## Contents

Preface xiii

Acknowledgments xv

1. Introduction 1

1.1 Displacement Formulation-Based Finite Element Method 2

1.1.1 Derivation of element equation of motion 2

1.1.2 Mass and stiffness matrices of uniform beam element 7

1.1.3 Mass and stiffness matrices of tapered beam element 9

1.2 Element Equations of Motion for Temporally and Spatially Stochastic Systems 13

1.3 Hybrid Stress-Based Element Equations of Motion 14

1.3.1 Derivation of element equation of motion 15

1.3.2 Mass and stiffness matrices of uniform beam element 16

1.4 Incremental Variational Principle and Mixed Formulation-Based Nonlinear Element Matrices 18

1.4.1 Incremental variational principle and linearization 19

1.4.2 Linear and nonlinear element stiffness matrices 23

1.5 Constitutive Relations and Updating of Configurations and Stresses 36

1.5.1 Elastic materials 36

1.5.2 Elasto-plastic materials with isotropic strain hardening 39

1.5.3 Configuration and stress updatings 45

1.6 Concluding Remarks 48

References 49

2. Spectral Analysis and Response Statistics of Linear Structural Systems 53

2.1 Spectral Analysis 53

2.1.1 Theory of spectral analysis 54

2.1.2 Remarks 56

2.2 Evolutionary Spectral Analysis 56

2.2.1 Theory of evolutionary spectra 56

2.2.2 Modal analysis and evolutionary spectra 57

2.3 Evolutionary Spectra of Engineering Structures 60
3. **Direct Integration Methods for Linear Structural Systems**

3.1 Stochastic Central Difference Method

3.2 Stochastic Central Difference Method with Time Co-ordinate Transformation

3.3 Applications

3.3.1 Beam structures under base random excitations

3.3.2 Plate structures

3.3.3 Remarks

3.4 Extended Stochastic Central Difference Method and Narrow-band Force Vector

3.4.1 Extended stochastic central difference method

3.4.2 Beam structure under narrow-band excitations

3.4.3 Concluding remarks

3.5 Stochastic Newmark Family of Algorithms

3.5.1 Deterministic Newmark family of algorithms

3.5.2 Stochastic version of Newmark algorithms

3.5.3 Responses of square plates under transverse random forces

References

<table>
<thead>
<tr>
<th>Section</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1 Modal Analysis of Temporally Stochastic Quasi-linear Systems</td>
<td>131</td>
</tr>
<tr>
<td>4.1.1 Modal analysis and bi-modal approach</td>
<td>132</td>
</tr>
<tr>
<td>4.1.2 Response statistics by Cumming's approach</td>
<td>137</td>
</tr>
<tr>
<td>4.2 Response Analysis Based on the Melosh-Zienkiewicz-Cheung Bending Plate Finite Element</td>
<td>141</td>
</tr>
<tr>
<td>4.2.1 Simply-supported plate structure</td>
<td>142</td>
</tr>
<tr>
<td>4.2.2 Square plate clamped at all sides</td>
<td>150</td>
</tr>
<tr>
<td>4.2.3 Remarks</td>
<td>152</td>
</tr>
<tr>
<td>4.3 Response Analysis Based on High Precision Triangular Plate Finite Element</td>
<td>156</td>
</tr>
<tr>
<td>4.3.1 Simply-supported plate structures</td>
<td>157</td>
</tr>
<tr>
<td>4.3.2 Square plate clamped at all sides</td>
<td>159</td>
</tr>
<tr>
<td>4.4 Concluding Remarks</td>
<td>166</td>
</tr>
</tbody>
</table>

References 166


<table>
<thead>
<tr>
<th>Section</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1 Stochastic Central Difference Method for Quasi-linear Structural Systems</td>
<td>169</td>
</tr>
<tr>
<td>5.1.1 Derivation of covariance matrix of displacements</td>
<td>169</td>
</tr>
<tr>
<td>5.1.2 Column under external and parametric random excitations</td>
<td>171</td>
</tr>
<tr>
<td>5.2 Recursive Covariance Matrix of Displacements of Cantilever Pipe Containing Turbulent Fluid</td>
<td>174</td>
</tr>
<tr>
<td>5.2.1 Recursive covariance matrix of displacements</td>
<td>174</td>
</tr>
<tr>
<td>5.2.2 Cantilever pipe containing turbulent fluid</td>
<td>178</td>
</tr>
<tr>
<td>5.3 Quasi-linear Systems under Narrow-band Random Excitations</td>
<td>184</td>
</tr>
<tr>
<td>5.3.1 Recursive covariance matrix of pipe with mean flow and under narrow-band random excitation</td>
<td>184</td>
</tr>
<tr>
<td>5.3.2 Responses of pinned pipe with mean flow and under narrow-band random excitation</td>
<td>186</td>
</tr>
<tr>
<td>5.4 Concluding Remarks</td>
<td>188</td>
</tr>
</tbody>
</table>

References 190
6. Direct Integration Methods for Temporally Stochastic Nonlinear Structural Systems

   6.1 Statistical Linearization Techniques

   6.2 Symplectic Algorithms of Newmark Family of Integration Schemes
       6.2.1 Deterministic symplectic algorithms
       6.2.2 Symplectic members of stochastic version of Newmark family of algorithms
       6.2.3 Remarks

   6.3 Stochastic Central Difference Method with Time Co-ordinate Transformation and Adaptive Time Schemes
       6.3.1 Issues in general nonlinear analysis of shells
       6.3.2 Time-dependent variances and mean squares of responses
       6.3.3 Time co-ordinate transformation and adaptive time schemes

   6.4 Outline of steps in computer program

   6.5 Large Deformations of Plate and Shell Structures
       6.5.1 Responses of cantilever plate structure
       6.5.2 Responses of clamped spherical cap

   6.6 Concluding Remarks

References

7. Direct Integration Methods for Temporally and Spatially Stochastic Nonlinear Structural Systems

   7.1 Perturbation Approximation Techniques and Stochastic Finite Element Methods
       7.1.1 Stochastic finite element method
       7.1.2 Statistical moments of responses
       7.1.3 Solution procedure and computational steps
       7.1.4 Concluding remarks

   7.2 Stochastic Central Difference Methods for Temporally and Spatially Stochastic Nonlinear Systems
       7.2.1 Temporally and spatially homogeneous stochastic nonlinear systems
7.2.2 Temporally and spatially non-homogeneous stochastic nonlinear systems 248
7.3 Finite Deformations of Spherical Shells with Large Spatially Stochastic Parameters 251
  7.3.1 Spherical cap with spatially homogeneous properties 252
  7.3.2 Spherical cap with spatially non-homogeneous properties 254
7.4 Closing Remarks 255
References 257

Appendices
1A Mass and Stiffness Matrices of Higher Order
  Tapered Beam Element 261

1B Consistent Stiffness Matrix of Lower Order
  Triangular Shell Element 267
  1B.1 Inverse of Element Generalized Stiffness Matrix 267
  1B.2 Element Leverage Matrices 268
  1B.3 Element Component Stiffness Matrix Associated with Torsion 271
References 276

1C Consistent Mass Matrix of Lower Order
  Triangular Shell Element 277
Reference 280

2A Eigenvalue Solution 281
References 282

2B Derivation of Evolutionary Spectral Densities and Variances of Displacements 283
  2B.1 Evolutionary Spectral Densities Due to Exponentially Decaying Random Excitations 283
  2B.2 Evolutionary Spectral Densities Due to Uniformly Modulated Random Excitations 286
  2B.3 Variances of Displacements 288
References 297
2C Time-dependent Covariances of Displacements 299
2D Covariances of Displacements and Velocities 311
2E Time-dependent Covariances of Velocities 317
2F Cylindrical Shell Element Matrices 323
  Reference 325
3A Deterministic Newmark Family of Algorithms 327
  Reference 331
Index 333