INTRODUCTION

Outline

1.1 Mechanics of Materials 2
1.2 Scope of the Book 3
1.3 Methods of Analysis 4
1.4 Engineering Design 5
1.5 Review of Static Equilibrium 6
1.6 Internal Force Resultants 10
1.7 Problem Formulation and Solution 13
1.8 Application to Simple Structures 15
Problems 21
Chapter Summary 26
References 27

George Washington Bridge, spanning the Hudson River between New Jersey and New York. This book is devoted to the study of forces, stresses, and deformations occurring in many of the members contained in such a structure, including bars, beams, frames, columns, and connections. The applications of mechanics of materials are almost endless and can be found in every engineering field. © The Port Authority of NY & NJ
1.1 MECHANICS OF MATERIALS

The subject **mechanics of materials** is a branch of applied mechanics that deals with the internal behavior of variously loaded solid bodies. This field of study is also frequently called **strength of materials**, **mechanics of deformable bodies**, or simply **mechanics of solids**. The “solid bodies” referred to here include shafts, bars, beams, plates, shells, and columns, as well as structures and machines that are assemblies of these components. **Stress analysis** and the **mechanical properties of materials** are the main aspects of the mechanics of materials.

The study of mechanics of materials is based upon an understanding of the equilibrium of bodies under the action of forces. Whereas **statics** treats the external behavior of bodies that are assumed to be ideally rigid and at rest, mechanics of materials is concerned with the relationships between external loads (forces and moments) and internal forces and deformations or displacements induced in the body. **Stress** and **strain** are fundamental quantities connected with the former and the latter, respectively.

Complete analysis of a structure under load requires the determination of stress, strain, and deformation through the use of three fundamental principles, which will be outlined in Section 1.3: the laws of forces, the laws of material deformation, and the conditions of geometric compatibility. An introduction to engineering design is also included. We consider here the first principle, static equilibrium, and its application to a loaded body. The remaining principles are studied in Chapter 3.

The equilibrium conditions are reviewed in Section 1.5. This is followed by a discussion on the internal force determination on the cross sections of the loaded members. The problem solution procedures are outlined in Section 1.7. The International System of Units (SI) and the U.S. Customary system that will be utilized in this book are described. Finally, applications to simple structures are illustrated by a variety of examples. Problems, a brief chapter summary, and references appear at the end of the chapter. Answers to selected even-numbered problems are listed at the end of the text.

HISTORICAL DEVELOPMENT

The development of mechanics of materials is a blend of experiment and theory. Experimental investigation of the behavior of bars under loads began with Leonardo da Vinci (1452–1519) and Galileo Galilei (1564–1642). For a proper understanding, however, it was necessary to establish an accurate experimental description of a material’s properties. Robert Hooke (1615–1703) was the first to show that a body is deformed if a force acts upon it. Sir Isaac Newton (1642–1727) developed the concepts of Newtonian mechanics that became key elements of the strength of materials.

Leonard Euler (1707–1783) presented the mathematical theory of columns in 1744. Thomas Young (1773–1829) established a coefficient of elasticity, Young’s modulus. The advent of railroads in the late 1800s provided the impetus for much of the basic work in this area. Many famous scientists and engineers, including Coulomb, Poisson, Navier, St. Venant, and Cauchy, were responsible for advances in mechanics of materials during the nineteenth century.

Over the years, most fundamental problems of solid mechanics had been solved. Stephan P. Timoshenko (1878–1972) made many original contributions to the field of applied mechanics and wrote pioneering textbooks on the mechanics of materials, theory of elasticity, and theory of elastic stability. The theoretical basis for modern strength of materials had been developed by the end of the nineteenth century. Since then, problems associated with the design of aircraft, space vehicles, and nuclear reactors have led to numerous studies of the more advanced aspects of the subject. As a result, the mechanics of materials is being expanded into the theories of elasticity and plasticity.
Research in the preceding areas is ongoing, not only to meet demands for treating complex problems, but also to justify further use and limitations on which the theory of mechanics of solids is based. Although a vast body of knowledge exists at present, mechanics of materials remains a fascinating subject by continuously expanding its areas of application.

The literature related to solid mechanics is voluminous. A number of selected references are identified in parentheses and compiled at the end of each chapter of the text for those seeking more extensive information.

1.2 SCOPE OF THE BOOK

The usual objective of mechanics of materials is the examination of the load-carrying capacity of a body from three standpoints: strength, stiffness, and stability. These quantities relate, respectively, to the ability of a member to resist permanent deformation or fracture, to resist deflection, and to retain its equilibrium configuration. Failure can be defined, in very general terms, as any action that results in an inability on the part of the structure to function in the manner intended. For instance, when loading produces an abrupt shape change of a member, instability occurs; in like manner, an inelastic deformation or an excessive magnitude of deflection in a member will cause malfunction in normal service. Each of these examples indicates a type of failure.

The main concern in the study of mechanics of materials is analysis of stress and deformation within a loaded body, which is accomplished by application of one of the

---

I cannot doubt that these things, which now seem to be mysterious, will be no mysteries at all; that the scales will fall from our eyes; that we shall learn to look on things in a different way—when that which is now a difficulty will be the only common sense and intelligible way of looking at the subject.

W. T. KELVIN (1824–1907)

Research in the preceding areas is ongoing, not only to meet demands for treating complex problems, but also to justify further use and limitations on which the theory of mechanics of solids is based. Although a vast body of knowledge exists at present, mechanics of materials remains a fascinating subject by continuously expanding its areas of application. The literature related to solid mechanics is voluminous. A number of selected references are identified in parentheses and compiled at the end of each chapter of the text for those seeking more extensive information.

---

I cannot doubt that these things, which now seem to be mysterious, will be no mysteries at all; that the scales will fall from our eyes; that we shall learn to look on things in a different way—when that which is now a difficulty will be the only common sense and intelligible way of looking at the subject.

W. T. KELVIN (1824–1907)

The usual objective of mechanics of materials is the examination of the load-carrying capacity of a body from three standpoints: strength, stiffness, and stability. These quantities relate, respectively, to the ability of a member to resist permanent deformation or fracture, to resist deflection, and to retain its equilibrium configuration. Failure can be defined, in very general terms, as any action that results in an inability on the part of the structure to function in the manner intended. For instance, when loading produces an abrupt shape change of a member, instability occurs; in like manner, an inelastic deformation or an excessive magnitude of deflection in a member will cause malfunction in normal service. Each of these examples indicates a type of failure.

The main concern in the study of mechanics of materials is analysis of stress and deformation within a loaded body, which is accomplished by application of one of the

---

1 The history of mechanics of materials is given in References 1.1 and 1.2.
methods to be described in Section 1.3. It is to this consideration that this book is primarily directed. The analysis of loads is essential in obtaining the stress or deformation of a member. A structure or machine cannot be satisfactory unless its design is based on realistic operating loads. Clearly, determination of the appropriate loads, often a difficult and challenging task, is the initial step in the analysis and design of a component. Design relies upon the performance of the stress analysis, and will be discussed in Section 1.4.

The ever-increasing industrial demand for more sophisticated structural and machine components calls for a good grasp of the concepts of stress and strain and of the behavior of materials—and a considerable degree of ingenuity. This text will, in the very least, provide the reader with the ideas and information necessary for a basic understanding of the mechanics of deformable bodies and will encourage the creative process based on that understanding. Very few basic formulas are actually derived in this volume; rather, the student will master these formulas through their repeated application. It is important, however, that the reader visualize the nature of the quantities being computed. Complete, carefully drawn free-body diagrams facilitate visualization, and these we have provided, all the while knowing that the subject matter can be learned best by solving practical problems.

### 1.3 METHODS OF ANALYSIS

The approaches in widespread employment for determining the influence of loads upon deformable bodies are the **mechanics of materials theory** (also known as technical theory), which is presented in this text, and the **theory of elasticity**. The difference between these theories lies primarily in the extent to which strains are described and in the nature of simplifications used. The mechanics of materials approach uses assumptions, based upon experimental evidence and the lessons of engineering practice, to make a reasonable solution of the basic problem possible.

Note that the theory of elasticity establishes every step rigorously from the mathematical point of view and hence seeks to verify the validity of the assumptions introduced. This technique can provide “exact” results where configurations of loading and shape are simple. In general, however, the theory of elasticity yields solutions with considerable difficulty. We shall refer to stresses obtained by elasticity for noncircular shafts in Section 5.11 and discuss the limitations of the shear formula for beams developed in Sections 7.8 and 7.10.

The complete analysis of structural members by the so-called **method of equilibrium** requires consideration of a number of conditions relating to certain laws of forces, laws of material deformation, and geometry. These essential relationships, referred to as **basic principles of analysis**, should be outlined in summary form before we proceed:

1. **Equilibrium Conditions.** The equations of equilibrium of forces must be satisfied throughout the member.
2. **Material Behavior.** The stress–strain or force–deformation relations (for example, Hooke’s law) must apply to the behavior of the material of which the member is constructed.
3. **Geometry of Deformation.** The conditions of geometric fit or compatibility of deformations must be satisfied: that is, each deformed portion of the member must fit together with adjacent portions.
The stress and deformation obtained by applying these principles must be such as to conform to the conditions of loading imposed at the boundaries of a member. This is known as satisfying the boundary conditions. Applications of the foregoing procedure will be shown in the problems presented as the subject unfolds. We note here, however, that it is not always necessary to execute an analysis in the exact order of the steps listed above.

As an alternative to the equilibrium methods, the analysis of stress and deformation can be accomplished through the use of energy methods, which are based upon the concept of strain energy. The role of both the equilibrium and energy methods is twofold. These methods can provide solutions of acceptable accuracy where configurations of loading and member shape are regular, and they can be employed as the basis of numerical methods (see Chapter 13) in the solution of more complex problems.

A final point to be noted is that a degree of caution is necessary when employing formulas for which there is uncertainty in applicability and restriction of use. The relatively simple form of many formulas often results from idealizations made in their derivations—idealizations such as simplified boundary conditions and loading on a member and approximation of shape or material. Designers and stress analysts must be aware of such limitations.

### 1.4 ENGINEERING DESIGN

Engineering design is the process of applying science and engineering methods to define a structure or system in detail to permit its realization. The main objective of a mechanical design process includes determination of proper materials, dimensions, and shapes of the components of a structure or machine so that they will support given loads and function without failure. Machine design involves devising new or improved machines to accomplish a specific purpose. Generally, structural design interacts with any engineering discipline that requires a structural member or system.

Design is the essence, art, and intent of engineering. A good design meets performance, cost, and safety requirements. An optimum design is the best solution to a design problem within given constraint(s). Efficiency of the optimization may be gaged by such criteria as minimum weight or volume, minimum cost, and/or any other standard deemed appropriate. When faced with a design problem with many choices, a designer may make decisions based on experience, reducing the problem to a single variable. A solution to obtain the optimum result is often straightforward in such a situation, as is illustrated in Case Study 2A.

### DESIGN PROCEDURE

The role of analysis in design may be observed best in examining the phases of a design process. The following is a rational procedure in the design for strength of a load-carrying member:

1. Evaluate the mode of possible failure of the member.
2. Determine a relationship between the applied load and the resulting effect such as stress or deformation.
3. Determine the maximum usable value of a significant quantity such as stress or deformation that could conceivably cause failure. Employ this value in connection with the equation found in step 2 or, if required, in any of the formulas associated with the various theories of failure, which will be discussed in Sections 9.8 and 9.9.
4. Select the factor of safety as outlined in Section 2.7.
The foregoing procedure must be tailored somewhat to each individual case, since some steps may be regarded as unnecessary or as obvious for a certain member.

Suffice it to say here that complete design solutions are not unique, involve a consideration of many factors, and often require a trial-and-error process (Ref. 1.3). This text provides an elementary treatment of the concept of “design to meet strength requirements” as those requirements relate to individual component parts. That is, the geometrical configuration and material of a member are preselected and the applied loads are specified. Then the basic formulas, to be developed in Chapters 2 through 7, are used to select members of adequate size in each case.

Stress is only one consideration in design. Other aspects of the design of members are the prediction of the service ability of a component for deflection under given loading and the consideration of buckling. The methods of determining deformation are discussed in several chapters throughout the text. In Chapter 11, we shall be concerned with the buckling of slender members loaded axially in compression. To conclude, we note that there is a very close relationship between analysis and design, and the examples, case studies, and problems that appear throughout this book illustrate that connection.

### 1.5 REVIEW OF STATIC EQUILIBRIUM

Statics plays a significant role in both development and applications of mechanics of materials. For this reason, we will review some of the basic principles of statics here and in Chapter 6. The analysis and design of structural and machine components require a knowledge of the distribution of forces within such members. Fundamental concepts and conditions of static equilibrium provide the necessary background for the determination of internal as well as external forces. In the next section we shall see that components of internal-force resultants have special meaning in terms of the type of deformations they cause, as applied, for example, to slender members.

#### EXTERNAL LOADS

All forces acting on a body, including the reactive forces caused by supports, are considered external forces. These forces are classified as surface forces and body forces. A surface force is concentrated when it acts at a point, but it may also be distributed over a finite area. A body force acts on a volumetric element rather than on a surface and is attributable to fields such as gravity and magnetism. The force of the Earth on an object at or near the surface is termed the weight of the object. Internal forces in a body can be considered as forces of interaction between the constituent material particles of the body.

The loading type on a structure may be divided into several classes based on the character of the applied forces and the presence or absence of system motion. Once the structure configuration is defined, the next step is to ascertain the magnitudes and senses of all the forces and couples present in the various components. The forces may be uniform or may be varying with time. The structural members may be stationary or moving. The loads on bodies may be concentrated forces (or couples), and/or distributed forces. Any force applied to an area that is relatively small compared with the size of the loaded structural member is assumed to be a concentrated load.

Line loads and concentrated forces are considered to act along a line and at a single point, respectively. Both of these forces are thus idealizations. Nevertheless, they permit accurate analysis of a loaded member except in the immediate vicinity of the loads (see Section 4.7). A load slowly and steadily applied is regarded as a static load, while a rapidly applied load is called an impact load. Multiple applications and removals of load, usually
measured in thousands of episodes or more, are referred to as **repeated loading.** Having said all that, we add that, unless otherwise stated, we assume in this text that the weight of the body can be neglected and that the load is static.

Loads and internal forces may be further classified with respect to location and method of application: normal, shear, bending, and torsion loads, and combined loadings. There are only a few types of loading that may commonly occur on machine or structural members. The coordinate direction must be set up before the sign of the preceding load types is established (see Section 1.6). As is discussed in Section 1.7 in the International System of Units (SI), force is measured in newtons (N). But because the newton is a small quantity, the kilonewton (kN) is often used in practice. In the U.S. Customary system, force is expressed in pounds (lb) or kilopounds (kips).

### SUPPORT REACTIONS

The surface forces that develop at support points of a structure are called **reactions.** They equilibrate the effects of the applied loads on the structures. This text is mainly concerned with the loading and support that lie in a single plane. Frequently, however, we consider three-dimensional loading and support. For **planar structures,** the restraint is provided by the support in the xy plane. In the case of **nonplanar structures,** the same consideration must be taken into account in three mutually perpendicular xy, xz, and yz planes.

Table 1.1 illustrates some types of supports and connections in common usage for structures. Observe that for convenience, most two-dimensional members (cases 1 to 5) are depicted in horizontal positions. A **hinge** or **pin support** restrains the member from translating (moving) in any direction of the plane, but it does not prevent rotation. At a **roller support,** translation is prevented in the vertical direction (y) but not in the horizontal direction (x). Just as at a pin, there is no restraint offered to rotation at a roller. So, a pinned support resists tangential and normal forces to the surface, while a roller support withstands forces normal to the surface only. Both are called **simple supports.**

At the **fixed** or **clamped support,** the bar can neither translate nor rotate. Therefore, a fixed support resists both a moment and a force in any direction. This force may have components normal and tangential to the surface. A guided support restrains the member from translating in the horizontal direction and rotating. The restraint conditions that are provided by the support in nonplanar structures (cases 6 and 7) are similar to those for planar structures. To distinguish between the applied loads and the reactions, a **slash** is drawn across the reaction (force \( R \) or \( F \) and moment \( M \)) vectors.

### CONDITIONS OF EQUILIBRIUM

When a system of forces acting upon a body has zero resultant, the body is said to be in **force equilibrium.** From another viewpoint, equilibrium of forces is the state in which the forces applied on a body are in balance. Newton’s first law states that if the resultant force acting on a particle (the simplest body) is zero, the particle will remain at rest or will move with constant velocity. Statics, as its name implies, deals essentially with the case in which the particle or body remains at rest.

Consider the **equilibrium** of a body in space. The **equations of statics** require that the following **conditions** be satisfied:

\[
\begin{align*}
\sum F_x &= 0 \\
\sum F_y &= 0 \\
\sum F_z &= 0 \\
\sum M_x &= 0 \\
\sum M_y &= 0 \\
\sum M_z &= 0
\end{align*}
\]

(1.1)

In words, we are saying that the sum of all forces acting upon a body in any direction must be zero and that the sum of all moments about any axis must be zero.
TABLE 1.1 Reactions at Supports and Connections

<table>
<thead>
<tr>
<th>Type of Support or Connection</th>
<th>Reaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Hinge or pin</td>
<td>$\vec{R}<em>{Ax}$, $\vec{R}</em>{Ay}$</td>
</tr>
<tr>
<td></td>
<td>$\vec{M}_A$</td>
</tr>
<tr>
<td>2. Roller</td>
<td>$\vec{R}_{Ay}$</td>
</tr>
<tr>
<td></td>
<td>$\vec{R}_{b}$</td>
</tr>
<tr>
<td>3. Cable or link</td>
<td>$\vec{F}_{AB}$</td>
</tr>
<tr>
<td>4. Fixed or clamped</td>
<td>$\vec{R}_{Ax}$</td>
</tr>
<tr>
<td></td>
<td>$\vec{M}_A$</td>
</tr>
<tr>
<td>5. Guided</td>
<td>$\vec{R}_{Ax}$</td>
</tr>
<tr>
<td></td>
<td>$\vec{M}_A$</td>
</tr>
<tr>
<td>6. Ball and socket (nonplanar)</td>
<td>$\vec{R}_x$, $\vec{R}_y$, $\vec{R}_z$</td>
</tr>
<tr>
<td></td>
<td>$\vec{M}_x$, $\vec{M}_y$</td>
</tr>
<tr>
<td>7. Fixed (nonplanar)</td>
<td>$\vec{R}_x$, $\vec{R}_y$, $\vec{R}_z$</td>
</tr>
<tr>
<td></td>
<td>$\vec{M}_x$, $\vec{M}_y$, $\vec{M}_z$</td>
</tr>
</tbody>
</table>

*Usually the reaction will be identified in this text with a slash drawn through its vector as shown.*
If the forces act on a body in equilibrium in a single (xy) plane, three expressions—\( \sum F_z = 0, \sum M_x = 0, \) and \( \sum M_y = 0 \)—in Eqs. (1.1), while still valid, are trivial. This leaves only three independent conditions of equilibrium for planar problems:

\[
\begin{align*}
\sum F_x &= 0 \\
\sum F_y &= 0 \\
\sum M_A &= 0
\end{align*}
\]  

(1.2)

In words then, the sum of all forces in any two \((x, y)\) directions must be zero, and the resultant moment with respect to any axis \(z\) or any point \(A\) in the plane must be zero.

Alternative sets of conditions may be applied with some limitations. That is,

\[
\begin{align*}
\sum F_x &= 0 \\
\sum M_A &= 0 \\
\sum M_B &= 0
\end{align*}
\]  

(1.3a)

provided that the line connecting the points \(A\) and \(B\) is not perpendicular to the \(x\) axis, and

\[
\begin{align*}
\sum M_A &= 0 \\
\sum M_B &= 0 \\
\sum M_C &= 0
\end{align*}
\]  

(1.3b)

where points \(A\), \(B\), and \(C\) are not collinear. Clearly the alternative sets are obtained by replacing a force summation by an equivalent moment summation. We often find it convenient to employ Eqs. (1.3a) or even Eqs. (1.3b). The judicious selection of points for taking moments can generally simplify algebraic computations.

It is noted that, for a body in accelerated motion, additional forces must be included for the equations of statics to be applicable. These additional forces are the inertia forces; for purposes of structural analysis, incorporating them allows us to consider the body as being acted upon by a set of forces in equilibrium. This is the so-called d’Alembert principle.

Common engineering problems involve machines and structures in equilibrium. Certain forces—usually loads—are specified, and the problem is to obtain the unknown forces—usually reactions—balancing the loads. If it is possible to determine all forces by using the equilibrium conditions alone, the system is called statically determinate. However, there are problems for which equations of statics are not sufficient to ascertain the unknown forces on the member. Such problems are called statically indeterminate. The degree of static indeterminacy is equal to the difference between the number of unknown forces and the number of pertinent equilibrium equations. Any reaction that is in excess of those that can be obtained by statics alone is said to be redundant. Thus the number of redundants is the same as the degree of indeterminacy.

**FREE-BODY DIAGRAMS**

Effective application of equilibrium conditions requires complete specification of all loads and reactions that act on the body. So, the first step in the solution of an equilibrium problem should consist of drawing a free-body diagram of the body under consideration. A free-body diagram (FBD) is simply a sketch of a body, with all of the appropriate forces, both known and unknown, acting on it. The diagram may be of an entire structure or a portion of a larger structure. The general procedure in drawing a complete free-body diagram includes the following steps:

1. Select the free body to be used.
2. Detach this body from its supports and separate it from any other bodies. (If internal force resultants are to be found, use the method discussed in Section 1.6.)
3. Show on the sketch all of the external forces acting on the body. Location, magnitude, and direction of each force should be marked on the sketch.
4. Label significant points and include dimensions. Any other detail, however, should be omitted.
Obviously, the prudent selection of the free body to be used (item 1) is of primary importance. The reader is strongly urged to adopt the habit of drawing clear and complete free-body diagrams in the solution of problems concerning equilibrium. Examples 1.1 through 1.3 will illustrate the construction of free-body diagrams and the use of equations of statics.

1.6 INTERNAL FORCE RESULTANTS

A body responds to the application of external forces by deforming and by developing an internal force system (forces and couples or moments) that holds together the particles forming the body. These internal force resultants represent the internal resultant loadings. We shall now deal with one of the principal problems of mechanics of materials: the investigation of internal forces by the familiar approach of statics, or the method of sections. The steps involved in applying the method of sections may be summarized as follows:

1. Isolate the bodies. Sketch the isolated body and all external forces acting on it: draw a free-body diagram. Since in practice the allowable deformations are negligible as compared with the size of the member, free-body diagrams contain the initial dimensions and ignore the deformations.

2. Apply the equations of equilibrium to the diagram to determine the unknown external forces.

3. Cut the body at a section of interest by an imaginary plane (Fig. 1.1a), isolate one of the segments, and repeat step 2 for that segment. If the entire body is in equilibrium, any part of it must be in equilibrium. That is, there must be internal forces transmitted across the cut sections (Fig. 1.1b).

It may be concluded from the foregoing discussion that the external forces or loads are balanced by internal forces. The former can thus be regarded as a continuation of the distribution of the latter, though the precise distribution of the forces within a member depends upon the imaginary plane selected to separate the two parts. Note that load-carrying

FIGURE 1.1 Method of sections: (a) body acted on by external forces; (b) internal forces acting on a plane.
members appear in such diverse forms as bars, beams, columns, plates, and frames in bridges and buildings (see the chapter opener photo and photo shown below).

The section on which internal forces produce the largest stress is called the **critical section** of the loaded member. When a single load is acting on a member, the critical section and its orientation are usually evident by inspection. With combined loads, however, the angular position of the cutting plane is determined by approaches that will be given in Sections 8.3 and 8.5.

**COMPONENTS OF INTERNAL FORCES**

Many structural elements can be classified as **slender members**. According to the criterion often used to define a slender member, the length of such a member should be at least 5 times greater than its largest cross-sectional dimension. In general, forces within a slender member can be represented by a statically equivalent set consisting of a force vector and moment or couple vectors acting *at the centroid C* of the cross section. These internal-force resultants, also called **stress resultants**, are usually resolved into components normal and tangent to the section.

The preceding is seen in Fig. 1.2, and the **right-handed coordinate system** shown in the figure will be used throughout this text. It is observed that the $x$ axis coincides with the longitudinal axis of the member; the $y$ axis is taken as the vertical upward axis, and the $z$ axis points toward the reader. Note that the sense of the moment components follows the **right-hand screw rule**—that is, a right-hand screw advances with the sense of the vector when it is twisted in the sense indicated by the moment couple. For convenience, the moment components are also represented by double-headed vectors, as depicted in Fig. 1.2.

A building in construction. To design members of this structure, it is necessary to first determine the internal load resultants at their critical sections. *Courtesy Prof. W. Konon, NJIT.*

---

*The centroid of an area is explained in Section A.2 of Appendix A.*
Each internal force and moment component reflects a different effect of the applied loading on the member. These effects can be described as follows (Fig. 1.2):

The axial force $F_x$ tends to elongate (or contract) the member and is often identified by the letter $F$, $N$, or $P$. If the force acts away from the cut, it is termed axial tension: if toward the cut, it is called axial compression.

The shear forces $F_y$ and $F_z$ act parallel to the cross section. They tend to cause two parts of the member to slide over one another and are often designated by the letters $V_y$, $V_z$, or $V$.

The twisting moment or torque $M_x$ is responsible for twisting the member about its axis and is identified by the letter $T$.

The bending moments $M_y$ and $M_z$ cause the member to bend and are often designated by the letter $M$.

In Chapters 4, 5, 7, 9, and 10, we shall investigate the relation of such force and moment components to the stresses and deformations in the members. Our main concern now is with the calculation of the magnitude of the components.

A structural element may be subject to any combination of or all of these four modes of force transmission simultaneously, though the modes are usually treated separately, and, if appropriate, the results are combined to obtain the final solution. Therefore the method of sections is counted on as a first step in all problems where the internal forces, and thus the corresponding stresses and strains, are being investigated.

In practice, the problem is often considerably simpler in that all forces act in a single plane, here taken as $xy$. In planar problems, we find only three components acting across a section: the axial force $F_x = P$, the shear force $F_y = V$, and the bending moment $M_z = M$. The diagrammatic representations of these components, as used in this text, are shown in Fig. 1.3. It is noted that the cross-sectional face, or plane, is defined as positive when its outward normal points in a positive coordinate direction and as negative when its outward normal points in the negative coordinate direction. According to Newton’s third law, the axial forces, shear forces, and bending moments acting on these faces at a cut section are equal and opposite.

### GENERAL SIGN CONVENTION

To assure consistency among the various analytical approaches, the following sign convention is established for axial force, shear force, twisting moment, and bending moment. When both the outer normal and the internal force or moment vector component point in a positive (or negative) coordinate direction, the force or moment is defined as positive. When a negatively directed component acts on a positive face (or vice versa), the force or moment is negative. Figures 1.2 and 1.3 show positive internal force and moment components. Hence, the tensile force at a section is positive. Observe that the sense of a positive twisting moment vector is the same as that of the positive axial force vector.

The foregoing sign convention adopted for forces and moments is also associated with the behavior of a member under load. For example, a straight bar elongates when subjected to positive axial forces at its ends. Interestingly, the general sign convention applies to the stress components as well (see Section 2.10). A final point to be noted is that the sense of
the reaction at a support of a structure is arbitrarily assumed; the positive (negative) sign of the answer calculated by applying the equations of static equilibrium will mean that the assumption is correct (incorrect).

**Beam Sign Convention for Shear Force.** Although the choice of the sign convention is arbitrary, in this book we will adopt the one often used for beams in engineering practice as illustrated in Section 6.4. Accordingly, the direction shown in Fig. 1.3 represents a negative shear force for a beam. That is, a sign convention for shear force in a beam which is opposite to that given in the figure will be used.

### 1.7 PROBLEM FORMULATION AND SOLUTION

A basic method of attack for structural analysis problems is to define (or understand) the problem. Formulation of the problem requires consideration of the physical situations and an idealized description by the relevant diagrams that approximate the actual member involved. The following outline may help in the formulation and solution of a problem:

1. **Given:** Define the problem and state briefly what is known.
2. **Find:** State consistently what is to be determined.
3. **Assumptions:** List simplifying idealizations to be made.
4. **Solution:** Apply the appropriate equations to determine the unknowns.
5. **Comments:** Discuss the results briefly.

The preceding steps will be used in examples and case studies throughout this text. The problem statements should indicate clearly and precisely what information is required. Free-body diagrams must be complete, showing all essential quantities involved. Assumptions expand on the given information to further constrain the problem. For example, one might take the effects of friction to be negligible, or ignore the weight of the member in a particular case. The student needs to understand what assumptions are made in solving a problem. Solutions must be based on the principles of statics and mechanics of materials, formulas, tables, charts, and diagrams. Comments present the key aspects of the solution and discuss how better results might be obtained by making different analysis decisions, relaxing the assumptions, and so on.

### SIGNIFICANT DIGITS

In engineering problems of practical importance, the data are seldom known with an accuracy greater than 0.2%. So, it is rarely justified to write the answers to such problems with an accuracy greater than 0.2%. Because calculations are usually performed by electronic calculators and computers (usually carrying eight or nine digits) the possibility exists that the numerical results will be reported to an accuracy that has no physical meaning. For consistency throughout this book, we generally follow a common engineering rule to report the final results of calculations:

- Numbers beginning with 1 are recorded to four significant digits.
- All other numbers (that begin with 2 through 9) are recorded to three significant digits.
Accordingly, a force of 14 lb, for instance, should read 14.00 lb, and a force of 52 lb should read 52.0 lb. Intermediate results, if retained for further calculations, are recorded to several additional digits to preserve the numerical accuracy. The values of π and trigonometric functions are calculated to many significant digits (ten or more) within the calculator or computer.

**COMPUTATIONAL TOOLS**

A variety of computational tools may be used to perform analysis calculations with success. A quality scientific calculator may be the best tool for solving most of the problems in this book. General purpose analysis tools such as spreadsheets and equation solvers have particular merit for certain computational tasks. Mathematical software packages of these types include MATLAB, TK Solver, and MathCAD. The tools have the advantage of allowing the user to document and save completed work in detailed form. The importance of computations in engineering cannot be overemphasized.

Computer-aided drafting or design (CAD) software packages can produce realistic three-dimensional representations of a member. Most CAD packages provide an interface to one or more finite element analysis (FEA) programs. They permit direct transfer of the member’s geometry to an FEA package for analysis of stress and vibration, as well as fluid and thermal analysis. The FEA techniques will be discussed in Chapter 13.

The preceding computer-based software may be used as tools to assist students with lengthy homework assignments. However, computer output providing analysis results must not be accepted on faith alone; the analyst must always make checks of computer solutions. It is essential that fundamentals of analysis be thoroughly understood.

**SYSTEMS OF UNITS**

The units of the physical quantities used in engineering calculations are of major importance. The most recent universal system is the International System of Units (SI). The U.S. Customary system of units has long been used by engineers in the United States. Both systems of units, reviewed briefly here, will be employed in this text. However, greater emphasis is placed on SI units in line with international convention. Typical fundamental quantities in SI and U.S. Customary systems of units are given in Table 1.2.

We see from Table 1.2 that in SI, force \( F \) is a derived quantity (obtained by multiplying the mass \( m \) with the acceleration \( a \), using Newton’s second law, \( F = ma \)). However, in the U.S. Customary system, the situation is reversed, with mass being the derived quantity. It is obtained by Newton’s second law, as \( \text{lb}-\text{s}^2/\text{ft} \), occasionally termed the slug. Temperature is expressed in SI by a unit termed kelvin (K), but for common purposes the degree Celsius (°C) is used (as shown in the table). The relationship between the two units is as follows: temperature in Celsius = temperature in kelvins – 273.5. The temperature is expressed in U.S. units by the degree Fahrenheit (°F). The conversion formula for the temperature scales is expressed in the form:

\[
t_C = \frac{5}{9}(t_F - 32)
\]  

(1.4a)

and

\[
t_K = \frac{5}{9}(t_F - 32) + 273.15
\]  

(1.4b)

Here \( t \) represents the temperature, and the subscripts \( C, F, \) and \( K \) denote Celsius, Fahrenheit, and kelvin, respectively.
It is sufficiently accurate to assume that the acceleration of gravity, designated by the letter \( g \), near Earth’s surface equals

\[
g = 9.81 \text{ m/s}^2 \quad \text{(or 32.2 ft/s}^2)\tag{a}
\]

By Newton’s second law, it follows that in SI, the weight \( W \) of a body of mass 1 kg is 

\[
W = mg = (1 \text{ kg})(9.81 \text{ m/s}^2) = 9.81 \text{ N}.
\]

In the U.S. customary system, weight is expressed in pounds (lb). The unit of force is of particular importance in engineering analysis and design, because it is involved in calculations of force, moment, torque, stress (or pressure), work (or energy), power, and elastic modulus. Interestingly, in SI units, a newton is approximately the weight of (or Earth’s gravitational force on) an average apple.

Tables B.1 through B.3 list principal units used in mechanics, standard prefixes for SI units, and some conversion factors. The prefixes avoid unusually large or small numbers. We also note that a dot is to be used to separate units that are multiplied together. Therefore, for example, a newton meter is written \( \text{N} \cdot \text{m} \) and must not be confused with \( \text{mN} \), which stands for millinewtons. The reader is cautioned always to check the units in any equation written for a problem solution. If properly written, an equation should cancel all units across the equal sign.

### 1.8 APPLICATION TO SIMPLE STRUCTURES

An extensive variety of structures are used in many fields of engineering. A structure is a unit composed of interconnected members supported in such a manner that it is capable of resisting applied forces in static equilibrium. Structures may be considered in four broad categories: frames, trusses, machines, and thin-walled structures whose thicknesses are slight compared to their other dimensions. Adoption of thin-walled structure behavior permits certain simplifying assumptions to be made in the analysis (see Section 8.8). Most bridges and buildings include bars, beams, frames, cables, and columns. The chapter opening photo shows a classic case in which a bridge is highly functional and also aesthetically pleasing.

Each structure should be designed to meet its own set of demands. The American Society of Civil Engineers (ASCE) furnishes design loads for most common structures (Refs. 1.4 and 1.5). Frames and machines are structures containing multforce members. Frames support loads and are usually stationary, fully restrained structures. Machines transmit and modify forces (or power); they may or may not be stationary and will always contain moving parts. The truss is one of the major types of structures. It provides both a practical and economical solution to many engineering situations, particularly in the design of bridges and buildings.
Figure 1.4 illustrates the basic triangular truss ABC. It is assumed that the connection between the members is pinned and friction in the joints is neglected (Fig. 1.4b).

When the truss is loaded only at its joints, the only force in each member is an axial force, either tensile or compressive. A two-bar truss (V structure) is pinned to a rigid foundation at A and C (Fig. 1.4c). A larger plane truss (Fig. 1.5) may be obtained by adding two members at a time to the basic plane truss ABC. Trusses that cannot be constructed from the basic truss are called compound trusses. When several straight members are joined together at their ends to form a three-dimensional configuration, the structure is known as a space truss. In a space truss, the members are taken to be interconnected by smooth ball-and-socket joints. These supports prevent translation in the three directions and hence involve three unknown forces.

Structural analysis involves the determination of the forces and deflections within a structure or its members. This comprises the study of the influence of various types and positions of loads on the member forces so that the structural design may be performed. The earliest demands for sophisticated structural analysis led to a host of so-called classical methods. Note that the advent and subsequent development of the digital computer obviated the need for particular techniques. The specialization of the classical methods was replaced by generalities of the modern matrix methods. A number of problems involving typical assemblies and components will be solved by the latter approach in Chapter 13.

The procedure in load analysis of a pin-connected structure (see Example 1.1) may be outlined as follows. To begin with, consider the whole structure as a free body and write the equilibrium conditions. Next, dismember the structure and label the various members as either two-force (axially loaded) members or multiforce members. Pins are assumed to form an integral part of one of the members they attach. Sketch the free-body diagrams of each member. Obviously, when two-force members are connected to the same member, they are acted upon by that member with equal and opposite forces of unknown magnitude but known direction. Then, the equilibrium conditions applied to the free-body diagrams of the members are solved to obtain various internal forces.
Two methods are in common use for analyzing trusses. The **method of joints** consists of analyzing the structure joint by joint to obtain the forces in members by applying the equilibrium conditions at each joint. The **method of sections** is employed for problems where the forces in only a few members are to be found. It permits calculation of the axial force conveniently in any particular bar in a plane truss if that truss can be split into two sections by cutting through no more than three bars. The preceding approaches are demonstrated in Example 1.2. Once the internal force resultants are obtained, we may readily compute the stresses as discussed in the next chapter.

---

**EXAMPLE 1.1**

**Member Forces in a Pin-Connected Frame**

*Given:* An assembly of two beams $ABCD$ and $CEF$, and one bar $BE$, connected by pins, carries a vertical load $P = 6$ kips at point $F$ (Fig. 1.6a); it is supported by a pin at $A$ and a cable $DG$. Dimensions are in inches.

*Find:*

(a) The components of the forces acting on joints $B$, $C$, and $E$.

(b) The internal force and moment resultants acting on the cross section at point $O$.

*Assumptions:* Friction forces in the pin joints will be neglected. All forces are coplanar and two dimensional. Weights of the members are negligible compared to the applied load and can be ignored.

*SOLUTION:* See Fig. 1.6 and Table 1.1.

**FIGURE 1.6** (a) Structural assembly; (b and c) free-body diagrams of members and part $CO$ of beam $CEF$.

**Free-Body: Entire Frame.** There are two components $R_{Ax}$ and $R_{Ay}$ of the reaction at $A$ and the force $T$ exerted by the cable at $D$. So, we can find the reactions by considering the free-body diagram of the entire frame (Fig. 1.6a):

\[
\sum M_A = -6(125) + T \sin 30^\circ (200) = 0 \quad T = 7.5 \text{ kips}
\]

\[
\sum F_x = R_{Ax} - 7.5 \sin 30^\circ = 0 \quad R_{Ax} = 3.75 \text{ kips}
\]

\[
\sum F_y = R_{Ay} - 7.5 \cos 30^\circ - 6 = 0 \quad R_{Ay} = 12.5 \text{ kips}
\]

(a) The frame is now dismembered. Inasmuch as only two members are connected at each joint, equal and opposite components are shown on each
member at each joint (Figure 1.6b). Observe that $BE$ is a two-force member—that is, a bar subjected to forces having the same line of action, same magnitude, and opposite senses at only two points.

**Free-Body: Member CEF**

\[
\begin{align*}
\sum M_C &= -6(125) + \frac{2}{\sqrt{13}} F_{BE}(75) = 0 \quad F_{BE} = 18.03 \text{ kips (C)} \\
\sum M_E &= -6(50) + F_{Cy}(75) = 0 \quad F_{Cy} = 4 \text{ kips} \\
\sum F_x &= \frac{3}{\sqrt{13}} (18.03) - F_{Cx} = 0 \quad F_{Cx} = 15 \text{ kips}
\end{align*}
\]

**Free-Body: Member ABCD.** All internal force resultants have been found. To check the results, by applying equations of statics we verify that the member $ABCD$ is in equilibrium.

**Comment:** The positive values found indicate that the directions shown for the force components are correct; member $BE$ is in compression (C).

**(b) Free-Body: Part CO.** The beam $CEF$ is cut at point $O$. Selecting the free-body diagram of the portion $CO$ (Fig. 1.6c), we obtain

\[
M_O = 4(50) = 200 \text{ kip-in.} \quad F_O = 15 \text{ kips} \quad V_O = 4 \text{ kips}
\]

**Comment:** The internal load resultants at $O$ are equivalent to a couple, an axial force, and a shear force, acting as shown in the figure.

---

**EXAMPLE 1.2**

**Forces in Members of a Truss**

**Given:** The truss shown in Fig. 1.7a is constructed of seven bars, each having length $L$. The loads $P_1 = 100 \text{ N}$ and $P_2 = 50 \text{ N}$ act at joints $B$ and $C$, respectively.

![Figure 1.7](a) A seven-member truss; (b and c) free-body diagrams of joint $A$ and joint $B$. 

\[P_1 = 100 \text{ N} \quad P_2 = 50 \text{ N} \]

\[F_{AB} \quad F_{BC} \quad F_{BE}\]
Find:
(a) The axial forces $F_{AB}$, $F_{AE}$, $F_{BC}$, and $F_{BE}$ using the method of joints.
(b) The axial force $F_{BC}$ using the method of sections.

Assumption: Friction in pinned joints and the weight of the members are neglected.

SOLUTION: The support reactions are indicated in the figure.

Free-Body: Entire Truss. Reactions at the supports can be calculated by applying the equilibrium conditions to the free-body diagram of the entire truss (Fig. 1.7a):

- $\sum F_x = 0 : \quad -R_{Ax} + 50 = 0 \quad R_{Ax} = 50 \text{ N}$
- $\sum M_A = 0 : \quad -100 \left(\frac{L}{2}\right) - 50 \left(\frac{\sqrt{3}}{2} L\right) + R_{Dy}(2L) = 0 \quad R_{Dy} = 46.6 \text{ N}$
- $\sum M_D = 0 : \quad -R_{Ay}(2L) + 100 \left(\frac{3}{2} L\right) + 50 \left(\frac{\sqrt{3}}{2} L\right) = 0 \quad R_{Ay} = 53.4 \text{ N}$

Note, as a check, that $\sum F_y = 0$.

(a) Method of Joints. We first draw free-body diagrams of the joints. The unknown forces are usually shown as being directed away from a joint if they are in tension and toward the joint for compression. Occasionally, it is possible to tell by inspection if the forces are in tension or in compression. If the assumed direction is correct, the force calculated from equations of equilibrium will be positive; when the direction is not correct, the force will be negative. In the method of joints, the equations of statics are applied to each joint separately. Great care must be exercised in handling the signs of the forces obtained for a joint when setting up the free-body diagrams of subsequent joints.

Free Body: Joint A (Fig. 1.7b)

- $\sum F_y = 0 : \quad 53.4 - F_{AB} \sin 60^\circ = 0 \quad F_{AB} = 61.5 \text{ N}$
- $\sum F_x = 0 : \quad -50 + F_{AE} - F_{AB} \cos 60^\circ = 0 \quad F_{AE} = 80.8 \text{ N}$

Free Body: Joint B (Fig. 1.7c)

- $\sum F_y = 0 : \quad -100 + 61.5 \sin 60^\circ - F_{BE} \sin 60^\circ = 0 \quad F_{BE} = -54 \text{ N}$
- $\sum F_x = 0 \quad F_{BC} + 61.5 \cos 60^\circ + F_{BE} \cos 60^\circ = 0 \quad F_{BC} = -3.8 \text{ N}$

Comment: The minus sign means that the directions of $F_{BE}$ and $F_{BC}$ are opposite to that assumed in Fig. 1.7c; members $BE$ and $BC$ are in compression (C).

(b) Method of Sections. An imaginary cut through $a-a$ (Fig. 1.7a) exposes the unknown forces $F_{AE}$, $F_{BC}$, and $F_{BE}$ as shown in Fig. 1.8. To eliminate the two forces passing through the point $E$, we write

- $\sum M_E = 0 : \quad -53.3 L - F_{BC} \left(\frac{\sqrt{3}L}{2}\right) + 100 \left(\frac{L}{2}\right) = 0 \quad F_{BC} = -3.8 \text{ N}$

This corresponds to the result found in part (a).
Comment: Observe that cut a-a has been judiciously selected so that the summation of moments about point E immediately yielded the desired force $F_{BC}$.

EXAMPLE 1.3

Internal Load Resultants at a Section of a Pipe Assembly

Given: An L-shaped three-dimensional pipe assembly consisting of two perpendicular parts $AB$ and $BC$ connected by an elbow at $B$ is bolted to a rigid frame at $C$ (Fig. 1.9a). The pipe supports a vertical load $P_A = 80$ N and a torque $T_A = 20$ N·m at $A$ as well as its own weight. Each pipe is made of steel with 50-mm nominal diameter.

Find: What are the axial force, shear forces, and moments acting on the cross section at point $O$?

Assumptions: The weight of the pipe assembly is uniformly distributed over its entire length. The mass of the pipe is taken as 2.45 kg/m.

SOLUTION: See Figs. 1.2 and 1.9, and Eqs. (1.1). The weights of pipes $AB$ and $BO$ are

$$W_{AB} = (2.45)(0.5)(9.81) = 12.02 \text{ N} \quad W_{BO} = (2.45)(0.4)(9.81) = 9.61 \text{ N}$$

**Free-Body: Part $AB$.** There are six conditions of equilibrium for the three-dimensional force system of six unknowns (Fig. 1.9b). The first three
of Eqs. (1.1) relate the applied loads to the internal forces on the pipe at point $O$:

$$
\sum F_x = 0 : \quad F = 0
$$

$$
\sum F_y = 0 : \quad V_y - 12.02 - 9.61 - 80 = 0 \quad V_y = 101.6 \text{ N}
$$

$$
\sum F_z = 0 : \quad V_z = 0
$$

Using the last three of Eqs. (1.1), the moments about point $O$ are

$$
\sum M_x = 0 : \quad T + (12.02)(0.25) + 80(0.5) = 0 \quad T = -43 \text{ N} \cdot \text{m}
$$

$$
\sum M_y = 0 : \quad M_y = 0
$$

$$
\sum M_z = 0 : \quad M_z - 20 - 80(0.4) - (12.02)(0.4) - (9.61)(0.2) = 0 \quad M_z = 58.7 \text{ N} \cdot \text{m}
$$

**Comment:** The negative value obtained for $T$ indicates that the torque vector is directed opposite to that shown in the figure.

**PROBLEMS**

*The following problems are to be solved using these assumptions, as needed:* Weights of members (unless otherwise specified) are insignificant compared to the applied forces and neglected, all loads are static, and friction in the pin joints is disregarded.

**Sections 1.1 through 1.8**

**1.1** A frame formed by joining a bent beam $ACD$ with a bar $BC$ by a hinge is supported at $A$ and $B$ as shown in Fig. P1.1. A vertical load $P = 150 \text{ kN}$ is applied at point $D$. For each member, determine:

(a) The components of support reactions at $A$ and $B$.

(b) The axial force, shear force, and bending moment acting on the cross section at point $O$.

![Figure P1.1](image)

**1.2** A structure constructed of two pin-connected members $ADB$ and $BC$ carries a vertical load $P = 90 \text{ kips}$ at point $D$ (Fig. P1.2). Calculate:

(a) Reactions at supports $A$ and $C$.

(b) The internal force and moment resultants on the cross section at point $O$.

![Figure P1.2](image)

**1.3** A frame formed by joining two members $ADB$ and $BC$ with a hinge at $B$ supports a vertical load $P = 90 \text{ kips}$ at point $E$ (Fig. P1.2). *Note:* No load acts at $D$. Determine:

(a) Reactions at supports $A$ and $C$.

(b) The internal force and moment resultants on the cross section at point $O$.

![Figure P1.3](image)

**1.4** An engine system consists of a piston attached to a connecting rod $AB$, which in turn is connected to a crank arm $BC$ as shown in Fig. P1.4. The piston slides without friction in a cylinder and is subjected to a force $P$. The crank arm is exerting a torque $T = 2 \text{ kN} \cdot \text{m}$. For the position shown, calculate:

(a) The force $P$ required to hold the system in equilibrium.

(b) The axial force in the rod $AB$.

![Figure P1.4](image)
1.5 The frame shown in Fig. P1.5 is constructed of four pin-connected members \( ABC \) of length \( 3a \), \( DEF \) of length \( 2a \), \( BE \), and \( CF \). A vertical load \( P \) acts at point \( C \). What are the axial forces in members \( CF \) and \( BE \)?

![Figure P1.5](image1)

1.6 A frame \( AB \) and a simple beam \( CD \) of length \( 6a \) are supported as illustrated in Fig. P1.6. A roller fits snugly between the two members at point \( E \). The structure is subjected to an inclined concentrated load \( P \) at point \( B \). Determine the support reactions at \( A \) and \( C \).

![Figure P1.6](image2)

1.7 A frame constructed of three members \( ABC \) of length \( 3a \), \( ADE \), and \( BD \) is subjected to a vertical load \( W \) at point \( E \) (Fig. P1.7). Determine the internal force and moment resultants acting on the cross sections:

(a) At point \( G \).

(b) At point \( H \).

![Figure P1.7](image3)

1.8 A structure constructed by joining a beam \( ABC \) with bar \( BD \) by a hinge carries a vertical load \( P_1 = 5 \) kips and a horizontal load \( P_2 = 10 \) kips as shown in Fig. P1.8. Compute:

(a) The reactions at support \( C \).

(b) The internal force and moment resultants on a cross section at point \( O \).

![Figure P1.8](image4)

1.9 A frame \( AC \) and a simple beam \( BCD \) are pin connected and supported as shown in Fig. P1.9. The structure is subjected to a vertical load \( P = 10 \) kips at point \( D \). Determine:

(a) The reactions at support \( B \) and on joint \( C \).

(b) The internal force and moment resultants on the cross section at point \( O \).

![Figure P1.9](image5)

1.10 A structure formed by joining members \( ABC, AD, BE, \) and \( DEF \) with pins is subjected to a horizontal load \( P = 30 \) kN at point \( A \), as shown in Fig. P1.10. Determine:

(a) The reactions at supports \( C \) and \( F \).

(b) The axial forces in members \( AD \) and \( BE \).

![Figure P1.10](image6)
*1.11 A structure constructed of four rigidly fixed members $AB$, $BC$, $BD$, and $DE$ is loaded as shown in Fig. P1.11. Determine:
(a) The reactions at supports $C$ and $E$.
(b) The internal force and moment resultants on a cross section at point $O$.

![Figure P1.11](image)

1.12 A frame $ABC$ and a bar $CD$ are pin connected at point $C$ and supported as shown in Fig. P1.12. The structure is subjected to a horizontal load $P_1 = 20$ kN at point $B$ and a vertical load $P_2 = 15$ kN at point $C$. Calculate the axial force in bar $CD$.

![Figure P1.12](image)

1.13 The truss $ABCD$ of height 1.5 m and span 3.5 m supports a vertical load $P = 10$ kN at joint $D$ (Fig. P1.13). What are the axial forces in each member? Use the method of joints.

![Figure P1.13](image)

1.14 A truss $ABCD$ constructed of five pin-connected members is subjected to two vertical loads $P = 9$ kN applied at joints $B$ and $C$ as shown in Fig. P1.14. Calculate:
(a) Reactions at supports $A$ and $D$.
(b) The axial forces in each member of the truss, using the method of joints.

![Figure P1.14](image)

1.15 A truss consisting of seven members is subjected to a vertical load $P = 100$ N at joint $B$ (Fig. P1.15). Determine:
(a) Reactions at supports $A$ and $B$.
(b) The axial forces in each member of the truss, using the method of joints.

![Figure P1.15](image)

1.16 A truss that has height $H$ and span $L$ supports the loads $P$ and $2P$ as shown in Fig. P1.16. Find:
(a) Reactions at supports $A$ and $G$.
(b) The axial forces in members $CE$, $DE$, and $DF$, using the method of sections.

![Figure P1.16](image)
1.17 The truss shown in Fig. P1.17 is constructed of eleven members. A vertical load \( P_1 = 90 \) kips and a horizontal load \( P_2 = 120 \) kips act at joint B. Compute:
(a) Reactions at supports A and E.
(b) The axial forces in members CD, CG, and FG, using the method of sections.

![Figure P1.17](image)

1.18 The structure shown in Fig. P1.17 consists of eleven members. A vertical load \( P_1 = 30 \) kips and a horizontal load \( P_2 = 40 \) kips act at joint B. Determine the axial forces in members BC, CF, and FG. Apply the method of sections.

1.19 A truss constructed of nine pin-connected members supports loads \( P_1 = 15 \) kN and \( P_2 = 5 \) kN at points B and C, respectively (Fig. P1.19). Using the method of sections, determine the axial forces in members BC, EF, and EC of the truss, respectively.

![Figure P1.19](image)

1.20 A truss of height \( H \) and span \( L \) supports vertical loads \( P_1 \) and \( P_2 \) at joints C and D, respectively, as shown in Fig. P1.20. Take \( H = 3 \) m, \( L = 8 \) m, \( P_1 = 25 \) kN, and \( P_2 = 20 \) kN. Calculate:
(a) Reactions at supports A and E.
(b) The axial forces in members AB, AF, BC, and BF, using the method of joints.

1.21 A truss formed by nine pin-connected members is subjected to vertical loads \( P_1 \) and \( P_2 \) at joints C and D, respectively (Fig. P1.20). Let \( H = 6 \) ft, \( L = 16 \) ft, \( P_1 = 5 \) kips, and \( P_2 = 2 \) kips. Using the method of sections, determine the axial forces in members CD, DF, and FE of the truss.

![Figure P1.20](image)

1.22 A truss of 5-m height and 11.25-m span carries a vertical load \( P_1 = 40 \) kN and a horizontal load \( P_2 = 20 \) kN at joints D and B, respectively (Fig. P1.22). Find:
(a) Reactions at supports C and F.
(b) The axial forces in members AC, CD, AD, and DE, using the method of joints.

![Figure P1.22](image)

1.23 A truss of 5-m height and 11.25-m span, constructed of nine pin-connected members (Fig. P1.22), supports a vertical load \( P_1 = 20 \) kN and a horizontal load \( P_2 = 10 \) kN at joints D and B, respectively. Employing the method of sections, determine the axial forces in members AB, DE, and AE of the truss.

1.24 A pipe ABCD having three perpendicular arms AB, BC, and CD carries a vertical load \( P = 50 \) lb applied at point E by a wrench as shown in Fig. P1.24. What are the reactions at the fixed support A?

![Figure P1.24](image)
**1.25** A 4-in. nominal diameter standard steel pipe $ABCD$ formed by three perpendicular arms $AB$, $BC$, and $CD$ is subjected to a vertical load $P = 50$ lb applied at point $E$ by a wrench (Fig. P1.24). Calculate the reactions at the clamped support $A$.

*Assumption:* The weight of the pipe (see Table B.7) will be considered.

**1.26** A hollow transmission shaft $AB$ is supported at $A$ and $E$ by bearings and subjected to loadings applied to the pulleys $B$, $C$, and $D$ as shown in Fig. P1.26. Note that the 300-lb and 600-lb forces act parallel to the $y$ direction; the 200-lb and 800-lb forces act parallel to the $z$ direction. Determine:

(a) The torque $T$ required for equilibrium.

(b) The reactions at the bearings.

*Assumption:* The bearings act as simple supports.

**1.27** A hollow transmission shaft $AB$ is supported at the ends $A$ and $B$ by bearings and carries loadings applied to the pulleys $C$ and $D$ as shown in Fig. P1.27. Observe that the 80-lb forces and the 100-lb forces act in the $y$ and $z$ directions, respectively. Calculate:

(a) The components of the reactions at $A$ and $B$.

(b) The internal force resultants acting on the cross section at point $O$.

*Assumption:* The bearings act as simple supports.

**1.28** A 3.2-m-long boom or bar $AB$ and cable $BC$ assembly carries a weight $W = 40$ kN as shown in Fig. P1.28. The bar is hinged at left end $A$. Determine the reactions at $A$ for the boom and the tension in the cable.

*Assumption:* The boom will not fail by buckling.

**1.29** A sign of weight $W = 6$ kN acting at the center of gravity $G$ is supported by a pipe as shown in Fig. P1.29. The dimensions of the sign are 4 m by 2 m. The wind pressure against the sign is $p = 2$ kPa. What are the internal resultant forces and moments acting on the cross section at the point $O$ of the pipe?
1.30 A bent rod is supported in the $xz$ plane by bearings at $B$, $C$, $D$, and subjected to $P_1 = 40\text{ lb}$ and $P_2 = 60\text{ lb}$ acting in the $y$ direction (Fig. P1.30). Determine the internal load resultants acting on the cross section at point $O$.

![Figure P1.30](image1.png)

1.31 A pipe formed by four perpendicular arms $AB$, $BC$, $CD$, and $DE$ lying in the $x$, $z$, $y$, and $x$ directions, respectively, is fixed at left end $A$ (Fig. P1.31). The loads $P_1 = 50\text{ lb}$ and $P_2 = 100\text{ lb}$ act at the right end $E$ in the $x$ and $z$ directions, respectively. Calculate the internal forces and moments on the cross section at point $O$.

![Figure P1.31](image2.png)
KEY CHAPTER EQUATIONS

Equations of Equilibrium

Three-Dimensional Problems

\[
\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0
\]
\[
\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0
\]

Two-Dimensional Problems

\[
\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_A = 0
\]

Section 1.5

Alternative Sets:

\[
\sum F_x = 0 \quad \sum M_A = 0 \quad \sum M_B = 0
\]

or

\[
\sum M_A = 0 \quad \sum M_B = 0 \quad \sum M_C = 0
\]

See the referenced section for information on the derivation and proper use of these equations.

REFERENCES