Time 228
     12.1.2 Sufficient Number of Signal Samples 231
     12.1.3 Influence of Quantization Errors 231
     12.1.4 Estimation of Periodic Signal Parameters 233
  12.2 Optimal Estimation 234
     12.2.1 Minimizing the Number of Signal Samples 234
     12.2.2 Simplifying Hardware 236
     12.2.3 Minimizing Bit Flow 237
     12.2.4 Deviations from Optimal Conditions 240
     12.2.5 Comments 241
  12.3 Specifics Related to Pseudo-randomized Quantizing 242
     12.3.1 Avoiding Processing of the Dither Process 243
     12.3.2 Simplified Processing of the Dither Process 244
  12.4 Estimation of the Absolute Mean Value 246
     12.4.1 Electronic Device 246
     12.4.2 Estimation Errors 247
  12.5 Estimation of the Mean Power 249
     12.5.1 Estimation Efficiency 250
  12.6 Errors Due to Randomized Sampling 251
     12.6.1 Absolute Mean Value Estimate 252
     12.6.2 Mean Power Estimate 252
     12.6.3 Overall Estimation Errors 252
  Bibliography 253
## Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>13 Estimation of Correlation Functions</strong></td>
<td></td>
</tr>
<tr>
<td>13.1 Multiplication of Quantized Signals</td>
<td>256</td>
</tr>
<tr>
<td>13.1.1 Expected Value of Multiplied Quantized Signals</td>
<td>256</td>
</tr>
<tr>
<td>13.1.2 Variance of Multiplication Results</td>
<td>257</td>
</tr>
<tr>
<td>13.1.3 Optional Approaches</td>
<td>259</td>
</tr>
<tr>
<td>13.2 Correlation Analysis of Pseudo-randomly Quantized Signals</td>
<td>260</td>
</tr>
<tr>
<td>13.2.1 Estimation Procedure</td>
<td>261</td>
</tr>
<tr>
<td>13.2.2 Essential Relationships</td>
<td>261</td>
</tr>
<tr>
<td>13.2.3 Implementation Issues</td>
<td>263</td>
</tr>
<tr>
<td>13.3 Correlation Analysis of Pseudo-randomly Sampled Signals</td>
<td>264</td>
</tr>
<tr>
<td>13.4 Comments</td>
<td>267</td>
</tr>
<tr>
<td>Bibliography</td>
<td>268</td>
</tr>
<tr>
<td><strong>14 Signal Transforms</strong></td>
<td></td>
</tr>
<tr>
<td>14.1 Problem of Matching Signal Processing to Sampling</td>
<td>269</td>
</tr>
<tr>
<td>14.2 Bases of Signal Transforms</td>
<td>271</td>
</tr>
<tr>
<td>14.2.1 Required Properties of the Transform Bases</td>
<td>271</td>
</tr>
<tr>
<td>14.2.2 Transforms by Means of a Finite Number of Basis Functions</td>
<td>272</td>
</tr>
<tr>
<td>14.3 Orthogonal Transforms</td>
<td>274</td>
</tr>
<tr>
<td>14.3.1 Analog Processing</td>
<td>275</td>
</tr>
<tr>
<td>14.3.2 Digital Processing</td>
<td>276</td>
</tr>
<tr>
<td>14.4 Discrete Unorthogonal Transforms</td>
<td>277</td>
</tr>
<tr>
<td>14.5 Conversion of Unorthogonal Transforms</td>
<td>279</td>
</tr>
<tr>
<td>Bibliography</td>
<td>281</td>
</tr>
<tr>
<td><strong>15 DFT of Nonuniformly Sampled Signals</strong></td>
<td></td>
</tr>
<tr>
<td>15.1 Problems Related to Sampling Irregularities</td>
<td>284</td>
</tr>
<tr>
<td>15.1.1 Alternative Approaches to DFT</td>
<td>284</td>
</tr>
<tr>
<td>15.1.2 Best-fitting Procedure Versus Direct DFT</td>
<td>285</td>
</tr>
<tr>
<td>15.1.3 Sample Values Partly Fitting to Any Frequency</td>
<td>288</td>
</tr>
<tr>
<td>15.2 Cross-interference Corrupting DFT</td>
<td>289</td>
</tr>
<tr>
<td>15.3 Exploitation of FFT</td>
<td>291</td>
</tr>
<tr>
<td>15.3.1 Application of FFT for Processing Nonuniformly Sampled Signals</td>
<td>291</td>
</tr>
<tr>
<td>15.3.2 Fast Transforms of Signals Sampled at Sine-Wave Crossing Instants</td>
<td>294</td>
</tr>
<tr>
<td>15.4 Revealing the Essence of the Fourier Coefficient Estimation</td>
<td>299</td>
</tr>
<tr>
<td>Bibliography</td>
<td>306</td>
</tr>
</tbody>
</table>
16 Complexity-reduced DFT

16.1 Potential Gains from Application of Rectangular Function Sets

16.1.1 Use of Orthogonal Rectangular Functions 308

16.1.2 Reduction in the Computational Burden for DFT 309

16.2 Complexity-reduced DFT Exploiting Rectangular Functions 310

16.2.1 Essentials of the Method 310

16.2.2 Mathematical Description 312

16.2.3 Digital Implementation 316

16.3 Computer Simulations of the Rectangular Function-based DFT 318

16.4 Fast DFT of Sine-Wave Crossings 322

17 Spatial Data Acquisition and Processing

17.1 Sensor Array Model 326

17.2 Temporal and Spatial Spectra of Array Signals

17.2.1 When Signal Source Frequencies Do Not Overlap 330

17.2.2 When Signal Source Frequencies Overlap 331

17.2.3 Aliasing in the Spatial Domain 332

17.3 Beamforming 335

17.4 Signal Direction of Arrival Estimation 337

17.5 Pseudo-randomization of Sensor Arrays

17.5.1 Complexity Reduction of Arrays 343

17.5.2 Pseudo-randomization of Array Signal Processing 345

18 Adapting Signal Processing to Sampling Nonuniformities

18.1 Cross-interference Coefficients

18.1.1 Definition 348

18.1.2 Interpretation 349

18.1.3 Approximation 350

18.2 Taking the Cross-interference into Account 353

18.3 Achievable Improvement and Typical Problems 356

18.4 Parallel Computing Approach

18.4.1 Decomposition of the Signal Sample Value Sequence 359

18.4.2 Adapting the Estimation for Each Signal Sample Value Subset 361
18.4.3 Data Aggregation

18.5 Mapping of the Cross-interference Coefficients
  18.5.1 Required Frequency Resolution
  18.5.2 Coefficient Mapping Versus On-line Calculations

19 Estimation of Object Parameters
  19.1 Measuring the Frequency Response of Objects
  19.2 Test Signal Synthesis from a Sparsely Periodically Sampled Basis Function
    19.2.1 Synthesis in the Case of Monoharmonic Basis
    19.2.2 Synthesis in the Case of Multifrequency Basis
  19.3 Test Signal Synthesis from a Nonuniformly Sampled Basis Function
    19.3.1 Spectrum of the Synthesized Signal
    19.3.2 Multifrequency Signal Synthesis
    19.3.3 Amplitude Equalization
  19.4 Synthesis of Narrowband and Wideband Signals
  19.5 Measuring Small Delays and Switching Times
  19.6 Bioimpedance Signal Demodulation in Real-time
    19.6.1 Typical Conditions for Bioimpedance Signal Forming
    19.6.2 Complexity Reduction of Bioimpedance Signal Demodulation

Bibliography

20 Encapsulating DASP Technology
  20.1 Linking Digital Alias-free Signal Processing with Traditional Methods
    20.1.1 Generic Model of the Embedded DASP Systems
    20.1.2 Various DASP System Embedding Conditions
  20.2 Algorithm Options in the Development of Firmware
    20.2.1 Sequential Exclusion of Signal Components
    20.2.2 Iterative Variable Threshold Calculations of DFT and IDFT
    20.2.3 Algorithms Adapted to the Sampling Irregularities
    20.2.4 Comparison of Algorithm Performance
  20.3 Dedicated Services of the Embedded DASP Systems
  20.4 Dedicated Services Related to Processing of Digital Inputs
    20.4.1 Approach to Data Compression
### Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>20.4.2 Data Compression for One-Dimensional Signals</td>
<td>414</td>
</tr>
<tr>
<td>20.4.3 Data Compression for Two-Dimensional Signals</td>
<td>415</td>
</tr>
<tr>
<td>20.4.4 Providing for Fault Tolerance</td>
<td>416</td>
</tr>
<tr>
<td>20.5 Reducing the Quantity of Sensors in Large-aperture Arrays</td>
<td>419</td>
</tr>
<tr>
<td>20.5.1 Adapting Signal Processing to Pseudo-random Positions of Sensors</td>
<td>420</td>
</tr>
<tr>
<td>Bibliography</td>
<td>425</td>
</tr>
</tbody>
</table>

### Index

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Index</td>
<td>427</td>
</tr>
</tbody>
</table>