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CHARACTERISTICS OF AIRCRAFT STRUCTURES AND MATERIALS

1.1 INTRODUCTION

The main difference between aircraft structures and materials and civil engineering structures and materials lies in their weight. The main driving force in aircraft structural design and aerospace material development is to reduce weight. In general, materials with high stiffness, high strength, and light weight are most suitable for aircraft applications.

Aircraft structures must be designed to ensure that every part of the material is used to its full capability. This requirement leads to the use of shell-like structures (monocoque constructions) and stiffened shell structures (semimonocoque constructions). The geometrical details of aircraft structures are much more complicated than those of civil engineering structures. They usually require the assemblage of thousands of parts. Technologies for joining the parts are especially important for aircraft construction.

The size and shape of an aircraft structural component are usually determined based on nonstructural considerations. For instance, the airfoil is chosen according to aerodynamic lift and drag characteristics. Then the solutions for structural problems in terms of global configurations are limited. Often, the solutions resort to the use of special materials developed for applications in aerospace vehicles.

Because of their high stiffness/weight and strength/weight ratios, aluminum and titanium alloys have been the dominant aircraft structural materials for many decades. However, the recent advent of advanced fiber-reinforced composites has changed the outlook. Composites may now
achieve weight savings of 30 to 40 percent over aluminum or titanium counterparts. As a result, composites have been used increasingly in aircraft structures. Figure 1.1 shows the key materials on the Boeing-McDonnell-Douglas F/A–18E fighter jet. On the latest Boeing commercial airliner, the 787, composites account for up to 50 percent of structural weight.

1.2 BASIC STRUCTURAL ELEMENTS IN AIRCRAFT STRUCTURE

Major components of aircraft structures are assemblages of a number of basic structural elements, each of which is designed to take a specific type of load, such as axial, bending, or torsional load. Collectively, these elements can efficiently provide the capability for sustaining loads on an airplane. The governing equations for these basic structural elements are introduced in the first course in mechanics of solids. In the following subsections, the governing equations are reviewed briefly and their behavior discussed.

1.2.1 Axial Member

Axial members are used to carry extensional or compressive loads applied in the direction of the axial direction of the member. The resulting stress is uniaxial:

$$\sigma = E \varepsilon$$  

(1.1)

where $E$ and $\varepsilon$ are the Young’s modulus and normal strain, respectively, in the loading direction. The total axial force $F$ provided by the member is

$$F = A \sigma = EA \varepsilon$$  

(1.2)

where $A$ is the cross-sectional area of the member. The quantity $EA$ is termed the axial stiffness of the member, which depends on the modulus of the material and the cross-sectional area of the member. It is obvious that the axial stiffness of axial members cannot be increased (or decreased) by changing the shape of the cross-section. In other words, a circular rod and a channel (see Figs. 1.2a and 1.2b) can carry the same axial load as long as they have the same cross-sectional area.

Axial members are usually slender and are susceptible to buckling failure when subjected to compression. Buckling strength can be increased by increasing the bending stiffness and by shortening the length of the buckle mode. For buckling, the channel section is better since it has higher bending stiffness than the circular section. However, because of the slenderness
Figure 1.1 Materials used on Boeing-McDonnell-Douglas F/A–18E jet. (Courtesy of the Boeing Company.)
of most axial members used in aircraft (such as stringers), the bending stiffness of these members is usually very small and is not sufficient to achieve the necessary buckling strength. In practice, the buckling strength of axial members is enhanced by providing lateral supports along the length of the member with more rigid ribs (in wings) and frames (in fuselage).

### 1.2.2 Shear Panel

A shear panel is a thin sheet of material used to carry in-plane shear load. Consider a shear panel of uniform thickness $t$ under uniform shear stress $\tau$ as shown in Fig. 1.3. The total shear force in the $x$-direction provided by the panel is given by

$$V_x = \tau t a = G \gamma t a$$

(1.3)

where $G$ is the shear modulus, and $\gamma$ is the shear strain. Thus, for a flat panel, the shear force $V_x$ is proportional to its thickness and the lateral dimension $a$.

For a curved panel under a state of constant shear stress $\tau$ (see Fig 1.4), the resulting shear force of the shear stress on the thin-walled section may be

![Figure 1.3](image-url)  
**Figure 1.3** Shear panel under uniform shear stress.
decomposed into a horizontal component $V_x$ and a vertical component $V_y$ as

$$V_x = \tau ta$$  \hspace{1cm} (1.4)$$
$$V_y = \tau tb$$  \hspace{1cm} (1.5)$$

Thus, the components of the resultant force of the shear stress $\tau$ have the relation

$$\frac{V_x}{V_y} = \frac{a}{b}$$

Since this relation does not depend on the contour shape of the section of the panel, a flat panel would be the most efficient (in material usage) in providing a shear force for given values of $a$ and $b$.

1.2.3 Bending Member (Beam)

A structural member that can carry bending moments is called a beam. A beam can also act as an axial member carrying longitudinal tension and compression. According to simple beam theory, bending moment $M$ is related to beam deflection $w$ as

$$M = -EI \frac{d^2 w}{dx^2}$$  \hspace{1cm} (1.6)$$

where $EI$ is the bending stiffness of the beam. The area moment of inertia $I$ depends on the geometry of the cross-section.

Except for pure moment loading, a beam is designed to carry both bending moments and transverse shear forces as the latter usually produce the former. For a beam of a large span/depth ratio, the bending stress is usually more
critical than the transverse shear stress. This is illustrated by the example of a cantilever beam shown in Fig. 1.5.

It is easy to see that the maximum bending moment and bending stress occur at the fixed root of the cantilever beam. We have

$$\sigma_{\text{max}} = \frac{M_{\text{max}}(h/2)}{I} = \frac{VL(h/2)}{bh^2/12} = \frac{6VL}{bh^2}$$ (1.7)

The transverse shear stress distribution is parabolic over the beam depth with maximum value occurring at the neutral plane, i.e.,

$$\tau_{\text{max}} = \frac{3V}{2bh}$$ (1.8)

From the ratio

$$\frac{\sigma_{\text{max}}}{\tau_{\text{max}}} = \frac{4L}{h}$$ (1.9)

it is evident that bending stress plays a more dominant role than transverse shear stress if the span-to-depth ratio is large (as in wing structure). For such beams, attention is focused on optimizing the cross-section to increase bending stiffness.

In the elastic range, bending stress distribution over depth is linear with maximum values at the farthest positions from the neutral axis. The material near the neutral axis is underutilized. Thus, the beam with a rectangular cross-section is not an efficient bending member.

In order to utilize the material to its full capacity, material in a beam must be located as far as possible from the neutral axis. An example is the wide flange beam shown in Fig. 1.6a. Although the bending stress distribution is still linear over the depth, the bending line force (bending stress times the width) distribution is concentrated at the two flanges as shown in Fig. 1.6b because \( b \gg t_w \). For simplicity, the small contribution of the vertical web to bending can be neglected.

The transverse shear stress distribution in the wide flange beam is shown in Fig. 1.6c. The vertical web is seen to carry essentially all the transverse shear stress.
load; its variation over the web is small and can be practically assumed to be constant. For all practical purposes, the wide flange beam can be regarded as two axial members (flanges) connected by a flat shear panel.

1.2.4 Torsion Member

Torque is an important form of load to aircraft structures. In a structural member, torque is formed by shear stresses acting in the plane of the cross-section. Consider a hollow cylinder subjected to a torque $T$ as shown in Fig. 1.7. The torque-induced shear stress $\tau$ is linearly distributed along the radial direction. The torque is related to the twist angle $\theta$ per unit length as

$$T = GJ\theta$$

(1.10)
where $J$ is the torsional constant. For hollow cylinders, $J$ is equal to the polar moment of inertia of the cross-section, i.e.,

$$J = I_p = \frac{1}{2} \pi (b^4 - a^4) = \frac{1}{2} \pi (b - a)(b + a)(b^2 + a^2) \quad (1.11)$$

The term $GJ$ is usually referred to as torsional stiffness.

If the wall thickness $t = b - a$ is small compared with the inner radius, then an approximate expression of $J$ is given by

$$J = 2t \pi \bar{r}^3 \quad (1.12)$$

where $\bar{r} = (a + b)/2$ is the average value of the outer and inner radii. Thus, for a thin-walled cylinder, the torsional stiffness is proportional to the $3/2$ power of the area ($\pi \bar{r}^2$) enclosed by the wall.

Note that the material near the inner cavity in a thick-walled cylinder is underutilized. It is obvious that a thin-walled tube would be more efficient for torques than a solid cylinder or a thick-walled hollow cylinder. Figure 1.8 shows the cross-sections of a solid cylinder (Fig. 1.8a) and a tube (Fig. 1.8b), both having the same amount of material. Using (1.11) or (1.12), it is easy to show that the torsional stiffness of the tube is almost 50 times that of the solid cylinder. This example illustrates that a thin-walled structure can be made into a very efficient torsion member.

1.3 WING AND FUSELAGE

The wing and fuselage are the two major airframe components of an airplane. The horizontal and vertical tails bear close resemblance to the wing. Hence, these two components are taken for discussion to exemplify the principles of structural mechanics employed in aircraft structures.
1.3.1 Load Transfer

Wing and fuselage structures consist of a collection of basic structural elements. Each component, as a whole, acts like a beam and a torsion member. For illustrative purposes, let us consider the box beam shown in Fig. 1.9. The box beam consists of stringers (axial members) that are located at the maximum allowable distance from the neutral axis to achieve the most bending capability, and the thin skin (shear panel), which encloses a large area to provide a large torque capability. The design of Fig. 1.9 would be fine if the load is directly applied in the form of global torque $T$ and bending moment $M_x$. In reality, aircraft loads are in the form of air pressure (or suction) on the skin, concentrated loads from the landing gear, power plants, passenger seats, etc. These loads are to be “collected” locally and transferred to the major load-carrying members. Without proper care, these loads may produce excessive local deflections that are not permissible from aerodynamic considerations.

Using the box beam of Fig. 1.9 as an example, we assume that a distributed air pressure is applied on the top and bottom surfaces of the beam. The skin (shear panel) is thin and has little bending stiffness to resist the air pressure. To avoid incurring large deflections in the skin, longitudinal stringers (stiffeners) can be added, as shown in Fig. 1.10, to pick up the air loads. These stiffeners are usually slender axial members with a moderate amount of bending stiffness. Therefore, the transverse loads picked up by the stiffeners must be transferred “quickly” to more rigid ribs or frames at sections A and B (see Fig. 1.9) to avoid excessive deflections. The ribs collect all transverse loads from the stiffeners and transfer them to the two wide-flange beams (spars) that are designed to take transverse shear loads.
The local-to-global load transfer is thus complete. Note that besides serving as a local load distributor, the stiffeners also contribute to the total bending capability of the box beam.

### 1.3.2 Wing Structure

The main function of the wing is to pick up the air and power plant loads and transmit them to the fuselage. The wing cross-section takes the shape of an airfoil, which is designed based on aerodynamic considerations. The wing as a whole performs the combined function of a beam and a torsion member. It consists of axial members in stringers, bending members in spars and shear panels in the cover skin and webs of spars. The spar is a heavy beam running spanwise to take transverse shear loads and spanwise bending. It is usually composed of a thin shear panel (the web) with a heavy cap or flange at the top and bottom to take bending. A typical spar construction is depicted in Fig. 1.11. A multiple-spar wing construction is shown in Fig. 1.1.

Wing ribs are planar structures capable of carrying in-plane loads. They are placed chordwise along the wing span. Besides serving as load redistributors, ribs also hold the skin stringer to the designed contour shape. Ribs
reduce the effective buckling length of the stringers (or the stringer-skin system) and thus increase their compressive load capability. Figure 1.12 shows a typical rib construction. Note that the rib is supported by spanwise spars.

The cover skin of the wing together with the spar webs form an efficient torsion member. For subsonic airplanes, the skin is relatively thin and may be designed to undergo postbuckling. Thus, the thin skin can be assumed to make no contribution to bending of the wing box, and the bending moment is taken by spars and stringers. Figure 1.13 presents two typical wing cross-sections for two-spar subsonic aircraft. One type (Fig. 1.13a) consists only of spars (the concentrated flange type) to take bending. The other type (the distributed flange type, Fig. 1.13b) uses both spars and stringers to take bending.

Supersonic airfoils are relatively thin compared with subsonic airfoils. To withstand high surface air loads and to provide additional bending capability of the wing box structure, thicker skins are often necessary. In addition, to increase structural efficiency, stiffeners can be manufactured (either by forging or machining) as integral parts of the skin.

1.3.3 Fuselage

Unlike the wing, which is subjected to large distributed air loads, the fuselage is subjected to relatively small air loads. The primary loads on the fuselage

![Figure 1.13](image_url)  
**Figure 1.13** Typical two-spar wing cross-sections for subsonic aircraft: (a) spars only; (b) spars and stringers.
include large concentrated forces from wing reactions, landing gear reactions, and pay loads. For airplanes carrying passengers, the fuselage must also withstand internal pressures. Because of internal pressure, the fuselage often has an efficient circular cross-section. The fuselage structure is a semimonocoque construction consisting of a thin shell stiffened by longitudinal axial elements (stringers and longerons) supported by many transverse frames or rings along its length; see Fig. 1.14. The fuselage skin carries the shear stresses produced by torques and transverse forces. It also bears the hoop stresses produced by internal pressures. The stringers carry bending moments and axial forces. They also stabilize the thin fuselage skin.

Fuselage frames often take the form of a ring. They are used to maintain the shape of the fuselage and to shorten the span of the stringers between supports in order to increase the buckling strength of the stringer. The loads on the frames are usually small and self-equilibrated. Consequently, their constructions are light. To distribute large concentrated forces such as those from the wing structure, heavy bulkheads are needed.

Figure 1.15 shows the fuselage of a Boeing 777 under construction.

1.4 AIRCRAFT MATERIALS

Traditional metallic materials used in aircraft structures are aluminum, titanium, and steel alloys. In the past three decades, applications of advanced fiber composites have rapidly gained momentum. To date, some new commercial jets, such as the Boeing 787, already contain composite materials up to 50 percent of their structural weight.
Figure 1.5 Fuselage of a Boeing 777 under construction. (Courtesy of the Boeing Company.)
Selection of aircraft materials depends on many considerations which can, in general, be categorized as cost and structural performance. Cost includes initial material cost, manufacturing cost and maintenance cost. The key material properties that are pertinent to maintenance cost and structural performance are

- Density (weight)
- Stiffness (Young’s modulus)
- Strength (ultimate and yield strengths)
- Durability (fatigue)
- Damage tolerance (fracture toughness and crack growth)
- Corrosion

Seldom is a single material able to deliver all desired properties in all components of the aircraft structure. A combination of various materials is often necessary. Table 1.1 lists the basic mechanical properties of some metallic aircraft structural materials.

**Steel Alloys** Among the three metallic materials, steel alloys have highest densities, and are used only where high strength and high yield stress are critical. Examples include landing gear units and highly loaded fittings. The high strength steel alloy 300 M is commonly used for landing gear components. This steel alloy has a strength of 1.9 GPa (270 ksi) and a yield stress of 1.5 GPa (220 ksi).

**TABLE 1.1 Mechanical properties of metals at room temperature in aircraft structures**

<table>
<thead>
<tr>
<th>Material</th>
<th>Property $^a$</th>
<th>$E$ (GPa (msi))</th>
<th>$\nu$</th>
<th>$\sigma_u$ (MPa (ksi))</th>
<th>$\sigma_Y$ (MPa (ksi))</th>
<th>$\rho$ (g/cm$^3$ (lb/in$^3$))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2024-T3</td>
<td></td>
<td>72 (10.5)</td>
<td>0.33</td>
<td>449 (65)</td>
<td>324 (47)</td>
<td>2.78 (0.10)</td>
</tr>
<tr>
<td>7075-T6</td>
<td></td>
<td>71 (10.3)</td>
<td>0.33</td>
<td>538 (78)</td>
<td>490 (71)</td>
<td>2.78 (0.10)</td>
</tr>
<tr>
<td>Titanium</td>
<td></td>
<td>110 (16.0)</td>
<td>0.31</td>
<td>925 (134)</td>
<td>869 (126)</td>
<td>4.46 (0.16)</td>
</tr>
<tr>
<td>Steel</td>
<td></td>
<td>200 (29.0)</td>
<td>0.32</td>
<td>1790 (260)</td>
<td>1483 (212)</td>
<td>7.8 (0.28)</td>
</tr>
<tr>
<td>AISI4340</td>
<td></td>
<td>200 (29.0)</td>
<td>0.32</td>
<td>1860 (270)</td>
<td>1520 (220)</td>
<td>7.8 (0.28)</td>
</tr>
<tr>
<td>300 M</td>
<td></td>
<td>200 (29.0)</td>
<td>0.32</td>
<td>1860 (270)</td>
<td>1520 (220)</td>
<td>7.8 (0.28)</td>
</tr>
</tbody>
</table>

$^a$ $\sigma_u = $ tensile ultimate stress; $\sigma_Y = $ tensile yield stress.
Besides being heavy, steel alloys are generally poor in corrosion resistance. Components made of these alloys must be plated for corrosion protection.

**Aluminum Alloys**  Aluminum alloys have played a dominant role in aircraft structures for many decades. They offer good mechanical properties with low weight. Among the aluminum alloys, the 2024 and 7075 alloys are perhaps the most used. The 2024 alloys (2024-T3, T42) have excellent fracture toughness and slow crack growth rate as well as good fatigue life. The code number following T for each aluminum alloy indicates the heat treatment process. The 7075 alloys (7075-T6, T651) have higher strength than the 2024 but lower fracture toughness. The 2024-T3 is used in the fuselage and lower wing skins, which are prone to fatigue due to applications of cyclic tensile stresses. For the upper wing skins, which are subjected to compressive stresses, fatigue is less of a problem, and 7075-T6 is used.

The recently developed aluminum lithium alloys offer improved properties over conventional aluminum alloys. They are about 10 percent stiffer and 10 percent lighter and have superior fatigue performance.

**Titanium Alloys**  Titanium such as Ti-6Al-4V (the number indicates the weight percentage of the alloying element) with a density of 4.5 g/cm\(^3\) is lighter than steel (7.8 g/cm\(^3\)) but heavier than aluminum (2.7 g/cm\(^3\)). See Table 1.1. Its ultimate and yield stresses are almost double those of aluminum 7075-T6. Its corrosion resistance in general is superior to both steel and aluminum alloys. While aluminum is usually not for applications above 350\(^\circ\)F, titanium, on the other hand, can be used continuously up to 1000\(^\circ\)F.

Titanium is difficult to machine, and thus the cost of machining titanium parts is high. Near net shape forming is an economic way to manufacture titanium parts. Despite its high cost, titanium has found increasing use in military aircraft. For instance, the F-15 contained 26 percent (structural weight) titanium.

**Fiber-Reinforced Composites**  Materials made into fiber forms can achieve significantly better mechanical properties than their bulk counterparts. A notable example is glass fiber versus bulk glass. The tensile strength of glass fiber can be two orders of magnitude higher than that of bulk glass. In this century, fiber science has made gigantic strides, and many high-performance fibers have been introduced. Listed in Table 1.2 are the mechanical properties of some high-performance manufactured fibers.

Fibers alone are not suitable for structural applications. To utilize the superior properties of fibers, they are embedded in a matrix material that holds the fibers together to form a solid body capable of carrying complex loads.
Matrix materials that are currently used for forming composites include three
major categories: polymers, metals, and ceramics. The resulting composites
are usually referred to as polymer matrix composites (PMCs), metal matrix
composites (MMCs), and ceramic matrix composites (CMCs). Table 1.3
presents properties of a list of composites. The range of service temperature
of a composite is often determined by its matrix material. Polymer matrix
composites are usually for lower temperature (less than 300°F) applications,
and ceramic matrix composites are intended for applications in hot (higher
than 1500°F) environments, such as jet engines.

Fiber composites are stiff, strong, and light and are thus most suitable for
aircraft structures. They are often used in the form of laminates that consist of
a number of unidirectional laminae with different fiber orientations to provide
multidirectional load capability. Composite laminates have excellent fatigue
life, damage tolerance, and corrosion resistance. Laminate constructions offer
the possibility of tailoring fiber orientations to achieve optimal structural
performance of the composite structure.

### TABLE 1.2  Mechanical properties of fibers

<table>
<thead>
<tr>
<th>Material</th>
<th>Property</th>
<th>E (GPa (msi))</th>
<th>σ_u (GPa (ksi))</th>
<th>ρ (g/cm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-glass</td>
<td></td>
<td>77.0 (11)</td>
<td>2.50 (350)</td>
<td>2.54</td>
</tr>
<tr>
<td>S-glass</td>
<td></td>
<td>85.0 (12)</td>
<td>3.50 (500)</td>
<td>2.48</td>
</tr>
<tr>
<td>Silicon carbide (Nicalon)</td>
<td></td>
<td>190.0 (27)</td>
<td>2.80 (400)</td>
<td>2.55</td>
</tr>
<tr>
<td>Carbon (Hercules AS4)</td>
<td></td>
<td>240.0 (35)</td>
<td>3.60 (510)</td>
<td>1.80</td>
</tr>
<tr>
<td>Carbon (Hercules HMS)</td>
<td></td>
<td>360.0 (51)</td>
<td>2.20 (310)</td>
<td>1.80</td>
</tr>
<tr>
<td>Carbon (Toray T300)</td>
<td></td>
<td>240.0 (35)</td>
<td>3.50 (500)</td>
<td>1.80</td>
</tr>
<tr>
<td>Boron</td>
<td></td>
<td>385.0 (55)</td>
<td>3.50 (500)</td>
<td>2.65</td>
</tr>
<tr>
<td>Kevlar-49 (Aramid)</td>
<td></td>
<td>130.0 (18)</td>
<td>2.80 (400)</td>
<td>1.45</td>
</tr>
<tr>
<td>Kevlar-29</td>
<td></td>
<td>65.0 (9.5)</td>
<td>2.80 (400)</td>
<td>1.45</td>
</tr>
</tbody>
</table>

### TABLE 1.3  Longitudinal mechanical properties of fiber composites

<table>
<thead>
<tr>
<th>Material</th>
<th>Type</th>
<th>E (GPa (msi))</th>
<th>σ_u (GPa (ksi))</th>
<th>ρ (g/cm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon/epoxy</td>
<td>T300/5208</td>
<td>140.0 (20)</td>
<td>1.50 (210)</td>
<td>1.55</td>
</tr>
<tr>
<td></td>
<td>IM6/3501-6</td>
<td>177.0 (25.7)</td>
<td>2.86 (414)</td>
<td>1.55</td>
</tr>
<tr>
<td></td>
<td>AS4/3501-6</td>
<td>140.0 (20)</td>
<td>2.10 (300)</td>
<td>1.55</td>
</tr>
<tr>
<td>Boron/aluminum</td>
<td>B/Al 2024</td>
<td>210.0 (30)</td>
<td>1.50 (210)</td>
<td>2.65</td>
</tr>
<tr>
<td>Glass/epoxy</td>
<td>S2 Glass/epoxy</td>
<td>43.0 (6.2)</td>
<td>1.70 (245)</td>
<td>1.80</td>
</tr>
<tr>
<td>Aramid/epoxy</td>
<td>Kev 49/epoxy</td>
<td>70.0 (10)</td>
<td>1.40 (200)</td>
<td>1.40</td>
</tr>
</tbody>
</table>
PROBLEMS

1.1 The beam of a rectangular thin-walled section (i.e., \( t \) is very small) is designed to carry both bending moment \( M \) and torque \( T \). If the total wall contour length \( L = 2(a + b) \) (see Fig. 1.16) is fixed, find the optimum \( b/a \) ratio to achieve the most efficient section if \( M = T \) and \( \sigma_{\text{allowable}} = 2\tau_{\text{allowable}} \). Note that for closed thin-walled sections such as the one in Fig. 1.16, the shear stress due to torsion is

\[
\tau = \frac{T}{2abt}
\]

Figure 1.16  Closed thin-walled section.

1.2 Do Problem 1.1 with \( M = \alpha T \) where \( \alpha = 0 \) to \( \infty \).

1.3 The dimensions of a steel (300 M) I-beam are \( b = 50 \text{ mm}, t = 5 \text{ mm}, \) and \( h = 200 \text{ mm} \) (Fig. 1.17). Assume that \( t \) and \( h \) are to be fixed for an aluminum (7075-T6) I-beam. Find the width \( b \) for the aluminum beam so that its bending stiffness \( EI \) is equal to that of the steel beam. Compare the weights-per-unit length of these two beams. Which is more efficient weightwise?

1.4 Use AS4/3501-6 carbon/epoxy composite to make the I-beam as stated in Problem 1.3. Compare its weight with that of the aluminum beam.

1.5 Derive the relations given by (1.4) and (1.5).

1.6 The sign convention (positive directions of resultants) used in the beam theory depends on the coordinate system chosen. Consider the moment–curvature relation

\[
M = -EI \frac{d^2w}{dx^2}
\]
in reference to the coordinate system shown in Fig. 1.18. If \( w \) is regarded as a positive displacement (or deflection) in the positive \( y \)-direction, find the positive direction of the bending moment. State the reason.

1.7 Compare the load-carrying capabilities of two beams having the respective cross-sections shown in Fig. 1.19. Use bending rigidity as the criterion for comparison. It is given that \( a = 4 \) cm, \( t = 0.2 \) cm, and the two cross-sections have the same area.