Introduction

1.1 A Short History of Antennas

Work on antennas started many years ago. The first well-known satisfactory antenna experiment was conducted by the German physicist Heinrich Rudolf Hertz (1857–1894), pictured in Figure 1.1. The SI (International Standard) frequency unit, the Hertz, is named after him. In 1887 he built a system, as shown in Figure 1.2, to produce and detect radio waves. The original intention of his experiment was to demonstrate the existence of electromagnetic radiation.

In the transmitter, a variable voltage source was connected to a dipole (a pair of one-meter wires) with two conducting balls (capacity spheres) at the ends. The gap between the balls could be adjusted for circuit resonance as well as for the generation of sparks. When the voltage was increased to a certain value, a spark or break-down discharge was produced. The receiver was a simple loop with two identical conducting balls. The gap between the balls was carefully tuned to receive the spark effectively. He placed the apparatus in a darkened box in order to see the spark clearly. In his experiment, when a spark was generated at the transmitter, he also observed a spark at the receiver gap at almost the same time. This proved that the information from location A (the transmitter) was transmitted to location B (the receiver) in a wireless manner – by electromagnetic waves.

The information in Hertz’s experiment was actually in binary digital form, by tuning the spark on and off. This could be considered the very first digital wireless system, which consisted of two of the best-known antennas: the dipole and the loop. For this reason, the dipole antenna is also called the Hertz (dipole) antenna.

Whilst Heinrich Hertz conducted his experiments in a laboratory and did not quite know what radio waves might be used for in practice, Guglielmo Marconi (1874–1937, pictured in Figure 1.3), an Italian inventor, developed and commercialized wireless technology by introducing a radiotelegraph system, which served as the foundation for the establishment of numerous affiliated companies worldwide. His most famous experiment was the transatlantic transmission from Poldhu, UK to St Johns, Newfoundland in Canada in 1901, employing untuned systems. He shared the 1909 Nobel Prize for Physics with Karl Ferdinand Braun ‘in recognition of their contributions to the development of wireless telegraphy’. Monopole antennas (near quarter-wavelength) were widely used in Marconi’s experiments; thus vertical monopole antennas are also called Marconi antennas.
During World War II, battles were won by the side that was first to spot enemy aeroplanes, ships or submarines. To give the Allies an edge, British and American scientists developed radar technology to ‘see’ targets from hundreds of miles away, even at night. The research resulted in the rapid development of high-frequency radar antennas, which were no longer just wire-type antennas. Some aperture-type antennas, such as reflector and horn antennas, were developed, an example is shown in Figure 1.4.

![Figure 1.1 Heinrich Rudolf Hertz](image1)

![Figure 1.2 1887 experimental set-up of Hertz’s apparatus](image2)
Introduction

Figure 1.3  Guglielmo Marconi

Broadband, circularly polarized antennas, as well as many other types, were subsequently developed for various applications. Since an antenna is an essential device for any radio broadcasting, communication or radar system, there has always been a requirement for new and better antennas to suit existing and emerging applications.

More recently, one of the main challenges for antennas has been how to make them broadband and small enough in size for wireless mobile communications systems. For example, WiMAX (worldwide interoperability for microwave access) is one of the latest systems aimed at providing high-speed wireless data communications (>10 Mb/s) over long distances from point-to-point links to full mobile cellular-type access over a wide frequency band. The original WiMAX standard in IEEE 802.16 specified 10 to 66 GHz as the WiMAX band; IEEE 802.16a

Figure 1.4  World War II radar (Reproduced by permission of CSIRO Australia Telescope National Facility)
was updated in 2004 to 802.16-2004 and added 2 to 11 GHz as an additional frequency range. The frequency bandwidth is extremely wide although the most likely frequency bands to be used initially will be around 3.5 GHz, 2.3/2.5 GHz and 5 GHz.

The UWB (ultra-wide band) wireless system is another example of recent broadband radio communication systems. The allocated frequency band is from 3.1 to 10.6 GHz. The beauty of the UWB system is that the spectrum, which is normally very expensive, can be used free of charge but the power spectrum density is limited to $-41.3 \text{ dBm/MHz}$. Thus, it is only suitable for short-distance applications. The antenna design for these systems faces many challenging issues.

The role of antennas is becoming increasingly important. In some systems, the antenna is now no longer just a simple transmitting/receiving device, but a device which is integrated with other parts of the system to achieve better performance. For example, the MIMO (multiple-in, multiple-out) antenna system has recently been introduced as an effective means to combat multipath effects in the radio propagation channel and increase the channel capacity, where several coordinated antennas are required.

Things have been changing quickly in the wireless world. But one thing has never changed since the very first antenna was made: the antenna is a practical engineering subject. It will remain an engineering subject. Once an antenna is designed and made, it must be tested. How well it works is not just determined by the antenna itself, it also depends on the other parts of the system and the environment. The standalone antenna performance can be very different from that of an installed antenna. For example, when a mobile phone antenna is designed, we must take the case, other parts of the phone and even our hands into account to ensure that it will work well in the real world. The antenna is an essential device of a radio system, but not an isolated device! This makes it an interesting and challenging subject.

### 1.2 Radio Systems and Antennas

A radio system is generally considered to be an electronic system which employs radio waves, a type of electromagnetic wave up to GHz frequencies. An antenna, as an essential part of a radio system, is defined as a device which can radiate and receive electromagnetic energy in an efficient and desired manner. It is normally made of metal, but other materials may also be used. For example, ceramic materials have been employed to make dielectric resonator antennas (DRAs). There are many things in our lives, such as power leads, that can radiate and receive electromagnetic energy but they cannot be viewed as antennas because the electromagnetic energy is not transmitted or received in an efficient and desired manner, and because they are not a part of a radio system.

Since radio systems possess some unique and attractive advantages over wired systems, numerous radio systems have been developed. TV, radar and mobile radio communication systems are just some examples. The advantages include:

- **mobility**: this is essential for mobile communications;
- **good coverage**: the radiation from an antenna can cover a very large area, which is good for TV and radio broadcasting and mobile communications;
- **low pathloss**: this is frequency dependent. Since the loss of a transmission line is an exponential function of the distance (the loss in dB = distance × per unit loss in dB) and the loss
of a radio wave is proportional to the distance squared (the loss in dB = 20 log_{10} (distance)), the pathloss of radio waves can be much smaller than that of a cable link. For example, assume that the loss is 10 dB for both a transmission line and a radio wave over 100 m; if the distance is now increased to 1000 m, the loss for the transmission line becomes 10 × 10 = 100 dB but the loss for the radio link is just 10 + 20 = 30 dB! This makes the radio link extremely attractive for long-distance communication. It should be pointed out that optical fibers are also employed for long-distance communications since they are of very low loss and ultra-wide bandwidth.

Figure 1.5 illustrates a typical radio communication system. The source information is normally modulated and amplified in the transmitter and then passed on to the transmit antenna via a transmission line, which has a typical characteristic impedance (explained in the next chapter) of 50 ohms. The antenna radiates the information in the form of an electromagnetic wave in an efficient and desired manner to the destination, where the information is picked up by the receive antenna and passed on to the receiver via another transmission line. The signal is demodulated and the original message is then recovered at the receiver.

Thus, the antenna is actually a transformer that transforms electrical signals (voltages and currents from a transmission line) into electromagnetic waves (electric and magnetic fields), or vice versa. For example, a satellite dish antenna receives the radio wave from a satellite and transforms it into electrical signals which are output to a cable to be further processed. Our eyes may be viewed as another example of antennas. In this case, the wave is not a radio wave but an optical wave, another form of electromagnetic wave which has much higher frequencies.

Now it is clear that the antenna is actually a transformer of voltage/current to electric/magnetic fields, it can also be considered a bridge to link the radio wave and transmission line. An antenna system is defined as the combination of the antenna and its feed line. As an antenna is usually connected to a transmission line, how to best make this connection is a subject of interest, since the signal from the feed line should be radiated into the space in an efficient and desired way. Transmission lines and radio waves are, in fact, two different subjects in engineering. To understand antenna theory, one has to understand transmission lines and radio waves, which will be discussed in detail in Chapters 2 and 3 respectively.

In some applications where space is very limited (such as hand-portables and aircraft), it is desirable to integrate the antenna and its feed line. In other applications (such as the reception of TV broadcasting), the antenna is far away from the receiver and a long transmission line has to be used.

Unlike other devices in a radio system (such as filters and amplifiers), the antenna is a very special device; it deals with electrical signals (voltages and currents) as well as electromagnetic waves (electric fields and magnetic fields), making antenna design an interesting and difficult
subject. For different applications, the requirements on the antenna may be very different, even for the same frequency band.

In conclusion, the subject of antennas is about how to design a suitable device which will be well matched with its feed line and radiate/receive the radio waves in an efficient and desired manner.

1.3 Necessary Mathematics

To understand antenna theory thoroughly requires a considerable amount of mathematics. However, the intention of this book is to provide the reader with a solid foundation in antenna theory and apply the theory to practical antenna design. Here we are just going to introduce and review the essential and important mathematics required for this book. More in-depth study materials can be obtained from other references [1, 2].

1.3.1 Complex Numbers

In mathematics, a complex number, $Z$, consists of real and imaginary parts, that is

$$Z = R + jX$$

(1.1)

where $R$ is called the real part of the complex number $Z$, i.e. Re($Z$), and $X$ is defined as the imaginary part of $Z$, i.e. Im($Z$). Both $R$ and $X$ are real numbers and $j$ (not the traditional notation $i$ in mathematics to avoid confusion with a changing current in electrical engineering) is the imaginary unit and is defined by

$$j = \sqrt{-1}$$

(1.2)

Thus

$$j^2 = -1$$

(1.3)

Geometrically, a complex number can be presented in a two-dimensional plane where the imaginary part is found on the vertical axis whilst the real part is presented by the horizontal axis, as shown in Figure 1.6.

In this model, multiplication by $-1$ corresponds to a rotation of 180 degrees about the origin. Multiplication by $j$ corresponds to a 90-degree rotation anti-clockwise, and the equation $j^2 = -1$ is interpreted as saying that if we apply two 90-degree rotations about the origin, the net result is a single 180-degree rotation. Note that a 90-degree rotation clockwise also satisfies this interpretation.

Another representation of a complex number $Z$ uses the amplitude and phase form:

$$Z = Ae^{j\phi}$$

(1.4)
\[ Z = R + jX = Ae^{j\varphi}, \]
\[ A = \sqrt{R^2 + X^2}, \quad \varphi = \tan^{-1}(X/R) \]
\[ R = A \cos \varphi, \quad X = A \sin \varphi \]  

1.3.2 Vectors and Vector Operation

A scalar is a one-dimensional quantity which has magnitude only, whereas a complex number is a two-dimensional quantity. A vector can be viewed as a three-dimensional (3D) quantity, and a special one – it has both a magnitude and a direction. For example, force and velocity are vectors (whereas speed is a scalar). A position in space is a 3D quantity, but it does not have a direction, thus it is not a vector. Figure 1.7 is an illustration of vector \( A \) in Cartesian coordinates.
coordinates. It has three orthogonal components \((A_x, A_y, A_z)\) along the \(x, y\) and \(z\) directions, respectively. To distinguish vectors from scalars, the letter representing the vector is printed in bold, for example \(\mathbf{A}\) or \(\mathbf{a}\), and a unit vector is printed in bold with a hat over the letter, for example \(\hat{x}\) or \(\hat{n}\).

The magnitude of vector \(\mathbf{A}\) is given by

\[
|\mathbf{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2} \tag{1.6}
\]

Now let us consider two vectors \(\mathbf{A}\) and \(\mathbf{B}\):

\[
\mathbf{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z}
\]
\[
\mathbf{B} = B_x \hat{x} + B_y \hat{y} + B_z \hat{z}
\]

The addition and subtraction of vectors can be expressed as

\[
\mathbf{A} + \mathbf{B} = (A_x + B_x) \hat{x} + (A_y + B_y) \hat{y} + (A_z + B_z) \hat{z}
\]
\[
\mathbf{A} - \mathbf{B} = (A_x - B_x) \hat{x} + (A_y - B_y) \hat{y} + (A_z - B_z) \hat{z} \tag{1.7}
\]

Obviously, the addition obeys the **commutative law**, that is \(\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}\).

Figure 1.8 shows what the addition and subtraction mean geometrically. A vector may be multiplied or divided by a scalar. The magnitude changes but its direction remains the same. However, the multiplication of two vectors is complicated. There are two types of multiplication: the dot product and the cross product.

The **dot product** of two vectors is defined as

\[
\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}| |\mathbf{B}| \cos \theta = A_x B_x + A_y B_y + A_z B_z \tag{1.8}
\]

where \(\theta\) is the angle between vector \(\mathbf{A}\) and vector \(\mathbf{B}\) and \(\cos \theta\) is also called the direction cosine. The dot \(\cdot\) between \(\mathbf{A}\) and \(\mathbf{B}\) indicates the dot product, which results in a scalar; thus, it is also called a **scalar product**. If the angle \(\theta\) is zero, \(\mathbf{A}\) and \(\mathbf{B}\) are in parallel – the dot product is
Right-Hand Rule

Figure 1.9 The cross product of vectors $\mathbf{A}$ and $\mathbf{B}$

maximized – whereas for an angle of 90 degrees, i.e. when $\mathbf{A}$ and $\mathbf{B}$ are orthogonal, the dot product is zero.

It is worth noting that the dot product obeys the **commutative law**, that is, $\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$.

The cross product of two vectors is defined as

$$
\mathbf{A} \times \mathbf{B} = \hat{n} |\mathbf{A}| |\mathbf{B}| \sin \theta = \mathbf{C} \\
= \hat{x}(A_y B_z - A_z B_y) + \hat{y}(A_z B_x - A_x B_z) + \hat{z}(A_x B_y - A_y B_x)
$$

(1.9)

where $\hat{n}$ is a unit vector normal to the plane containing $\mathbf{A}$ and $\mathbf{B}$. The cross $\times$ between $\mathbf{A}$ and $\mathbf{B}$ indicates the cross product, which results in a vector $\mathbf{C}$; thus, it is also called a **vector product**. The vector $\mathbf{C}$ is orthogonal to both $\mathbf{A}$ and $\mathbf{B}$, and the direction of $\mathbf{C}$ follows a so-called right-hand rule, as shown in Figure 1.9. If the angle $\theta$ is zero or 180 degrees, that is, $\mathbf{A}$ and $\mathbf{B}$ are in parallel, the cross product is zero; whereas for an angle of 90 degrees, i.e. $\mathbf{A}$ and $\mathbf{B}$ are orthogonal, the cross product of these two vectors reaches a maximum. Unlike the dot product, the cross product does not obey the commutative law.

The cross product may be expressed in determinant form as follows, which is the same as Equation (1.9) but may be easier for some people to memorize:

$$
\mathbf{A} \times \mathbf{B} =
\begin{vmatrix}
\hat{x} & \hat{y} & \hat{z} \\
A_x & A_y & A_z \\
B_x & B_y & B_z
\end{vmatrix}
$$

(1.10)

Another important thing about vectors is that any vector can be decomposed into three orthogonal components (such as $x$, $y$, and $z$ components) in 3D or two orthogonal components in a 2D plane.

**Example 1.1: Vector operation.** Given vectors $\mathbf{A} = 10\hat{x} + 5\hat{y} + 1\hat{z}$ and $\mathbf{B} = 2\hat{y}$, find:

$\mathbf{A} + \mathbf{B}$; $\mathbf{A} - \mathbf{B}$; $\mathbf{A} \cdot \mathbf{B}$; and $\mathbf{A} \times \mathbf{B}$
Solution:
\[ A + B = 10\hat{x} + (5 + 2)\hat{y} + 1\hat{z} = 10\hat{x} + 7\hat{y} + 1\hat{z}; \]
\[ A - B = 10\hat{x} + (5 - 2)\hat{y} + 1\hat{z} = 10\hat{x} + 3\hat{y} + 1\hat{z}; \]
\[ A \cdot B = 0 + (5 \times 2) + 0 = 10; \]
\[ A \times B = 10 \times 2\hat{z} + 1 \times 2\hat{x} = 20\hat{z} + 2\hat{x}. \]

1.3.3 Coordinates

In addition to the well-known Cartesian coordinates, spherical coordinates \((r, \theta, \phi)\), as shown in Figure 1.10, will also be used frequently throughout this book. These two coordinate systems have the following relations:

\[
\begin{align*}
    x &= r \sin \theta \cos \phi \\
    y &= r \sin \theta \sin \phi \\
    z &= r \cos \theta
\end{align*}
\]

and

\[
\begin{align*}
    r &= \sqrt{x^2 + y^2 + z^2} \\
    \theta &= \cos^{-1} \frac{z}{\sqrt{x^2 + y^2 + z^2}}; \quad 0 \leq \theta \leq \pi \\
    \phi &= \tan^{-1} \frac{y}{x}; \quad 0 \leq \phi \leq 2\pi
\end{align*}
\]

Figure 1.10 Cartesian and spherical coordinates
Introduction

The dot products of unit vectors in these two coordinate systems are:

\[
\begin{align*}
\hat{x} \cdot \hat{r} &= \sin \theta \cos \phi; \\
\hat{y} \cdot \hat{r} &= \sin \theta \sin \phi; \\
\hat{z} \cdot \hat{r} &= \cos \theta
\end{align*}
\]

\[
\begin{align*}
\hat{x} \cdot \hat{\theta} &= \cos \theta \cos \phi; \\
\hat{y} \cdot \hat{\theta} &= \cos \theta \sin \phi; \\
\hat{z} \cdot \hat{\theta} &= -\sin \theta
\end{align*}
\]

\[
\hat{x} \cdot \hat{\phi} = -\sin \phi; \quad \hat{y} \cdot \hat{\phi} = \cos \phi; \quad \hat{z} \cdot \hat{\phi} = 0
\]

Thus, we can express a quantity in one coordinate system using the known parameters in the other coordinate system. For example, if \( A_r, A_\theta, A_\phi \) are known, we can find

\[
A_x = A \cdot \hat{x} = A_r \sin \theta \cos \phi + A_\theta \cos \theta \cos \phi - A_\phi \sin \phi
\]

1.4 Basics of Electromagnetics

Now let us use basic mathematics to deal with antennas or, more precisely, electromagnetic (EM) problems in this section.

EM waves cover the whole spectrum; radio waves and optical waves are just two examples of EM waves. We can see light but we cannot see radio waves. The whole spectrum is divided into many frequency bands. Some radio frequency bands are listed in Table 1.1.

Although the whole spectrum is infinite, the useful spectrum is limited and some frequency bands, such as the UHF, are already very congested. Normally, significant license fees have to be paid to use the spectrum, although there are some license-free bands: the most well-known ones are the industrial, science and medical (ISM) bands. The 433 MHz and 2.45 GHz are just two examples. Cable operators do not need to pay the spectrum license fees, but they have to pay other fees for things such as digging out the roads to bury the cables.

The wave velocity, \( v \), is linked to the frequency, \( f \), and wavelength, \( \lambda \), by this simple equation:

\[
v = \frac{\lambda}{f}
\]

It is well known that the speed of light (an EM wave) is about \( 3 \times 10^8 \) m/s in free space. The higher the frequency, the shorter the wavelength. An illustration of how the frequency is linked

<table>
<thead>
<tr>
<th>Table 1.1</th>
<th>EM spectrum and applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>Band</td>
</tr>
<tr>
<td>3–30 kHz</td>
<td>VLF</td>
</tr>
<tr>
<td>30–300 kHz</td>
<td>LF</td>
</tr>
<tr>
<td>0.3–3 MHz</td>
<td>MF</td>
</tr>
<tr>
<td>3–30 MHz</td>
<td>HF</td>
</tr>
<tr>
<td>30–300 MHz</td>
<td>VHF</td>
</tr>
<tr>
<td>0.3–3 GHz</td>
<td>UHF</td>
</tr>
<tr>
<td>3–30 GHz</td>
<td>SHF</td>
</tr>
<tr>
<td>30–300 GHz</td>
<td>EHF</td>
</tr>
<tr>
<td>0.3–3 THz</td>
<td>THz</td>
</tr>
</tbody>
</table>
to the wavelength is given in Figure 1.11, where both the frequency and wavelength are plotted on a logarithmic scale. The advantage of doing this is that we can see clearly how the function is changed, even over a very large scale.

Logarithmic scales are widely used in RF (radio frequency) engineering and the antennas community since the signals we are dealing with change significantly (over 1000 times in many cases) in terms of the magnitude. The signal power is normally expressed in dB and is defined as

\[ P(dBW) = 10 \log_{10} \frac{P(W)}{1W} \quad P(dBm) = 10 \log_{10} \frac{P(W)}{1mW} \]  

Thus, 100 watts is 20 dBW, just expressed as 20 dB in most cases. 1 W is 0 dB or 30 dBm and 0.5 W is −3 dB or 27 dBm. Based on this definition, we can also express other parameters in dB. For example, since the power is linked to voltage \( V \) by \( P = \frac{V^2}{R} \) (so \( P \propto V^2 \)), the voltage can be converted to dBV by

\[ V(dBV) = 20 \log_{10} \left( \frac{V(V)}{1V} \right) \]  

Thus, 3 kVolts is 70 dBV and 0.5 Volts is −6 dBV (not −3 dBV) or 54 dBmV.

### 1.4.1 The Electric Field

The electric field (in V/m) is defined as the force (in Newtons) per unit charge (in Coulombs). From this definition and Coulomb's law, the electric field, \( E \), created by a single point
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charge $Q$ at a distance $r$ is

$$E = \frac{F}{Q} = \frac{Q}{4\pi \varepsilon r^2} \hat{r} \quad (V/m) \quad (1.17)$$

where
- $F$ is the electric force given by Coulomb’s law ($F = \frac{Q_1 Q_2}{4\pi \varepsilon r^2}$);
- $\hat{r}$ is a unit vector along the $r$ direction, which is also the direction of the electric field $E$;
- $\varepsilon$ is the electric permittivity (it is also called the dielectric constant, but is normally a function of frequency and not really a constant, thus permittivity is preferred in this book) of the material. Its SI unit is Farads/m. In free space, it is a constant:

$$\varepsilon_0 = 8.85419 \times 10^{-12} \text{ F/m} \quad (1.18)$$

The product of the permittivity and the electric field is called the electric flux density, $D$, which is a measure of how much electric flux passes through a unit area, i.e.

$$D = \varepsilon E = \varepsilon_r \varepsilon_0 E (C/m^2) \quad (1.19)$$

where $\varepsilon_r = \varepsilon / \varepsilon_0$ is called the relative permittivity or relative dielectric constant. The relative permittivities of some common materials are listed in Table 1.2. Note that they are functions of frequency and temperature. Normally, the higher the frequency, the smaller the permittivity in the radio frequency band. It should also be pointed out that almost all conductors have a relative permittivity of one.

The electric flux density is also called the electric displacement, hence the symbol $D$. It is also a vector. In an isotropic material (properties independent of direction), $D$ and $E$ are in the same direction and $\varepsilon$ is a scalar quantity. In an anisotropic material, $D$ and $E$ may be in different directions if $\varepsilon$ is a tensor.

If the permittivity is a complex number, it means that the material has some loss. The complex permittivity can be written as

$$\varepsilon = \varepsilon' - j \varepsilon'' \quad (1.20)$$

The ratio of the imaginary part to the real part is called the loss tangent, that is

$$\tan \delta = \frac{\varepsilon''}{\varepsilon'} \quad (1.21)$$

It has no unit and is also a function of frequency and temperature.

The electric field $E$ is related to the current density $J$ (in A/m$^2$), another important parameter, by Ohm’s law. The relationship between them at a point can be expressed as

$$J = \sigma E \quad (1.22)$$

where $\sigma$ is the conductivity, which is the reciprocal of resistivity. It is a measure of a material’s ability to conduct an electrical current and is expressed in Siemens per meter (S/m). Table 1.3
### Table 1.2  
Relative permittivity of some common materials at 100 MHz

<table>
<thead>
<tr>
<th>Material</th>
<th>Relative permittivity</th>
<th>Material</th>
<th>Relative permittivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABS (plastic)</td>
<td>2.4–3.8</td>
<td>Polypropylene</td>
<td>2.2</td>
</tr>
<tr>
<td>Air</td>
<td>1</td>
<td>Polyvinylchloride (PVC)</td>
<td>3</td>
</tr>
<tr>
<td>Alumina</td>
<td>9.8</td>
<td>Porcelain</td>
<td>5.1–5.9</td>
</tr>
<tr>
<td>Aluminum silicate</td>
<td>5.3–5.5</td>
<td>PTFE-teflon</td>
<td>2.1</td>
</tr>
<tr>
<td>Balsa wood</td>
<td>1.37 @ 1 MHz</td>
<td>PTFE-ceramic</td>
<td>10.2</td>
</tr>
<tr>
<td></td>
<td>1.22 @ 3 GHz</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Concrete</td>
<td>~8</td>
<td>PTFE-glass</td>
<td>2.1–2.55</td>
</tr>
<tr>
<td>Copper</td>
<td>1</td>
<td>RT/Duroid 5870</td>
<td>2.33</td>
</tr>
<tr>
<td>Diamond</td>
<td>5.5–10</td>
<td>RT/Duroid 6006</td>
<td>6.15 @ 3 GHz</td>
</tr>
<tr>
<td>Epoxy (FR4)</td>
<td>4.4</td>
<td>Rubber</td>
<td>3.0–4.0</td>
</tr>
<tr>
<td>Epoxy glass PCB</td>
<td>5.2</td>
<td>Sapphire</td>
<td>9.4</td>
</tr>
<tr>
<td>Ethyl alcohol (absolute)</td>
<td>24.5 @ 1 MHz</td>
<td>Sea water</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>6.5 @ 3 GHz</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FR-4(G-10)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>– low resin</td>
<td>4.9</td>
<td>Silicon</td>
<td>11.7–12.9</td>
</tr>
<tr>
<td>– high resin</td>
<td>4.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GaAs</td>
<td>13.0</td>
<td>Soil</td>
<td>~10</td>
</tr>
<tr>
<td>Glass</td>
<td>~4</td>
<td>Soil (dry sandy)</td>
<td>2.59 @ 1 MHz</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2.55 @ 3 GHz</td>
</tr>
<tr>
<td>Gold</td>
<td>1</td>
<td>Water (32˚F)</td>
<td>88.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(68˚F)</td>
<td>80.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(212˚F)</td>
<td>55.3</td>
</tr>
<tr>
<td>Ice (pure distilled water)</td>
<td>4.15 @ 1 MHz</td>
<td>Wood</td>
<td>~2</td>
</tr>
<tr>
<td></td>
<td>3.2 @ 3 GHz</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 1.3  
Conductivities of some common materials at room temperature

<table>
<thead>
<tr>
<th>Material</th>
<th>Conductivity (S/m)</th>
<th>Material</th>
<th>Conductivity (S/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silver</td>
<td>$6.3 \times 10^7$</td>
<td>Graphite</td>
<td>$\approx 10^3$</td>
</tr>
<tr>
<td>Copper</td>
<td>$5.8 \times 10^7$</td>
<td>Carbon</td>
<td>$\approx 10^4$</td>
</tr>
<tr>
<td>Gold</td>
<td>$4.1 \times 10^7$</td>
<td>Silicon</td>
<td>$\approx 10^3$</td>
</tr>
<tr>
<td>Aluminum</td>
<td>$3.5 \times 10^7$</td>
<td>Ferrite</td>
<td>$\approx 10^2$</td>
</tr>
<tr>
<td>Tungsten</td>
<td>$1.8 \times 10^7$</td>
<td>Sea water</td>
<td>$\approx 5$</td>
</tr>
<tr>
<td>Zinc</td>
<td>$1.7 \times 10^7$</td>
<td>Germanium</td>
<td>$\approx 2$</td>
</tr>
<tr>
<td>Brass</td>
<td>$1 \times 10^7$</td>
<td>Wet soil</td>
<td>$\approx 1$</td>
</tr>
<tr>
<td>Phosphor bronze</td>
<td>$1 \times 10^7$</td>
<td>Animal blood</td>
<td>0.7</td>
</tr>
<tr>
<td>Tin</td>
<td>$9 \times 10^6$</td>
<td>Animal body</td>
<td>0.3</td>
</tr>
<tr>
<td>Lead</td>
<td>$5 \times 10^6$</td>
<td>Fresh water</td>
<td>$\approx 10^{-2}$</td>
</tr>
<tr>
<td>Silicon steel</td>
<td>$2 \times 10^6$</td>
<td>Dry soil</td>
<td>$\approx 10^{-3}$</td>
</tr>
<tr>
<td>Stainless steel</td>
<td>$1 \times 10^6$</td>
<td>Distilled water</td>
<td>$\approx 10^{-4}$</td>
</tr>
<tr>
<td>Mercury</td>
<td>$1 \times 10^6$</td>
<td>Glass</td>
<td>$\approx 10^{-12}$</td>
</tr>
<tr>
<td>Cast iron</td>
<td>$\approx 10^{9}$</td>
<td>Air</td>
<td>0</td>
</tr>
</tbody>
</table>
lists conductivities of some common materials linked to antenna engineering. The conductivity is also a function of temperature and frequency.

1.4.2 The Magnetic Field

Whilst charges can generate an electric field, currents can generate a magnetic field. The magnetic field, $\mathbf{H}$ (in A/m), is the vector field which forms closed loops around electric currents or magnets. The magnetic field from a current vector $\mathbf{I}$ is given by the Biot–Savart law as

$$ \mathbf{H} = \frac{\mathbf{I} \times \hat{r}}{4\pi r^2} (\text{A/m}) $$

(1.23)

where

$\hat{r}$ is the unit displacement vector from the current element to the field point and $r$ is the distance from the current element to the field point.

$I$, $\hat{r}$ and $\mathbf{H}$ follow the right-hand rule; that is, $\mathbf{H}$ is orthogonal to both $\mathbf{I}$ and $\hat{r}$, as illustrated by Figure 1.12.

Like the electric field, the magnetic field exerts a force on electric charge. But unlike an electric field, it employs force only on a moving charge, and the direction of the force is orthogonal to both the magnetic field and the charge’s velocity:

$$ \mathbf{F} = Q\mathbf{v} \times \mu \mathbf{H} $$

(1.24)

where

$\mathbf{F}$ is the force vector produced, measured in Newtons;

$Q$ is the electric charge that the magnetic field is acting on, measured in Coulombs (C);

$\mathbf{v}$ is the velocity vector of the electric charge $Q$, measured in meters per second (m/s);

$\mu$ is the magnetic permeability of the material. Its unit is Henries per meter (H/m). In free space, the permeability is

$$ \mu_0 = 4\pi \times 10^{-7} \text{H/m} $$

(1.25)

In Equation (1.24), $Q\mathbf{v}$ can actually be viewed as the current vector $\mathbf{I}$ and the product of $\mu \mathbf{H}$ is called the magnetic flux density $\mathbf{B}$ (in Tesla), the counterpart of the electric flux density.

Figure 1.12  Magnetic field generated by current $I$
Thus

\[ B = \mu H \]  (1.26)

Again, in an isotropic material (properties independent of direction), \( B \) and \( H \) are in the same direction and \( \mu \) is a scalar quantity. In an anisotropic material, \( B \) and \( E \) may be in different directions and \( \mu \) is a tensor.

Like the relative permittivity, the relative permeability is given as

\[ \mu_r = \frac{\mu}{\mu_0} \]  (1.27)

The relative permeabilities of some materials are given in Table 1.4. Permeability is not sensitive to frequency or temperature. Most materials, including conductors, have a relative permeability very close to one.

Combining Equations (1.17) and (1.24) yields

\[ F = Q(E + v \times \mu H) \]  (1.28)

This is called the Lorentz force. The particle will experience a force due to the electric field \( QE \), and the magnetic field \( Qv \times B \).

### 1.4.3 Maxwell’s Equations

Maxwell’s equations are a set of equations first presented as a distinct group in the latter half of the nineteenth century by James Clerk Maxwell (1831–1879), pictured in Figure 1.13. Mathematically they can be expressed in the following differential form:

\[ \nabla \times E = -\frac{dB}{dt} \]

\[ \nabla \times H = J + \frac{dD}{dt} \]  (1.29)

\[ \nabla \cdot D = \rho \]

\[ \nabla \cdot B = 0 \]

where

\( \rho \) is the charge density;

\( \nabla = \frac{\partial}{\partial x} \hat{x} + \frac{\partial}{\partial y} \hat{y} + \frac{\partial}{\partial z} \hat{z} \) is a vector operator;

Table 1.4 Relative permeabilities of some common materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Relative permeability</th>
<th>Material</th>
<th>Relative permeability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Superalloy</td>
<td>( \approx 1 \times 10^6 )</td>
<td>Aluminum</td>
<td>( \approx 1 )</td>
</tr>
<tr>
<td>Purified iron</td>
<td>( \approx 2 \times 10^3 )</td>
<td>Air</td>
<td>1</td>
</tr>
<tr>
<td>Silicon iron</td>
<td>( \approx 7 \times 10^3 )</td>
<td>Water</td>
<td>( \approx 1 )</td>
</tr>
<tr>
<td>Iron</td>
<td>( \approx 5 \times 10^3 )</td>
<td>Copper</td>
<td>( \approx 1 )</td>
</tr>
<tr>
<td>Mild steel</td>
<td>( \approx 2 \times 10^3 )</td>
<td>Lead</td>
<td>( \approx 1 )</td>
</tr>
<tr>
<td>Nickel</td>
<td>600</td>
<td>Silver</td>
<td>( \approx 1 )</td>
</tr>
</tbody>
</table>
\[ \nabla \times \mathbf{E} = \frac{d\mathbf{B}}{dt} \]  
(1.30)

This equation simply means that the induced electromotive force is proportional to the rate of change of the magnetic flux through a coil. In layman’s terms, moving a conductor (such as a metal wire) through a magnetic field produces a voltage. The resulting voltage is directly proportional to the speed of movement. It is apparent from this equation that a time-varying magnetic field \( \mu \frac{d\mathbf{H}}{dt} \neq 0 \) will generate an electric field, i.e. \( \mathbf{E} \neq 0 \). But if the magnetic field is not time-varying, it will NOT generate an electric field.
1.4.3.2 Ampere’s Circuital Law

\[ \nabla \times H = J + \frac{dD}{dt} \quad (1.31) \]

This equation was modified by Maxwell by introducing the displacement current \( \frac{dD}{dt} \). It means that a magnetic field appears during the charge or discharge of a capacitor. With this concept, and Faraday’s law, Maxwell was able to derive the wave equations, and by showing that the predicted wave velocity was the same as the measured velocity of light, Maxwell asserted that light waves are electromagnetic waves.

This equation shows that both the current \( J \) and time-varying electric field \( \varepsilon \frac{dE}{dt} \) can generate a magnetic field, i.e. \( H \neq 0 \).

1.4.3.3 Gauss’s Law for Electric Fields

\[ \nabla \cdot D = \rho \quad (1.32) \]

This is the electrostatic application of Gauss’s generalized theorem, giving the equivalence relation between any flux, e.g. of liquids, electric or gravitational, flowing out of any closed surface and the result of inner sources and sinks, such as electric charges or masses enclosed within the closed surface. As a result, it is not possible for electric fields to form a closed loop. Since \( D = \varepsilon E \), it is also clear that charges \( \rho \) can generate electric fields, i.e. \( E \neq 0 \).

1.4.3.4 Gauss’s Law for Magnetic Fields

\[ \nabla \cdot B = 0 \quad (1.33) \]

This shows that the divergence of the magnetic field \( \nabla \cdot B \) is always zero, which means that the magnetic field lines are closed loops; thus, the integral of \( B \) over a closed surface is zero.

For a time-harmonic electromagnetic field (which means a field linked to time by factor \( e^{j\omega t} \) where \( \omega \) is the angular frequency and \( t \) is the time), we can use the constitutive relations

\[ D = \varepsilon E, \quad B = \mu H, \quad J = \sigma E \quad (1.34) \]

to write Maxwell’s equations in the following form

\[ \begin{align*}
\nabla \times E &= -j\omega \mu H \\
\nabla \times H &= J + j\omega \varepsilon E = j\omega \varepsilon \left(1 - j\frac{\sigma}{\omega \varepsilon}\right) E \\
\n\nabla \cdot E &= \rho / \varepsilon \\
\n\nabla \cdot H &= 0
\end{align*} \quad (1.35) \]

where \( B \) and \( D \) are replaced by the electric field \( E \) and magnetic field \( H \) to simplify the equations and they will not appear again unless necessary.
Introduction

It should be pointed out that, in Equation (1.35), \( \varepsilon(1 - j \sigma / \omega \varepsilon) \) can be viewed as a complex permittivity defined by Equation (1.20). In this case, the loss tangent is

\[
\tan \delta = \frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega \varepsilon}
\]  

(1.36)

It is hard to predict how the loss tangent changes with the frequency, since both the permittivity and conductivity are functions of frequency as well. More discussion will be given in Chapter 3.

1.4.4 Boundary Conditions

Maxwell’s equations can also be written in the integral form as

\[
\oint_{C} \mathbf{E} \cdot d\mathbf{l} = -\int_{S} \frac{d\mathbf{B}}{dt} \cdot d\mathbf{s}
\]

\[
\oint_{C} \mathbf{H} \cdot d\mathbf{l} = \int_{S} (\mathbf{J} + \frac{d\mathbf{D}}{dt}) \cdot d\mathbf{s}
\]

\[
\int_{S} \mathbf{D} \cdot d\mathbf{s} = \iiint_{V} \rho dV = Q
\]

\[
\int_{S} \mathbf{B} \cdot d\mathbf{s} = 0
\]

(1.37)

Consider the boundary between two materials shown in Figure 1.14. Using these equations, we can obtain a number of useful results. For example, if we apply the first equation of Maxwell’s equations in integral form to the boundary between Medium 1 and Medium 2, it is not difficult to obtain [2]:

\[
\hat{n} \times \mathbf{E}_1 = \hat{n} \times \mathbf{E}_2
\]

(1.38)

where \( \hat{n} \) is the surface unit vector from Medium 2 to Medium 1, as shown in Figure 1.14. This condition means that the tangential components of an electric field (\( \hat{n} \times \mathbf{E} \)) are continuous across the boundary between any two media.

![Figure 1.14](boundary.png)  

**Figure 1.14**  Boundary between Medium 1 and Medium 2
Similarly, we can apply the other three Maxwell equations to this boundary to obtain:

\[
\begin{align*}
\hat{n} \times (H_1 - H_2) &= J_s \\
\hat{n} \cdot (\varepsilon_1 E_1 - \varepsilon_2 E_2) &= \rho_s \\
\hat{n} \cdot (\mu_1 H_1 - \mu_2 H_2) &= 0
\end{align*}
\] (1.39)

where \( J_s \) is the surface current density and \( \rho_s \) is the surface charge density. These results can be interpreted as

- the change in tangential component of the magnetic field across a boundary is equal to the surface current density on the boundary;
- the change in the normal component of the electric flux density across a boundary is equal to the surface charge density on the boundary;
- the normal component of the magnetic flux density is continuous across the boundary between two media, whilst the normal component of the magnetic field is not continuous unless \( \mu_1 = \mu_2 \).

Applying these boundary conditions on a perfect conductor (which means no electric and magnetic field inside and the conductivity \( \sigma = \infty \)) in the air, we have

\[
\begin{align*}
\hat{n} \times E &= 0; \ \hat{n} \times H = J_s; \ \hat{n} \cdot E = \rho_s / \varepsilon; \ \hat{n} \cdot H = 0
\end{align*}
\] (1.40)

We can also use these results to illustrate, for example, the field distribution around a two-wire transmission line, as shown in Figure 1.15, where the electric fields are plotted as the solid lines and the magnetic fields are shown as broken lines. As expected, the electric field is from positive charges to negative charges, whilst the magnetic field forms loops around the current.
Introduction

1.5 Summary

In this chapter we have introduced the concept of antennas, briefly reviewed antenna history and laid down the mathematical foundations for further study. The focus has been on the basics of electromagnetics, which include electric and magnetic fields, electromagnetic properties of materials, Maxwell’s equations and boundary conditions. Maxwell’s equations have revealed how electric fields, magnetic fields and sources (currents and charges) are interlinked. They are the foundation of electromagnetics and antennas.

References


Problems

Q1.1 What wireless communication experiment did H. Hertz conduct in 1887? Use a diagram to illustrate your answer.

Q1.2 Use an example to explain what a complex number means in our daily life.

Q1.3 Given vectors $\mathbf{A} = 10\hat{x} + 5\hat{y} + 12\hat{z}$ and $\mathbf{B} = 5\hat{z}$, find
   a. the amplitude of vector $\mathbf{A}$;
   b. the angle between vectors $\mathbf{A}$ and $\mathbf{B}$;
   c. the dot product of these two vectors;
   d. a vector which is orthogonal to $\mathbf{A}$ and $\mathbf{B}$.

Q1.4 Given vector $\mathbf{A} = 10\sin(10t + 10z)\hat{x} + 5\hat{y}$, find
   a. $\nabla \cdot \mathbf{A}$;
   b. $\nabla \times \mathbf{A}$;
   c. $(\nabla \cdot \nabla) \mathbf{A}$;
   d. $\nabla \nabla \cdot \mathbf{A}$

Q1.5 Vector $\mathbf{E} = 10e^{j(10t - 10z)}\hat{x}$.
   a. find the amplitude of $\mathbf{E}$;
   b. plot the real part of $\mathbf{E}$ as a function of $t$;
   c. plot the real part of $\mathbf{E}$ as a function of $z$;
   d. explain what this vector means.

Q1.6 Explain why mobile phone service providers have to pay license fees to use the spectrum. Who is responsible for the spectrum allocation in your country?

Q1.7 Cellular mobile communications have become part of our daily life. Explain the major differences between the 1st, 2nd and 3rd generations of cellular mobile systems in terms of the frequency, data rate and bandwidth. Further explain why their operational frequencies have increased.

Q1.8 Which frequency bands have been used for radar applications? Give an example.

Q1.9 Express 1 kW in dB, 10 kV in dBV, 0.5 dB in W and 40 dB$\mu$V/m in V/m and $\mu$V/m.

Q1.10 Explain the concepts of the electric field and magnetic field. How are they linked to the electric and magnetic flux density functions?
Q1.11 What are the material properties of interest to our electromagnetic and antenna engineers?

Q1.12 What is the Lorentz force? Name an application of the Lorentz force in our daily life.

Q1.13 If a magnetic field on a perfect conducting surface $z = 0$ is $H = 10 \cos(10t - 5z)\hat{x}$, find the surface current density $\mathbf{J}_s$.

Q1.14 Use Maxwell’s equations to explain the major differences between static EM fields and time-varying EM fields.

Q1.15 Express the boundary conditions for the electric and magnetic fields on the surface of a perfect conductor.