1

Introduction

1.1 What is Reinsurance?

A reinsurance contract is an agreement in which one party (the reinsurer) agrees to indemnify another party (the reinsured, the first-line insurer or also the ceding company, cedent) for specified parts of its underwritten insurance risk. In turn, the cedent pays to the reinsurer a reinsurance premium for this service. That is, in reinsurance the principle of insurance is lifted up one level, so an insurance company seeks itself the possibility of replacing parts of its future loss by a fixed premium payment (much like a policyholder does when entering an insurance contract). There are many reasons why such a risk transfer from the insurer to the reinsurer can be desirable for both parties, as well as for the economy in general, and we will outline a number of them in Section 1.2.

While reinsurance can be seen as a particular form of insurance, and naturally shares various common features with it, reinsurance is also quite distinct from primary insurance in a number of aspects. These include the type and magnitude of risks under consideration, the type of data available for the risk analysis, the diversification possibilities, demand/supply peculiarities of contracts quite different from the primary insurance market, and also the fact that reinsurance is a form of risk sharing among two “professional” insurance entities, so that the necessary guidelines for regulation can be quite different.

(Non-life) reinsurance contracts are typically written for one year, and one distinguishes between obligatory treaties, where a binding agreement is specified that applies to all risks of a specified risk class, and facultative arrangements, which are negotiated on each individual risk. A facultative treaty is then a contract where the cedent has the option to cede and the reinsurer has the option to decline or accept classified risks of a particular business line. In practice many contracts actually involve several reinsurers (e.g., the contract is negotiated with a primary reinsurer, and other reinsurers then participate proportionally in the reinsurance coverage, or a second reinsurance contract with another reinsurer is written for parts of the remaining risk of the cedent after a first contract). The relationship between insurer and reinsurer is often of a long-term nature, which also has an effect on the way reinsurance premiums are negotiated. Finally, there is no relation between a reinsurer and the individual policyholders of the
underlying risks. A reinsurer may itself enter a reinsurance contract with another insurance company on parts of the reinsured risk, and such a procedure is called retrocession.

Table 1.1 gives a feeling for the size of the global reinsurance market in comparison to the primary insurance market. One sees that in terms of premium volume, reinsurance is only employed for a small fraction of the primary insurance risk. However, typically the reinsured risk is the one that is complicated to assess and handle (this is one of the main reasons why it is reinsured!), which makes this type of risk particularly challenging for actuaries, statisticians, and other risk professionals. Worldwide, there are about 200 reinsurance companies today, and many of these are also acting as primary insurers in the market.

1.2 Why Reinsurance?

Let us look at why an insurance company is interested in buying reinsurance. The main function of insurance companies is to take risk. This is similar to the business model of other financial organizations, and both types leverage the capital provided by shareholders through raising debt. However, insurers raise debt by selling policies to insureds, which makes the debt very risky (due to uncertainty around the timing and severity of claims), whereas financial debt would typically rather have pre-determined expiry and face value (severity). This leveraging activity is a competitive advantage, but also makes the companies vulnerable to distress and insolvency, creating the demand for risk management. Among the available risk management tools, risk transfer through reinsurance then plays an important role in improving the company’s overall risk profile. Let us look at some of the main motivations for the insurer to buy reinsurance as a means of risk transfer (several of which are not independent of each other):

- **Reducing the probability to suffer losses that are hard to digest**
  This is a rather general statement and many of the items below are in fact refinements of this criterion. It should be kept in mind that for an insurance company buying reinsurance means passing on some of its insurance business (i.e., its core activity), and hence typically the goal is to keep the reinsured part small. However, reduction of risk exposure can be desirable or necessary for the reasons outlined below.

- **Stabilizing business results**
  Entering a reinsurance contract reduces the volatility of the cedent’s financial result, as random losses are replaced by a (typically deterministic) premium payment. That is, reinsurance can be a means to steer the volatility of an insurance company towards a
desired level, and the latter can have particular advantages (e.g., with respect to taxes, capital requirements and market expectations).

- **Reducing required capital**
  Reducing the aggregate risk will reduce the required capital to bear such risks, and in view of capital costs this may be desirable. Concretely, if the reinsurance premium (together with the administration costs) is smaller than the gain resulting from the corresponding reduction of capital, the reinsurance contract is desirable. In fact, due to the ongoing shift towards risk-based regulation, the notion of capital and its management becomes a central issue for insurance companies, and reinsurance then should be understood as a tool in this context. This corresponds to an important *finance function* of reinsurance as a substitute for capital, freeing up capacity.

- **Increasing underwriting capacity**
  In the presence of a reinsurance contract, only a certain part of the risk is assumed by the insurer, and hence under otherwise identical conditions an insurance company can afford to underwrite more and larger policies (see Chapter 2 for details), which may be desirable for various reasons, including market share targets, testing and entering of new markets, gaining (data) experience in certain business lines or regions etc. It also can lead to enhanced liquidity.

- **Accessing benefits from larger diversification pools**
  Often the primary insurers’ business model is restricted to a local area, in which case attempts to look on their own for diversification possibilities outside of that market for the more dangerous part of the risks would be very costly and inefficient. Reinsurers, on the other hand, typically act on an international level and therefore have more possibilities for diversifying such risks. Consequently the amount of capital needed to safeguard these risks in the portfolio can be considerably lower for a reinsurer and so the risk transfer produces economic gain through attractive reinsurance premiums.

We mention a few further motivations:

- **Reducing tax payments**
  Equalization reserves (i.e., reserves for volatility of claims and their arrivals over longer time periods, which is, for instance, particularly important for catastrophe risks) of insurance companies are taxed in most legislations. If such reserves are paid to a reinsurance company in the form of a reinsurance premium (or, alternatively, into a respectively created captive structure, cf. Section 10.2), then the taxation pattern becomes more favorable, as for reinsurers and captives (often located in tax-favorable countries) different tax rules may apply.

- **Other legal issues**
  Reinsurance can be a helpful tool to resolve legal constraints such as regulatory compliance. For instance, if an insurance company does not have a formal license to write business in a certain country, a solution can be to find a local insurer with such a license and act as a reinsurer for this local company.

- **Financial solutions**
  The reinsurer can serve as a facilitator for financial solutions. Examples include reducing (expected) financial distress costs by providing run-off solutions (cf. Section 10.2) and portfolio transfers to other companies or the capital markets as well as setting up securitization transactions like issuing bonds.
• **Protection against model risk**
  Insurance activities are designed on the basis of stochastic models for the underlying risks. For the aggregate performance, both the understanding of the marginal risks as well as of the dependence between them is important.\(^1\) However, every model is an imperfect description of reality, and the less experience and data one has, the higher the uncertainty about whether the model underlying the business plan is appropriate. Reinsurance is a way to mitigate model inadequacy (e.g., concerning the tails of the risks or their dependence).

• **Support in risk assessment, pricing, and management**
  In certain situations an insurance company does not have enough data points or manpower available to analyze the risks (in particular their tails), and passing on those risks to an entity with respective experience is a natural procedure, which is often much cheaper than dealing with such risks by other means. This also includes business expansions to new regions or business lines, in which the reinsurer may already have experience from earlier activities. In fact, reinsurance contracts often have a certain consultancy component, as the reinsurer may share its expertise and data on the respective risks with the cedent.

On the society level, reinsurance allows insurers to write more business, which makes insurance more broadly available and affordable. This can foster economic growth and increase stability at large. Reinsurance enables risks to be insured that otherwise would not be insurable, and assigning premiums to (i.e., quantifying) risks can also provide incentives for more risk-adequate behavior and possibly risk prevention.

For all these reasons, reinsurance serves as a tool to increase the efficiency of the marketplace. When designing reinsurance contracts, all these aspects will play some role. The goal of this book is to focus on the actuarial elements involved in the process as well as the statistical challenges that appear in this context.

### 1.3 Reinsurance Data

As for primary insurance, in the reinsurance business one will be interested in the statistical analysis of claim information for different types of business lines (such as car liability insurance or fire insurance), where one can obtain claim information on the individual claim level. Due to the nature of the reinsurance contract, there are, however, additional challenges with respect to the type of claim data.

Consider, for instance, the case of non-proportional reinsurance where the reinsurer will pay (parts of) the excesses over some threshold, say \(M\). The ceding company then does not need to provide all claim information to the reinsurer. For example, information may be provided on those events only for which the incurred claim amount \(I\) (i.e., the estimate of the amount of outstanding liabilities) is larger than a certain percentage of \(M\). Then, as long as \(I\) stays below that reporting threshold during the development process, the claim will not be known to the reinsurer and hence the

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\(^1\) Here, dependence can be causal (e.g., the occurrence of a claim triggers another claim) or due to common risk drivers. An appropriate modelling of dependence can be a considerable challenge, particularly when only few data points are available and the number of dependent risks is high.
incurred value is left truncated in such a case. For some lines of business, development times can be quite large (up to several decades) so that, at the time of evaluation, the cumulative payments are still a lower bound for the ultimate claim amount. The data then are censored. In practice, companies use claim development methods to forecast the ultimate claim amounts. Of course these also yield uncertain information, which hampers the statistical analysis. Hence, in reinsurance we face incomplete information, due to incurred but not reported (IBNR) and reported but not settled (RBNS) claims (the latter are also frequently called open case estimates). This is illustrated in Figure 1.1.

The development of claims progresses with calendar time, and when the notification does not arrive before the present evaluation time (e.g., because the incurred value is too low), the data are left truncated (IBNR). If the claim is notified to the reinsurer but not completely settled before the evaluation time, the information is censored (RBNS).

Throughout the development of the book we will make use of the real data examples described in the following sections to illustrate the practical statistical side of implementing reinsurance treaties.

### 1.3.1 Case Study I: Motor Liability Data

We here present a data set on motor third-party liability (MTPL) data, gathering information about two direct insurance companies operating in the EU, named A and B hereafter. The data come from an observation period between 1991 and 2010, with evaluation date at January 1, 2011. All amounts are corrected in order to reflect costs in calendar year 2011, with inflation and super-inflation taken into account. For every claim, the payments in a given year were aggregated in a single observation. For Company A 16 years and for Company B 20 years of data are available. In the subsequent chapters we will analyze the two data sets separately: the statistical analysis of the losses will show different characteristics for these companies. For Company A, the exact
occurrence dates of the claims are also available, so the analysis of the counting process can be performed more accurately for that claim data set.

Per development year and per claim the *aggregate payment* and *incurred loss* are given. The incurred loss at a given moment in time is the sum of the already paid amount and a reserve for further payments, proposed by company experts at that moment. A claim enters the database from the moment the incurred value exceeds the reporting threshold as given in Figure 1.2. Once a claim has been reported, it stays in the data set even if the associated incurred loss falls below the reporting threshold at some point later. When estimating the loss experienced by the reinsurer, one needs to model the *reporting delay* between the accident time and the year where the claim was first reported to the reinsurer, that is, when the incurred loss $I_t$ first exceeds the reporting threshold. Indeed, claims that have occurred close to the evaluation time at the end of 2010 can still be IBNR to the reinsurer. In Figure 1.3 the histogram of the reporting delays is given. Given that the accident dates were only reported for Company A, we restrict the plot to this data set. The delay time is then obtained from the difference between the reporting year and accident date, rounded off in years, using the reporting threshold of the particular accident year (see Figure 1.2).

For Company A one has 849 claims of which 340 are completely developed, while the sample size for Company B is 560 of which 225 are fully developed. In Figure 1.4 we show the development of four selected claims. The cumulative payments (aggregated on a yearly basis) are indicated by a full line, while the incurred values are given by dashed lines. When payments and incurred meet, the claim is closed. The characteristics of the four depicted claims are given in Table 1.2.

Note that the information concerning the loss values and development periods is right censored since for the claims which are not fully developed at the end of 2010, the loss as well as the development time at the end of 2010 are only lower bounds for their final value. In Table 1.3 the observed numbers of claims per accident year and per

![Figure 1.2 Indexed reporting thresholds of Companies A and B.](image-url)
development time up to 2010 (in years, $DY_{2010}$) are given for Company A. Clearly the amount of censoring increases with increasing accident year.

In Figures 1.5 and 1.6, time plots of the incurred loss data of Company A and Company B, respectively, are given as a function of accident year.
Table 1.2 Company A: characteristics of the claims from Figure 1.4

<table>
<thead>
<tr>
<th>Claim</th>
<th>Reporting year</th>
<th>Closing year</th>
<th>Development time (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top left</td>
<td>1995</td>
<td>2005</td>
<td>10</td>
</tr>
<tr>
<td>Top right</td>
<td>1996</td>
<td>-</td>
<td>≥ 15</td>
</tr>
<tr>
<td>Bottom left</td>
<td>1997</td>
<td>2002</td>
<td>5</td>
</tr>
<tr>
<td>Bottom right</td>
<td>1998</td>
<td>-</td>
<td>≥ 13</td>
</tr>
</tbody>
</table>

Table 1.3 Company A: observed number of claims per accident year and per number of development years in 2010 \((DY_{2010})\)

<table>
<thead>
<tr>
<th></th>
<th>(DY_{2010})</th>
<th>Nr. censored</th>
<th>Prop. Total non-censored</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1995</td>
<td>0 0 2 3 4 8 1 1 2 2 2 1 1 2 2 11</td>
<td>44</td>
<td>0.75</td>
</tr>
<tr>
<td>1996</td>
<td>0 1 0 0 4 1 6 2 3 4 3 4 2 3 0</td>
<td>14</td>
<td>0.7</td>
</tr>
<tr>
<td>1997</td>
<td>0 1 3 3 1 3 6 6 4 3 2 2 5 1</td>
<td>10</td>
<td>0.8</td>
</tr>
<tr>
<td>1998</td>
<td>0 0 0 6 3 8 8 7 5 4 4 4 0</td>
<td>21</td>
<td>0.7</td>
</tr>
<tr>
<td>1999</td>
<td>0 0 0 3 2 4 3 2 1 4 4 3</td>
<td>17</td>
<td>0.6</td>
</tr>
<tr>
<td>2000</td>
<td>0 1 1 1 4 6 8 6 2 7 2</td>
<td>19</td>
<td>0.67</td>
</tr>
<tr>
<td>2001</td>
<td>0 0 1 2 5 4 6 4 9 3</td>
<td>23</td>
<td>0.6</td>
</tr>
<tr>
<td>2002</td>
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</tr>
<tr>
<td>2003</td>
<td>0 2 2 5 7 6 5 1</td>
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<td>0.42</td>
</tr>
<tr>
<td>2004</td>
<td>0 0 1 6 8 5 4</td>
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</tr>
<tr>
<td>2005</td>
<td>0 0 1 2 6 1</td>
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<td>2006</td>
<td>0 0 2 2 2</td>
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<td>0.12</td>
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<tr>
<td>2007</td>
<td>0 0 0 4</td>
<td>65</td>
<td>0.06</td>
</tr>
<tr>
<td>2008</td>
<td>0 0 0</td>
<td>69</td>
<td>0.00</td>
</tr>
<tr>
<td>2009</td>
<td>0 0</td>
<td>55</td>
<td>0.00</td>
</tr>
<tr>
<td>2010</td>
<td>0</td>
<td>10</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Censored: 10 55 69 65 43 46 41 38 27 23 19 17 21 10 14 11 509
Total: 10 60 83 104 94 93 91 69 54 50 36 32 29 15 16 13 849 0.60

The classical statistical procedure to estimate the distribution of right censored random variables is given by the Kaplan–Meier estimator of the distribution function. This estimator is discussed in more detail in Chapter 4. Note from these plots that about half of the claims are expected to demand a development period of at least 10 years.
Alongside the aggregate payment and the incurred loss, when analysing the risk many companies compute *ultimate loss amounts* for claims that are still under development. These ultimates are statistical estimates of the final loss. The ultimate value of course equals the final aggregate payment in case the claim is closed during the period of study. In practice, the ultimate estimates for non-closed claims are often primarily based on chain ladder development factors based on paid and incurred loss triangles (e.g., see Wüthrich and Merz [797] and Radtke et al. [638]), but then applied on the individual loss data, see also Drieskens et al. [308]. In Figure 1.7 scatterplots of the ultimate against the incurred losses for the data of the two companies are given. Note that the regression fits on these scatterplots for the claims that are still open at the end of the observation period indicate a linear relation between ultimate and incurred values with a negligible intercept: ultimate = $a \times$ incurred, for some $a > 1$.

Finally, in Figure 1.8 we plot the daily cluster sizes for the claims of Company A. Up to three claims per day were observed.
1.3.2 Case Study II: Dutch Fire Insurance Data

We will use claim data from the Dutch fire insurance market between 2000 and 2015, provided by a reinsurance company. The date for every fire is known, together with the type of building and regional information. Here the development times are short. Figure 1.9 depicts the logarithm of the claim sizes as a function of time as well as the daily cluster sizes (one sees up to five claims per day). The loss data are indexed to 2015. The reporting threshold equals a value equivalent to 2 million Dutch guilders up to 2002, after which 1 million Euros is used.

1.3.3 Case Study III: Austrian Storm Claim Data

Sometimes individual claim data are not available, and instead claims aggregated over time or regions have to be used. As an illustration, we will use data from historical storm losses of residential buildings in Austria in the period 1998–2009, aggregated over two-digit postcode regions. This data set contains 36 storm events and was provided by the
and the province of Upper Austria. Here one studies the distribution of the losses of the 36 storms in the data set as a function of this wind index \( W \) for Vienna and the province of Upper Austria. Here one studies the distribution of the loss data as a function of \( W \) in a regression setting.

### 1.3.4 Case Study IV: European Flood Risk Data

Floods rank amongst the most wide-reaching natural hazards. Losses from floods show an increasing trend which (to a considerable extent) is attributable to socio-economic factors, including population growth, economic development and construction
activities in vulnerable areas. In Pretthnaler et al. [632] (indexed) flood loss data across Europe (provided by Munich Re NatCatSERVICE, 2014) were transformed into losses expressed as a percentage of building stock value, and then used to determine loss quantiles as required for flood risk management. Figure 1.11 depicts the respective aggregate annual losses for the period 1980–2013 for Germany and the UK.

1.3.5 Case Study V: Groningen Earthquakes

Next to loss amount data, reinsurers also need to analyze the physical phenomena causing damage. A classical example is earthquake risk. We discuss the Groningen earthquakes caused by gas extraction. The Groningen field is the largest gas field of Western Europe, with 2800 billion cubic metres available and 800 billion cubic metres left. The pressure inside the gas layers decreases due to the extraction, and the layers on top collapse. This collapse does not happen homogeneously, which causes the earthquakes. Hundreds of earthquakes have been detected since 1986 with magnitudes between 2 and 3 on the Richter scale, and 14 larger than 3 (Figure 1.12). The damage to houses and public buildings was substantial, with many buildings needing reinforcements. The largest observed magnitude was 3.6 (Huizinge, August 2012). In this context, the estimation of the maximum possible magnitude is the main goal. Depending on the research team, maximum magnitudes between 3.9 and 5 were predicted, see for instance Bourne et al. [157].

1.3.6 Case Study VI: Danish Fire Insurance Data

It is quite common to combine reinsurance forms across various lines of business (LoB), so modelling the dependence of the different LoB is important. To illustrate the appropriate multivariate models and statistical methodology, we will use the Danish fire insurance data set containing information on 2167 fire losses over the period
1980–1990. The data have been adjusted for inflation to reflect 1985 values and are expressed in millions of Danish kroner. The total loss amount $X_i$ of the $i$th claim is subdivided into damage to building ($X_{i,1}$), damage to content ($X_{i,2}$) (e.g., furniture and personal property) and loss of profits ($X_{i,3}$). A claim is only registered if the total loss exceeds 1 million kroner, that is, $X_{i,1} + X_{i,2} + X_{i,3} \geq 1$. This data set was collected at the Copenhagen Reinsurance Company and can nowadays be seen as a folklore example as it has been studied extensively over the years in the academic literature (e.g., see Embrechts et al. [329]). In Figure 1.13 a scatterplot matrix is given for the log-transformed data. On the diagonal, histograms of the logarithm of the marginal losses are given. Note that several claims exhibit losses in only one or two of the components (for only 517 claims there is a loss in each of the three components).
Figure 1.10 Normalized loss data against wind index $W^*$ for Vienna (top) and Upper Austria (bottom); original scale (left) and log-scale (right).
Figure 1.11 Flood risk: aggregate annual losses (in log scale) by percentage of building value for Germany and the UK.

Figure 1.12 Induced (dark points) earthquakes in the northern part of the Netherlands with magnitudes larger than 1.5.
The occurrence dates are also given and hence simultaneous occurrences of claims for the three components can be observed. Figure 1.14 illustrates the occurrences and the cluster sizes when all portfolio components were affected.

1.4 Notes and Bibliography

There are a number of classical textbooks available which provide a general introduction to reinsurance, for example Carter [185], Gerathewohl [382], Grossmann [409], Strain [710], Gastel [375], Schwepecke [686], and Walhin [765]. A number of articles in Teugels and Sundt [741] also deal with the topic. For the role of reinsurance in risk management, see D’Outreville [598]. More recent and shorter overviews can be found in Liebwein [543], Albrecher [13], Outreville [599], Bernard [120], and
Deelstra et al. [267]. Furthermore, a number of basic textbooks on risk theory contain sections on reinsurance. Examples include (in alphabetic order) Beard et al. [93], Beekman [94], Borch [149], Bowers et al. [157], Bühlmann [165], Cramér [234], Daykin et al. [254], De Vylder [263], Gerber [383], Goovaerts et al. [402], Heilmann [435], Kaas et al. [475], Klugman et al. [491], Lundberg [551], Mack [555], Rolski et al. [652], Schmidt [677], Seal [690], Straub [711], and Sundt [716]. For a discussion on the challenges and opportunities of reinsurance as an international business, see Göbel [392]. A recent overview from a practical perspective can be found in Swiss Re [725]. The increasing role of the notion of capital and capital management in running insurance and reinsurance companies, which can be seen as an ongoing change of paradigm in the insurance industry, is highlighted in Dacorogna [243], see also [689] and Krvavych [510, 512].

The number of 200 reinsurance companies can be compared with the more than 10,000 primary insurance companies in the market today (using economic arguments, Powers and Shubik [627] in fact claim that the “optimal” number of reinsurers in the market is connected to the number of primary insurers by a square-root rule).

Historically, the first documented reinsurance contract dates back to 1370, when the cargo of a ship sailing from Genoa to Sluis (near Bruges in Flanders) was reinsured by the direct insurer for the more dangerous part of the journey from Cadiz to Sluis (interestingly, the contract did not state the premium, which most likely was done to avoid usury discussions). The first reinsurance company was founded much later, in 1846, in Cologne after the big fire of Hamburg in 1842, and the first retrocession contract seems to date back to 1854, involving Le Globe Compagnie d’Assurance contre L’incendie. Soon the (nowadays) major European reinsurance companies were founded, and the American Life Reinsurers followed in the early 20th century. For a detailed account of the history of reinsurance, see Kopf [496], Holland [451], and Borscheid et al. [154].