1

Introduction

1.1 WHY CONSIDER FINITE ARRAYS?

The short answer to this question is, Because they are the only ones that really exist.

However, there are more profound reasons. Consider, for example, the infinite × infinite array shown in Fig. 1.1. It consists of straight elements of length $2l$, and the interelement spacings are denoted $D_x$ and $D_z$ as shown. Such an infinite periodic structure was investigated in great detail in my earlier book, Frequency Selective Surfaces, Theory and Design [1]. There the underlying theory and notation for the Periodic Moment Method (PMM) is described. It became the basis for the computer program PMM written by Dr. Lee Henderson as part of his doctoral dissertation in 1983 [2, 3].

In the intervening years it has stood its test and has become the standard in the industry.

Consider next the finite × infinite array shown in Fig. 1.2. It consists, like the infinite × infinite case in Fig. 1.1, of columns that are infinite in the Z direction, however, there is only a finite number of these columns in the X direction. Such arrays have been investigated by numerous researchers [4–23]—in particular, by Usoff, who wrote the computer program SPLAT (Scattering from a Periodic Linear Array of Thin wire elements) as part of his doctoral dissertation in 1993 [24, 25].

Let us now apply the PMM program to obtain the element currents for an infinite × infinite FSS array of dipoles with $D_x = 0.9$ cm and $D_z = 1.6$ cm, while
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Fig. 1.1 An "infinite x infinite" truly periodic structure with interelement spacing $D_x$ and $D_z$ and element length $2l$.

Incident wave in direction $\hat{s} = \hat{x}s_x + \hat{y}s_y + \hat{z}s_z$:

$$I_{qm} = I_0 e^{-j\beta D_z s_x} e^{-j\beta_m D_z s_z}$$

Floquet's Theorem

$m = 0$

$m = 1$

$m = -1$

Finite x Infinite Array

Fig. 1.2 An array that has a finite number of element columns in the X direction and is infinite in the Z direction. It is truly periodic in the latter direction but not in the former. Thus, Floquet's Theorem applies only to the Z direction, not the X direction.
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Fig. 1.3 Various cases of a plane wave incident upon infinite as well as finite arrays at $45^\circ$ from normal in the H plane. Element length $2l = 1.5$ cm, load impedance $Z_L = 0$ and frequencies as indicated. (a) Element currents for an infinite $\times$ infinite array at $10$ GHz as obtained by the PMM program (close to resonance). (b) Element currents for a finite $\times$ infinite array of 25 columns at $10$ GHz (close to resonance). (c) Element currents for a finite $\times$ infinite array of 25 columns at $7.8$ GHz (~25% below resonance).

$\lambda_0 = 0.9$ cm $= 0.3 \lambda_0$

$\lambda_0$ = Wavelength at dipole resonant frequency of $10$ GHz
the element length $2l = 1.5$ cm; that is, the array will resonate around 10 GHz. The angle of incidence is $45^\circ$ in the orthogonal plane (H plane). The current magnitudes are plotted column by column in Fig. 1.3a at $f = 10$ GHz.

Similarly we apply the SPLAT program to obtain the current magnitudes in an finite $\times$ infinite array of 25 columns as depicted in Fig. 1.3b. We notice that the infinite case in Fig. 1.3a agrees pretty well with the finite case in Fig. 1.3b, except for the very ends of the finite array. This observation is typical in general for large arrays and is simply the basis for using the infinite array program to solve large finite array problems as encountered in practice. The deviation between the two cases (namely the departure from Floquet’s theorem [26] in the finite case) is usually of minor importance as long as the array is used as a frequency selective surface (FSS) like here [27]. However, if the array instead is designed to be an active array in front of a groundplane and each element is loaded with identical load resistors (representing the receiver or transmitter impedances), the situation may change dramatically. As shown in Chapters 2 and 5, we can in that case adjust the load impedances such that no reradiation takes place in the specular direction from all the elements except the edge elements. However, as also discussed in Chapter 5, we may change the loads for the edge elements such that no scattering in the specular direction takes place from these as well.

So far we have merely tacitly approved of the standard practice, namely the use of infinite array theory to solve finite periodic structure problems, at least in the case of an FSS with no loads and no groundplane. However, even in that case we may encounter a strong departure from the infinite array approach. In short, we may encounter phenomena that shows up only in a finite periodic structure and never in an infinite as will be discussed next.

1.2 SURFACE WAVES UNIQUE TO FINITE PERIODIC STRUCTURES

We have calculated the element currents only at $f = 10$ GHz—that is, close to the resonant frequency of the array. Let us now explore the situation at a frequency approximately 25% lower, namely at $f = 7.8$ GHz. From the SPLAT program we obtain the element currents shown in Fig. 1.3c, while the PMM program gives us element currents equal to 0.045 mA as shown in Fig. 1.3c, close to what would be expected based on the resonant value of 0.055 mA (see Fig. 1.3a).

We observe in Fig. 1.3c that the element currents for the finite array not only fluctuate dramatically from column to column but also exhibit an average current that can be estimated to be somewhat higher than the currents even for resonance condition (0.055 mA).

We shall investigate this phenomena in detail in Chapter 4. It will there be shown that the element currents are composed of three components:

1. The Floquet currents as observed in an infinite $\times$ infinite array—that is, currents with equal magnitude and a phase matching that of the incident plane wave.
2. Two surface waves, each of them propagating in opposite directions along the x axis. They will in general have different amplitudes but the same phase velocities that differ greatly from those of the Floquet currents. Thus, the surface waves and the Floquet currents will interfere with each other, resulting in strong variations of the current amplitudes as seen in Fig. 1.3c.

3. The so-called end currents. These are prevalent close to the edges of the finite array and are usually interpreted as reflections of the two surface waves as they arrive at the edges.

We emphasize that these surface waves are unique for finite arrays. They will not appear on an infinite array and will consequently not be printed out by, for example, the PMM program that deals strictly with infinite arrays. Nor should they be confused with what is sometimes referred to as edge waves [28]. The propagation constant of these equals that of free space, and they die out as you move away from the edges. See also Section 1.5.3.

Furthermore, the surface waves here are not related to the well-known surface waves that can exist on infinite arrays in a stratified medium next to the elements. These will readily show up in PMM calculations. These are simply grating lobes trapped in the stratified medium and will consequently show up only at higher frequencies, typically above resonance but not necessarily so in a poorly designed array. In contrast, the surface waves associated with finite arrays will typically show up below resonance (20–30%) and only if the interelement spacing $D_x$ is $<0.5\lambda$.

From a practical point of view, the question is of course whether these surface waves can hurt the performance of a periodic structure when used either passively as an FSS or actively as a phased array. And if so, what can be done about it.

We will discuss these matters next and in more detail in Chapters 4 and 5.

### 1.3 EFFECTS OF SURFACE WAVES

The most prevalent effects of the new type of surface waves associated with finite periodic structures depend to an extent upon whether they are used passively as an FSS or actively as a phased array.

In the first case we will observe a significant increase in the bistatic scattering. In the second case we will observe a variation of the terminal impedance as we move from column to column. Let us look upon these two phenomena separately.

#### 1.3.1 Surface Wave Radiation from an FSS

Surface waves on a finite FSS will radiate just like the Floquet currents will radiate. These matters—and, in particular, how they are being excited—will be the subject of detailed discussions in Chapter 4. It suffices in this introduction to present a typical example as shown in Fig. 1.4. We show here 25 columns with the same element dimension as earlier (see insert). The angle of incidence is
The bistatic scattered field in the $H$ plane from a finite $\times$ infinite array of 25 columns at $f = 7.7$ GHz. (--) Scattering pattern calculated by using merely the Floquet currents—that is, simply by truncating an infinite structure. (---) Scattering pattern calculated by using the actual element currents (exact).

67.5° as also indicated in the insert. The Floquet currents alone are producing a bistatic scattering pattern as indicated by the full line in Fig. 1.4 (this corresponds to simple truncation of an infinite FSS). Also shown is the bistatic scattering pattern as obtained by using the total currents on the finite FSS—that is, the sum of the Floquet currents, the two surface waves, and the end currents as obtained by direct calculation from the SPLAT program (see the broken line pattern). The pattern obtained from the Floquet currents only are of course merely a pattern of the $\sin x/x$ type. However, when using the total calculated current we observe
no perceptible change of the two main beams while the sidelobe level in this case is raised by about 10 dB (the exception is the sector between the two main beams where it actually is lower than the Floquet pattern). Later in Chapter 4 we will show more examples where the sidelobe level can be raised by more than 10 dB. In other words, the RCS of a finite FSS could be raised by that amount unless treated.

The encouraging conclusion is of course that even if the surface waves might actually be stronger than the Floquet currents (see, for example, Fig. 1.3c), they apparently radiate less efficiently than the Floquet currents. These facts will be discussed in detail in Chapter 4.

1.3.2 Variation of the Scan Impedance from Column to Column

If our periodic structure is fed as a phased array from constant voltage generators without generator impedances, the relative current magnitudes at the terminals will be like those shown in Fig. 1.3. Since the scan impedance is equal to the terminal voltage (namely the constant generator voltages) divided by the terminal currents, it is clear that the scan impedance will vary inversely to the currents in Figs. 1.3b and 1.3c.

Obviously it would be too much of a challenge to match an impedance with precision to the fluctuating scan impedance of Fig. 1.3c—in particular, when we realize that the maximum and minimum will start moving around with scan angle and frequency. Thus, we must simply look for ways to get rid of the surface waves or at least reduce them. We will discuss these matters next and in more detail in Chapter 4.

1.4 HOW DO WE CONTROL THE SURFACE WAVES?

1.4.1 Phased Array Case

In the previous section we considered phased arrays fed from constant voltage generators with the generator impedance equal to zero. We saw how this scenario could lead to disastrous variations in the scan impedance. Fortunately, a more realistic situation would be to feed the individual elements from constant voltage generators with generator impedances similar to the scan impedances as obtained from the infinite array case (i.e., approximating conjugate match). Thus, we show in Fig. 1.5a the same case as shown earlier in Fig. 1.3c but with load resistors equal to 100 ohms in order to simulate the generator impedances.

Several features are worth observing. First of all the fluctuations from element to element have been greatly reduced but obviously not completely eradicated. Second, the Floquet currents in Fig. 1.3c have been reduced from 0.045 mA to ~0.032 in Fig. 1.5a—that is, a reduction of approximately 0.032/0.045 = 0.71.

This reduction is easy to explain by inspection of the equivalent circuit shown in Fig. 1.6a.

Here the voltage generator $V^g$ is connected in series with its generator impedance $Z_G$ and the scan impedance $Z_A$. The ratio between the currents
Fig. 1.5 The actual element currents in each column of a finite array of 25 columns when exposed to an incident plane wave at 45° from normal or fed like a phased array from individual voltage generators with a linear phase delay. (a) All elements loaded with $R_L = 100$ ohms. (b) Only the outer columns (each side) are loaded with 200 ohms, the next inner columns with 100 ohms and finally the third inner ones with 50 ohms. All other elements have no load resistances. (c) All elements loaded with $R_L = 20$ ohms.
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**Fig. 1.6** The approximate equivalent circuit for an infinite periodic structure when used as: (a) A phased array with the individual elements fed from generators \( V^g \) with generator impedances \( Z_G \). (b) A frequency selective surface when a voltage \( V^i \) is induced by an incident wave and \( Z_L \) is a load impedance (in general purely reactive).

without and with the generator impedance is seen to be \( Z_A/(Z_A + Z_G) \). For the array considered here a rough estimate of the average \( Z_A \) would be around 200 ohms. Thus, for \( Z_G = 100 \) ohms the reduction would be approximately \( 200/(200 + 100) = 0.67 \), which is in fair engineering agreement with the observation above (namely 0.71).

We emphasize that this reduction is by no means "embarrassing." It is in basic agreement with the conjugate matched case where the current ratio would be 0.50 and the efficiency 50%. See also the discussion in Appendix B.9.

But how do we explain the much stronger reduction of the ripples associated with the surface waves? Well, we shall later in Chapter 4 investigate surface waves in much more detail. It will there be shown that the terminal impedance associated with the surface waves is quite low, say of the order of \( Z_{surf} \sim 10 \) ohms for each of the two surface waves. Thus, by the same reasoning as for the Floquet currents above, we find for each surface wave a reduction equal to \( 10/(10 + 100) = 0.091 \). This is of course an average value but explains the strong ripple reduction observed in Fig. 1.5a.

This observation is quite noteworthy. It shows that by matching an antenna in the neighborhood of maximum power transfer (i.e., conjugate matching) we obtain an added benefit, namely a potential strong reduction of the ripples of the scan impedance even at a frequency where the surface waves are dominating.

Incidentally, the low value of the terminal surface impedance \( Z_{surf} \) is just another manifest of what has been observed earlier (see Fig. 1.4)—namely, that in spite of the fact that the surface wave currents may be stronger than the Floquet currents (see Fig. 1.3c), their radiation intensity will in general be considerably below that of the Floquet currents. Several actual calculated examples illustrating this statement will be given in Chapter 4.

### 1.4.2 The FSS Case

When a periodic structure is intended to work as a wire FSS, it would lead to unacceptably high reflection loss if each element was loaded with resistors
comparable to the terminal impedance $Z_A$ (about 3 dB). To gain further insight, let us consider the equivalent circuit for an FSS as shown in Fig. 1.6b. Here the generator voltages $V^i$ are no longer produced by man-made generators $V^g$ but are instead induced by the incident plane wave. The objective at resonance is now simply to get as high a current as possible to flow through $Z_A$ and $Z_L$ in order to obtain lossless reflection from the surface. Thus, any load impedance $Z_L$ should ideally be purely imaginary and serve merely to cancel any imaginary components of $Z_A$.

So how do we control surface waves on an FSS?

One approach is to simply have no resistors anywhere over the entire surface, with the exception of a few columns at the edges. An example is shown in Fig. 1.5b, where the two outer columns have been loaded with 200 ohms, the next ones toward the center with 100 ohms, and finally the third column with 50 ohms. We observe a significant reduction of the ripple amplitudes as compared to the unloaded case in Fig. 1.3c. It should be noted that no parametric study was done on the resistive values of the loads at this point. More in Chapter 4.

We also show in Fig. 1.5c a case where each element over the entire surface has been loaded very lightly, namely with 20 ohms. We observe a strong reduction of the ripples from column to column—in particular, in the right half of the array.

The transmission loss at resonance due to the 20-ohm load resistors is obtained from the equivalent circuit in Fig. 1.6b. The reduction of current is equal to $Z_A/(Z_A + Z_L) = 200/(200 + 20) = 0.9$, or about 1 dB (just barely permissible).

Alternatively we may instead of the 20-ohm loss resistors obtain a moderate loss by simply using a slightly lossy dielectric next to the elements or simply a resistive sheet close to the elements.

Finally, many possibilities are open by combinations of the various approaches listed above. More about this in Chapter 4.

1.5 COMMON MISCONCEPTIONS

1.5.1 On Common Misconceptions

In my first book, *Frequency Selective Surfaces, Theory and Design* [1], I introduced at the end of each chapter a section called *Common Misconceptions*. It was intended to eradicate some of the many myths and misunderstandings that seem so prevalent "out there." It was also intended to form the basis for further discussion in class. It soon became very popular. In fact, I became aware that these sections were often read with great glee before the text preceding them. This was manifested in well-meaning comments like: "Well, it is fine that you tell us what will and will not work. But you must also tell us why." It slowly dawned on me that a new misconception had arrived: You just had to read the sections about common misconceptions and you would be up to speed and not make a fool out of yourself.

Furthermore, it was often implied that the design examples were the results of either a parametric study or an optimization process or were based on "many years of experience."
While I will admit to some parametric observations where no specific theoretical background could be established right away, we are basically using an analytic approach\(^1\) that not only leads to a clear understanding of the problems but also establishes whether solutions exist and what they are.

I think it was Edison that once stated, “There is no substitute for hard work.”

1.5.2 On Radiation from Surface Waves

This title will undoubtedly raise a few eyebrows. As stated in many respectable textbooks, surface waves do not radiate—period. What is not always emphasized is the fact that the theory for surface waves in general is based on a two-dimensional model like for example an infinitely long dielectric coated wire. And as discussed in this chapter infinite array theory may reveal many fundamental properties about arrays in general but there are phenomena that occur only when the array is finite. The fact is that surfaces waves are associated with element currents. They will radiate on a finite structure in the same manner an antenna radiates, namely by adding the fields from each column in an end-fire array. Numerous examples of this kind of radiation pattern will be shown in Chapter 4. They are typically characterized by having a “mainbeam” in the direction of the \(X\) axis that is lower than the “sidelobe” level. The reason for this “abnormality” is simply that the phase delay from column to column exceeds that of the Hansen–Woodyard condition by a considerable amount \([29]\). They also have a much lower radiation resistance.

An alternative approach is to assume that the radiation from a finite array is associated entirely with the edge currents. While Maxwell’s equations do not state specifically that radiation or scattering takes place from neither edges or element tips, it is nevertheless an observation that has proven valuable in classical electromagnetic theory. It is a convenient way to handle scattering properties from perfectly conducting half-planes, strips, wedges, and more, even when made of dielectric.

However, in the case of finite arrays of loaded wire elements the approach loses some of its appeal by the fact that surface waves exist only in a limited frequency range inside which the amplitude and phase vary considerably with frequency. Consequently, the scattering properties must be calculated numerically at each frequency and will actually also depend on array size in a somewhat complicated way.

At this point, this approach therefore is primarily of academic interest.

1.5.3 Should the Surface Waves Encountered Here Be Called Edge Waves?

It has been suggested to denote the surface waves introduced in this chapter as “edge waves” for no reason other than they originate at the edges of the array. This confronts us with certain problems.

\(^1\) By analytic we mean to separate a problem into components and study each of these individually before we put them back together.
First of all, the term edge wave has been used to denote a wave that propagates along and not orthogonal to an edge \[30\]. In other words, we are talking about two entirely different kinds of waves.

Second, the term edge wave has been used by Ufimtsev and many others to denote waves that originate on the edges and propagate orthogonal to these all right \[28\]. However, that kind of edge wave dies down as you move away from the edge and their propagation constant is that of free space. The surface waves encountered here are basically not attenuated (except by radiation and ohmic losses) as they move away from the edge and propagate over the entire array.

Furthermore, the propagation constants of the waves encountered have been determined in Chapter 4 to be precisely equal to that of surface waves propagating along arrays of dipoles. These propagation constants are of course vastly different than that of free space. Thus, the surface waves encountered here should be called surface waves because that is what they are.

One is of course entitled to wonder why this phenomenon has gotten so sparse attention in the literature if any. The main reason is probably that the interelement spacing should be less than 0.5\(\lambda\) and the frequency \(~20–30\%\) below resonance (see Chapter 4 for details). Typically, many researchers choose a borderline spacing of \(D_x = 0.5\lambda\) and concentrate their attention around the resonance frequency \[31–33\]. As can be seen in Fig. 1.3, this basically precludes the existence of any strong surface waves.

1.6 CONCLUSION

We have demonstrated the presence of surface waves that can exist only on a finite periodic structure. It is quite different from the well-known types of surface waves that can exist in a stratified medium next to a periodic structure often referred to as Type 1. These merely represent grating lobes trapped inside the stratified medium. Thus, they will readily manifest themselves in computations based on infinite array theory at frequencies so high that grating lobes can be launched.

In contrast, the new type of surface wave (Type 2) can exist only if the interelement spacing \(D_x\) is so small that no grating lobe can exist. In addition, the frequency must typically be 20–30\% below the resonance frequency of the periodic structure.

The presence of this new type of surface wave manifests itself in various ways:

1. If used as an FSS, it can lead to a significant increase in the bistatic scattering. In particular, we may observe a sizeable increase in the RCS of objects comprised of FSS without treatment.

2. If the structure is used as a phased array, it can lead to dramatic variations of the terminal or scan impedance from column to column. Under these circumstances it would be very difficult to design a high-quality matching network in particular since the maxima and minima of the scan impedance will move significantly with frequency and scan angle.
We also indicated that this type of surface wave could be controlled in various ways. One approach is to load each element resistively. If used as an FSS, the resistors should have a low value in order not to significantly attenuate the reflected signal. In case of phased arrays a resistive loading could be obtained by simply feeding the elements from constant voltage generators with realistic generator impedances.

Alternatively, we could use no resistors at any of the elements across the surface but only at a few columns at the edges of the periodic structure. Slightly lossy dielectric slabs or even resistive sheets can also be used.

Finally, many possibilities are left open by combinations of some or all of the approaches above. See also Chapter 4 for details.

One might well ask the question, Why not just operate in a frequency range between the two types of surface waves? Well, in the case of an FSS it has been demonstrated numerous times that stability with angle of incidence can be obtained only for small interelement spacings (see, for example, reference 34). And basically the same is true for phased arrays in particular if designed for broad bandwidth. See Chapter 6 for details.

This introduction has merely pointed out the presence and treatment of surface waves that may exist below resonance for finite periodic structures. An in-depth investigation will be given in Chapter 4 where we will rely entirely on rigorously calculated examples.

**PROBLEMS**

1.1 Consider a phased array with scan impedance \( Z_A = 200 \) ohms. It is being fed from a generator with impedance \( Z_G \) as shown in Fig. 1.6a. Assume conjugate match—that is, \( Z_G = Z_A = 200 \) ohms.

As shown in Chapter 4, each of the two surface waves are generated from semi-infinite arrays located adjacent to the finite array. We will assume the equivalent circuit to consist of surface wave generators at each end of the finite array with surface wave generator impedances for the left- and right-going surface wave denoted \( Z_{SW_L} \) and \( Z_{SW_R} \), respectively. We will assume that these impedances depend on angle of incidence.

Furthermore, we will assume that the generator impedances \( Z_G \) are connected in series with \( Z_{SW_L} \) and \( Z_{SW_R} \), separately; that is, \( Z_G \) will reduce the surface waves as observed for example in Fig. 1.5a.

Given the surface wave impedances \( Z_{SW_L} \) and \( Z_{SW_R} \) and the generator impedance \( Z_G \):

1. Find the reduction of the surface waves compared to the no-load case \( Z_G = 0 \) for \( Z_G = 200 \) ohms, in decibels for \( Z_{SW_L} \) equal to
   (a) 2.5 ohms
   (b) 5.0 ohms
   (c) 10.0 ohms
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(d) 20.0 ohms
(e) 40.0 ohms

2. If the generator loads are increased to 400 ohms, state approximately how many decibels the reduction will change (up, down?).