Acceptance event, 174–175
Acceptance/rejection method, 174–175
Akaike information criterion (AIC), 240
log-likelihood values, contrast, 242
Algorithms, usage, 257–259
Alias method, 172
Alpha-algebra, generation, 311
Alpha-quantile, 34–35
Alpha-stable, generation, 180
Alpha-stable densities, illustration, 60f
Alpha-stable distribution, 58–65
characteristic function, 127
fitting, 126–131
flexibility/suitability, 59
incorporation, 57
Lévy measure, 78
simulation, 181
Alpha-stable motions, 170
Alpha-stable Paretian distribution, 5
Alpha-stable processes, 93, 192–193
Lévy measure, 93
Alpha-stable random measure, 193
Alpha-stable random variable
convergence, 62
definition, 58–61
parameters, 93
properties, 61–62
sum, 7
tails, tempering/truncating, 8–9
Alpha-stable sub-Gaussian
distributions, radial symmetry, 226
Alpha-stable subordinator, 100
American option, 141
European option, contrast, 357
holding/exercising decision, 359
price, obtaining, 359
American option pricing
discrete time, 358–359
empirical illustration, 365–371
Monte Carlo methods, usage, 357
Anderson-Darling statistic
(AD-statistic), 277, 296, 343
usage, 127, 129
values, 135
Annualized estimated spot variance, 321
Annualized volatilities, estimation, 301t
Annual $\sigma_t$ process, 345f
Antithetic sampling, 211
Arbitrage opportunity, 111
illustration, 147t
Archimedean copulas, 50
Arithmetic Brownian motion, 99
Lévy process, 105
usage, 100
Arithmetic sampling, 211–212
Asian Financial Crisis (1997), 1
Asset prices
financial modeling, improvement, 1–4
log-normality assumption, 204
modeling, geometric Brownian motion (usage), 10
Asset-pricing theory, involvement, 39
Asset-return behavior, assessment, 237
Asset return distributions, conditionally leptokurtic characteristic, 309
INDEX

Asset returns
- empirical distribution, skewed/fat-tailed properties, 57
- financial time series, 2
- multivariate non-normal distributions, modeling, 225
- unconditional multivariate distribution, estimation, 237

Asymmetric alpha-stable distribution, 292
Asymmetric GJR model, 274
Asymmetric NGARCH stock price model, forecasting properties, 291

Asymmetry, 31
- measure, 30
- Atlantic options, 142
- At-the-money options
  - pricing error, 352
  - volatilities, 152

Autoregressive moving average (ARMA), 259
- dynamics, 261, 263
- parameters, 265
- Average absolute error (AAE), 155, 302, 348, 370
- Average prediction error (APE), 155, 370
- Average relative pricing error (ARPE), 155, 302, 348, 370
- Average value-at-risk (AVaR), 253–255
  - closed-form formula, computation, 254
  - computation, 266
  - defining, 254
  - evaluation, conversion, 257
  - usage, 256

B
- Backward induction, usage, 358–359
- Backward iteration, 359
- Barndorff-Nielsen model, 165
- Barrier option, 142
- Bayes formula, 40

Bell curve, 1–2
- Benoulli-distributed random variable, 21
- Bermuda options, 142
- Bernoulli distribution, 21
- Big jumps, tempering (absence), 196
- Bilateral gamma (IG) distribution, 206
- Bilateral gamma (IG) random variable, 328
- Binomial-distributed random variable, 21
- Binomial distribution, 21–22
- Bivariate scatterplot, 239f, 243f
- Black-Scholes call option price, obtaining, 148
- Black-Scholes formula, failure, 4
- Black-Scholes framework, limitations, 309
- Black-Scholes model, 9, 350
  - benchmark, 370
  - overcoming, 9–10
  - risk-neutral parameters, calibration, 153, 155
  - usage, 148–149
- BMW, bivariate scatterplot, 239f, 243f
- Borel measure, 77
- Boundary conditions, tightening, 142–143
- Box-Muller algorithm, variant, 178
- Box plots, usage, 236f, 262f, 263f
- Brown, Robert, 87
- Brownian motion, 95–104, 203–209
  - paths, possibility, 96f
  - popularity, 3
  - variance level, 185
- Brownian subordination, 204
- Business time, 204

C
- X² test, p-value, 263f
- Call option, 141, 142
  - arbitrage-free price, 314–315
  - prices
    - calculation, 150
    - condition, 152t
Index

value, 144
worth, 143
Cash flow, 360t
generation, payoff function (usage), 359–360
Cauchy distribution, 6–7
CBK
bivariate scatterplot, 243f
returns, 242
Center, 77
Central Limit Theorem, 7, 178
application, absence, 62
distribution, 99
statement, 27
Central location, μ measure, 5
Central value, 31
Change of measures, 107, 111–121, 319
impact, 120t
usage, 113–121
Characteristic exponent, 37–38
Characteristic function, 35–39
calculation, 126, 324
example, 36t
exponential part, 37
expression, 59
fitting, 126
form, 70
formula, 76
method, 176
modification, 177
nth derivative, 37
properties, 37
second derivative, 176–177
usage, 176–177
Chicago Board Options Exchange
(CBOE), options trading, 11
Chi-square statistic (X²-statistic),
usage, 127, 128
Clark model, usage, 165
Classical tempered stable (CTS)
distribution, 8, 65–68, 365
characteristic function, 68
convergence, 67
extension, KRTS distribution, 73
Lévy measure, 78, 79
P/Q equivalence, 119t
probability density, 66f, 68f, 74f
sample, simulation, 200f
Classical tempered stable GARCH
model (CTS-GARCH model), 316–317
in-sample option pricing results, 349f
out-of-sample option pricing results, 374f
Classical tempered stable GARCH
option pricing model
(CTS-GARCH option pricing model), 367
Classical tempered stable (CTS) Lévy
measure, 118
Classical tempered stable (CTS) model, 310, 315–317
Classical tempered stable (CTS)
process, 198–199
description, 205–206
jumps, 206
parameters, 118
process X, 94
simulation, 187f, 207f
time-changing Brownian motion,
usage, 206
Classical tempered stable (CTS)
random number
generator, usage, 216–217
obtaining, 205
Clayton copula, 50
Closed-form analytical solutions,
unavailability, 210
Closed-form solution, absence, 58–59
Collateralized debt obligations (CDOs),
pricing, 12
Completion of (Ω, F, P), 108
Compounded Poisson distribution,
approximation, 13–14
Compounded Poisson process, 89
Compound Poisson process, 183
drift, presence, 184
Concentration in tails, 30, 32
Conditional expectation, 107, 109–111
Conditional log-likelihood function, 293
Conditional probability, 39–40
computation, 40
properties, usage, 101
Conditional variance, 326
assumption, 274, 289
definition, 275
evaluation, 315
form, 278, 290
process, 273
recursive collection, 293
updating, 276
Confluent hypergeometric function, 75,
82–84
Continue to hold, 361, 363
Continuous nonuniform distributions,
172–177
Continuous probability distributions,
22–30
understanding, 24
Continuous random variable, 22–23
density level, 37
Continuous time modeling, 312–313
Continuous-time models, advantage, 11
Continuous-time processes, change of
measure, 113–117
Continuous-time stochastic processes,
87
Continuous-time stochastic volatility,
169
Control variates, usage, 210
Convergence speed, determination, 190
Converge result, 189–190
Copula, 251
density, consideration, 53
usage, 48
advantage, 48–49
Copula functions, 47–54
connection, 49
consideration, 247
usage, 51
Correlation
covariance, relationship, 43
drawback, 47–48
matrix, diagonalization, 227–228
Counting measure, 219
Covariance
correlation, relationship, 42–43
denotation, 43
matrix, 51, 238
estimation, 265
Cox-Ingersoll-Ross (CIR) process, 165
subordinator process, 166
time integral, 10
Cross-sectional regression, usage, 359
Cumulative distribution, 24
Cumulative distribution function
(c.d.f.), 23–26, 40, 149
form, 28
inversion, 175
numerical approximation, 135–139
Cumulative probability functions, 24
Cutting points, stable parameters
(influence), 64f
D
Daily log-returns, definition, 131
DaimlerChrysler, bivariate scatterplot,
239f, 243f
DAX
ML estimates, 245
multi-tail t model check, 242–244
DAX 30 Index, 225
behavior, 266
composition, 260
DAX index
composition, 259, 260
expected return, 264
DBK
bivariate scatterplot, 243f
returns, 242
d-dimensional hypercube, probability
distribution function, 251
Decomposition, 220
Deep in-the-money options (DITM
options), 339, 351
volatilities, 152
Deep in-the-money put option (DITM
put option price), exercise
payoff (relationship), 358
Index

Deep out-of-the-money options (DOTM options), 339, 351
prices, explanation, 353
volatilities, 152
Default, annualized probability, 21
Default event, 21
Density function
behavior, 8
denotation, 23–24
discrete evaluation, 341
example, 316f
expression, 44
form, 28
numerical approximation
derivation, 126
maximum likelihood estimation,
usage, 127
obtaining, 183
Density generator, 47
Density-generator ML-approach,
usage, 233–235
Density process, 313–314
Dependence
local structure, 53, 251
structure, 247
modeling, 259
Dependency, modeling, 48
Derivative
instruments, infinite price, 63
position, calculation, 272
Deterministic methods, 170
Differentiation, involvement, 24
Dirichlet distribution, 179
Discounted cash flows, 360, 361t, 362t
Discounted simulated process, 214
Discount stock price process, 314
Discrete distributions, 172
AVaR, 254–255
Discrete Fourier-transform, usage, 176
Discrete numerical integration, 139
Discrete probability distributions,
20–22
formula, 33
Discrete random variable, 20–21
i.i.d. sequence, 198
impact, 20
Discrete-time autoregressive
conditionally heteroscedastic
(discrete-time ARCH) model, 10
Discrete-time Black-Scholes log normal
model, constant volatility, 290
Discrete-time GARCH models, 169
usage, 10
Discrete-time model, 326–327
Disjoint sets, 218
Dispersion matrix, 231
eigenvector-eigenvalue pairs,
denotation, 228
estimation, 233
normalized version, estimation, 233
usage, 232
Dispersion measure, 30
Dispersion parameter, estimation, 244
Distributions
density, closed-form, 35–36
function, characteristic function, 36
parameters, 27
simulation, 177–182
symmetry, 26
tails, 2
theory, foundation, 12–13
Dividend return, continuously
compounded risk-free rate, 289
Domain of attraction, 27
Dot-com bubble (2000), 1
Dow Jones Industrial Average (DJIA)
index, 131
significance level, 134
parameter fit, 133t
Drift, 77
Driving process, tempered stable
parameters, 131
E
Efficient market hypothesis, 110
Eigenvector-eigenvalue pairs,
denotation, 228
Eigenvectors, portfolio representation,
228
Elliptical copulas, 49
Empirical distributions, skew, 236
**INDEX**

| Equivalent martingale measure (EMM), 9, 147, 166 | asymptotic independence, 252 |
| determination, 148, 366 | joint capture, 215 |
| set, possibilities, 313 | joint probability, 40 |
| usage, 210, 312 | Ex-ante perspective, 338 |
| Equivalent probability measure, 111–113 | Excess kurtosis, 28, 33 |
| Equivalent random variable (obtaining), Esscher transform (usage), 325 | incorporation, 215 |
| Error estimates, risk-neutral parameters (relationship), 162t–163t | Excess skewness, incorporation, 215 |
| Error estimators, 158t–159t | Exercise/continue to hold, 361t, 363t |
| OEX put option, 370t | Exercise payoff, DITM put option price (relationship), 358 |
| Esscher transform, 328, 332–334 | Exercise price, 142 |
| consideration, 324 | Expected value, 32 |
| exponential tilting, 216 | Expiration date, 141 |
| tilting, relationship, 213, 333 | Explicit copulas, 252 |
| Estimator accuracy, 236 | Exponential alpha-stable model, 124 |
| European call options, 143 | Exponential Brownian motion, 4 model, 123 |
| consideration, 278 | Exponential distribution, 28 |
| fair value, determination, 281–282 | Exponential KRTS model, 124 |
| implied volatility, 152, 350f, 354f | Exponential Lévy models, 122–125 |
| price, calculation, 145 | option pricing, 141 |
| pricing performances, 351t | proposal, 9–10 |
| trading, consideration, 339 | Exponential moments, 80–82, 311 |
| value, 164 | absence, 80 |
| European options, 141 | Exponential process, 92 |
| American options, contrast, 357 | Exponential random variable, 174 |
| closed-form solution, 210 | Exponential tempered stable models, 125t |
| no-arbitrage price, 147 | exponential Lévy model, relationship, 124 |
| prices, closed-form solution (availability), 11 | usage, 149–163 |
| pricing | Exponential tilting, 207–208 |
| exponential tempered stable models, usage, 149–163 | density transformation, 213 |
| formula, 152 | Esscher transform, relationship, 216 |
| European put option, 143–144 | Ex-post perspective, 338 |
| European put option computation | Extended Girsanov’s theorem, usage, 113 |
| Fourier transform method, usage, 216t | Extreme returns, probabilities (computation), 5 |
| Monte Carlo simulation, usage, 216f | Extreme values, probability, 2 |

**F**

Factors, 248–249

Fast Box-Muller algorithm, 178
Index

Fast Fourier transform (FFT), 127
  procedure, 341
  usage, 139
Fat tails (heavy tails), 2, 292
  effects, 204
  property, 343, 345
Filtering historical probability (FHS)
  approach, 214
  risk-neutral GARCH parameters,
  consideration, 353
Filtering historical probability GARCH
  (FHS-GARCH) model, 310, 341–342, 346–348
  risk-neutral estimated parameters,
  348
Filtration, 107–108
  generation, 114
Filtration generated by X, 108
Financial derivatives pricing, Monte
  Carlo methods (usage), 212–213
Financial instruments, pricing, 271
Financial models, failure, 1–2
Financial time series, log returns, 351
Finite activity, 91
Finite discrete process, consideration, 111
Finite measure space, 186
Finite variance random variables, sum
  (consideration), 178
Finite variation, 88
First principal factor, scatter plots, 261f
First quartile, 35
Fisher’s kurtosis, 33
Fisher’s skewness, 32–33
Fitted stdCTS residuals, 263f
fmincon (function), 280–281
Fourier-inversion formula, 127
Fourier transform, 135–136
  numerical method, 139
Fourth central moment, 32
Frank copula, 50
Fundamental approach
  consideration, 338
  trader approach, contrast, 282
Fundamental framework, 339
G
Gamma distribution, 28–29, 179–180
  consideration, 92
Gamma fields, 107–108
Gamma function
  definition, 29
  denotation, 29
  integral representation, 29
Gamma process, 92
Gamma variables, 328
Gamma variates, generation, 179
GAMS (optimization software), 257
Gaussian copula, 49, 252
Gaussian distribution, 1–2, 26
Gaussian one-factor copula model,
  failure, 12
Generalized autoregressive conditional
  heteroscedasticity (GARCH), 271
  components, importance, 292–293
  dynamics, 12, 261, 263
  normal innovation, 272–275
  market estimation, 275–277
  methodology, usefulness, 11
  option pricing model, LSM method
  (usage), 364–365
  parameters, 264
  estimation, 284
  set, estimation, 280–281
  stock price models, 288–289
  structures, 210
Generalized autoregressive conditional
  heteroscedasticity (GARCH) models
  explicit-form solutions, 13
  fitting, 11–12
  TS/TID innovations, combination,
  170
Generalized classical tempered stable
  (GTS) distribution, 68–69,
  138
Generalized classical tempered stable
  (GTS) process, P/Q equivalence,
  119t
Generalized CTS distribution, 68–69
INDEX

Generalized elliptical random vector, location parameter (computation), 238–239
Generalized hyperbolic (GH) density, 238
Generalized hyperbolic (GH) distribution, 237
Generalized hyperbolic (GH) framework, 227
Generalized NGARCH option pricing model, 288–291
Generalized tempered stable (GTS) model, 317–319
Generalized tempered stable (GTS) process, 94, 110
Geometric Brownian motion, 99–100 model, 123 stock price process, relationship, 124
German Treasury bill rate, market value, 265
Girsanov theorem, 148 usage, 113–114
Glosten-Jagannathan-Runkle GARCH model (GJR-GARCH model), 279, 353 in-sample option pricing results, 280f
Glosten-Jagannathan-Runkle (GJR) model, 274
Goodness of fit assessment, 127–131 consideration, 338 statistics, 277t, 343t calculation, 277 test, 369t statistic, formulas (computation), 130t
Gumbel copula, 50

H
Heavier-tailed distribution, 62
Heavy-tailed elliptical distributions, 226
Heavy-tailed innovations, 337–338
Heavy tails (fat tails), 2, 292

Heston model, obtaining, 165
Heston-Nandi form, 278
Heston-Nandi GARCH model, in-sample option pricing results, 280f
High-yield bond portfolio defaults, 28
HN-GRACH, 282
Hypergeometric function, 69, 75, 82, 83

I
ID-GARCH paths, simulation, 332
Implicit copulas, 251
Implicit skewed t copula distribution function, derivation, 252–253
Implied volatility, 9 curve, 370 correspondence, 348, 350 illustration, 152–153 surface, 153 example, 174f usage, 350f variation, 4
Importance measure, 212
Importance sampling, 212–213
Independence, meaning, 21
Independent and identically distributed (i.i.d.) random variables, 58 finite sequences, 366–367
sum, 62, 99, 105 usage, 76, 89
Independent and identically distributed (i.i.d.) real random variables, 365–366
Independent and identically distributed (i.i.d.) sequence, 188, 194
Independent and identically distributed (i.i.d.) uniform distributions, 170–171 sequence, 170
Independent and identically distributed (i.i.d.) uniform variables, 173
Independent exponential random variable, 89
Independent increments, 88
Index

Independently distributed random variables, 42
Independent random variables, 49
Infinite activity, 91
Infinitely divisible (ID) asymmetric distributions, fat tails (presence), 310–311
Infinitely divisible (ID) distributed i.i.d. real random variables, 312
Infinitely divisible (ID) distributions, 76–82, 310
characteristic function, 76
illustration, 77t
simulation, 189
Infinitely divisible GARCH model (ID-GARCH model), 309, 312, 318
consideration, 322–323
risk-neutral dynamic, 312–315
risk-neutral parameters, 348
simulation, 331–332
stock price dynamic, 311–312
Infinitely divisible GARCH paths (ID-GARCH paths), simulation, 332
Infinitely divisible (ID) random variable, 80, 287
Infinite variation, 88
Innovation
defining, 324
density function, 341
distribution, 368t
skewness, admission, 296
marginal distribution, 290
process
estimation, 345f
usage, 344f
In-sample analysis, 346–354, 368
In-sample European call option pricing
pricing errors, 281t, 302t
results, 349t
In-sample option pricing results, 280f, 304f, 349f
Integral calculus, 24
Inter arrival times, 28
Interest rate
analysis, 338–339
representation, 273
In-the-money European call option (ITM European call option), 357
In-the-money options (ITM options), 339, 351
implied volatility, 4
In-the-money stock prices, 361t, 362t
Intrinsic time process, 100
Inverse Gaussian (IG) distribution, 30
Inverse Gaussian GARCH model (IG-GARCH model), 325–326
Inverse Gaussian (IG) innovations, 12
Inverse Gaussian (IG) model, 165, 310, 324–326
Inverse Gaussian (IG) process, 92, 102
parameters, 209
Inverse Gaussian (IG) time-changed Brownian motion, 329
Inverse transform algorithm, 172–174
Ivy DB (Option Metrics), 11

J
Jöhnk’s method, 179
Joint characteristic function, 38
Joint multivariate distribution, modeling, 12–13
Joint probability, 41
Joint probability distributions, 39–44
definition, 40–41
Jumps
acceptance, 208
consideration, 195–196
counting measure, 218–219
processes, 91
Jump sizes
discrete subset, 90
set, extension, 90

K
Kim-Rachev (KR) distribution, definition, 196
Kim-Rachev (KR) model, 319–321
INDEX

Kim-Rachev (KR) process, 196–198
parameters, 197
Kim-Rachev tempered stable (KRTS)
distribution, 73–74, 138
characteristic function, 73
CTS distribution extension, 73
cumulants, 73–74
Lévy measure, 78
Kim-Rachev tempered stable (KRTS)
process, 94, 110
P/Q, equivalence, 119
Kolmogorov-Smirnov distance statistic
(KS-statistic), 277, 296, 343
p-values, 368–369
usage, 127–129, 131, 134
Kolmogorov-Smirnov GARCH
(KR-GARCH) model, 319
Kurtosis
approximation, 343
obtaining, 38
value, 5

L
Laplace distributions, 47
Laplace transform, shape, 300
Least squares Monte Carlo (LSM)
method, 359–364
usage, 364–365
Lehman Brothers, bankruptcy, 1, 283
Leverage effect, 274
Lévy-based models, deficiency,
overcoming, 10
Lévy distribution, 6–7
probability mass, concentration, 6
Lévy-Ito decomposition, 188, 218–220
impact, 204
Lévy-Khinchine formula, 77, 325,
332–334
usage, 79
Lévy measures, 77, 78t, 90
assumption, 103
inversion, 183
representation, 190–191
spherical coordinates, 190
usage, 79
Lévy process, 87, 104–105
consideration, 182
construction, 65
generation, 191–192
jump counting measure, 218–219
jumps, 218
Lévy triplet, inclusion, 117–118
series representation, 183
simulation, 14
techniques, usage, 182–193
subordination, 204
supposition, 114
Lévy stable distribution, 2–3
Lévy subordinator, usage, 10
Lévy triplet, 77
process supposition, 116–117
usage, 114, 333
Likelihood ratio test, 240
Limited exercise options, 142
Linear dependence structure, 226
Linear interpolation, usage, 97
Linear regression, estimated parameters
(box plots), 262
Locally risk-neutral valuation
relationship (LNVR), 291
Local semimartingale, 203
Location, 31
measure, 30
parameter, 26, 59
Location-scale invariance, 26–27
Log characteristic function, 37–38
Logistic distributions, 47
Log-Laplace model, obtaining, 165
Log-Laplace transform, 81, 311, 328
function, 289
Log-likelihood, analytic form, 340
Log-likelihood function
analytic form, 276
maximization, 275
obtaining, 234
Log-likelihood values, AIC (contrast),
242
Log-normality assumption, 204
Log-return, 47–48, 289
series, denotation, 293
Index

Log-Student-t model, 166
London Interbank Offered Rate (LIBOR), percentage (change), 19
Long-term maturity, pricing, 350
Loss function, impact, 304, 306
Loss tail, 351
Lower-tail AD-statistic (AD_{down}), 129–130

M
Marginal distribution, 40–41
collection, 48
Marginal probability distributions, 40–41
Market-based approach, 282
Market estimation, 275–277, 339–346
Market measure, 297
Market option prices
  implied volatilities, 160f
  simulated options prices, square-root error (minimization), 321
Market parameters, 149
  calibration, risk-neutral parameter
  calibration (combination), 161–163
  estimation, MLE approach (usage), 277t
Market price dynamic, modeling, 365–366
Market put option prices, implied volatilities, 371f
Market returns, empirical evidence, 8
Market volatility curve, matching, 159
Markov process, 110
  usage, 364
Markov properties, 107
Markowitz mean-variance analysis, 264
  portfolio, scaling, 265
  problem, 256
Markowitz portfolio theory, assumptions, 39
Marsaglia method, 179
Martingale measure, no-arbitrage pricing (equivalence), 145–147
Martingale process, 110
  absence, 114
Martingale properties, 107
Mathematical function,
  differentiation/integration, 24
Maturity calibration, 161
Maturity date, 141
Maximum likelihood estimation (MLE), 127, 297, 337
  approach, usage, 277t
  method, usage, 337
  methodology, 288
  MLE/fitted methodology, 288, 297–298
  out-of-sample performance, 306
  procedure, usage, 275
  usage, 259
Maximum likelihood method, application, 232
Mean absolute deviation, 31
Mean-AVaR portfolios
  behavior, 266f, 267f
  weights, box plots, 265f
Mean measure, 219
Mean-variance analysis, 255–256
Mean-variance optimization, 249
Mean-variance portfolios
  behavior, 266f, 267f
  problem, 249–250
  weights, box plots, 265f
Measure, change, 107, 111–121
ML-ratio test, p-value, 242
Model dependencies, 12–13
Modeling dependencies, 251–253
Modified Bessel function (of the second kind), 30
Modified Bessel function (of the third kind), 252–253
Modified tempered stable (MTS)
distributed random variable, denotation, 69–70
Modified tempered stable (MTS) distribution, 8, 69–70, 138, 365
characteristic function, 69
exponential tilting, 329
Lévy measure, 78
parameters, role, 70
Modified tempered stable (MTS) process, 110, 202–203
parameters, 94
P/Q, equivalence, 119t
simulation, 202–203
Moments, 32
sample, 35
Moments of arbitrary order, 64–65
Money market account, risk-free return rate, 145
Moneyness, defining, 155, 278
Monotonicity, 212
Monte Carlo integration methods, 13–14, 169
Monte Carlo integration procedure, usage, 299
Monte Carlo methods, 209–217
example, 216–217
simulation, relationship, 209–210
usage, 13, 337, 357
Monte Carlo prices, obtaining, 216
Monte Carlo procedure, 346–347
antithetic sampling, 211
Monte Carlo simulation, 213, 279–280
basis, 256
risk-neutral scenarios, usage, 272
usage, 259
Monte Carlo variance, reduction, 213
Monty Python method, 178
Multidimensional random variable, local dependence structure, 53–54
Multifactor linear model, 248–250
usage, 258
Multifactor model, consideration, 258
Multi-tail distribution estimation, procedure, 232
Multi-tail generalized elliptical distribution, 225
dispersion matrix, 231
empirical investigation, 237
tail function, 228
Multi-tail generalized elliptical model, comparison, 237–238
Multi-tail generalized elliptical random vector, 228
dispersion matrix, 233
Multi-tail t distribution
density generator ML-approach, application, 235
Multi-tail t-distribution, 225
empirical results, 237–244
parameters, estimation, 232–236
Multi-tail t distribution, density, 231
Multi-tail t model check, 242–244
Multi-tail t random variable, density, 231
Multi-tail t random vector, 231
density, usage, 233–234
Multivariate alpha-stable distributions, 225–226
Multivariate asset returns, modeling, 225–226
Multivariate distribution, 39
consideration, 225
dependence structure, 247
Multivariate generalized hyperbolic (GH) distribution, 237
Multivariate non-normal distributions, modeling, 225
Multivariate normal distribution, 43–46
density, 237
Multivariate probability distribution function, 49
Multivariate random variable, 40
Multivariate skewed Student’s t-distribution, skewed t copula, 252
Multivariate stable distributions, 47
Multivariate t-distributions, 47
Index

N
- dimensional hypercube, 48–49
- dimensional random vector, 46
Negatively skewed distribution, density graphs, 32f
Negative skewness, 31
Newton-Raphson method, 174
NGARCH model, 274
form, 279
in-sample option pricing results, 280f
NGARCH stock price model, 290
conditional log-likelihood function, 293
No-arbitrage pricing, martingale measure (equivalence), 145–147
No-arbitrage structure, 146
Non compound Lévy process, non zero Lévy measure, 183
Non-decreasing process, 88
Non dividend-paying stock, American call options, 357
Non-Gaussian GARCH models, 11–12
Non-Gaussian innovations, 12
Non-Gaussian portfolio allocation, 247
algorithm, usage, 257–259
empirical test, 259–268
modeling dependencies, 251–253
Nonlinear asymmetric GARCH (NGARCH) dynamic, enhancement, 288
Nonlinear asymmetric GARCH (NGARCH) stock price model, 288
Nonlinear dependencies, 268
allowance, 250
Nonlinear least squares (NLS), 298
approach, 299
methodology, 288
parameters, determination, 304, 306
usage, 155, 346–347
Non-Markovian GARCH models, results, 364–365
Non-normal distributions, usage, 345
Non-normal GARCH model, 292
Non-normal ID distributed innovations, 310–311
Non-normal infinitely divisible GARCH, 315–331
Non-normal models, 4
Nonparametric FHS-GARCH model, 310, 341–342
in-sample option pricing results, 349f
out-of-sample option pricing results, 354f
Nonparametric Monte Carlo method, 214–216
Nonsymmetric distribution, 2
Non zero Lévy measure, 183
Normal-based models, 296
Normal distribution, 26–28, 177–178
exponential moment, 80
parameters, selection, 63
usage, 27
yield, 6–7
Normal-GARCH models, 271–273, 343, 345, 366
in-sample option pricing results, 280f, 349f
out-of-sample option pricing results, 354f
Normal-GARCH parameters (determination), MLE (usage), 342–343
Normal innovation, Monte Carlo simulations, 279–280
Normal inverse Gaussian (NIG) distribution, 72, 138
obtaining, 329–330
sample, simulation, 209f
Normal inverse Gaussian (NIG) model, 165, 310, 329–331
Normal inverse Gaussian (NIG) process, 102
process X, 94
usage, 208–209
Normality assumption, 3
Normalized dispersion matrix, denotation, 231
Normal model, 1
Normal-NGARCH model
in-sample option pricing results, 304f
out-of-sample option pricing results, 305f
Normal tempered stable (NTS)
distributed random variable, 71
Normal tempered stable (NTS)
distribution, 70–73, 138
definition, substitution, 72
Lévy measure, 78
probability density, 72f
Normal tempered stable (NTS) process,
103–104, 110
collection, 207–208
Lévy measure, 116
obtaining, 104
parameters, 94
symmetric MTS process,
relationship, 115–116
Nth central moment, definition, 33
Nth cumulant, 38
nth derivative, usage, 65
Nth raw moments, 33
Null hypothesis
definition, 134
rejection, 134
O
Objective probability measure, results,
292–296
OEX put option
error estimators, 370t
model prices, 371f
One-dimensional infinitely divisible
distribution, 77
One-dimensional time-series process,
13
One-period ahead conditional variance,
311–312
One-sided alpha/2-stable subordinator,
205
Open-source software, usage, 170
Optimal portfolios, 255–257
Optimization, 155, 232, 234, 347
inequality, 256
Option prices, 141
boundary conditions, 142–144
computation, GARCH processes
(usage), 210
convergence, obtaining, 215
generation, NGARCH model
(usage), 298
obtaining, backward induction
(usage), 358–359
Option pricing
Black-Scholes model, usage,
148–149
data set, 338–346
market estimation, 339–346
models, 290
in-sample analysis, 346–352
out-of-sample analysis, 352–354
performance, 346–354
Monte Carlo methods, usage, 337
volatility clustering, usage, 9–12
Options
absolute valuation errors, 302, 304
arbitrage pricing, martingale basis, 9
cash flows, change, 361
conditional expectation, current
value, 360
contract, 141–142
market, pricing/hedging theories, 4
premium, 141
Ordinary least squares (OLS) method,
usage, 258, 261
Ornstein-Uhlenbeck process, 10
Outcome, 19–20
concentration, 32
limitation, discrete random variable
(usage), 20
Out-of-sample analysis, 352–354
formula, 369
spot variances, 321–322
Out-of-sample European call option
pricing
errors, 284t, 303t
results, 352t
Out-of-sample option pricing results,
306f, 354f
Index

Out-of-the-money options (OTM options), 339, 351
implied volatility, 4
Over-the-counter (OTC) options, valuation, 337

P
Parameter probability distribution family, formation, 30
Parameters
estimation, 232–236
goodness of fit test, 369t
illustration, 131–135, 369t
fit, 132t–133t
formula, 33t
role, 65
Parameter set (Θ), estimation, 233–236
Parametrization, insertion, 102
Pareto power-type decay, 2
Path-dependent options, pricing, 13
Payoff function, 211
usage, 359–360
Pearson’s kurtosis, 32–33
Pearson’s skewness, 32–33
Percentiles, 35
Physical measure, 147
Plain vanilla derivatives, pricing, 272
Pochhammer symbol, 83
Poisson-distributed random variable, 22, 88
probability distribution, 22
Poisson distribution, 22
description, 28
Poisson point process, 194
Poisson process, 88–89
arrival times sequence, 193
Poisson random measure, 218–220
Portfolios
expected return, 250
no-arbitrage structure, 146
payoff example, 143t
rebalancing, 259
sensitivity, 250
value, 144
calculation, 264
variance, 227–228
calculation, 250
Portfolio selection theory, involvement, 39
Positively skewed distribution, density graphs, 32f
Positive skewness, 31
Price-driving process, skewness/heavy-tail properties, 9
Principal component analysis (PCA), 227–231, 258
Principal component tail function (pc-tail function), 228–230
Probabilities
calculation, 23
computation, 23–24
Probability density functions, 113
Fourier transform, 135–136
numerical approximation, 135–139
Probability density function, 23–26
illustration, 25f
knowledge, 5
mathematical connection, 24–25
Probability distribution, 19, 173
concepts, 19–20
function, 23–26
Probability Integral Transformation formula, 130
Probability measure equivalence, 111–113
random variable, usage, 112
Probability space, 219
sequence, 311
Pseudorandom, term (usage), 170
Pure fundamental framework, 339
Pure jump process, 89–95
Lévy process, 105
path behavior, 91
Poisson process, relationship, 91
process Z, 91
properties, change, 115
INDEX

Purely non-Gaussian distribution, 78
characteristic functions, 78
GTS distribution, comparison, 79
Purely non-Gaussian infinitely divisible
distribution, 91
Purely non-Gaussian infinitely divisible
random variable, obtaining, 92
Put-call parity, 144
Put option, 141, 142
prices, 148
calculation, 150
condition, 152
p-value, 128, 134, 263f

Q
Q-martingale, 147
usage, 314
Quantile-quantile plots (QQ-plots),
137f, 343
innovation process, usage, 344f
usage, 127, 128
Quantiles
asymmetry, 31
dispersion, 31
estimation, 235t
location, 31
measure, 30
statistical moments, 32–34
tails, concentration, 32
usage, 30, 34–35
Quasi-random numbers, usage, 210

R
Radial symmetry, 226
Radon-Nikodym derivative, 111
calculation, 115, 117
defining, 212
usage, 313–314
rand function (Matlab), usage, 171f
Random number
approximation, 176
generation, 172

importance, 13–14
problem, 174–175
generators, 170–182
Random variables, 19
cumulative distribution function, 41
density function, denotation, 23–24
dependence, 41–42
distribution, 311
expectation, 109
exponential moment, 80
infinite division, 76
example, 138
multivariate distribution, 251
outcome, 19–20
probability distribution, 41
provision, 49
probability measure, 212
values, implication, 41–42
weak convergence, 7
Random variates
generation, 317
simulation, 323
Random vector, density function, 44
Rapidly decreasing tempered stable
(RDTS) distribution, 75–76,
138, 365
Lévy measure, 78
data, simulation, 202f
Rapidly decreasing tempered stable
(RDTS) model, 310, 322–323
RDTS-GARCH model
in-sample option pricing results,
349f
out-of-sample option pricing
results, 354f
Rapidly decreasing tempered stable
(RDTS) process, 110, 201–202
parameters, 94
P/Q, equivalence, 119t
simulation, 201
Rapidly decreasing tempered stable
(RDTS) random variable, 322
Raw moments, 61
Raw moments of order, 33
Real random numbers, preference, 170
Index

Reflected Lévy distribution, 7
Relative errors, correspondence, 235t
Replicating portfolio, 147
Return
  continuously compounded risk-free rate, 289
  dynamic, form, 324
Reward-to-risk measure, 264
Reward-to-risk problem, solution, 255–256
Risk
  factors, modeling, 251
  market price, 148
  measure, 253
  calculation, 272
Risk-free asset, 266
Risk-neutral dynamic, 312–315
  form, 278
Risk-neutral estimates, obtaining, 279
Risk-neutral estimation, 278–285
Risk-neutral measure, 9, 147–149, 288
  asymmetry parameter, 321–322
  driving process, distribution, 150
  form, 297
Risk-neutral parameter calibration
  illustration, 153–161
  market parameter calibration,
    combination, 161–163
  results, 156t–157t
Risk-neutral parameters
  determination, change of measure
    (usage), 319
  error estimates, relationship,
    162t–163t
  estimation, 281t, 283, 3091t
  errors, minimization, 272
Risk-neutral paths, simulation, 369
Risk-neutral probability measure, 213
Risk-neutral scenarios, usage, 272
Risk-neutral valuation relationship
  (RNVR), 291
Root mean-square error (RMSE), 155, 302, 348, 370
Rosinski rejection method, 191–192, 205
  application, 206, 208
Russell global indexes, usage, 268
Sample moments, 35
  calculation, 36t
Sample path, 88
Sample space, 19–20
  Brownian motion, function,
    113–114
  subsets, partitioning, 20
Scale, σ measure, 5
Scale parameter, 26, 59, 65, 233
  estimation, 232, 240, 244
Scaling parameter, assumption,
  234–235
Scenario-based portfolio optimization,
  255
Scenario matrix
  generation, Monte Carlo simulation
    (usage), 259
  Monte Carlo simulation basis, 256
Second central moment, 32
Second quartile, 35
Series representation, 13–14, 217
  example, 200f
  framework, 186–191
Shape parameter, 26, 59
Sharpe ratio (SR), 266
  annualized value, 268t
  selection, 264
Short-term maturity, pricing, 350
Shot noise representation, 217
  usage, 199
Sigma field (σ-field), 108, 111
  adapted process, 110
Sigma field (σ-field) by X, 108
Sigma field (σ-field) generated by G,
  108
Simulated option prices/market option
  prices, square-root error
    (minimization), 321
INDEX

Simulation, 169
  techniques, usage, 182–193
Skewed distributions, allowance, 5
Skewed $t$ copula, 252
Skewed variance gamma (SVG) distribution
  parameters, selection, 327
  sample, simulation, 208f
  innovation, 12, 348
  model, 310, 326–329
  process, 206–207
Skewed variance gamma GARCH model (SVG-GARCH model), 329
Skew-elliptical distributions, 226
Skewness
  approximation, 343
  obtaining, 38
  parameter, 59
Skorokhod embedding problem, 203–204
Small jumps
  Brownian approximation, 185
  Brownian motion, 186
  intensity, 185
  truncation, 183–186
  example, 200f
Smoothly truncated stable (STS) distribution, 63–65
  assumption, 64
  density function, 63
  log Laplace transform, shape, 300f
  parameters, 298
Smoothly truncated stable GARCH (STS-GARCH) models, 287
  empirical analysis, 291–306
  models, consideration, 294t
  pricing process, risk-neutral version, 290
Smoothly truncated stable GARCH (STS-GARCH) option pricing model, pricing performance, 296–297
Smoothly truncated stable (STS) innovations, 12
Smoothly truncated stable NGARCH (STS-NGARCH) model
  estimation results, 321
  in-sample option pricing results, 304f
  out-of-sample option pricing results, 305f
Smoothly truncated stable NGARCH (STS-NGARCH) option pricing model, absolute performance, 302
Smoothly truncated stable (STS) random variable, 287
  simulation, 299
Spectral estimator, 233
Spherically distributed random vector, 46–47
Square integrable martingale, 220
Stability index, 59, 62
Stability property, 7
  usefulness, 62
Stable density, 6f
Stable distribution, 5, 57, 180–182
  alternative representation, 217–218
  density function
    behavior, 8
    numerical approximation, 126
    family, 5–9
    parameterization, 5–7
    properties, 7
    representation, 217
    skew, 6
    types, 6–7
  usage, considerations, 8–9
Stable parameters, influence, 64f
Stable Paretian distribution, 2
  reference, 5
Stable processes, values (differences), 192f
Stable tail-adjusted return ratio (STARR), 266–267
  annualized value, 268t
Index

Standard Brownian motion, 114
Lévy process, 105
paths, possibility, 96f
properties, 96
Standard CTS distribution, 67
Standard CTS process, 94
Standard deviation, 27
collection, 31
Standard GTS distribution, 69, 81
Standard GTS process, 94
Standardized STS distribution, tail
probabilities (comparison), 64f
Standard KRTS distributed variable, 74
Standard KRTS process, 94
Standard MTS distribution, 70, 81
Standard MTS process, 94
Standard NIG distribution, 73, 82
Standard normal distribution, 26
cumulative distribution function, 149
density functions, 316f, 318f, 321f, 323f, 327f, 330f
Standard normal innovation process, estimation, 345f
Standard normal STS distribution
log Laplace transform, shape, 300f
tail probabilities, comparison, 64f
Standard NTS distribution, 71, 81
Standard NTS process, 94
Standard Poisson process, arrival times
sequence, 193
Standard & Poor’s 100 index (OEX) option, 11, 368
Standard & Poor’s 500 (S&P500) analysis, 268
market parameters (estimation), MLE approach (usage), 277t, 342t
market performance, market models (comparison), 295t
monthly return, fluctuation, 27
option prices, explanation, 296–306
returns, empirical density (bar plots), 136f

Standard & Poor’s 500 (S&P500) index (SPX), 131
European call options, implied volatility, 350f, 354f
index, return, 20
log returns, 310f
calculation, 275
market parameters, 161
parameter fit, 132t
significance level, 134
study, 291
values, 300f
Standard RDTS distribution, 75, 82
Standard TS processes, change of measures, 120t
Stationary distribution, parameters, 237
Stationary increments, definition, 88
Statistical analysis, 234–236
Statistical moments, 32–34
illustration, 34t
Statistic moments
asymmetry, 31
dispersion, 31
location, 31
usage, 30–35
stdCTS, 75, 316
density function, 316f
noise, 259, 263
stdGTS
density function, 318f
distribution, defining, 317–318
stdID, distribution, 313
stdKR
density, 320
distribution, 320
stdNIG, 330–331
density function, 330f
stdRDTS, 75
density function, 323f
distribution, 322
stdSVG
density functions, 327f
random variable, parameters, 329
INDEX

Stochastic model, 10
Stochastic process
derivation, 114
continuous time, 87
terms, 88
definition, 88
description, 14
Stochastic volatility Lévy process
definition, 166
model, 166
Stock log returns, definition, 273
Stock market returns, 255
empirical findings, 226
modeling, 248–249
Stock price
dynamics, representation, 314
log returns, 274
form, assumption, 273
process, 123
Strike prices, 142, 153
Student- t distribution, 179
Sub-additivity property, 253
Subordinated stock price model,
164–166
defining, 164
Subordinator, 100
definition, 203–204
Lévy measure, 205
models, 204
process, 166
Subprime mortgage crisis (2007), 1
Summation stability, 27
Symmetric alpha-stable distribution, 292
Symmetric CTS distribution,
probability density, 67f
Symmetric distribution, 2
Symmetric MTS distribution
characteristic function, 70
exponential tilting, 207–208, 329
parameters, roles, 71
Symmetric MTS process, NTS process
(relationship), 115–116
T
Tail
collection, 32
dependence, 226
dependencies, capture, 261
function, 240
parameters, 229–230, 234–235
interval, 228
weights, 67
Tail-adjusted return ratio, 266
t-copula, 50
t-distributions, 226
Tempered infinitely divisible (TID)
distributed innovation, 322
Tempered infinitely divisible (TID)
distribution, 170
obtaining, 180
simulation, shot noise representation
(usage), 199
Tempered infinitely divisible (TID)
processes, 199–203
Tempered stable (TS) distribution, 8–9,
57, 65–76
exponential moment, existence
region, 138
fitting, 126–131
introduction (Rosiński), 315
log-Laplace transform, 82
obtaining, 180, 287
parameter estimation, example,
131–135
reference, 79
Tempered stable GARCH models
(TS-GARCH models)
innovation distribution, 368t
option pricing model, 366
Tempered stable (TS) model
prices, implied volatilities, 160f
Tempered stable (TS) option pricing
models, 151t
Tempered stable (TS) process, 94–95,
193–199
change of measure, 149
impact, 120t
usage, 117–121
Index

characteristic exponents, 95t
Lévy measure, 95
series representation, 200
Tempered stable (TS) subordinator, 103
generation, 208
parameters, 104
Tempering functions, 79t
generalization, 79
multiplication, 78–79
t-GARCH model, defining, 292
Third central moment, 32
Third quartile, 35
Tilting procedure, Esscher transform (relationship), 333
Time-changed Brownian motion, 100–104, 203–209
consideration, 216–217
expression, 203–204
framework, 205
procedure, 205
usage, 70
Time-changing Brownian motion, description, 207f
Time-continuity models, 11
Time horizon, 314
Time series, goodness of fit statistics, 277t, 343t
Trader approach, 169
consideration, 338
fundamental approach, contrast, 282
Truncated Lévy flight, obtaining, 68–69
Truncated stable distributions, usage, 63
Truncation function, 332
Two-dimensional analysis, 238–242
Two-dimensional density function, plot, 45
Two-dimensional ML estimates, 241t
Two-dimensional normal distribution copula density, 51, 52f
contour lines, 52f
covariance matrix, inclusion, 44–45
plot, level lines, 46f
probability density function, 25f, 45f
Two-step MLE method, usage, 337
U
Uncorrelated random variables, 43
Underlying asset, price, 145
Uniform distributions, 170–172
Uniform random variable, 173
i.i.d. sequence, 194
Uniform variates (generation), rand function (usage), 171f
Unit hypercube, 51–52
Unit interval, presentation (restriction), 97
Univariate distribution, 39
Upper-tail AD-statistic (AD_up), 129–130
Upper tail dependence, 226
U.S. Treasury bill index (IRX), 155, 161, 368
U.S. Treasury bill rates, 283f
time window, 275
V
Value-at-risk (VaR)
deficiencies, 253
measure, adoption, 253
Vanilla call option, 142
Vanilla put option, 142
Variance-covariance matrix, diagonalization, 227–228
Variance gamma (VG) distribution, 29–30
parameters, determination, 29
Variance gamma (VG) path, 206
Variance gamma (VG) process, 92–93, 101–102
presentation, 166
usage, 206–207
Variance reduction presence, 211
techniques, 210–214
Variance-reduction techniques, 210
Volatility, 204
definition, 9
dynamic, 273
parameters, estimation, 215
effects, 247
filtering, 298
INDEX

Volatility (Continued)
- GARCH structure, 346
- skew, 4
- smile, 4, 350
- surface, fit, 10

Volatility clustering, 2, 345
- behavior, 2
- one-dimensional time-series process, 13
- usage, 9–12

W
- Weighting functions, usage, 229
- Weron’s algorithm, 181
- Wiener, Norbert, 87

Z
- Zero Lévy measure, 187
- Zero truncation function, usage, 184
- Ziggurat method, 178