1 Reproducibility, Objectivity, Invariance
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Abstract. The independent reproducibility of experiments and their results, for instance, the determination of fundamental constants of nature, belongs to the most important conditions for the possibility of nomological empirical science. Two aspects are particularly significant: (1) laws of nature could not be empirically tested and confirmed if experiments and measurements were not reproducible, and (2) the independent reproducibility of experiments and measurements guarantees the objectivity of scientific results that otherwise would amount to mere subjective beliefs of individual researchers, neither comprehensible nor verifiable by others.
The nomological character and the intersubjectivity of propositions in science share a common basic root, namely, the technical applicability of research – and thereby control over nature as one of the most significant goals of modern occidental science. This goal hinges on the requirement of reproducibility. It will be discussed which structural features of our theories, e.g., conservation laws, actually are consequences of the demand that research results be reproducible.

1.1 Introduction

Results of scientific investigations must be reproducible. This is what scientists expect from their findings. It is also what laypeople expect, however vaguely. How can this requirement and expectation be justified?

To demand that scientific results should be reproducible is not to demand something minor. The reproducibility of scientific results is at the core of every scientific inquiry. Science lays claim to truth.\textsuperscript{1} Science cannot give up this claim to truth without giving up itself. However, the concept of truth is to be understood in science or in the individual sciences, their claim to truth would be amiss if every researcher in one and the same scientific discipline would take something different to be true. Instead of the objective truth of scientific propositions, we would merely have subjective opinions of individual scientists. Without sufficient intersubjective consensus among the representatives of a scientific discipline, it would be impossible to uphold a claim to objective truth in science.

What is a sufficient intersubjective consensus in the context of philosophy of science? Science distinguishes itself by trying to justify its claims about what

\textsuperscript{1}For the following considerations compare Tetens (2013), especially pp. 17–28.
is the case in the world. The methods and procedures of justification can differ substantially across individual scientific disciplines. This plurality of methods of justification is grounded in the fact that the different sciences deal with vastly different segments of reality (cf. Tetens 2013, pp. 34–38). For the sciences, in a broad sense of the term, also include the social sciences and even many of the humanities.

But all methods of justification share the feature that different scientists or scientific teams can apply a method of justification to the pertinent matters of their discipline over and over again. And here, only one principle is constitutive: If scientists or scientific teams independently and correctly apply the same method of justification to the same subject matter, then they must reach the same conclusions about this subject matter. This is what the intersubjective reproducibility of scientific results means in general. In order to hold its results to be true, a scientific discipline must fulfil this principle categorically.

 Aren’t scientists often in disagreement? Isn’t the principle of the intersubjective reproducibility of scientific results often unfulfilled? This is certainly the case, in some scientific disciplines to a greater, in others to a smaller extent. But as long as scientists are in disagreement, as long as scientific propositions are disputed because researchers cannot comprehend their justifications or because they achieve different results, one cannot reasonably raise a claim to scientific truth. Scientists will then try to make their propositions more precise, to modify and improve their methods, and to scrupulously control the correct application of these methods to reach an unanimously accepted result.

 Where this does not work out, they will at least have to agree that they disagree, where they disagree and why they disagree – especially whether the disagreement has a foundation in the subject matter itself and in the difficulties of approaching it with the established methods. But this shows: The principle of the intersubjective reproducibility of scientific results is generally accepted, and the lack of its fulfilment must remain an exception. Even the exceptions (e.g., anomalies, see Atmanspacher 2009), always annoying and problematic for science, are dealt with in a way that itself tries to live up to the principle of reproducibility.

1.2 Reproducibility in the Empirical Sciences

As mentioned above, the principle of the intersubjective reproducibility of scientific results must certainly be spelled out differently for the different sciences and their different methods of justification. And it should turn out that the principle can be redeemed more or less strictly in the different sciences. It looks
very different in mathematics compared with molecular genetics, and again very different compared with history. But however limited the intersubjective reproducibility of the results of a particular science may be, no discipline can wholly renounce the reproducibility of its results without seeing its claim to objective truth dissolve and, as a consequence, disappearing as a science.

Since there is not much more to be said in general about the principle of intersubjective reproducibility, let us turn to a special class of sciences that, however, count among the most important ones: the empirical laboratory sciences. Here, the demand of the reproducibility of results takes a special and an especially important form. First, natural processes are experimented with, and, second, characteristics of natural processes are measured in experiments. I assume that readers are sufficiently familiar with the fundamental structural features of experiments and measurements, and will therefore not comment on those here.\(^2\)

But there is one question concerning the empirical sciences which I must briefly touch upon. Why has the measuring experiment become such an important method in the sciences? There is a general answer to this question which can be well supported by the history of science (see Tetens 2013, pp. 28–34). What happens in a measuring experiment is structurally the same as in our efforts to technically control natural processes and thereby to redesign nature with regard to human purposes. If we have experimentally investigated a natural process successfully, then we know how we can technically create a corresponding process with the aid of artificial devices. Every successful scientific experiment provides a prototype of such a device, with which we can change and manipulate the process concerned with respect to certain parameters.

We can only act directly and purposefully if we can successfully plan our actions in advance. To do so, we must know in advance under which circumstances we generate which effects with which action. Such knowledge is produced, among other things, in scientific experiments. Every kind of experiment has the goal to make sure that, under the same boundary and initial conditions, executing the same action leads to the same results. In other words: The principle of the reproducibility of scientific results is indispensable from the point of view of our technical control over nature. Insofar as we want to learn how certain natural processes can be technically controlled, we have to investigate them experimentally in the laboratory (see also Zimmerli, this volume).

Tailoring the principle of the intersubjective reproducibility of scientific results to the empirical laboratory sciences means that scientists must be able to independently reproduce measurements and their results in the context of exper-

\(^2\)See Tetens (1987) and Tetens (2006) for more extensive and detailed discussion.
iments. We can, therefore, formulate the general principle of the intersubjective reproducibility of scientific results as follows: *Under the same boundary and initial conditions, an experiment must always proceed in the same way with respect to the quantitative parameters measured within its measurement accuracy.*

The principle of the intersubjective reproducibility of scientific results only gives a necessary condition for empirical laboratory science. If it were not satisfied, then laboratory science would be impossible with objective results, so it would be impossible at all. Therefore, we know *a priori* that the principle of reproducibility is mandatory for any successful empirical laboratory science.

### 1.3 Objectivity

The principle of the intersubjective reproducibility of scientific results has a truth-theoretic aspect that aims at the *objectivity* of scientific results. This truth-theoretic aspect is supplemented by the technological aspect mentioned above. This supplementation means that both aspects can be distinguished, but do not exclude each other. Instead, both are significant features in empirical laboratory science. This will be the topic of this section.

What follows from the fulfilment of the principle of reproducibility, i.e., if experiments and their results can be successfully reproduced? Wherever this principle can be redeemed for a specific experiment or a class of experiments, this experimental practice corresponds to true propositions of the form: Whenever boundary and initial conditions of the kind $U$ fix certain results of a measurement by particular values at an initial time and location, then after some spatial and/or temporal shift, one measures particular (other) values as the results of a measurement. Such general propositions can be called experimental-scientific laws of nature. Now we see that the reproducibility of experiments in science and empirically adequate experimental laws of nature are merely two different sides of one and the same scientific effort.

At this point, we have to turn to the fact that experiments do not always succeed in reproducing certain values for measurement results. This holds es-

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3 For a comprehensive account of the concept of scientific objectivity, how it should be defined, whether it is desirable, and to what extent scientists can achieve it, see Reiss and Sprenger (2014).

4 Compare Giambattista Vico’s famous statement that the true and the made are convertible: *verum et factum convertuntur*. See Miner (1998) for more discussion.

5 A proposition $L$ which involves an experimental law of nature is empirically adequate if and only if, when $B_1, \ldots, B_n$ are true observation statements that entail, together with $L$, the observation statement $B$, then $B$ is also true.
especially for a large class of experiments in quantum mechanics, where one can only reproduce the relative frequencies of different measurements (though with impressive reliability). If not even relative frequencies were reproducible, this would be the end of physics as we know it. Furthermore, there are three things to note in the context of quantum mechanics.

1. If physicists could choose between experiments that reproduce fixed measurements and those that only reproduce relative frequencies of measurements reliably, they would choose experiments of the first kind. The reason is obvious given the goal of controlling nature.

2. Quantum mechanics also fulfils the general principle of the intersubjective reproducibility of scientific results, yet in a slightly modified form: Under the same boundary and initial conditions, the probabilities of the different measurement results are always the same. The probabilities of the measurement results are reproducible.

3. The principle of the reproducibility of quantum experiments and their results depends on the strict reproducibility of the boundary and initial conditions under which they are repeatedly executed in order to gradually stabilize the relative frequencies.

### 1.4 Invariance and Symmetry

After this excursion into quantum mechanics, let us return to more general considerations. Every experiment is defined by certain boundary and initial conditions that experimenters can reproduce in principle. Certain realizations of a measurement \( m_1, ..., m_n \) yield certain reproducible values depending on each other, on time \( t \), and on location \( x, y, z \).

If the course of an experiment can be reproduced under the boundary and initial conditions \( U_i \), then it can in principle be described by an equation of the form \( f_i(m_1, ..., m_n, x, y, z, t) = 0 \), where \( f \) characterizes the state of the system.

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\(^6\) This is an aspect which especially the proponents of the so-called Copenhagen interpretation of quantum mechanics have been stressing. Quantum mechanics relies on experiments in which certain values of classical measurements are registered using devices that have been and must have been reliably built and reproduced in the framework of classical physics.

\(^7\) In quantum mechanics, the probabilities of the values of measurement results must be the same. This entails that in long series of tests, the relative frequencies of the different measurements deviate the less from the probabilities, the longer one continues the series of tests. The distribution converges in the limit of infinitely many tests (see also Stahel, this volume). In what follows, we shall leave the peculiarities of quantum mechanics aside.
A further aim in science is to subsume as many different experiments as possible by deducing the respective functions \( f_i(m_1, ..., m_n, x, y, z, t) = 0 \), the state evolution, from the same differential equation or system of differential equations. The different boundary and initial conditions \( U_i \) classifying the experiments provide the parameters for determining the special function describing the state evolution from the general family of solutions of the differential equations. In this way, reproducible experiments correspond to (systems of) differential equations, the solutions of which quantitatively describe the courses and the results of the experiments.\(^8\)

Is the reproducibility of experiments reflected in corresponding laws which, as just elaborated, are to be expressed as differential equations? Well, regardless of when and where one experiments, whenever the relevant boundary and initial conditions are created, an experiment always proceeds in the same way. This is what it means that experiments can be reproduced. But this also means that the differential equations, the solutions of which describe the processes of the reproducible experiments, do not change under temporal translations, movements of the spatial coordinate system or rotations of its axes. Technically speaking: The differential equations must be invariant with respect to temporal and spatial translations and spatial rotations.

At this point, a much more general connection between the reproducibility of scientific results and their objectivity emerges. For the reproduction of an experiment, it must not matter where and when it is performed. One can also express this as follows: The result of an experiment must not depend on where and when the experimenters are situated. More generally: The scientific results must not differ arbitrarily from experimenter to experimenter, from observer to observer, from scientist to scientist. Otherwise, the results would lose their objective liability. Put the other way around: The less scientific results depend on and are influenced by differences between researchers, the more objective are they and the better can they be reproduced intersubjectively.

In the empirical sciences, most distinctly in physics, this is reflected by invariance principles. Most basic among them is the invariance of physical laws under temporal translations and spatial translations and rotations. But physics does not stop here. It is not only the time and location of observers that ought not influence the results but also the state of motion of the observers. This independence from the state of motion leads to the different versions of the

\(^8\)It would take us too far afield to elaborate more closely on the overwhelming success in describing different reproducible experiments and their processes uniformly by deducing the process functions from one and the same differential equation or system of differential equations. For the case of classical mechanics, this is elaborated more extensively in Tetens (1987).
principle of relativity, which plays a crucial role in classical mechanics and in the special and general theory of relativity.\footnote{In the special theory of relativity, this is the principle: If the laws of relativistic mechanics and Maxwell’s equations in electrodynamics apply in a reference frame \( I \), and \( I' \) is a reference frame which is in rectilinear and uniform motion with respect to \( I \), then the same laws of relativistic mechanics and Maxwell’s equations in electrodynamics apply in \( I' \).}

Another example is the cosmological principle in astrophysics, according to which the universe appears the same from every reference point. And there are many other invariances and symmetries without which physics would not be the same. From a mathematical point of view, they are all connected to corresponding conservation laws. The close connection between invariances, conservation laws, and symmetries can be formulated mathematically: A relation \( R \), defined on a set \( S \) of objects, is an equivalence relation (cf. Atmanspacher, this volume) if there is a mathematical group \( G \) of transformations \( T : S \rightarrow S \) from the set \( S \) onto itself such that two elements \( x, y \in S \) are mapped onto one another by \( T \). If a property stays the same (is invariant) under certain changes (transformations), this is called a \textit{symmetry}.\footnote{For some more background about symmetries and invariance principles in physics compare Scheibe (2001, Part VII), Wigner (1979, Secs. 1–5), or Weyl (1952).}

We have seen that some important symmetries or invariance principles can be derived immediately from the principle of the reproducibility of experimental results. Other symmetries are at least indirectly connected with the fundamental requirement of reproducibility. Generally speaking, symmetries can be identified by equivalence relations \( R \), which leave a property unchanged under the transformation of some variable.

If certain symmetries hold for processes in nature, then certain changes do not affect these processes and do not have to be considered, controlled and eliminated in the technical–empirical generation of these processes. The less circumstances must be considered, controlled, or, if necessary, eliminated in order to technically generate a process, the easier it can be reproduced technically–empirically.

\subsection*{1.5 Summary}

A natural science aimed at objectivity one the one hand and at technical applications on the other is an empirical laboratory science.

If it is successful, then, for methodological reasons, it necessarily fulfils the fundamental principle of the reproducibility of its results: \textit{Under the same boundary and initial conditions, an experiment must always proceed in the same way}
with respect to the quantitative parameters measured within its measurement accuracy.

Fulfilling this principle, the experimental investigation of nature in the laboratory will in the long run produce theories in which natural processes are described by differential equations that satisfy symmetry and invariance principles.

Some of these invariance principles can be directly deduced from the fundamental demand of the reproducibility of scientific experiments, while others support the reproducibility of experiments at least indirectly.

References


