APPLIED CALCULUS

Sixth Edition

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The inside back cover will contain printing identification and country of origin if omitted from this page. In addition, if the ISBN on the back cover differs from the ISBN on this page, the one on the back cover is correct.
We dedicate this book to Andrew M. Gleason.

His brilliance and the extraordinary kindness and dignity with which he treated others made an enormous difference to us, and to many, many people. Andy brought out the best in everyone.

Deb Hughes Hallett
for the Calculus Consortium
Calculus is one of the greatest achievements of the human intellect. Inspired by problems in astronomy, Newton and Leibniz developed the ideas of calculus 300 years ago. Since then, each century has demonstrated the power of calculus to illuminate questions in mathematics, the physical sciences, engineering, business, and the social and biological sciences.

Calculus has been so successful because of its extraordinary power to reduce complicated problems to simple rules and procedures. Therein lies the danger in teaching calculus: it is possible to teach the subject as nothing but the rules and procedures—thereby losing sight of both the mathematics and of its practical value. This edition of *Applied Calculus* continues our effort to promote courses in which understanding reinforces computation.

**Embracing e-Learning**

Paper books are playing a smaller role in courses than in the past and are being replaced by electronic materials. This Sixth Edition provides opportunities for students to experience the concepts of calculus in ways that would not be possible in a traditional textbook. The enhanced e-text of *Applied Calculus*, powered by VitalSource, provides embedded videos and the complete solutions from the Student Solutions Manual. The enhanced e-text also contains additional content not found in the print edition:

- Worked example videos, which provide students the opportunity to see and hear over one hundred of the book’s examples being explained and worked out in detail, have been created to accompany the sixth edition.
- Strengthen Your Understanding true/false problems that focus on conceptual understanding.
- Appendices that extend ideas in the course.
- Chapter 10, on Geometric Series.
- Chapter summaries, giving a concise overview of each chapter.

**WileyPLUS**

In addition to the enhanced e-text, Students and instructors can access a wide variety of resources through WileyPLUS with ORION, Wiley’s digital learning environment. ORION Learning provides an adaptive, personalized learning experience that delivers easy-to-use analytics so instructors and students can see exactly where they’re excelling and where they need help. WileyPLUS with ORION features the following resources:

- Homework management tools, which enable the instructor to assign questions easily and grade them automatically, using a rich set of options and controls.
- QuickStart pre-designed reading and homework assignments. Use them as-is or customize them to fit the needs of your classroom.
- Intelligent Tutoring questions, in which students are prompted for responses as they step through a problem solution and receive targeted feedback based on those responses.
- Algebra Refresher material, delivered through ORION, provides students with an opportunity to brush up on material necessary to master Calculus, as well as to determine areas that require further review.
- Graphing Calculator Manual, to help students get the most out of their graphing calculator, and to show how they can apply the numerical and graphing functions of their calculators to their study of calculus.

**Flexible Balance: Concepts and Modeling**

The first goal of a calculus course is to acquire a clear intuitive picture of the central ideas. After this foundation has been laid, there is a choice of direction. All students benefit from both mathematical concepts and
modeling, but the balance may differ for different groups of students. For instructors wishing to emphasize the connection between calculus and other fields, the text includes:

- A variety of problems and examples from the biological sciences, economics, and business.
- Models from the health sciences and of population growth.
- Problems on sustainability.
- Case studies on medicine by David E. Sloane, MD.

**Active Learning: Good Problems**

As instructors ourselves, we know that interactive classrooms and well-crafted problems promote student learning. Since its inception, the hallmark of our work has been its innovative and engaging problems. These problems probe student understanding in ways often taken for granted. Praised for their creativity and variety, the influence of these problems has extended far beyond the users of our textbook.

The Sixth Edition continues this tradition. Under our approach, which we called the “Rule of Four,” ideas are presented graphically, numerically, symbolically, and verbally, thereby encouraging students with a variety of learning styles to deepen their understanding. This edition continues to provide a wide variety of problem types:

- *End of Section Problems* reinforce the ideas of that section and make connections with earlier sections.
- *ConcepTests* promote active learning in the classroom. These can be used with or without any polling software, and have been shown to dramatically improve student learning. ConcepTests are particularly useful to instructors teaching in a flipped classroom. ConcepTests are available online for instructors in PPT or PDF format in WileyPLUS or on the Instructor Book Companion site at www.wiley.com/college/hughes-hallett.
- *Chapter Review Problems*, reserved for instructor-only use in WileyPLUS, provide opportunities to review ideas from the whole chapter.
- *Projects* for each chapter provide opportunities for a sustained investigation, often using skills from different parts of the course. These include business applications, issues in sustainability, and medical case studies based on clinical practice.
- *True-False Strengthen Your Understanding* questions, available online for every chapter, enable students to check their progress.
- *Spreadsheet Projects* in the online Appendix provide the opportunity for students to develop their spreadsheet skills while deepening their understanding of functions and calculus.
- *Focus on Practice* exercises at the end of Chapter 3 and 6 (Derivatives and Antiderivatives) build student skill and confidence.

**Origin of the Text: A Community of Instructors**

This text, like others we write, draws on the experience of a diverse group of authors and users. We have benefitted enormously from input from a broad spectrum of instructors—at research universities, four-year colleges, community colleges, and secondary schools. For *Applied Calculus*, the contributions of colleagues in biology, economics, medicine, business, and other life and social sciences have been equally central to the development of the text. It is the collective wisdom of this community of mathematicians, teachers, and natural and social scientists that forms the basis for the new edition.

**What Student Background is Expected?**

This book is intended for students in business, the social sciences, and the life sciences. A background in trigonometry is *not* required; the sections involving trigonometry are optional.

We have found the material to be thought-provoking for well-prepared students while still accessible to students with limited algebra backgrounds. Providing numerical and graphical approaches as well as the
algebraic gives students several ways of mastering the material. This approach encourages students to persist, thereby lowering failure rates. A pre-test over background material is available at the student book companion site: www.wiley.com/college/hughes-hallett. An ORION algebra refresher is available in WileyPLUS.

Mathematical Skills: A Balance Between Symbolic Manipulation and Technology

To use calculus effectively, students need familiarity with both symbolic manipulation and the use of technology. The balance between them may vary, depending on the needs of the students and the wishes of the instructor. The book is adaptable to many different combinations.

The book does not require any specific software or technology. Students may use whatever is readily available—a graphing calculator or online tools.

The Sixth Edition

Because different users often choose very different topics to cover in a one-semester applied calculus course, we have designed this book for either a one-semester course (with much flexibility in choosing topics) or a two-semester course. Sample syllabi are provided in the Instructor’s Manual.

The sixth edition has the same vision as previous editions. In preparing this edition, we solicited comments from a large number of mathematics instructors who had used the text. We continued to discuss with our colleagues in client disciplines the mathematical needs of their students. We were offered many valuable suggestions, which we have tried to incorporate, while maintaining our original commitment to a focused treatment of a limited number of topics. The changes we have made include:

- About 400 additional problems have been added to the WileyPLUS course, expanding instructors’ options for online homework assignments.
- Worked example videos have been added for every section in the text.
- Updated data and fresh applications throughout the book, including
  - Problems on sustainability.
  - Case studies on medicine by David E. Sloane, MD.
- Many new problems have been added, designed to build student confidence with basic concepts and to reinforce skills.
- As in the previous edition, a Pre-test is included for students whose skills may need a refresher prior to taking the course. It is available online at www.wiley.com/college/hughes-hallett.

Content

This content represents our vision of how applied calculus can be taught. It is flexible enough to accommodate individual course needs and requirements. Topics can easily be added or deleted, or the order changed.

Chapter 1: Functions and Change

Chapter 1 introduces the concept of a function and the idea of change, including the distinction between total change, rate of change, and relative change. All elementary functions are introduced here. Although the functions are probably familiar, the graphical, numerical, verbal, and modeling approach to them is likely to be new. We introduce exponential functions early, since they are fundamental to the understanding of real-world processes. The trigonometric functions are optional.

Chapter 2: Rate of Change: The Derivative

Chapter 2 presents the key concept of the derivative according to the Rule of Four. The purpose of this chapter is to give the student a practical understanding of the meaning of the derivative and its interpretation as an instantaneous rate of change. Students will learn how the derivative can be used to represent relative
rates of change. After finishing this chapter, a student will be able to approximate derivatives numerically by taking difference quotients, visualize derivatives graphically as the slope of the graph, and interpret the meaning of first and second derivatives in various applications. The student will also understand the concept of marginality and recognize the derivative as a function in its own right.

*Focus on Theory:* This section discusses limits and continuity and presents the symbolic definition of the derivative.

**Chapter 3: Short-Cuts to Differentiation**

The derivatives of all the functions in Chapter 1 are introduced, as well as the rules for differentiating products, quotients, and composite functions. Students learn how to find relative rates of change using logarithms.

*Focus on Theory:* This section uses the definition of the derivative to obtain the differentiation rules.

*Focus on Practice:* This section provides a collection of differentiation problems for skill-building.

**Chapter 4: Using the Derivative**

The aim of this chapter is to enable the student to use the derivative in solving problems, including optimization and graphing. It is not necessary to cover all the sections.

**Chapter 5: Accumulated Change: The Definite Integral**

Chapter 5 presents the key concept of the definite integral, in the same spirit as Chapter 2. The purpose of this chapter is to give the student a practical understanding of the definite integral as a limit of Riemann sums, and to bring out the connection between the derivative and the definite integral in the Fundamental Theorem of Calculus. We use the same method as in Chapter 2, introducing the fundamental concept in depth without going into technique. The student will finish the chapter with the ability to approximate a definite integral numerically and interpret it graphically. The chapter includes applications of definite integrals in a variety of contexts, including the average value of a function.

Chapter 5 can be covered immediately after Chapter 2 without difficulty.

*Focus on Theory:* This section presents the Second Fundamental Theorem of Calculus and the properties of the definite integral.

**Chapter 6: Antiderivatives and Applications**

This chapter covers antiderivatives from a graphical, numerical, and algebraic point of view. The Fundamental Theorem of Calculus is used to evaluate definite integrals.

Sections 6.4–6.7 are optional. Application sections are included on consumer and producer surplus and on present and future value; the integrals in these sections can be evaluated numerically or using the Fundamental Theorem. The chapter concludes with sections on integration by substitution and integration by parts.

*Focus on Practice:* This section provides a collection of integration problems for skill-building.

**Chapter 7: Probability**

This chapter covers probability density functions, cumulative distribution functions, the median and the mean.

**Chapter 8: Functions of Several Variables**

This chapter introduces functions of two variables from several points of view, using contour diagrams, formulas, and tables. It gives students the skills to read contour diagrams and think graphically, to read tables and think numerically, and to apply these skills, along with their algebraic skills, to modeling. The idea of the partial derivative is introduced from graphical, numerical, and symbolic viewpoints. Partial derivatives are then applied to optimization problems, ending with a discussion of constrained optimization using Lagrange multipliers.

*Focus on Theory:* This section uses optimization to derive the formula for the regression line.
Chapter 9: Mathematical Modeling Using Differential Equations

This chapter introduces differential equations. The emphasis is on modeling, qualitative solutions, and interpretation. This chapter includes applications of systems of differential equations to population models, the spread of disease, and predator-prey interactions.

Focus on Theory: This section explains the technique of separation of variables.

Chapter 10: Geometric Series (Available online and in the e-text)

This chapter covers geometric series and their applications to business, economics, and the life sciences.

Appendices (Available online and in the e-text)

Appendix A introduces the student to fitting formulas to data; Appendix B provides further discussion of compound interest and the definition of the number $e$. Appendix C contains selection of spreadsheet projects.

Supplementary Materials

Supplements for the instructor can be obtained online at the book companion site or by contacting your Wiley representative. The following supplementary materials are available for this edition:

- Instructor’s Manual containing teaching tips, sample syllabi, calculator programs, and overhead transparency masters, also available as PowerPoint slides.
- Instructor’s Solution Manual with complete solutions to all problems.
- Student’s Solution Manual with complete solutions to half the odd-numbered problems.
- Additional Material for Instructors, elaborating specially marked points in the text, lecture notes, and course notes, as well as password protected electronic versions of the instructor ancillaries, can be found on the web at the book companion site: www.wiley.com/college/hughes-hallett.
- Additional Material for Students, including an algebra refresher, is available via WileyPLUS.

ConcepTests

ConcepTests, or clicker questions, modeled on the pioneering work of Harvard physicist Eric Mazur, are questions designed to promote active learning during class, particularly (but not exclusively) in large lectures. Evaluation data showed that students taught with ConcepTests outperformed students taught by traditional lecture methods 73% versus 17% on conceptual questions, and 63% versus 54% on computational problems. A supplement to Applied Calculus, 6th edn, containing ConcepTests by section, is available through WileyPLUS and at the book companion site, www.wiley.com/college/hughes-hallett.

WileyPLUS Studio

The WileyPLUS studio is an online community that brings WileyPLUS users together in an engaging, virtual environment. It’s a space where you can share insights, identify best practices, provide product feedback, learn from peers and get rewarded for your efforts.

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First and foremost, we want to express our appreciation to the National Science Foundation for their faith in our ability to produce a revitalized calculus curriculum and, in particular, to Louise Raphael, John Kenelly, John Bradley, Bill Haver, and James Lightbourne. We also want to thank the members of our Advisory Board, Benita Albert, Lida Barrett, Bob Davis, Lovenia DeConge-Watson, John Dossey, Ron Douglas, Don Lewis, Seymour Parter, John Prados, and Steve Rodi for their ongoing guidance and advice.

1“Peer Instruction in Physics and Mathematics” by Scott Pilzer in Primus, Vol XI, No 2, June 2001. At the start of Calculus II, students earned 73% on conceptual questions and 63% on computational questions if they were taught with ConcepTests in Calculus I; 17% and 54% otherwise.
In addition, we want to thank all the people across the country who encouraged us to write this book and who offered so many helpful comments. We would like to thank the following people, for all that they have done to help our project succeed: Enrique Acosta, Ruth Baruth, Graeme Bird, Jeanne Bowman, Lucille Buonocore, Scott Clark, Jeff Edmunds, Sunny Fawcett, Lynn Garner, Sheldon P. Gordon, Ole Hald, Jenny Harrison, Adrian Iovita, Thomas Judson, Christopher Kennedy, Hannah Knight, Donna Krawczyk, Suzanne Lenhart, Madelyn Lesure, Kevin Martin, Rosalind Horn Martin, Georgia Kamvosoulis Mederer, Nolan Miller, David Muñoz Ramírez, Andrew Pasquale, Richard D. Porter, Laurie Rosatone, Kenneth Santor, Anne Scanlan-Rohrer, Alfred Schipke, Virginia Stallings, Ralph Teixeira, Joe B. Thrash, J. Jerry Uhl, Rachel Deyette Werkema, Hannah Winkler, and Hung-Hsi Wu.

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To Students: How to Learn from this Book

- This book may be different from other math textbooks that you have used, so it may be helpful to know about some of the differences in advance. At every stage, this book emphasizes the meaning (in practical, graphical or numerical terms) of the symbols you are using. There is much less emphasis on “plug-and-chug” and using formulas, and much more emphasis on the interpretation of these formulas than you may expect. You will often be asked to explain your ideas in words or to explain an answer using graphs.

- The book contains the main ideas of calculus in plain English. Success in using this book will depend on reading, questioning, and thinking hard about the ideas presented. It will be helpful to read the text in detail, not just the worked examples.

- There are few examples in the text that are exactly like the homework problems, so homework problems can’t be done by searching for similar-looking “worked out” examples. Success with the homework will come by grappling with the ideas of calculus.

- For many problems in the book, there is more than one correct approach and more than one correct solution. Sometimes, solving a problem relies on common sense ideas that are not stated in the problem explicitly but which you know from everyday life.

- Some problems in this book assume that you have access to a graphing calculator or computer. There are many situations where you may not be able to find an exact solution to a problem, but you can use a calculator or computer to get a reasonable approximation.

- This book attempts to give equal weight to four methods for describing functions: graphical (a picture), numerical (a table of values), algebraic (a formula), and verbal (words). Sometimes it’s easier to translate a problem given in one form into another. For example, you might replace the graph of a parabola with its equation, or plot a table of values to see its behavior. It is important to be flexible about your approach: if one way of looking at a problem doesn’t work, try another.

- Students using this book have found discussing these problems in small groups helpful. There are a great many problems which are not cut-and-dried; it can help to attack them with the other perspectives your colleagues can provide. If group work is not feasible, see if your instructor can organize a discussion session in which additional problems can be worked on.

- You are probably wondering what you’ll get from the book. The answer is, if you put in a solid effort, you will get a real understanding of one of the crowning achievements of human creativity—calculus—as well as a real sense of the power of mathematics in the age of technology.
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