CHAPTER 1
Introduction

1.1 Introduction to Fluid Mechanics
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1.3 Methods of Analysis
1.4 Dimensions and Units
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Case Study

At the beginning of each chapter we present a case study that shows how the material in the chapter is incorporated into modern technology. We have tried to present novel developments that show the ongoing importance of the field of fluid mechanics. Perhaps, as a creative new engineer, you’ll be able to use the ideas you learn in this course to improve current fluid-mechanics devices or invent new ones!

Wind Power

According to the July 16, 2009, edition of the New York Times, the global wind energy potential is much higher than previously estimated by both wind industry groups and government agencies. Using data from thousands of meteorological stations, the research indicates that the world’s wind power potential is about 40 times greater than total current power consumption; previous studies had put that multiple at about seven times! In the lower 48 states, the potential from wind power is 16 times more than total electricity demand in the United States, the researchers suggested, again much higher than a 2008 Department of Energy study that projected wind could supply a fifth of all electricity in the country by 2030. The findings indicate the validity of the often-made claim that “the United States is the Saudi Arabia of wind.” The new estimate is based on the idea of deploying 2.5- to 3-megawatt (MW) wind turbines in rural areas that are neither frozen nor forested and also on shallow offshore locations, and it includes a conservative 20 percent estimate for capacity factor, which is a measure of how much energy a given turbine actually produces. It has been estimated that the total power from the wind that could conceivably be extracted is about 72 terawatts (TW 72 $\times 10^{12}$ W). Bearing in mind that the total power consumption by all humans was about 16 TW (as of 2006), it is clear that wind energy could supply all the world’s needs for the foreseeable future!

One reason for the new estimate is due to the increasingly common use of very large turbines that rise to almost 100 m, where wind speeds are greater. Previous wind studies were based on the use of 50- to 80-m turbines. In addition, to reach even higher elevations (and hence wind speed), two approaches have been proposed. In a recent paper, Professor Archer at California State University and Professor Caldeira at the Carnegie Institution of Washington, Stanford, discussed some possibilities. One of these is a design of KiteGen (shown in the figure), consisting of tethered airfoils (kites) manipulated by a control unit and connected to a ground-based, carousel-shaped generator; the kites are maneuvered so that they drive the carousel, generating power, possibly as much as 100 MW. This approach would be best for the lowest few kilometers of the atmosphere. An approach using further increases in elevation...
is to generate electricity aloft and then transmit it to the surface with a tether. In the design proposed by Sky Windpower, four rotors are mounted on an airframe; the rotors both provide lift for the device and power electricity generation. The aircraft would lift themselves into place with supplied electricity to reach the desired altitude but would then generate up to 40 MW of power. Multiple arrays could be used for large-scale electricity generation.

1.1 Introduction to Fluid Mechanics

We decided to title this textbook “Introduction to …” for the following reason: After studying the text, you will not be able to design the streamlining of a new car or an airplane, or design a new heart valve, or select the correct air extractors and ducting for a $100 million building; however, you will have developed a good understanding of the concepts behind all of these, and many other applications, and have made significant progress toward being ready to work on such state-of-the-art fluid mechanics projects.

To start toward this goal, in this chapter we cover some very basic topics: a case study, what fluid mechanics encompasses, the standard engineering definition of a fluid, and the basic equations and methods of analysis. Finally, we discuss some common engineering student pitfalls in areas such as unit systems and experimental analysis.

Note to Students

This is a student-oriented book: We believe it is quite comprehensive for an introductory text, and a student can successfully self-teach from it. However, most students will use the text in conjunction with one or two undergraduate courses. In either case, we recommend a thorough reading of the relevant chapters. In fact, a good approach is to read a chapter quickly once, then reread more carefully a second and even a third time, so that concepts develop a context and meaning. While students often find fluid mechanics quite challenging, we believe this approach, supplemented by your instructor’s lectures that will hopefully amplify and expand upon the text material (if you are taking a course), will reveal fluid mechanics to be a fascinating and varied field of study.

Other sources of information on fluid mechanics are readily available. In addition to your professor, there are many other fluid mechanics texts and journals as well as the Internet (a recent Google search for “fluid mechanics” yielded 26.4 million links, including many with fluid mechanics calculators and animations!).

There are some prerequisites for reading this text. We assume you have already studied introductory thermodynamics, as well as statics, dynamics, and calculus; however, as needed, we will review some of this material.

It is our strong belief that one learns best by doing. This is true whether the subject under study is fluid mechanics, thermodynamics, or soccer. The fundamentals in any of these are few, and mastery of them comes through practice. Thus it is extremely important that you solve problems. The numerous problems included at the end of each chapter provide the opportunity to practice applying fundamentals to the solution of problems. Even though we provide for your convenience a summary of useful equations at the end of each chapter (except this one), you should avoid the temptation to adopt a so-called plug-and-chug approach to solving problems. Most of the problems are such that this approach simply will not work. In solving problems we strongly recommend that you proceed using the following logical steps:

1. State briefly and concisely (in your own words) the information given.
2. State the information to be found.
3. Draw a schematic of the system or control volume to be used in the analysis. Be sure to label the boundaries of the system or control volume and label appropriate coordinate directions.
4. Give the appropriate mathematical formulation of the basic laws that you consider necessary to solve the problem.
5. List the simplifying assumptions that you feel are appropriate in the problem.
6 Complete the analysis algebraically before substituting numerical values.
7 Substitute numerical values (using a consistent set of units) to obtain a numerical answer.
   (a) Reference the source of values for any physical properties.
   (b) Be sure the significant figures in the answer are consistent with the given data.
8 Check the answer and review the assumptions made in the solution to make sure they are reasonable.
9 Label the answer.

In your initial work this problem format may seem unnecessary and even long-winded. However, it is
our experience that this approach to problem solving is ultimately the most efficient; it will also prepare
you to be a successful professional, for which a major prerequisite is to be able to communicate informa-
tion and the results of an analysis clearly and precisely. This format is used in all examples presented
in this text; answers to examples are rounded to three significant figures.

Finally, we strongly urge you to take advantage of the many Excel tools available for this book on
the text website for use in solving problems. Many problems can be solved much more quickly using
these tools; occasional problems can only be solved with the tools or with an equivalent computer
application.

**Scope of Fluid Mechanics**

As the name implies, fluid mechanics is the study of fluids at rest or in motion. It has traditionally been
applied in such areas as the design of canal, levee, and dam systems; the design of pumps, compressors, and
piping and ducting used in the water and air conditioning systems of homes and businesses, as well as the
piping systems needed in chemical plants; the aerodynamics of automobiles and sub- and supersonic air-
planes; and the development of many different flow measurement devices such as gas pump meters.

While these are still extremely important areas (witness, for example, the current emphasis on
automobile streamlining and the levee failures in New Orleans in 2005), fluid mechanics is truly a
“high-tech” or “hot” discipline, and many exciting areas have developed in the last quarter-century.
Some examples include environmental and energy issues (e.g., containing oil slicks, large-scale wind
turbines, energy generation from ocean waves, the aerodynamics of large buildings, and the fluid
mechanics of the atmosphere and ocean and of phenomena such as tornadoes, hurricanes, and tsunamis);
biomechanics (e.g., artificial hearts and valves and other organs such as the liver; understanding of the
fluid mechanics of blood, synovial fluid in the joints, the respiratory system, the circulatory system, and
the urinary system); sport (design of bicycles and bicycle helmets, skis, and sprinting and swimming
clothing, and the aerodynamics of the golf, tennis, and soccer ball); “smart fluids” (e.g., in automobile
suspension systems to optimize motion under all terrain conditions, military uniforms containing a fluid
layer that is “thin” until combat, when it can be “stiffened” to give the soldier strength and protection,
and fluid lenses with humanlike properties for use in cameras and cell phones); and microfluids (e.g., for
extremely precise administration of medications).

These are just a small sampling of the newer areas of fluid mechanics. They illustrate how the dis-
cipline is still highly relevant, and increasingly diverse, even though it may be thousands of years old.

**Definition of a Fluid**

We already have a common-sense idea of when we are working with a fluid, as opposed to a solid: Fluids
tend to flow when we interact with them (e.g., when you stir your morning coffee); solids tend to deform
or bend (e.g., when you type on a keyboard, the springs under the keys compress). Engineers need a more
formal and precise definition of a fluid: A fluid is a substance that deforms continuously under the appli-
cation of a shear (tangential) stress no matter how small the shear stress may be. Because the fluid motion
continues under the application of a shear stress, we can also define a fluid as any substance that cannot
sustain a shear stress when at rest.

Hence liquids and gases (or vapors) are the forms, or phases, that fluids can take. We wish to dis-
tinguish these phases from the solid phase of matter. We can see the difference between solid and fluid
behavior in Fig. 1.1. If we place a specimen of either substance between two plates (Fig. 1.1a) and then
apply a shearing force \( F \), each will initially deform (Fig. 1.1b); however, whereas a solid will then be
at rest (assuming the force is not large enough to go beyond its elastic limit), a fluid will continue to
deform (Fig. 1.1c, Fig. 1.1d, etc) as long as the force is applied. Note that a fluid in contact with a solid surface does not slip—it has the same velocity as that surface because of the no-slip condition, an experimental fact.

The amount of deformation of the solid depends on the solid’s modulus of rigidity $G$; in Chapter 2 we will learn that the rate of deformation of the fluid depends on the fluid’s viscosity $\mu$. We refer to solids as being elastic and fluids as being viscous. More informally, we say that solids exhibit “springiness.” For example, when you drive over a pothole, the car bounces up and down due to the car suspension’s metal coil springs compressing and expanding. On the other hand, fluids exhibit friction effects so that the suspension’s shock absorbers (containing a fluid that is forced through a small opening as the car bounces) dissipate energy due to the fluid friction, which stops the bouncing after a few oscillations. If your shocks are “shot,” the fluid they contained has leaked out so that there is almost no friction as the car bounces, and it bounces several times rather than quickly coming to rest. The idea that substances can be categorized as being either a solid or a liquid holds for most substances, but a number of substances exhibit both springiness and friction; they are viscoelastic. Many biological tissues are viscoelastic. For example, the synovial fluid in human knee joints lubricates those joints but also absorbs some of the shock occurring during walking or running. Note that the system of springs and shock absorbers comprising the car suspension is also viscoelastic, although the individual components are not. We will have more to say on this topic in Chapter 2.

1.2 Basic Equations

Analysis of any problem in fluid mechanics necessarily includes statement of the basic laws governing the fluid motion. The basic laws, which are applicable to any fluid, are:

1. The conservation of mass
2. Newton’s second law of motion
3. The principle of angular momentum
4. The first law of thermodynamics
5. The second law of thermodynamics

Not all basic laws are always required to solve any one problem. On the other hand, in many problems it is necessary to bring into the analysis additional relations that describe the behavior of physical properties of fluids under given conditions.

For example, you probably recall studying properties of gases in basic physics or thermodynamics. The ideal gas equation of state

$$p = \rho RT$$

is a model that relates density to pressure and temperature for many gases under normal conditions. In Eq. 1.1, $R$ is the gas constant. Values of $R$ are given in Appendix A for several common gases; $p$ and $T$ in Eq. 1.1 are the absolute pressure and absolute temperature, respectively; $\rho$ is density (mass per unit volume). Example 1.1 illustrates use of the ideal gas equation of state.

It is obvious that the basic laws with which we shall deal are the same as those used in mechanics and thermodynamics. Our task will be to formulate these laws in suitable forms to solve fluid flow problems and to apply them to a wide variety of situations.
We must emphasize that there are, as we shall see, many apparently simple problems in fluid mechanics that cannot be solved analytically. In such cases we must resort to more complicated numerical solutions and/or results of experimental tests.

1.3 Methods of Analysis

The first step in solving a problem is to define the system that you are attempting to analyze. In basic mechanics, we made extensive use of the free-body diagram. We will use a system or a control volume, depending on the problem being studied. These concepts are identical to the ones you used in thermodynamics (except you may have called them closed system and open system, respectively). We can use either one to get mathematical expressions for each of the basic laws. In thermodynamics they were mostly used to obtain expressions for conservation of mass and the first and second laws of thermodynamics; in our study of fluid mechanics, we will be most interested in conservation of mass and

\[ \text{Example 1.1 FIRST LAW APPLICATION TO CLOSED SYSTEM} \]

A piston-cylinder device contains 0.95 kg of oxygen initially at a temperature of 27°C and a pressure due to the weight of 150 kPa (abs). Heat is added to the gas until it reaches a temperature of 627°C. Determine the amount of heat added during the process.

**Given:** Piston-cylinder containing O\(_2\), \( m = 0.95 \) kg.

\( T_1 = 27°C \quad T_2 = 627°C \)

**Find:** \( Q_{1\rightarrow 2} \).

**Solution:** \( p = \text{constant} = 150 \text{kPa (abs)} \)

We are dealing with a system, \( m = 0.95 \) kg.

**Governing equation:** First law for the system, \( Q_{1\rightarrow 2} - W_{1\rightarrow 2} = E_2 - E_1 \)

**Assumptions:**
1. \( E = U \), since the system is stationary.
2. Ideal gas with constant specific heats.

Under the above assumptions,

\[ E_2 - E_1 = U_2 - U_1 = m(u_2 - u_1) = mc_v(T_2 - T_1) \]

The work done during the process is moving boundary work

\[ W_{1\rightarrow 2} = \int_{V_1}^{V_2} pdV = p(V_2 - V_1) \]

For an ideal gas, \( pV = mRT \). Hence \( W_{1\rightarrow 2} = mR(T_2 - T_1) \). Then from the first law equation,

\[ Q_{1\rightarrow 2} = E_2 - E_1 + W_{1\rightarrow 2} = mc_v(T_2 - T_1) + mR(T_2 - T_1) \]

\[ Q_{1\rightarrow 2} = m(T_2 - T_1)(c_v + R) \]

\[ Q_{1\rightarrow 2} = mc_p(T_2 - T_1)\quad \{R = c_p - c_v\} \]

From the Appendix, Table A.6, for O\(_2\), \( c_p = 909.41\text{J/(kg \cdot K)} \). Solving for \( Q_{1\rightarrow 2} \), we obtain

\[ Q_{1\rightarrow 2} = 0.95 \text{ kg} \times 909.41\text{J/(kg \cdot K)} \times 600 \text{ K} = 518 \text{ kJ} \]

**This problem:**
- Was solved using the nine logical steps discussed earlier.
- Reviewed use of the ideal gas equation and the first law of thermodynamics for a system.

We must emphasize that there are, as we shall see, many apparently simple problems in fluid mechanics that cannot be solved analytically. In such cases we must resort to more complicated numerical solutions and/or results of experimental tests.

1.3 Methods of Analysis

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Newton’s second law of motion. In thermodynamics our focus was energy; in fluid mechanics it will
mainly be forces and motion. We must always be aware of whether we are using a system or a control
volume approach because each leads to different mathematical expressions of these laws. At this point
we review the definitions of systems and control volumes.

System and Control Volume

A system is defined as a fixed, identifiable quantity of mass; the system boundaries separate the system
from the surroundings. The boundaries of the system may be fixed or movable; however, no mass
crosses the system boundaries.

In the familiar piston-cylinder assembly from thermodynamics, Fig. 1.2, the gas in the cylinder is
the system. If the gas is heated, the piston will lift the weight; the boundary of the system thus moves.
Heat and work may cross the boundaries of the system, but the quantity of matter within the system
boundaries remains fixed. No mass crosses the system boundaries.

In mechanics courses you used the free-body diagram (system approach) extensively. This was log-
ical because you were dealing with an easily identifiable rigid body. However, in fluid mechanics we
normally are concerned with the flow of fluids through devices such as compressors, turbines, pipelines,
nozzles, and so on. In these cases it is difficult to focus attention on a fixed identifiable quantity of mass.
It is much more convenient, for analysis, to focus attention on a volume in space through which the fluid
flows. Consequently, we use the control volume approach.

A control volume is an arbitrary volume in space through which fluid flows. The geometric boundary
of the control volume is called the control surface. The control surface may be real or imaginary; it may be
at rest or in motion. Figure 1.3 shows flow through a pipe junction, with a control surface drawn on it.
Note that some regions of the surface correspond to physical boundaries (the walls of the pipe) and others
(at locations ①, ②, and ③) are parts of the surface that are imaginary (inlets or outlets). For the control
volume defined by this surface, we could write equations for the basic laws and obtain results such as the
flow rate at outlet ③ given the flow rates at inlet ① and outlet ② (similar to a problem we will analyze in
Example 4.1 in Chapter 4), the force required to hold the junction in place, and so on. Example 1.2 illus-
trates how we use a control volume to determine the mass flow rate in a section of a pipe. It is
always important to take care in selecting a control volume, as the choice has a big effect on the mathe-
matical form of the basic laws. We will illustrate the use of a control volume with an example.

Fig. 1.2 Piston–cylinder assembly.

Fig. 1.3 Fluid flow through a pipe junction.
Differential versus Integral Approach

The basic laws that we apply in our study of fluid mechanics can be formulated in terms of infinitesimal or finite systems and control volumes. As you might suspect, the equations will look different in the two cases. Both approaches are important in the study of fluid mechanics and both will be developed in the course of our work.

In the first case the resulting equations are differential equations. Solution of the differential equations of motion provides a means of determining the detailed behavior of the flow. An example might be the pressure distribution on a wing surface.

Frequently the information sought does not require a detailed knowledge of the flow. We often are interested in the gross behavior of a device; in such cases it is more appropriate to use integral formulations of the basic laws. An example might be the overall lift a wing produces. Integral formulations, using finite systems or control volumes, usually are easier to treat analytically. The basic laws of mechanics and thermodynamics, formulated in terms of finite systems, are the basis for deriving the control volume equations in Chapter 4.

Methods of Description

Mechanics deals almost exclusively with systems; you have made extensive use of the basic equations applied to a fixed, identifiable quantity of mass. On the other hand, attempting to analyze thermodynamic devices, you often found it necessary to use a control volume (open system) analysis. Clearly, the type of analysis depends on the problem.

Example 1.2 MASS CONSERVATION APPLIED TO CONTROL VOLUME

A reducing water pipe section has an inlet diameter of 50 mm and exit diameter of 30 mm. If the steady inlet speed (averaged across the inlet area) is 2.5 m/s, find the exit speed.

Given: Pipe, inlet $D_i = 50$ mm, exit $D_e = 30$ mm.
Inlet speed, $V_i = 2.5$ m/s.

Find: Exit speed, $V_e$.

Solution:

Assumption: Water is incompressible (density $\rho = \text{constant}$).

The physical law we use here is the conservation of mass, which you learned in thermodynamics when studying turbines, boilers, and so on. You may have seen mass flow at an inlet or outlet expressed as either $m = VA/v$ or $m = \rho VA$ where $V, A, u, \rho$ are the speed, area, specific volume, and density, respectively. We will use the density form of the equation.

Hence the mass flow is:

$$m = \rho VA$$

Applying mass conservation, from our study of thermodynamics,

$$\rho V_i A_i = \rho V_e A_e$$

(Note: $\rho_e = \rho_i = \rho$ by our first assumption.)
(Note: Even though we are already familiar with this equation from thermodynamics, we will derive it in Chapter 4.)

Solving for $V_e$,

$$V_e = \frac{V_i A_i}{A_e} = \frac{V_i \pi D_i^2/4}{\pi D_e^2/4} = V_i \left(\frac{D_i}{D_e}\right)^2$$

$$V_e = 2.7 \text{ m/s} \left(\frac{50}{30}\right)^2 = 7.5 \text{ m/s}$$

This problem:
- Was solved using the nine logical steps.
- Demonstrated use of a control volume and the mass conservation law.

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Where it is easy to keep track of identifiable elements of mass (e.g., in particle mechanics), we use a method of description that follows the particle. This sometimes is referred to as the Lagrangian method of description.

Consider, for example, the application of Newton’s second law to a particle of fixed mass. Mathematically, we can write Newton’s second law for a system of mass \( m \) as

\[
\sum F = m \ddot{a} = m \frac{d\dot{r}}{dt} = m \frac{d^2 r}{dt^2}
\]

In Eq. 1.2, \( \sum F \) is the sum of all external forces acting on the system, \( \ddot{a} \) is the acceleration of the center of mass of the system, \( \dot{V} \) is the velocity of the center of mass of the system, and \( r \) is the position vector of the center of mass of the system relative to a fixed coordinate system. In Example 1.3, we show how Newton’s second law is applied to a falling object to determine its speed.

**Example 1.3  FREE FALL OF BALL IN AIR**

The air resistance (drag force) on a 200 g ball in free flight is given by \( F_D = 2 \times 10^{-4} V^2 \), where \( F_D \) is in newtons and \( V \) is in meters per second. If the ball is dropped from rest 500 m above the ground, determine the speed at which it hits the ground. What percentage of the terminal speed is the result? (The terminal speed is the steady speed a falling body eventually attains.)

**Given:** Ball, \( m = 0.2 \) kg, released from rest at \( y_0 = 500 \) m.

Air resistance, \( F_D = kV^2 \), where \( k = 2 \times 10^{-4} \) N \( \cdot \) s\(^2\)/m\(^2\).

Units: \( F_D \) (N), \( V \) (m/s).

**Find:**
(a) Speed at which the ball hits the ground.
(b) Ratio of speed to terminal speed.

**Solution:**

**Governing equation:** \( \sum F = m \ddot{a} \)

**Assumption:** Neglect buoyancy force.

The motion of the ball is governed by the equation

\[
\sum F_y = ma_y = m \frac{dV}{dy} = mV \frac{dV}{dy}
\]

Since \( V = V(y) \), we write \( \sum F_y = mg = kV^2 - mg = mV \frac{dV}{dy} \) then,

\[
\sum F_y = F_D - mg = kV^2 - mg = mV \frac{dV}{dy}
\]

Separating variables and integrating,

\[
\int_{y_0}^{y} dy = \int_{0}^{V} \frac{mVdV}{kV^2 - mg}
\]

\[
y - y_0 = \left[ \frac{m}{2k} \ln(kV^2 - mg) \right]_{y_0}^{V} = \frac{m}{2k} \ln \left( \frac{kV^2 - mg}{-mg} \right)
\]

Taking antilogarithms, we obtain

\[
kV^2 - mg = -mg \ e^{\left(\frac{2k}{m}\right)(y - y_0)}
\]

Solving for \( V \) gives

\[
V = \left\{ \frac{mg}{k} \left( 1 - e^{\left(\frac{2k}{m}\right)(y - y_0)} \right) \right\}^{1/2}
\]
We could use this Lagrangian approach to analyze a fluid flow by assuming the fluid to be composed of a very large number of particles whose motion must be described. However, keeping track of the motion of each fluid particle would become a horrendous bookkeeping problem. Consequently, a particle description becomes unmanageable. Often we find it convenient to use a different type of description. Particularly with control volume analyses, it is convenient to use the field, or Eulerian, method of description, which focuses attention on the properties of a flow at a given point in space as a function of time. In the Eulerian method of description, the properties of a flow field are described as functions of space coordinates and time. We shall see in Chapter 2 that this method of description is a logical outgrowth of the assumption that fluids may be treated as continuous media.

1.4 Dimensions and Units

Engineering problems are solved to answer specific questions. It goes without saying that the answer must include units. In 1999, NASA’s Mars Climate Observer crashed because the JPL engineers assumed that a measurement was in meters, but the supplying company’s engineers had actually made the measurement in feet! Consequently, it is appropriate to present a brief review of dimensions and units. We say “review” because the topic is familiar from your earlier work in mechanics.

We refer to physical quantities such as length, time, mass, and temperature as dimensions. In terms of a particular system of dimensions, all measurable quantities are subdivided into two groups—primary quantities and secondary quantities. We refer to a small group of dimensions from which all others can be formed as primary quantities, for which we set up arbitrary scales of measure. Secondary quantities are those quantities whose dimensions are expressible in terms of the dimensions of the primary quantities.

Units are the arbitrary names (and magnitudes) assigned to the primary dimensions adopted as standards for measurement. For example, the primary dimension of length may be measured in units of meters, feet, yards, or miles. These units of length are related to each other through unit conversion factors (1 mile = 5280 feet = 1609 meters).

Systems of Dimensions

Any valid equation that relates physical quantities must be dimensionally homogeneous; each term in the equation must have the same dimensions. We recognize that Newton’s second law \( \vec{F} \propto m \vec{a} \) relates
the four dimensions, \( F, M, L \) and \( t \). Thus force and mass cannot both be selected as primary dimensions without introducing a constant of proportionality that has dimensions (and units).

Length and time are primary dimensions in all dimensional systems in common use. In some systems, mass is taken as a primary dimension. In others, force is selected as a primary dimension; a third system chooses both force and mass as primary dimensions. Thus we have three basic systems of dimensions, corresponding to the different ways of specifying the primary dimensions.

(a) Mass \([M]\), length \([L]\), time \([t]\), temperature \([T]\)
(b) Force \([F]\), length \([L]\), time \([t]\), temperature \([T]\)
(c) Force \([F]\), mass \([M]\), length \([L]\), time \([t]\), temperature \([T]\)

In system (a), force \([F]\) is a secondary dimension and the constant of proportionality in Newton’s second law is dimensionless. In system (b), mass \([M]\) is a secondary dimension, and again the constant of proportionality in Newton’s second law is dimensionless. In system (c), both force \([F]\) and mass \([M]\) have been selected as primary dimensions. In this case the constant of proportionality, \( g_c \) (not to be confused with \( g \), the acceleration of gravity!) in Newton’s second law (written \( F = m a / g_c \)) is not dimensionless. The dimensions of \( g_c \) must in fact be \([ML/Ft^2]\) for the equation to be dimensionally homogeneous. The numerical value of the constant of proportionality depends on the units of measure chosen for each of the primary quantities.

**Systems of Units**

There is more than one way to select the unit of measure for each primary dimension. We shall present only the more common engineering systems of units for each of the basic systems of dimensions. Table 1.1 shows the basic units assigned to the primary dimensions for these systems. The units in parentheses are those assigned to that unit system’s secondary dimension. Following the table is a brief description of each of them.

**a. MLtT**

SI, which is the official abbreviation in all languages for the Système International d’Unités,\(^1\) is an extension and refinement of the traditional metric system. More than 30 countries have declared it to be the only legally accepted system.

In the SI system of units, the unit of mass is the kilogram (kg), the unit of length is the meter (m), the unit of time is the second (s), and the unit of temperature is the kelvin (K). Force is a secondary dimension, and its unit, the newton (N), is defined from Newton’s second law as

\[
1 \text{ N} \equiv 1 \text{ kg} \cdot \text{m/s}^2
\]

In the Absolute Metric system of units, the unit of mass is the gram, the unit of length is the centimeter, the unit of time is the second, and the unit of temperature is the kelvin. Since force is a secondary dimension, the unit of force, the dyne, is defined in terms of Newton’s second law as

\[
1 \text{ dyne} \equiv 1 \text{ g} \cdot \text{cm/s}^2
\]

**Table 1.1**

<table>
<thead>
<tr>
<th>System of Dimensions</th>
<th>Unit System</th>
<th>Force ( F )</th>
<th>Mass ( M )</th>
<th>Length ( L )</th>
<th>Time ( t )</th>
<th>Temperature ( T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. MLtT</td>
<td>Système International d’Unités (SI)</td>
<td>(N)</td>
<td>kg</td>
<td>m</td>
<td>s</td>
<td>K</td>
</tr>
<tr>
<td>b. FLtT</td>
<td>British Gravitational (BG)</td>
<td>lbf</td>
<td>(slug)</td>
<td>ft</td>
<td>s</td>
<td>°R</td>
</tr>
<tr>
<td>c. FMLtT</td>
<td>English Engineering (EE)</td>
<td>lbf</td>
<td>lbm</td>
<td>ft</td>
<td>s</td>
<td>°R</td>
</tr>
</tbody>
</table>

b. FLT
In the British Gravitational system of units, the unit of force is the pound \( \text{lbf} \), the unit of length is the foot \( \text{ft} \), the unit of time is the second, and the unit of temperature is the degree Rankine \( ^\circ R \). Since mass is a secondary dimension, the unit of mass, the slug, is defined in terms of Newton’s second law as \[ 1 \text{ slug} \equiv 1 \text{ lbf} \cdot \text{s}^2/\text{ft} \]

c. FMLT
In the English Engineering system of units, the unit of force is the pound force \( \text{lbf} \), the unit of mass is the pound mass \( \text{lbm} \), the unit of length is the foot, the unit of time is the second, and the unit of temperature is the degree Rankine. Since both force and mass are chosen as primary dimensions, Newton’s second law is written as \[ \vec{F} = m \dot{\vec{a}} \]

A force of one pound \( (1 \text{ lbf}) \) is the force that gives a pound mass \( (1 \text{ lbm}) \) an acceleration equal to the standard acceleration of gravity on Earth, \( 32 \frac{2}{3} \text{ ft/s}^2 \). From Newton’s second law we see that \[ 1 \text{ lbf} \equiv \frac{1 \text{ lbm} \times 32 \frac{2}{3} \text{ ft/s}^2}{g_c} \]

or \[ g_c \equiv 32 \frac{2}{3} \text{ ft}\cdot\text{lbm}/(\text{lbf}\cdot\text{s}^2) \]

The constant of proportionality, \( g_c \), has both dimensions and units. The dimensions arose because we selected both force and mass as primary dimensions; the units (and the numerical value) are a consequence of our choices for the standards of measurement.

Since a force of 1 lbf accelerates 1 lbm at \( 32 \frac{2}{3} \text{ ft/s}^2 \), it would accelerate 32.2 lbm at \( 1 \text{ ft/s}^2 \). A slug also is accelerated at \( 1 \text{ ft/s}^2 \) by a force of 1 lbf. Therefore, \[ 1 \text{ slug} \equiv 32.2 \text{ lbm} \]

Many textbooks and references use lb instead of lbf or lbm, leaving it up to the reader to determine from the context whether a force or mass is being referred to.

Preferred Systems of Units
In this text we shall use the SI system of units, but we will be referencing the British Gravitational system of units when necessary. In either case, the constant of proportionality in Newton’s second law is dimensionless and has a value of unity. Consequently, Newton’s second law is written as \( \vec{F} = m \dot{\vec{a}} \). In these systems, it follows that the gravitational force (the “weight”\(^2\)) on an object of mass \( m \) is given by \( W = mg \).

SI units and prefixes, together with other defined units and useful conversion factors, are on the inside cover of the book. In Example 1.4, we show how we convert between mass and weight in the different unit systems that we use.

Dimensional Consistency and “Engineering” Equations
In engineering, we strive to make equations and formulas have consistent dimensions. That is, each term in an equation, and obviously both sides of the equation, should be reducible to the same dimensions. For example, a very important equation we will derive later on is the Bernoulli equation \[ \frac{p_1}{\rho} + \frac{V_1^2}{2} + g z_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + g z_2 \]

\(^2\)Note that in the English Engineering system, the weight of an object is given by \( W = mg/g_c \).
Example 1.4 USE OF UNITS

The label on a jar of peanut butter states its net weight is 510 g. Express its mass and weight in SI, BG, and EE units.

**Given:** Peanut butter “weight,” \( m = 510 \text{ g} \).

**Find:** Mass and weight in SI, BG, and EE units.

**Solution:** This problem involves unit conversions and use of the equation relating weight and mass:

\[ W = mg \]

The given “weight” is actually the mass because it is expressed in units of mass:

\[ m_{SI} = 0.510 \text{ kg} \]

Using the conversions given inside the book cover,

\[ m_{EE} = m_{SI} \left( \frac{1 \text{ lbm}}{0.454 \text{ kg}} \right) = 0.510 \text{ kg} \left( \frac{1 \text{ lbm}}{0.454 \text{ kg}} \right) = 1.12 \text{ lbm} \]

Using the fact that 1 slug = 32.2 lbm,

\[ m_{BG} = m_{EE} \left( \frac{1 \text{ slug}}{32.2 \text{ lbm}} \right) = 1.12 \text{ lbm} \left( \frac{1 \text{ slug}}{32.2 \text{ lbm}} \right) = 0.0349 \text{ slug} \]

To find the weight, we use

\[ W = mg \]

In SI units, and using the definition of a newton,

\[ W_{SI} = 0.510 \text{ kg} \times 9.8 \frac{m}{s^2} = 5.00 \frac{N}{m^2} \frac{m}{s^2} = 5.00 \text{ N} \]

In BG units, and using the definition of a slug,

\[ W_{BG} = 0.0349 \text{ slug} \times 32.2 \frac{\text{ft}}{s^2} = 1.12 \frac{\text{slug} \cdot \text{ft}}{s^2} = 1.12 \text{ lbf} \]

In EE units, we use the form \( W = mg_c \), and using the definition of \( g_c \),

\[ W_{EE} = 1.12 \text{ lbm} \times 32.2 \frac{\text{ft}}{s^2} \times \frac{1}{g_c} = \frac{36.1 \text{ lbm} \cdot \text{ft}}{s^2} \]

This problem illustrates:

- Conversions from SI to BG and EE systems.
- Use of \( g_c \) in the EE system.

Notes:
The student may feel this example involves a lot of unnecessary calculation details (e.g., a factor of 32.2 appears, then disappears), but it cannot be stressed enough that such steps should always be explicitly written out to minimize errors—if you do not write all steps and units down, it is just too easy, for example, to multiply by a conversion factor when you should be dividing by it. For the weights in SI, BG, and EE units, we could alternatively have looked up the conversion from newton to lbf.

which relates the pressure \( p \), velocity \( V \), and elevation \( z \) between points 1 and 2 along a streamline for a steady, frictionless incompressible flow (density \( \rho \)). This equation is dimensionally consistent because each term in the equation can be reduced to dimensions of \( L^2/T^2 \) (the pressure term dimensions are \( FL/M \), but from Newton’s law we find \( F = M/LT^2 \), so \( FL/M = ML^2/MI^2 = L^2/T^2 \).

Almost all equations you are likely to encounter will be dimensionally consistent. However, you should be alert to some still commonly used equations that are not; these are often “engineering”
equations derived many years ago, or are empirical (based on experiment rather than theory), or are proprietary equations used in a particular industry or company. For example, civil engineers often use the semi-empirical Manning equation

$$V = R_h^{2/3} S_0^{1/2} n$$

which gives the flow speed $V$ in an open channel (such as a canal) as a function of the hydraulic radius $R_h$ (which is a measure of the flow cross-section and contact surface area), the channel slope $S_0$, and a constant $n$ (the Manning resistance coefficient). The value of this constant depends on the surface condition of the channel. For example, for a canal made from unfinished concrete, most references give $n \approx 0.014$. Unfortunately, the equation is dimensionally inconsistent! For the right side of the equation, $R_h$ has dimensions $L$, and $S_0$ is dimensionless, so with a dimensionless constant $n$, we end up with dimensions of $L^{2/3}$; for the left side of the equation the dimensions must be $L / t$. A user of the equation is supposed to know that the values of $n$ provided in most references will give correct results only if we ignore the dimensional inconsistency, always use $R_h$ in meters, and interpret $V$ to be in m/s! (The alert student will realize that this means that even though handbooks provide $n$ values as just constants, they must have units of $s / m^{1/3}$.) Because the equation is dimensionally inconsistent, using the same value for $n$ with $R_h$ in ft does not give the correct value for $V$ in ft/s.

A second type of problem is one in which the dimensions of an equation are consistent but use of units is not. The commonly used energy efficiency ratio (EER) of an air conditioner is

$$EER = \frac{\text{cooling rate}}{\text{electrical input}}$$

which indicates how efficiently the AC works—a higher $EER$ value indicates better performance. The equation is dimensionally consistent, with the $EER$ being dimensionless (the cooling rate and electrical input are both measured in energy/time). However, it is used, in a sense, incorrectly, because the units traditionally used in it are not consistent. For example, a good $EER$ value is 10, which would appear to imply you receive, say, 10 kW of cooling for each 1 kW of electrical power. In fact, an $EER$ of 10 means you receive 10 $\text{Btu/ hr}$ of cooling for each 1 $\text{W}$ of electrical power! Manufacturers, retailers, and customers all use the $EER$, in a sense, incorrectly in that they quote an $EER$ of, say, 10, rather than the correct way, of 10 $\text{Btu/ hr/ W}$. (The $EER$, as used, is an everyday, inconsistent unit version of the coefficient of performance, $\text{COP}$, studied in thermodynamics.)

The two examples above illustrate the dangers in using certain equations. Almost all the equations encountered in this text will be dimensionally consistent, but you should be aware of the occasional troublesome equation you will encounter in your engineering studies.

As a final note on units, we stated earlier that we will use SI units in this text. You will become very familiar with their use through using this text but should be aware that many of the units used, although they are scientifically and engineering-wise correct, are nevertheless not units you will use in everyday activities, and vice versa; we do not recommend asking your grocer to give you, say, 22 N, or 0.16 slugs, of potatoes; nor should you be expected to immediately know what, say, a motor oil viscosity of 5W20 means!

SI units and prefixes, other defined units, and useful conversions are given on the inside of the book cover.

### 1.5 Analysis of Experimental Error

Most consumers are unaware of it but, as with most foodstuffs, soft drink containers are filled to plus or minus a certain amount, as allowed by law. Because it is difficult to precisely measure the filling of a container in a rapid production process, a 12-fl-oz container may actually contain 12.1, or 12.7, fl oz. The manufacturer is never supposed to supply less than the specified amount; and it will reduce profits if it is unnecessarily generous. Similarly, the supplier of components for the interior of a car must satisfy minimum and maximum dimensions (each component has what are called tolerances) so that the final appearance of the interior is visually appealing. Engineers performing experiments must measure not just data but also the uncertainties in their measurements. They must also somehow determine how these uncertainties affect the uncertainty in the final result.
14 Chapter 1 Introduction

All of these examples illustrate the importance of experimental uncertainty, that is, the study of uncertainties in measurements and their effect on overall results. There is always a trade-off in experimental work or in manufacturing: We can reduce the uncertainties to a desired level, but the smaller the uncertainty (the more precise the measurement or experiment), the more expensive the procedure will be. Furthermore, in a complex manufacture or experiment, it is not always easy to see which measurement uncertainty has the biggest influence on the final outcome.

Anyone involved in manufacturing, or in experimental work, should understand experimental uncertainties. Appendix E provides details on this topic; there is a selection of problems on this topic at the end of this chapter.

1.6 Summary

In this chapter we introduced or reviewed a number of basic concepts and definitions, including:

✓ How fluids are defined, and the no-slip condition
✓ System/control volume concepts
✓ Lagrangian and Eulerian descriptions
✓ Units and dimensions (including SI, British Gravitational, and English Engineering systems)
✓ Experimental uncertainty

PROBLEMS

Definition of a Fluid: Basic Equations

1.1 Give a word statement of each of the five basic conservation laws stated in Section 1.4, as they apply to a system.

Methods of Analysis

1.2 Discuss the physics of skipping a stone across the water surface of a lake. Compare these mechanisms with a stone as it bounces after being thrown along a roadway.

1.3 Make a guess at the order of magnitude of the mass (e.g., 0.01, 0.1, 1.0, 10, 100, or 1000 lbm or kg) of standard air that is in a room 3 m by 3 m by 2.4 m, and then compute this mass in kg to see how close your estimate was.

1.4 A spherical tank of inside diameter 500 cm contains compressed oxygen at 7 MPa and 25°C. What is the mass of the oxygen?

1.5 Very small particles moving in fluids are known to experience a drag force proportional to speed. Consider a particle of net weight \( W \) dropped in a fluid. The particle experiences a drag force, \( F_D = kV \), where \( V \) is the particle speed. Determine the time required for the particle to accelerate from rest to 95 percent of its terminal speed, \( V_t \), in terms of \( k, W, \) and \( g \).

1.6 Consider again the small particle of Problem 1.5. Express the distance required to reach 95 percent of its terminal speed in percent terms of \( g, k, \) and \( W \).

1.7 A cylindrical tank must be designed to contain 5 kg of compressed nitrogen at a pressure of 200 atm (gage) and 20°C. The design constraints are that the length must be twice the diameter and the wall thickness must be 0.5 cm. What are the external dimensions?

1.8 In a combustion process, gasoline particles are to be dropped in air at 93°C. The particles must drop at least 25 cm in 1 s. Find the diameter \( d \) of droplets required for this. (The drag on these particles is given by \( F_D = 3\pi\mu Vd \), where \( V \) is the particle speed and \( \mu \) is the air viscosity. To solve this problem, use Excel’s Goal Seek.)

1.9 For a small particle of styrofoam (16 kg/m³) (spherical, with diameter \( d = 0.3 \) mm) falling in standard air at speed \( V \), the drag is given by \( F_D = 3\pi\mu Vd \), where \( \mu \) is the air viscosity. Find the maximum speed starting from rest, and the time it takes to reach 95 percent of this speed. Plot the speed as a function of time.

1.10 In a pollution control experiment, minute solid particles (typical mass \( 5 \times 10^{-11} \) kg) are dropped in air. The terminal speed of the particles is measured to be 5 cm. The drag of these particles is given by \( F_D = kV \), where \( V \) is the instantaneous particle speed. Find the value of the constant \( k \). Find the time required to reach 99 percent of terminal speed.

1.11 A sky diver with a mass of 70 kg jumps from an aircraft. The aerodynamic drag force acting on the sky diver is known to be \( F_D = kV^2 \), where \( k = 0.25 \) N·s²/m⁴. Determine the maximum speed of free fall for the sky diver and the speed reached after 100 m of fall. Plot the speed of the sky diver as a function of time and as a function of distance fallen.

1.12 For Problem 1.11, the initial horizontal speed of the sky diver is 70 m/s. As she falls, the \( k \) value for the vertical drag remains as before, but the value for horizontal motion is \( k = 0.05 \) N·s/m². Compute and plot the 2D trajectory of the sky diver.

Dimensions and Units

1.13 For each quantity listed, indicate dimensions using mass as a primary dimension, and give typical SI and English units:

(a) Power
(b) Pressure
(c) Modulus of elasticity
(d) Angular velocity
Energy in a canal made from unfinished concrete, value, but with

Derive the following conversion factors:

An important equation in the theory of vibrations is

Convert a speed of 30 m/s

What would be suitable units for

is the mass and specific gravity of the mercury.

F ft, the channel slope $S_0$, and a Manning resistance coefficient constant value $n \approx 0.014$. For a canal with $R_e = 7.5$ m and a slope of 1/10, find the flow speed. Compare this result with that obtained using the same $n$ value, but with $R_e$ first converted to ft, with the answer assumed to be in ft/s. Finally, find the value of $n$ if we wish to correctly use the equation for BG units (and compute V to check!).

The density of tetrabromomethane is 2950 kg/m$^3$. Calculate the specific volume in m$^3$/kg and specific gravity of the mercury.

Calculate the specific weight in N/m$^3$ on Earth and on the Mars. Acceleration of gravity on the moon is 3.7 ft/s$^2$.

The maximum theoretical flow rate (kg/s) through a supersonic nozzle is

$$m_{\text{max}} = 2.38 \frac{A_0 \rho_0}{\sqrt{T_0}}$$

where $A_0$ (m$^2$) is the nozzle throat area, $\rho_0$ (Pa) is the tank pressure, and $T_0$ (K) is the tank temperature. Is this equation dimensionally correct? If not, find the units of the 2.38 term.

In Chapter 9 we will study aerodynamics and learn that the drag force $F_D$ on a body is given by

$$F_D = \frac{1}{2} \rho V^2 AC_D$$

Hence the drag depends on speed $V$, fluid density $\rho$, and body size (indicated by frontal area $A$ and shape (indicated by drag coefficient $C_D$). What are the dimensions of $C_D$?

A container weighs 15.5 N when empty. When filled with water at 32°C, the mass of the container and its contents is 36.5 kg. Find the weight of water in the container, and its volume in cubic meters, using data from Appendix A.

An important equation in the theory of vibrations is

$$m \frac{d^2x}{dt^2} + c \frac{dx}{dt} + kx = f(t)$$

where $m$ (kg) is the mass and $x(m)$ is the position at time $t(s)$. For a dimensionally consistent equation, what are the dimensions of $c$, $k$, and $f$? What would be suitable units for $c$, $k$, and $f$ in the SI and BG systems?

A parameter that is often used in describing pump performance is the specific speed, $N_{sp}$, given by

$$N_{sp} = \frac{N(\text{rpm}) Q(\text{gpm})^{1/2}}{H(\text{ft})^{1/4}}$$

What are the units of specific speed? A particular pump has a specific speed of 3000. What will be the specific speed in SI units (angular velocity in rad/s)?
1.31 A particular pump has an “engineering” equation form of the performance characteristic equation given by \( H(m) = 0.46 - 9.57 \times 10^{-7} Q[\text{Lit/min}^2] \), relating the head \( H \) and flow rate \( Q \). What are the units of the coefficients 1.5 and \( 4.5 \times 10^{-5} \)? Derive an SI version of this equation.

**Analysis of Experimental Error**

1.32 Calculate the density of standard air in a laboratory from the ideal gas equation of state. Estimate the experimental uncertainty in the air density calculated for standard conditions (101.3 kPa and 15°C) if the uncertainty in measuring the barometer height is ±2.5 mm, of mercury and the uncertainty in measuring temperature is ±0.3°C.

1.33 Repeat the calculation of uncertainty described in Problem 1.32 for air in a hot air balloon. Assume the measured barometer height is 759 mm of mercury with an uncertainty of ±1 mm of mercury and the temperature is 60°C with an uncertainty of ±1°C. [Note that 759 mm of mercury corresponds to 101 kPa (abs).]

1.34 The mass of the standard American golf ball is 45.4 ± 0.3 g and its mean diameter is 43 ± 0.25 mm. Determine the density and specific gravity of the American golf ball. Estimate the uncertainties in the calculated values.

1.35 A can of pet food has the following internal dimensions: 105 mm height and 75 mm diameter (each ±1 mm at odds of 20 to 1). The label lists the mass of the contents as 398 g. Evaluate the magnitude and estimated uncertainty of the density of the pet food if the mass value is accurate to ±1 g at the same odds.

1.36 An inner cylinder of radius 0.2 m rotates concentrically inside a rigid cylinder of radius 0.202 m, and the height of both the cylinders are 0.4 m. It is known that a momentum of 2 N·m is required to manage an angular velocity of 32.6 revolution per second. Calculate the liquid viscosity used between the cylinders.

1.37 The mass of the standard British golf ball is 52.1 ± 0.3 g and its mean diameter is 43.1 ± 0.3 mm. Determine the density and specific gravity of the British golf ball. Estimate the uncertainties in the calculated values.

1.38 The estimated dimensions of a soda can are \( D = 66.0 \pm 0.5 \text{ mm} \) and \( H = 110 \pm 0.5 \text{ mm} \). Measure the mass of a full can and an empty can using a kitchen scale or postal scale. Estimate the volume of soda contained in the can. From your measurements estimate the depth to which the can is filled and the uncertainty in the estimate. Assume the value of SG = 1.055, as supplied by the bottler.

1.39 Using the nominal dimensions of the soda can given in Problem 1.38, determine the precision with which the diameter and height must be measured to estimate the volume of the can within an uncertainty of ±0.5 percent.

1.40 An enthusiast magazine publishes data from its road tests on the lateral acceleration capability of cars. The measurements are made using a 46-m diameter skid pad. Assume the vehicle path deviates from the circle by ±0.6 m and that the vehicle speed is read from a fifth-wheel speed-measuring system to ±0.8 km/h. Estimate the experimental uncertainty in a reported lateral acceleration of 0.7 g. How would you improve the experimental procedure to reduce the uncertainty?

1.41 The height of a building may be estimated by measuring the horizontal distance to a point on the ground and the angle from this point to the top of the building. Assuming these measurements are \( L = 30 \pm 0.15 \text{ m} \) and \( \theta = 30 \pm 0.2^\circ \), estimate the height \( H \) of the building and the uncertainty in the estimate. For the same building height and measurement uncertainties, use Excel’s Solver to determine the angle (and the corresponding distance from the building) at which measurements should be made to minimize the uncertainty in estimated height. Evaluate and plot the optimum measurement angle as a function of building height for \( 15 \leq H \leq 300 \text{ m} \).

1.42 A syringe pump is to dispense liquid at a flow rate of 100 mL/min. The design for the piston drive is such that the uncertainty of the piston speed is 0.0025 cm/min, and the cylinder bore diameter has a maximum uncertainty of 0.00125 cm. Plot the uncertainty in the flow rate as a function of cylinder bore. Find the combination of piston speed and bore that minimizes the uncertainty in the flow rate.