## Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preface to the Book</td>
<td>XVII</td>
</tr>
<tr>
<td>Preface to the Journal</td>
<td>XIX</td>
</tr>
<tr>
<td>List of Contributors</td>
<td>XXI</td>
</tr>
<tr>
<td>1</td>
<td>The Deterministic Generation of Photons by Cavity Quantum Electrodynamics</td>
</tr>
<tr>
<td>1.1</td>
<td>Introduction</td>
</tr>
<tr>
<td>1.2</td>
<td>Oscillatory Exchange of Photons Between an Atom and a Cavity Field</td>
</tr>
<tr>
<td>1.2.1</td>
<td>Experimental Set-up of the One-atom Maser</td>
</tr>
<tr>
<td>1.2.2</td>
<td>One-atom Maser as a Source of Non-classical Light</td>
</tr>
<tr>
<td>1.2.3</td>
<td>Review of Experiments on Basic Properties of the One-atom Maser</td>
</tr>
<tr>
<td>1.2.4</td>
<td>Statistics of Detector Clicks</td>
</tr>
<tr>
<td>1.2.5</td>
<td>Trapping States</td>
</tr>
<tr>
<td>1.2.6</td>
<td>Trapping State Stabilization</td>
</tr>
<tr>
<td>1.2.7</td>
<td>Fock States on Demand</td>
</tr>
<tr>
<td>1.2.8</td>
<td>Dynamical Preparation of n-photon States in a Cavity</td>
</tr>
<tr>
<td>1.2.9</td>
<td>The One-atom Maser Spectrum</td>
</tr>
<tr>
<td>1.3</td>
<td>Other Microwave Cavity Experiments</td>
</tr>
<tr>
<td>1.3.1</td>
<td>Collapse-and-revival of the Rabi Oscillations in an Injected Coherent Field</td>
</tr>
<tr>
<td>1.3.2</td>
<td>Atom-photon and Atom-atom Entanglement</td>
</tr>
<tr>
<td>1.3.3</td>
<td>Atom-photon Phase Gate</td>
</tr>
<tr>
<td>1.3.4</td>
<td>Quantum Nondestructive-measurement of a Photon</td>
</tr>
<tr>
<td>1.3.5</td>
<td>Wigner-function of a One-photon State</td>
</tr>
<tr>
<td>1.3.6</td>
<td>Multiparticle Entanglement</td>
</tr>
<tr>
<td>1.3.7</td>
<td>Schrödinger Cats and Decoherence</td>
</tr>
</tbody>
</table>

*Elements of Quantum Information.* Edited by Wolfgang P. Schleich and Herbert Walther
Copyright © 2007 WILEY-VCH Verlag GmbH & Co. KGaA, Weinheim
ISBN: 978-3-527-40725-5
1.4 Cavity QED Experiments in the Visible Spectral Region  30
1.4.1 The One-atom Laser  30
1.4.2 Atoms Pushed by a Few Photons  31
1.4.3 Single-photon Sources  33
1.4.4 Single-atom Laser using an Ion Trap  34
1.5 Conclusions and Outlook  38
References  39

2 Optimization of Segmented Linear Paul Traps and Transport of Stored Particles  45
Stephan Schulz, Ulrich Poschinger, Kilian Singer, and Ferdinand Schmidt-Kaler
2.1 Introduction  45
2.2 Optimization of a Two-layer Microstructured Ion Trap  46
2.2.1 Design Objectives  47
2.2.2 Operating Mode and Modeling of the Segmented Linear Paul Trap  49
2.2.3 Optimization of the Radial Potential  51
2.2.4 Optimization of the Axial Potential  52
2.3 Open Loop Control of Ion Transport  54
2.3.1 Non-adiabatic Heating Sources  54
2.3.2 Overview of the Applied Optimization Strategies  55
2.3.3 The Optimal Control Method  55
2.3.4 Optimization Results  58
2.3.5 Ion Heating due to Anharmonic Dispersion  59
2.3.6 Quantum Mechanical Estimate of Non-adiabatic Parametric Heating  59
2.3.7 Improved Initial Guess Function and Ultra-fast Transport  60
2.3.8 Discussion of the Open-loop Result  62
2.4 Outlook  64
A Appendix  65
References  66

3 Transport Dynamics of Single Ions in Segmented Microstructured Paul Trap Arrays  69
3.1 Introduction  69
3.2 Classical Equations of Motion  71
3.3 Classical Dynamics of Ion Transport  72
3.3.1 Homogeneous Solution  73
3.3.2 Green’s Function and General Solution  74
3.3.3 Adiabatic Limit 75
3.4 Quantum and Classical, Dragged Harmonic Oscillators with Constant Frequency 76
3.5 The Dragged Quantum Harmonic Oscillator 78
3.6 Transport Dynamics in a Well-controlled Regime 81
3.6.1 Two Analytical Examples 82
3.6.2 Near-optimum Transport Functions 86
3.6.3 High-frequency Limit, Adiabatic Transport, and Approximate Trajectories 86
3.7 Please supply a short title 87
3.7.1 Determination of Waveforms 87
3.7.2 Potential Fluctuations and Aspect-ratio Rule 90
3.8 Conclusions 95
A Appendix 96
References 96

4 Ensemble Quantum Computation and Algorithmic Cooling in Optical Lattices 99
M. Popp, K. G. H. Vollbrecht, and J. I. Cirac
4.1 Introduction 99
4.2 Physical System 102
4.2.1 Bose-Hubbard Model 102
4.2.2 Initial State Properties 103
4.2.3 Entropy as Figure of Merit 105
4.2.4 Basic Operations 106
4.3 Ensemble Quantum Computation 108
4.4 Cooling with Filtering 112
4.5 Algorithmic Ground State Cooling 114
4.5.1 The Protocol 114
4.5.2 Theoretical Description 115
4.6 Conclusion 118
References 119

5 Quantum Information Processing in Optical Lattices and Magnetic Microtraps 121
Philipp Treutlein, Tilo Steinmetz, Yves Colombe, Benjamin Lev, Peter Hommelhoff, Jakob Reichel, Markus Greiner, Olaf Mandel, Artur Widera, Tim Rom, Immanuel Bloch, and Theodor W. Hänsch
5.1 Introduction 121
5.2 Optical Lattices 122
5.2.1 Preparation of a Qubit Register 122
5.2.2 A Quantum Conveyer Belt for Neutral Atoms 123
7.4 Probing Long-range Order  176
7.4.1 Bragg Scattering  176
7.4.2 Heterodyned Bragg Spectra  178
7.4.3 Measuring the Bragg Scattering Phase  179
7.5 Conclusion  180
References  181

8 Detecting Neutral Atoms on an Atom Chip  185
Marco Wilzbach, Albrecht Haase, Michael Schwarz, Dennis Heine, Kai Wicker, Xiyuan Liu, Karl-Heinz Brenner, Sönke Groth, Thomas Fernholz, Björn Hessmo, and Jörg Schmiedmayer
8.1 Introduction  185
8.2 Detecting Single Atoms  186
8.2.1 Measuring the Scattered Light: Fluorescence Detection  186
8.2.2 Measuring the Driving Field  187
8.2.2.1 Absorption on Resonance  187
8.2.2.2 Refraction  189
8.2.3 Cavities  189
8.2.3.1 Absorption on Resonance  189
8.2.3.2 Refraction  190
8.2.3.3 Many Atoms in a Cavity  190
8.2.4 Concentric Cavity  191
8.2.5 Miniaturization  191
8.3 Properties of Fiber Cavities  192
8.3.1 Loss Mechanisms for a Cavity  193
8.3.2 Losses due to the Gap Length  194
8.3.3 Losses due to Transversal Misalignment  195
8.3.4 Losses due to Angular Misalignment  196
8.3.5 Fresnel Reflections  197
8.4 Other Fiber Optical Components for the Atom Chip  199
8.4.1 Fluorescence and Absorption Detectors  199
8.4.2 A Single Mode Tapered Lensed Fiber Dipole Trap  199
8.5 Integration of Fibers on the Atom Chip  201
8.5.1 Building Fiber Cavities  201
8.5.2 The SU-8 Resist  203
8.5.3 Test of the SU-8 Structure  204
8.6 Pilot Test for Atom Detection with Small Waists  205
8.6.1 Dropping Atoms through a Concentric Cavity  205
8.6.2 Detecting Magnetically Guided Atoms  207
8.7 Conclusion  208
References  209
9 High Resolution Rydberg Spectroscopy of Ultracold Rubidium Atoms 211
Axel Grabowski, Rolf Heidemann, Robert Löw, Jürgen Stuhler, and Tilman Pfau

9.1 Introduction 211
9.2 Experimental Setup and Cold Atom Preparation 212
9.2.1 Vacuum System and Magneto Optical Trap (MOT) 212
9.2.2 Rydberg Laser System and Rydberg Excitation 215
9.2.3 Detection of the Rydberg Atoms 216
9.2.4 Excitation Sequence 217
9.3 Spectroscopy of Rydberg States, |m_j| Splitting of the Rydberg States 219
9.4 Spatial and State Selective Addressing of Rydberg States 220
9.4.1 Spatial Selective Rydberg Excitation 220
9.4.2 Hyperfine Selective Rydberg Excitation 221
9.5 Autler-Townes Splitting 222
9.6 Conclusion and Outlook 224

References 224

10 Prospects of Ultracold Rydberg Gases for Quantum Information Processing 227
Markus Reetz-Lamour, Thomas Amthor, Johannes Deiglmayr, Sebastian Westermann, Kilian Singer, André Luiz de Oliveira, Luis Gustavo Marcassa, and Matthias Weidemüller

10.1 Introduction 227
10.2 Excitation of Rydberg Atoms from an Ultracold Gas 229
10.3 Van-der-Waals Interaction 230
10.3.1 Blockade of Excitation 231
10.3.2 Ionization 232
10.4 States with Permanent Electric Dipole Moments 234
10.5 Förster Resonances 236
10.6 Conclusion 239
References 241

11 Quantum State Engineering with Spins 243
Andreas Heidebrecht, Jens Mende, and Michael Mehring

11.1 Introduction 243
11.1.1 Quantum States of Spins 244
11.2 Deutsch-Josza Algorithm 246
11.2.1 The Deutsch-Josza Algorithm 246
11.2.2 Implementation of the 3-qubit Deutsch-Josza Algorithm Using Liquid State NMR 247
11.2.2.1 2,3,4-Trifluoroaniline 247
11.2.2.2 Preparation of Pseudo-pure States 248
11.2.2.3 Results on the 3-qubit DJ-algorithm 249
11.3 Entanglement of an Electron and Nuclear Spin in $^{15}N@C_60$ 251
11.4 Spin Quantum Computing in the Solid State: S-bus 253
11.4.1 The S-bus Concept 253
11.4.2 Single Crystal $\text{CaF}_2 : \text{Ce}^{3+}$ as an S-bus system 255
11.4.3 Experimental Details 256
11.4.4 3-qubit Pseudo-pure States 258
11.4.5 2-qubit Deutsch-Josza Algorithm 259
11.4.5.1 Controlling Nuclear Spin Decoherence in $\text{CaF}_2 : \text{Ce}$ 260
11.5 Summary and Outlook 263
References 263

12 Improving the Purity of One- and Two-qubit Gates 265
Sigmund Kohler and Peter Hänggi
12.1 Introduction 265
12.2 Quantum Gate with Bit-flip Noise 266
12.2.1 Bloch-Redfield Master Equation 267
12.2.2 Purity Decay 268
12.2.3 Numerical Solution 269
12.3 Coherence Stabilization for Single Qubits 270
12.3.1 Dynamical Decoupling by Harmonic Driving 271
12.3.2 Coherent Destruction of Tunneling 272
12.4 Coherence Stabilization for a CNOT Gate 275
12.4.1 Heisenberg vs. Ising Coupling 276
12.4.2 Coherence Stabilization by an AC Field 278
12.4.3 Numerical Solution 279
12.4.4 Implementation with Quantum Dots 282
12.5 Conclusions 282
A Appendix 283
References 284

13 How to Distill Entanglement from a Finite Amount of Qubits? 287
Stefan Probst-Schendzielorz, Thorsten Bschorr, and Matthias Freyberger
13.1 Introduction 287
13.2 Entanglement Distillation 288
13.2.1 The Protocol 289
13.3 CNOT Distillation for a Finite Set of Entangled Systems 293
13.3.1 Iterative Distillation 294
13.4 Example of the Iterative Distillation for Small Finite Sets 297
<table>
<thead>
<tr>
<th>13.5</th>
<th>Conclusions</th>
<th>299</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Appendix</td>
<td>300</td>
</tr>
<tr>
<td></td>
<td>References</td>
<td>301</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>14</th>
<th>Experimental Quantum Secret Sharing</th>
<th>303</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Christian Schmid, Pavel Trojan, Sascha Gaertner, Mohamed Bourennane, Christian Kurtsiefer, Marek Zukowski, and Harald Weinfurter</td>
<td></td>
</tr>
<tr>
<td>14.1</td>
<td>Introduction</td>
<td>303</td>
</tr>
<tr>
<td>14.2</td>
<td>Theory</td>
<td>304</td>
</tr>
<tr>
<td>14.2.1</td>
<td>The GHZ-protocol</td>
<td>304</td>
</tr>
<tr>
<td>14.2.2</td>
<td>The</td>
<td>Ψ⟩⁻₄-states protocol</td>
</tr>
<tr>
<td>14.2.3</td>
<td>The Single Qubit Protocol</td>
<td>306</td>
</tr>
<tr>
<td>14.2.4</td>
<td>Security of the Protocols</td>
<td>307</td>
</tr>
<tr>
<td>14.3</td>
<td>Experiment</td>
<td>309</td>
</tr>
<tr>
<td>14.3.1</td>
<td>The</td>
<td>Ψ⟩⁻₄-states protocol</td>
</tr>
<tr>
<td>14.3.2</td>
<td>The Single-qubit Protocol</td>
<td>310</td>
</tr>
<tr>
<td>14.4</td>
<td>Conclusion</td>
<td>312</td>
</tr>
<tr>
<td></td>
<td>References</td>
<td>314</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>15</th>
<th>Free Space Quantum Key Distribution: Towards a Real Life Application</th>
<th>315</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Henning Weier, Tobias Schmitt-Manderbach, Nadja Regner, Christian Kurtsiefer, and Harald Weinfurter</td>
<td></td>
</tr>
<tr>
<td>15.1</td>
<td>Introduction</td>
<td>315</td>
</tr>
<tr>
<td>15.2</td>
<td>Setup</td>
<td>316</td>
</tr>
<tr>
<td>15.2.1</td>
<td>Transmitter Unit</td>
<td>316</td>
</tr>
<tr>
<td>15.2.2</td>
<td>Free Space Link</td>
<td>317</td>
</tr>
<tr>
<td>15.2.3</td>
<td>Receiver Unit</td>
<td>318</td>
</tr>
<tr>
<td>15.2.4</td>
<td>Synchronisation and Automatic Alignment Control</td>
<td>319</td>
</tr>
<tr>
<td>15.2.5</td>
<td>Sifting, Error Correction and Privacy Amplification</td>
<td>319</td>
</tr>
<tr>
<td>15.2.6</td>
<td>Experimental Results</td>
<td>320</td>
</tr>
<tr>
<td>15.3</td>
<td>Conclusion</td>
<td>322</td>
</tr>
<tr>
<td></td>
<td>References</td>
<td>323</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>16</th>
<th>Continuous Variable Entanglement Between Frequency Modes</th>
<th>325</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Oliver Glöckl, Ulrik L. Andersen, and Gerd Leuchs</td>
<td></td>
</tr>
<tr>
<td>16.1</td>
<td>Introduction</td>
<td>325</td>
</tr>
<tr>
<td>16.2</td>
<td>Sideband Separation</td>
<td>327</td>
</tr>
<tr>
<td>16.2.1</td>
<td>Theory</td>
<td>328</td>
</tr>
<tr>
<td>16.2.2</td>
<td>Pictorial Description</td>
<td>331</td>
</tr>
<tr>
<td>16.3</td>
<td>Experiment and Results</td>
<td>331</td>
</tr>
</tbody>
</table>
16.4 Conclusion and Discussion 335
References 336

17 Factorization of Numbers with Physical Systems 339
Wolfgang Merkel, Ilya Sh. Averbukh, Bertrand Girard, Michael Mehring,
Gerhard G. Paulus, and Wolfgang P. Schleich
17.1 Introduction 339
17.2 Chirping a Two-photon Transition 340
17.2.1 Chirped Laser Pulses 340
17.2.2 Excitation Probability Amplitude 341
17.2.3 Example for Factorization 342
17.3 Driving a One-photon Transition 343
17.3.1 Model 344
17.3.2 Floquet Ladder 345
17.3.3 Pulse Train 346
17.4 Factorization 347
17.4.1 Factorization with Floquet Ladder 348
17.4.2 Factorization with a Pulse Train 349
17.5 NMR-experiment 350
17.6 Conclusions 352
References 353

18 Quantum Algorithms for Number Fields 355
Daniel Haase and Helmut Maier
18.1 Introduction 355
18.1.1 Outline of the Survey 355
18.1.2 Why Number Fields? 356
18.1.3 Some History of the Subject 356
18.2 Geometry of Numbers 357
18.2.1 Number Fields 357
18.2.2 Lattices 358
18.2.3 Integral Elements 359
18.2.4 The Class Number 360
18.2.5 The Regulator 361
18.2.6 Complexity Results 361
18.3 Reduction 362
18.3.1 Reduced Ideals 362
18.3.2 Infrastructure 363
18.3.3 Geometric Interpretation of G 364
18.4 Results from Analytic Number Theory 366
18.4.1 Distribution of Prime Numbers 366
18.4.2 Class Number Formulas 367
## Contents

18.5 Examples of Minima Distributions 368
18.6 Computing the Regulator 370
18.6.1 Real Quadratic Case 370
18.6.2 Hallgren’s Algorithm 371
18.6.3 Generalization of the Weak Periodicity Condition 372
18.7 Computation of Other Invariants 374
18.7.1 The Principal Ideal Problem 374
18.7.2 Computing the Class Number 375
References 376

19 Implementation Complexity of Physical Processes as a Natural Extension of Computational Complexity 377
Dominik Janzing
19.1 Introduction 377
19.2 Similar Complexity Bounds for Different Tasks 379
19.3 Relating Control Problems to Hard Computational Problems 385
19.4 The Need for a Control-theoretic Foundation of Complexity 388
19.5 Hamiltonians that Compute Autonomously 393
References 396

20 Implementation of Generalized Measurements with Minimal Disturbance on a Quantum Computer 399
Thomas Decker and Markus Grassl
20.1 Introduction 399
20.2 Minimal-disturbing Implementations of POVMs 401
20.2.1 Generalized Measurements of Quantum Systems 401
20.2.2 Positive-operator Valued Measures 402
20.2.3 Orthogonal Measurements 403
20.2.4 Disturbance of a Generalized Measurement 404
20.2.5 Minimal-disturbing Implementation of a POVM 405
20.3 Symmetric Matrices and their Structure 406
20.3.1 Representations of Finite Groups 407
20.3.2 Projective Representations 408
20.3.3 Symmetry of a Matrix and Schur’s Lemma 410
20.3.4 Symmetric POVMs Define Matrices with Symmetry 411
20.4 Implementation of Symmetric POVMs 413
20.5 Cyclic and Heisenberg-Weyl Groups 416
20.5.1 Cyclic Groups 416
20.5.2 Heisenberg-Weyl Groups 419
20.6 Conclusions and Outlook 423
References 424
21 Full Counting Statistics of Interacting Electrons 425
D. A. Bagrets, Y. Utsumi, D. S. Golubev, and Gerd Schön
21.1 Introduction 425
21.2 Concepts of FCS 428
21.3 Full Counting Statistics in Interacting Quantum Dots 435
21.3.1 FCS of a Set for Intermediate Strength Conductance 437
21.3.2 Non-Markovian Effects: Renormalization and Finite Lifetime Broadening of Charge States 440
21.3.3 Keldysh Action and CGF in Majorana Representation 442
21.4 FCS and Coulomb Interaction in Diffusive Conductors 443
21.4.1 Model and Effective Action 445
21.4.2 “Cold Electron” Regime 447
21.4.3 “Hot Electron” Regime 453
21.5 Summary 454
References 455

22 Quantum Limit of the Carnot Engine 457
Friedemann Tonner and Günter Mahler
22.1 Introduction 457
22.2 Spin-oscillator Model 458
22.2.1 Basic Definitions 458
22.2.2 Thermodynamic Variables for $G$ 461
22.3 Master Equation 462
22.3.1 Lindblad Superoperator 462
22.3.2 Time Slot Operators 463
22.4 Machine Cycles 465
22.4.1 Choice of Amplitudes $a_{\pm}^{(j)}$ and Control Functions $\theta^{(j)}(\tau)$ 465
22.4.2 Heat and Work 467
22.4.3 Energy Balance 468
22.4.4 Fluctuations 468
22.5 Numerical Results 470
22.5.1 Heat Engine 470
22.5.2 Heat Pump 474
22.5.3 Longtime Limit 475
22.5.4 Quantum Limit and Classical Limit 475
22.6 Summary and Conclusions 477
References 479

Appendix: Colour Plates 481

Index 491