1. The Nature of Physics

Physics is the most basic of the sciences, and it is at the very root of subjects like chemistry, engineering, astronomy, and even biology. The discipline of physics has developed over many centuries, and it continues to evolve. It is a mature science, and its laws encompass a wide scope of phenomena that range from the formation of galaxies to the interactions of particles in the nuclei of atoms. Perhaps the most visible evidence of physics in everyday life is the eruption of new applications that have improved our quality of life, such as new medical devices, and advances in computers and high-tech communications.

The exciting feature of physics is its capacity for predicting how nature will behave in one situation on the basis of experimental data obtained in another situation. Such predictions place physics at the heart of modern technology and, therefore, can have a tremendous impact on our lives. Rocketry and the development of space travel have their roots firmly planted in the physical laws of Galileo Galilei (1564–1642) and Isaac Newton (1642–1727). The transportation industry relies heavily on physics in the development of engines and the design of aerodynamic vehicles. Entire electronics and computer industries owe their existence to the invention of the transistor, which grew directly out of the laws of physics that describe the electrical behavior of solids. The telecommunications industry depends extensively on electromagnetic waves, whose existence was predicted by James Clerk Maxwell (1831–1879) in his theory of electricity and magnetism. The medical profession uses X-ray, ultrasonic, and magnetic resonance methods for obtaining images of the interior of the human body, and physics lies at the core of all these. Perhaps the most widespread impact in modern technology is that due to the laser. Fields ranging from space exploration to medicine benefit from this incredible device, which is a direct application of the principles of atomic physics.

Because physics is so fundamental, it is a required course for students in a wide range of major areas. We welcome you to the study of this fascinating topic. You will learn how to see the world through the “eyes” of physics and to reason as a physicist does. In the process, you will learn how to apply physics principles to a wide range of problems. We hope that you will come to recognize that physics has important things to say about your environment.

1.2 Units

Physics experiments involve the measurement of a variety of quantities, and a great deal of effort goes into making these measurements as accurate and reproducible as possible. The first step toward ensuring accuracy and reproducibility is defining the units in which the measurements are made.
Chapter 1 | Introduction and Mathematical Concepts

In this text, we emphasize the system of units known as SI units, which stands for the French phrase “Le Système International d’Unités.” By international agreement, this system employs the meter (m) as the unit of length, the kilogram (kg) as the unit of mass, and the second (s) as the unit of time. Two other systems of units are also in use, however. The CGS system utilizes the centimeter (cm), the gram (g), and the second for length, mass, and time, respectively, and the BE or British Engineering system (the gravitational version) uses the foot (ft), the slug (sl), and the second. Table 1.1 summarizes the units used for length, mass, and time in the three systems.

Originally, the meter was defined in terms of the distance measured along the earth’s surface between the north pole and the equator. Eventually, a more accurate measurement standard was needed, and by international agreement the meter became the distance between two marks on a bar of platinum–iridium alloy (see Figure 1.1) kept at a temperature of 0 °C. Today, to meet further demands for increased accuracy, the meter is defined as the distance that light travels in a vacuum in a time of 1/299 792 458 second. This definition arises because the speed of light is a universal constant that is defined to be 299 792 458 m/s.

The definition of a kilogram as a unit of mass has also undergone changes over the years. As Chapter 4 discusses, the mass of an object indicates the tendency of the object to continue in motion with a constant velocity. Originally, the kilogram was expressed in terms of a specific amount of water. Today, one kilogram is defined to be the mass of a standard cylinder of platinum–iridium alloy, like the one in Figure 1.2.

As with the units for length and mass, the present definition of the second as a unit of time is different from the original definition. Originally, the second was defined according to the average time for the earth to rotate once about its axis, one day being set equal to 86 400 seconds. The earth’s rotational motion was chosen because it is naturally repetitive, occurring over and over again. Today, we still use a naturally occurring repetitive phenomenon to define the second, but of a very different kind. We use the electromagnetic waves emitted by cesium-133 atoms in an atomic clock like that in Figure 1.3. One second is defined as the time needed for 9 192 631 770 wave cycles to occur.*

The units for length, mass, and time, along with a few other units that will arise later, are regarded as base SI units. The word “base” refers to the fact that these units are used along with various laws to define additional units for other important physical quantities, such as force and energy. The units for such other physical quantities are referred to as derived units, since they are combinations of the base units. Derived units will be introduced from time to time, as they arise naturally along with the related physical laws.

The value of a quantity in terms of base or derived units is sometimes a very large or very small number. In such cases, it is convenient to introduce larger or smaller units that are related to the normal units by multiples of ten. Table 1.2 summarizes the prefixes that are used to denote multiples of ten. For example, 1000 or 10^3 meters are referred to as 1 kilometer (km), and 0.001 or 10^-3 meter is called 1 millimeter (mm). Similarly, 1000 grams and 0.001 gram are referred to as 1 kilogram (kg) and 1 milligram (mg), respectively. Appendix A contains a discussion of scientific notation and powers of ten, such as 10^3 and 10^-3.

Table 1.1 Units of Measurement

<table>
<thead>
<tr>
<th>System</th>
<th>SI</th>
<th>CGS</th>
<th>BE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>Meter (m)</td>
<td>Centimeter (cm)</td>
<td>Foot (ft)</td>
</tr>
<tr>
<td>Mass</td>
<td>Kilogram (kg)</td>
<td>Gram (g)</td>
<td>Slug (sl)</td>
</tr>
<tr>
<td>Time</td>
<td>Second (s)</td>
<td>Second (s)</td>
<td>Second (s)</td>
</tr>
</tbody>
</table>

*See Chapter 16 for a discussion of waves in general and Chapter 24 for a discussion of electromagnetic waves in particular.
1.3 | The Role of Units in Problem Solving

The Conversion of Units

Since any quantity, such as length, can be measured in several different units, it is important to know how to convert from one unit to another. For instance, the foot can be used to express the distance between the two marks on the standard platinum–iridium meter bar. There are 3.281 feet in one meter, and this number can be used to convert from meters to feet, as the following example demonstrates.

**Table 1.2** Standard Prefixes Used to Denote Multiples of Ten

<table>
<thead>
<tr>
<th>Prefix</th>
<th>Symbol</th>
<th>Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>tera</td>
<td>T</td>
<td>$10^{12}$</td>
</tr>
<tr>
<td>giga</td>
<td>G</td>
<td>$10^9$</td>
</tr>
<tr>
<td>mega</td>
<td>M</td>
<td>$10^6$</td>
</tr>
<tr>
<td>kilo</td>
<td>k</td>
<td>$10^3$</td>
</tr>
<tr>
<td>hecto</td>
<td>h</td>
<td>$10^2$</td>
</tr>
<tr>
<td>deka</td>
<td>da</td>
<td>$10^1$</td>
</tr>
<tr>
<td>deci</td>
<td>d</td>
<td>$10^{-1}$</td>
</tr>
<tr>
<td>centi</td>
<td>c</td>
<td>$10^{-2}$</td>
</tr>
<tr>
<td>milli</td>
<td>m</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>micro</td>
<td>μ</td>
<td>$10^{-6}$</td>
</tr>
<tr>
<td>nano</td>
<td>n</td>
<td>$10^{-9}$</td>
</tr>
<tr>
<td>pico</td>
<td>p</td>
<td>$10^{-12}$</td>
</tr>
<tr>
<td>femto</td>
<td>f</td>
<td>$10^{-15}$</td>
</tr>
</tbody>
</table>

*Appendix A contains a discussion of powers of ten and scientific notation.*
In addition to their role in guiding the use of conversion factors, units serve a useful purpose in solving problems. They can provide an internal check to eliminate errors, if they are carried along during each step of a calculation and treated like any algebraic factor. In particular, remember that only quantities with the same units can be added or subtracted (Problem-Solving Insight). Thus, at one point in a calculation, if you find yourself adding 12 miles to 32 kilometers, stop and reconsider. Either miles must be converted into kilometers or kilometers must be converted into miles before the addition can be carried out.

A collection of useful conversion factors is given on the page facing the inside of the front cover. The reasoning strategy that we have followed in Examples 1 and 2 for converting between units is outlined as follows:

**Reasoning Strategy  Converting Between Units**

1. In all calculations, write down the units explicitly.
2. Treat all units as algebraic quantities. In particular, when identical units are divided, they are eliminated algebraically.
3. Use the conversion factors located on the page facing the inside of the front cover. Be guided by the fact that multiplying or dividing an equation by a factor of 1 does not alter the equation. For instance, the conversion factor of 3.281 feet = 1 meter might be applied in the form (3.281 feet)/(1 meter) = 1. This factor of 1 would be used to multiply an equation such as “Length = 5.00 meters” in order to convert meters to feet.
4. Check to see that your calculations are correct by verifying that the units combine algebraically to give the desired unit for the answer. Only quantities with the same units can be added or subtracted.

Sometimes an equation is expressed in a way that requires specific units to be used for the variables in the equation. In such cases it is important to understand why only certain units can be used in the equation, as the following example illustrates.

**EXAMPLE 3 | The Physics of the Body Mass Index**

The body mass index (BMI) takes into account your mass in kilograms (kg) and your height in meters (m) and is defined as follows:

\[
\text{BMI} = \frac{\text{Mass in kg}}{\text{(Height in m)}^2}
\]

However, the BMI is often computed using the weight* of a person in pounds (lb) and his or her height in inches (in.). Thus, the expression for the BMI incorporates these quantities, rather than the mass in kilograms and the height in meters. Starting with the definition above, determine the expression for the BMI that uses pounds and inches.

**Reasoning** We will begin with the BMI definition and work separately with the numerator and the denominator. We will determine the mass in kilograms that appears in the numerator from the weight in pounds by using the fact that 1 kg corresponds to 2.205 lb. Then, we will determine the height in meters that appears in the denominator from the height in inches with the aid of the facts that 1 m = 3.281 ft and 1 ft = 12 in. These conversion factors are located on the page facing the inside of the front cover of the text.

**Solution** Since 1 kg corresponds to 2.205 lb, the mass in kilograms can be determined from the weight in pounds in the following way:

\[
\text{Mass in kg} = \text{(Weight in lb)} \left( \frac{1 \text{ kg}}{2.205 \text{ lb}} \right)
\]

Since 1 ft = 12 in. and 1 m = 3.281 ft, we have

\[
\text{Height in m} = \text{(Height in in.)} \left( \frac{1 \text{ ft}}{12 \text{ in.}} \right) \left( \frac{1 \text{ m}}{3.281 \text{ ft}} \right)
\]

*Weight and mass are different concepts, and the relationship between them will be discussed in Section 4.7.*
Substituting these results into the numerator and denominator of the BMI definition gives

\[
\text{BMI} = \frac{\text{Mass in kg}}{(\text{Height in m})^2} = \frac{(\text{Weight in lb})(\frac{1 \text{ kg}}{2.205 \text{ lb}})}{(\text{Height in in.})^2(\frac{1 \text{ m}}{3.281 \text{ ft}})^2}
\]

\[
= \left( \frac{1 \text{ kg}}{2.205 \text{ lb}} \right) \left( \frac{12 \text{ in.}}{1 \text{ ft}} \right) \left( \frac{3.281 \text{ ft}}{1 \text{ m}} \right)^2 \frac{(\text{Weight in lb})}{(\text{Height in in.})^2}
\]

\[
\text{BMI} = \left( \frac{703.0 \text{ kg \cdot in.}^2}{\text{lb \cdot m}^2} \right) \frac{(\text{Weight in lb})}{(\text{Height in in.})^2}
\]

For example, if your weight and height are 180 lb and 71 in., your body mass index is 25 kg/m². The BMI can be used to assess approximately whether your weight is normal for your height (see Table 1.3).

### Dimensional Analysis

We have seen that many quantities are denoted by specifying both a number and a unit. For example, the distance to the nearest telephone may be 8 meters, or the speed of a car might be 25 meters/second. Each quantity, according to its physical nature, requires a certain type of unit. Distance must be measured in a length unit such as meters, feet, or miles, and a time unit will not do. Likewise, the speed of an object must be specified as a length unit divided by a time unit. In physics, the term dimension is used to refer to the physical nature of a quantity and the type of unit used to specify it. Distance has the dimension of length, which is symbolized as [L], while speed has the dimensions of length [L] divided by time [T], or [L/T]. Many physical quantities can be expressed in terms of a combination of fundamental dimensions such as length [L], time [T], and mass [M]. Later on, we will encounter certain other quantities, such as temperature, which are also fundamental. A fundamental quantity like temperature cannot be expressed as a combination of the dimensions of length, time, mass, or any other fundamental dimension.

Dimensional analysis is used to check mathematical relations for the consistency of their dimensions. As an illustration, consider a car that starts from rest and accelerates to a speed \( v \) in a time \( t \). Suppose we wish to calculate the distance \( x \) traveled by the car but are not sure whether the correct relation is \( x = \frac{1}{2} vt^2 \) or \( x = \frac{1}{2} vt \). We can decide by checking the quantities on both sides of the equals sign to see whether they have the same dimensions. If the dimensions are not the same, the relation is incorrect. For \( x = \frac{1}{2} vt^2 \), we use the dimensions for distance [L], time [T], and speed [L/T] in the following way:

\[
x = \frac{1}{2} vt^2
\]

**Dimensions**

\[
\text{[L]} = \left[ \frac{\text{L}}{\text{T}} \right]^3 = \text{[L][T]}
\]

Dimensions cancel just like algebraic quantities, and pure numerical factors like \( \frac{1}{2} \) have no dimensions, so they can be ignored. The dimension on the left of the equals sign does not match those on the right, so the relation \( x = \frac{1}{2} vt^2 \) cannot be correct. On the other hand, applying dimensional analysis to \( x = \frac{1}{2} vt \), we find that

\[
x = \frac{1}{2} vt
\]

**Dimensions**

\[
\text{[L]} = \left[ \frac{\text{L}}{\text{T}} \right] = \text{[L]}
\]

The dimension on the left of the equals sign matches that on the right, so this relation is dimensionally correct. If we know that one of our two choices is the right one, then \( x = \frac{1}{2} vt \) is it. In the absence of such knowledge, however, dimensional analysis cannot identify the
correct relation. It can only identify which choices may be correct, since it does not account for numerical factors like \( \frac{1}{2} \) or for the manner in which an equation was derived from physics principles.

### Check Your Understanding
*(The answers are given at the end of the book.)*

1. (a) Is it possible for two quantities to have the same dimensions but different units?
   (b) Is it possible for two quantities to have the same units but different dimensions?

2. You can always add two numbers that have the same units (such as 6 meters + 3 meters). Can you always add two numbers that have the same dimensions, such as two numbers that have the dimensions of length [L]?

3. The following table lists four variables, along with their units:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>Meters (m)</td>
</tr>
<tr>
<td>( v )</td>
<td>Meters per second (m/s)</td>
</tr>
<tr>
<td>( t )</td>
<td>Seconds (s)</td>
</tr>
<tr>
<td>( a )</td>
<td>Meters per second squared (m/s²)</td>
</tr>
</tbody>
</table>

These variables appear in the following equations, along with a few numbers that have no units. In which of the equations are the units on the left side of the equals sign consistent with the units on the right side?

(a) \( x = vt \)  
(b) \( x = vt + \frac{1}{2}at^2 \)  
(c) \( v = at \)  
(d) \( v = \sqrt{\frac{2x}{a}} \)  
(e) \( v^3 = 2ax^2 \)  
(f) \( t = \sqrt{\frac{2x}{a}} \)

4. In the equation \( y = c^na^2 \) you wish to determine the integer value (1, 2, etc.) of the exponent \( n \). The dimensions of \( y, a, \) and \( t \) are known. It is also known that \( c \) has no dimensions. Can dimensional analysis be used to determine \( n \)?

### 1.4 | Trigonometry

Scientists use mathematics to help them describe how the physical universe works, and trigonometry is an important branch of mathematics. Three trigonometric functions are utilized throughout this text. They are the sine, the cosine, and the tangent of the angle \( \theta \) (Greek theta), abbreviated as \( \sin \theta \), \( \cos \theta \), and \( \tan \theta \), respectively. These functions are defined below in terms of the symbols given along with the right triangle in Figure 1.5.

#### Definition of \( \sin \theta \), \( \cos \theta \), and \( \tan \theta \)

\[
\sin \theta = \frac{h_o}{h} \tag{1.1}
\]

\[
\cos \theta = \frac{h_a}{h} \tag{1.2}
\]

\[
\tan \theta = \frac{h_o}{h_a} \tag{1.3}
\]

\( h = \) length of the hypotenuse of a right triangle  
\( h_o = \) length of the side opposite the angle \( \theta \)  
\( h_a = \) length of the side adjacent to the angle \( \theta \)
The sine, cosine, and tangent of an angle are numbers without units, because each is the ratio of the lengths of two sides of a right triangle. Example 4 illustrates a typical application of Equation 1.3.

**EXAMPLE 4 | Using Trigonometric Functions**

On a sunny day, a tall building casts a shadow that is 67.2 m long. The angle between the sun’s rays and the ground is \(\theta = 50.0^\circ\), as Figure 1.6 shows. Determine the height of the building.

**Reasoning** We want to find the height of the building. Therefore, we begin with the colored right triangle in Figure 1.6 and identify the height as the length \(h_o\) of the side opposite the angle \(\theta\). The length of the shadow is the length \(h_a\) of the side that is adjacent to the angle \(\theta\). The ratio of the length of the opposite side to the length of the adjacent side is the tangent of the angle \(\theta\), which can be used to find the height of the building.

**Solution** We use the tangent function in the following way, with \(\theta = 50.0^\circ\) and \(h_a = 67.2\) m:

\[
\tan \theta = \frac{h_o}{h_a} \tag{1.3}
\]

\[h_o = h_a \tan \theta = (67.2 \text{ m})(\tan 50.0^\circ) = (67.2 \text{ m})(1.19) = 80.0 \text{ m}
\]

The value of \(\tan 50.0^\circ\) is found by using a calculator.

![Figure 1.6](image)

Problem-Solving Insight

A lakefront drops off gradually at an angle \(\theta\), as Figure 1.7 indicates. For safety reasons, it is necessary to know how deep the lake is at various distances from the shore. To provide some information about the depth, a lifeguard rows straight out from the shore a distance of 14.0 m and drops a weighted fishing line. By measuring the length of the line, the lifeguard determines the depth to be 2.25 m. (a) What is the value of \(\theta\)? (b) What would be the depth \(d\) of the lake at a distance of 22.0 m from the shore?
If the distance from the shore and the depth of the water at any one point are known, the angle $\theta$ can be found with the aid of trigonometry. Knowing the value of $\theta$ is useful, because then the depth $d$ at another point can be determined.

Reasoning Near the shore, the lengths of the opposite and adjacent sides of the right triangle in Figure 1.7 are $h_o = 2.25$ m and $h_a = 14.0$ m, relative to the angle $\theta$. Having made this identification, we can use the inverse tangent to find the angle in part (a). For part (b) the opposite and adjacent sides farther from the shore become $h_o = d$ and $h_a = 22.0$ m. With the value for $\theta$ obtained in part (a), the tangent function can be used to find the unknown depth. Considering the way in which the lake bottom drops off in Figure 1.7, we expect the unknown depth to be greater than 2.25 m.

Solution (a) Using the inverse tangent given in Equation 1.6, we find that

$$\theta = \tan^{-1}\left(\frac{h_o}{h_a}\right) = \tan^{-1}\left(\frac{2.25 \text{ m}}{14.0 \text{ m}}\right) = 9.13^\circ$$

(b) With $\theta = 9.13^\circ$, the tangent function given in Equation 1.3 can be used to find the unknown depth farther from the shore, where $h_o = d$ and $h_a = 22.0$ m. Since $\tan \theta = h_o/h_a$, it follows that

$$h_a = h_o \tan \theta$$

$$d = (22.0 \text{ m})(\tan 9.13^\circ) = 3.54 \text{ m}$$

which is greater than 2.25 m, as expected.

The right triangle in Figure 1.5 provides the basis for defining the various trigonometric functions according to Equations 1.1–1.3. These functions always involve an angle and two sides of the triangle. There is also a relationship among the lengths of the three sides of a right triangle. This relationship is known as the Pythagorean theorem and is used often in this text.

Pythagorean Theorem

The square of the length of the hypotenuse of a right triangle is equal to the sum of the squares of the lengths of the other two sides:

$$h^2 = h_o^2 + h_a^2$$ (1.7)

1.5 | Scalars and Vectors

The volume of water in a swimming pool might be 50 cubic meters, or the winning time of a race could be 11.3 seconds. In cases like these, only the size of the numbers matters. In other words, how much volume or time is there? The 50 specifies the amount of water in units of cubic meters, while the 11.3 specifies the amount of time in seconds. Volume and time are examples of scalar quantities. A scalar quantity is one that can be described with a single number (including any units) giving its size or magnitude. Some other common scalars are temperature (e.g., 20 °C) and mass (e.g., 85 kg).

While many quantities in physics are scalars, there are also many that are not, and for these quantities the magnitude tells only part of the story. Consider Figure 1.8, which
depicts a car that has moved 2 km along a straight line from start to finish. When describing
the motion, it is incomplete to say that “the car moved a distance of 2 km.” This statement
would indicate only that the car ends up somewhere on a circle whose center is at the starting
point and whose radius is 2 km. A complete description must include the direction along
with the distance, as in the statement “the car moved a distance of 2 km in a direction 30°
north of east.” A quantity that deals inherently with both magnitude and direction is called a
vector quantity. Because direction is an important characteristic of vectors, arrows are used to
represent them; the direction of the arrow gives the direction of the vector. The colored
arrow in Figure 1.8, for example, is called the displacement vector, because it shows how the
car is displaced from its starting point. Chapter 2 discusses this particular vector.

The length of the arrow in Figure 1.8 represents the magnitude of the displacement
vector. If the car had moved 4 km instead of 2 km from the starting point, the arrow would
have been drawn twice as long. By convention, the length of a vector arrow is propor-
tional to the magnitude of the vector.

In physics there are many important kinds of vectors, and the practice of using the
length of an arrow to represent the magnitude of a vector applies to each of them. All
forces, for instance, are vectors. In common usage a force is a push or a pull, and the direc-
tion in which a force acts is just as important as the strength or magnitude of the force. The
magnitude of a force is measured in SI units called newtons (N). An arrow representing a
force of 20 newtons is drawn twice as long as one representing a force of 10 newtons.

The fundamental distinction between scalars and vectors is the characteristic of di-
rection. Vectors have it, and scalars do not. Conceptual Example 6 helps to clarify this
distinction and explains what is meant by the “direction” of a vector.

CONCEPTUAL EXAMPLE 6 | Vectors, Scalars, and the Role of Plus and Minus Signs

There are places where the temperature is +20 °C at one time of the year and −20 °C at
another time. Do the plus and minus signs that signify positive and negative temperatures imply that
temperature is a vector quantity? (a) Yes (b) No

Reasoning A hallmark of a vector is that there is both a magnitude and a physical direction
associated with it, such as 20 meters due east or 20 meters due west.

Answer (a) is incorrect. The plus and minus signs associated with +20 °C and −20 °C do not
convey a physical direction, such as due east or due west. Therefore, temperature cannot be a
vector quantity.

Answer (b) is correct. On a thermometer, the algebraic signs simply mean that the temperature
is a number less than or greater than zero on the temperature scale being used and have nothing
to do with east, west, or any other physical direction. Temperature, then, is not a vector. It is a
scalar, and scalars can sometimes be negative.

Often, for the sake of convenience, quantities such as volume, time, displacement, ve-
locity, and force are represented in physics by symbols. In this text, we write vectors in bold-
face symbols (this is boldface) with arrows above them* and write scalars in italic symbols
(this is italic). Thus, a displacement vector is written as “\( \vec{A} = 750 \text{ m, due east,} \)” where the \( \vec{A} \)
is a boldface symbol. By itself, however, separated from the direction, the magnitude of this
vector is a scalar quantity. Therefore, the magnitude is written as “\( A = 750 \text{ m,} \)” where the \( A \)
is an italic symbol without an arrow.

Check Your Understanding

(The answer is given at the end of the book.)

5. Which of the following statements, if any, involves a vector? (a) I walked 2 miles along the
beach. (b) I walked 2 miles due north along the beach. (c) I jumped off a cliff and hit the water
traveling at 17 miles per hour. (d) I jumped off a cliff and hit the water traveling straight down
at a speed of 17 miles per hour. (e) My bank account shows a negative balance of −25 dollars.

*Vectors are also sometimes written in other texts as boldface symbols without arrows above them.
1.6 | Vector Addition and Subtraction

Addition

Often it is necessary to add one vector to another, and the process of addition must take into account both the magnitude and the direction of the vectors. The simplest situation occurs when the vectors point along the same direction—that is, when they are colinear, as in Figure 1.9. Here, a car first moves along a straight line, with a displacement vector $\mathbf{A}$ of 275 m, due east. Then the car moves again in the same direction, with a displacement vector $\mathbf{B}$ of 125 m, due east. These two vectors add to give the total displacement vector $\mathbf{R}$, which would apply if the car had moved from start to finish in one step. The symbol $\mathbf{R}$ is used because the total vector is often called the resultant vector. With the tail of the second arrow located at the head of the first arrow, the two lengths simply add to give the length of the total displacement. This kind of vector addition is identical to the familiar addition of two scalar numbers ($2 + 3 = 5$) and *can be carried out here only because the vectors point along the same direction*. In such cases we add the individual magnitudes to get the magnitude of the total, knowing in advance what the direction must be. Formally, the addition is written as follows:

$$\mathbf{R} = \mathbf{A} + \mathbf{B}$$

Perpendicular vectors are frequently encountered, and Figure 1.10 indicates how they can be added. This figure applies to a car that first travels with a displacement vector $\mathbf{A}$ of 275 m, due east, and then with a displacement vector $\mathbf{B}$ of 125 m, due north. The two vectors add to give a resultant displacement vector $\mathbf{R}$. Once again, the vectors to be added are arranged in a tail-to-head fashion, and the resultant vector points from the tail of the first to the head of the last vector added. The resultant displacement is given by the vector equation

$$\mathbf{R} = \mathbf{A} + \mathbf{B}$$

The addition in this equation cannot be carried out by writing $R = 275\text{ m} + 125\text{ m}$, because the vectors have different directions. Instead, we take advantage of the fact that the triangle in Figure 1.10 is a right triangle and use the Pythagorean theorem (Equation 1.7). According to this theorem, the magnitude of $\mathbf{R}$ is

$$R = \sqrt{(275\text{ m})^2 + (125\text{ m})^2} = 302\text{ m}$$

The angle $\theta$ in Figure 1.10 gives the direction of the resultant vector. Since the lengths of all three sides of the right triangle are now known, $\sin \theta$, $\cos \theta$, or $\tan \theta$ can be used to determine $\theta$. Noting that $\tan \theta = B/A$ and using the inverse trigonometric function, we find that:

$$\theta = \tan^{-1}\left(\frac{B}{A}\right) = \tan^{-1}\left(\frac{125\text{ m}}{275\text{ m}}\right) = 24.4^\circ$$

Thus, the resultant displacement of the car has a magnitude of 302 m and points north of east at an angle of 24.4$^\circ$. This displacement would bring the car from the start to the finish in Figure 1.10 in a single straight-line step.

When two vectors to be added are not perpendicular, the tail-to-head arrangement does not lead to a right triangle, and the Pythagorean theorem cannot be used. Figure 1.11a illustrates such a case for a car that moves with a displacement $\mathbf{A}$ of 275 m, due east, and then with a displacement $\mathbf{B}$ of 125 m, in a direction $55.0^\circ$ north of west. As usual, the resultant displacement vector $\mathbf{R}$ is directed from the tail of the first to the head of the last vector added. The vector addition is still given according to

$$\mathbf{R} = \mathbf{A} + \mathbf{B}$$

However, the magnitude of $\mathbf{R}$ is not $R = A + B$, because the vectors $\mathbf{A}$ and $\mathbf{B}$ do not have the same direction, and neither is it $R = \sqrt{A^2 + B^2}$, because the vectors are not perpendicular, so the Pythagorean theorem does not apply. Some other means must be used to find the magnitude and direction of the resultant vector.
One approach uses a graphical technique. In this method, a diagram is constructed in which the arrows are drawn tail to head. The lengths of the vector arrows are drawn to scale, and the angles are drawn accurately (with a protractor, perhaps). Then the length of the arrow representing the resultant vector is measured with a ruler. This length is converted to the magnitude of the resultant vector by using the scale factor with which the drawing is constructed. In Figure 1.11b, for example, a scale of one centimeter of arrow length for each 10.0 m of displacement is used, and it can be seen that the length of the arrow representing $\vec{R}$ is 22.8 cm. Since each centimeter corresponds to 10.0 m of displacement, the magnitude of $\vec{R}$ is 228 m. The angle $\theta$, which gives the direction of $\vec{R}$, can be measured with a protractor to be $\theta = 26.7^\circ$ north of east.

**Subtraction**

The subtraction of one vector from another is carried out in a way that depends on the following fact. *When a vector is multiplied by $-1$, the magnitude of the vector remains the same, but the direction of the vector is reversed.* Conceptual Example 7 illustrates the meaning of this statement.

**CONCEPTUAL EXAMPLE 7 | Multiplying a Vector by $-1$**

Consider two vectors described as follows:

1. A woman climbs 1.2 m up a ladder, so that her displacement vector $\vec{D}$ is 1.2 m, upward along the ladder, as in Figure 1.12a.
2. A man is pushing with 450 N of force on his stalled car, trying to move it eastward. The force vector $\vec{F}$ that he applies to the car is 450 N, due east, as in Figure 1.13a.

What are the physical meanings of the vectors $-\vec{D}$ and $-\vec{F}$?

(a) $-\vec{D}$ points upward along the ladder and has a magnitude of $-1.2$ m; $-\vec{F}$ points due east and has a magnitude of $-450$ N.
(b) $-\vec{D}$ points downward along the ladder and has a magnitude of $-1.2$ m; $-\vec{F}$ points due west and has a magnitude of $-450$ N.
(c) $-\vec{D}$ points downward along the ladder and has a magnitude of 1.2 m; $-\vec{F}$ points due west and has a magnitude of 450 N.

**Reasoning** A displacement vector of $-\vec{D}$ is $(-1)\vec{D}$. The presence of the $(-1)$ factor reverses the direction of the vector, but does not change its magnitude. Similarly, a force vector of $-\vec{F}$ has the same magnitude as the vector $\vec{F}$ but has the opposite direction.

**Answer (a) and (b) are incorrect.** While scalars can sometimes be negative, magnitudes of vectors are never negative.

**Answer (c) is correct.** The vectors $-\vec{D}$ and $-\vec{F}$ have the same magnitudes as $\vec{D}$ and $\vec{F}$, but point in the opposite direction, as indicated in Figures 1.12b and 1.13b.

**Related Homework:** Problem 67

![Figure 1.12](image_url)

**Figure 1.12** (a) The displacement vector for a woman climbing 1.2 m up a ladder is $\vec{D}$. (b) The displacement vector for a woman climbing 1.2 m down a ladder is $-\vec{D}$.

![Figure 1.13](image_url)

**Figure 1.13** (a) The force vector for a man pushing on a car with 450 N of force in a direction due east is $\vec{F}$. (b) The force vector for a man pushing on a car with 450 N of force in a direction due west is $-\vec{F}$.
In practice, vector subtraction is carried out exactly like vector addition, except that one of the vectors added is multiplied by a scalar factor of −1. To see why, look at the two vectors \( \mathbf{A} \) and \( \mathbf{B} \) in Figure 1.14a. These vectors add together to give a third vector \( \mathbf{C} \), according to \( \mathbf{C} = \mathbf{A} + \mathbf{B} \). Therefore, we can calculate vector \( \mathbf{A} \) as \( \mathbf{A} = \mathbf{C} - \mathbf{B} \), which is an example of vector subtraction. However, we can also write this result as \( \mathbf{A} = \mathbf{C} + (-\mathbf{B}) \) and treat it as vector addition. Figure 1.14b shows how to calculate vector \( \mathbf{A} \) by adding the vectors \( \mathbf{C} \) and \( -\mathbf{B} \). Notice that vectors \( \mathbf{C} \) and \( -\mathbf{B} \) are arranged tail to head and that any suitable method of vector addition can be employed to determine \( \mathbf{A} \).

Check Your Understanding

(Answers are given at the end of the book)

6. Two vectors \( \mathbf{A} \) and \( \mathbf{B} \) are added together to give a resultant vector \( \mathbf{R} = \mathbf{A} + \mathbf{B} \). The magnitudes of \( \mathbf{A} \) and \( \mathbf{B} \) are 3 m and 8 m, respectively, but the vectors can have any orientation. What are (a) the maximum possible value and (b) the minimum possible value for the magnitude of \( \mathbf{R} \)?

7. Can two nonzero perpendicular vectors be added together so their sum is zero?

8. Can three or more vectors with unequal magnitudes be added together so their sum is zero?

9. In preparation for this question, review Conceptual Example 7. Vectors \( \mathbf{A} \) and \( \mathbf{B} \) satisfy the vector equation \( \mathbf{A} + \mathbf{B} = 0 \). (a) How does the magnitude of \( \mathbf{B} \) compare with the magnitude of \( \mathbf{A} \)? (b) How does the direction of \( \mathbf{B} \) compare with the direction of \( \mathbf{A} \)?

10. Vectors \( \mathbf{A} \), \( \mathbf{B} \), and \( \mathbf{C} \) satisfy the vector equation \( \mathbf{A} + \mathbf{B} + \mathbf{C} = \mathbf{0} \), and their magnitudes are related by the scalar equation \( A^2 + B^2 = C^2 \). How is vector \( \mathbf{A} \) oriented with respect to vector \( \mathbf{B} \)?

11. Vectors \( \mathbf{A} \), \( \mathbf{B} \), and \( \mathbf{C} \) satisfy the vector equation \( \mathbf{A} + \mathbf{B} = \mathbf{C} \), and their magnitudes are related by the scalar equation \( A + B = C \). How is vector \( \mathbf{A} \) oriented with respect to vector \( \mathbf{B} \)?

### 1.7 The Components of a Vector

#### Vector Components

Suppose a car moves along a straight line from start to finish, as in Figure 1.15, the corresponding displacement vector being \( \mathbf{r} \). The magnitude and direction of the vector \( \mathbf{r} \) give the distance and direction traveled along the straight line. However, the car could also arrive at the finish point by first moving due east, turning through 90°, and then moving due north. This alternative path is shown in the drawing and is associated with the two displacement vectors \( \mathbf{x} \) and \( \mathbf{y} \). The vectors \( \mathbf{x} \) and \( \mathbf{y} \) are called the \( x \) component vector and the \( y \) vector component of \( \mathbf{r} \).

Vector components are very important in physics and have two basic features that are apparent in Figure 1.15. One is that the components add together to equal the original vector:

\[
\mathbf{r} = \mathbf{x} + \mathbf{y}
\]

The components \( \mathbf{x} \) and \( \mathbf{y} \), when added vectorially, convey exactly the same meaning as does the original vector \( \mathbf{r} \): they indicate how the finish point is displaced relative to the starting point. The other feature of vector components that is apparent in Figure 1.15 is that \( \mathbf{x} \) and \( \mathbf{y} \) are not just any two vectors that add together to give the original vector \( \mathbf{r} \); they are perpendicular vectors. This perpendicularity is a valuable characteristic, as we will soon see.

Any type of vector may be expressed in terms of its components, in a way similar to that illustrated for the displacement vector in Figure 1.15. Figure 1.16 shows an arbitrary vector \( \mathbf{A} \) and its vector components \( \mathbf{A}_x \) and \( \mathbf{A}_y \). The components are drawn parallel to convenient \( x \) and \( y \) axes and are perpendicular. They add vectorially to equal the original vector \( \mathbf{A} \):

\[
\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y
\]
There are times when a drawing such as Figure 1.16 is not the most convenient way to represent vector components, and Figure 1.17 presents an alternative method. The disadvantage of this alternative is that the tail-to-head arrangement of \( \mathbf{\hat{A}} \) and \( \mathbf{\hat{A}}' \) is missing, an arrangement that is a nice reminder that \( \mathbf{\hat{A}} \) and \( \mathbf{\hat{A}}' \) add together to equal \( \mathbf{A} \).

The definition that follows summarizes the meaning of vector components:

**Definition of Vector Components**

In two dimensions, the vector components of a vector \( \mathbf{A} \) are two perpendicular vectors \( \mathbf{\hat{A}}_x \) and \( \mathbf{\hat{A}}_y \) that are parallel to the \( x \) and \( y \) axes, respectively, and add together vectorially according to \( \mathbf{A} = \mathbf{\hat{A}}_x + \mathbf{\hat{A}}_y \).

**Problem-Solving Insight.** In general, the components of any vector can be used in place of the vector itself in any calculation where it is convenient to do so. The values calculated for vector components depend on the orientation of the vector relative to the axes used as a reference. Figure 1.18 illustrates this fact for a vector \( \mathbf{A} \) by showing two sets of axes, one set being rotated clockwise relative to the other. With respect to the black axes, vector \( \mathbf{A} \) has perpendicular vector components \( \mathbf{\hat{A}}_x \) and \( \mathbf{\hat{A}}_y \); with respect to the colored rotated axes, vector \( \mathbf{A} \) has different vector components \( \mathbf{\hat{A}}'_x \) and \( \mathbf{\hat{A}}'_y \). The choice of which set of components to use is purely a matter of convenience.

**Scalar Components**

It is often easier to work with the *scalar components*, \( A_x \) and \( A_y \) (note the italic symbols), rather than the vector components \( \mathbf{\hat{A}}_x \) and \( \mathbf{\hat{A}}_y \). Scalar components are positive or negative numbers (with units) that are defined as follows: The scalar component \( A_x \) has a magnitude equal to that of \( \mathbf{\hat{A}}_x \) and is given a positive sign if \( \mathbf{\hat{A}}_x \) points along the +\( x \) axis and a negative sign if \( \mathbf{\hat{A}}_x \) points along the −\( x \) axis. The scalar component \( A_y \) is defined in a similar manner. The following table shows an example of vector and scalar components:

<table>
<thead>
<tr>
<th>Vector Components</th>
<th>Scalar Components</th>
<th>Unit Vectors</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathbf{\hat{A}}_x ) = 8 meters, directed along the +( x ) axis</td>
<td>( A_x = +8 ) meters</td>
<td>( \mathbf{\hat{A}}_x = (+8 ) meters ( ) \hat{x} )</td>
</tr>
<tr>
<td>( \mathbf{\hat{A}}_y ) = 10 meters, directed along the −( y ) axis</td>
<td>( A_y = -10 ) meters</td>
<td>( \mathbf{\hat{A}}_y = (-10 ) meters ( ) \hat{y} )</td>
</tr>
</tbody>
</table>

In this text, when we use the term “component,” we will be referring to a scalar component, unless otherwise indicated.

Another method of expressing vector components is to use unit vectors. A *unit vector* is a vector that has a magnitude of 1, but no dimensions. We will use a caret (\(^\wedge\)) to distinguish it from other vectors. Thus,

\[ \mathbf{\hat{x}} \text{ is a dimensionless unit vector of length 1 that points in the positive } x \text{ direction, and} \]

\[ \mathbf{\hat{y}} \text{ is a dimensionless unit vector of length 1 that points in the positive } y \text{ direction.} \]

These unit vectors are illustrated in Figure 1.19. With the aid of unit vectors, the vector components of an arbitrary vector \( \mathbf{A} \) can be written as \( \mathbf{\hat{A}}_x = A_x \mathbf{\hat{x}} \) and \( \mathbf{\hat{A}}_y = A_y \mathbf{\hat{y}} \), where \( A_x \) and \( A_y \) are its scalar components (see the drawing and the third column of the table above). The vector \( \mathbf{A} \) is then written as \( \mathbf{A} = A_x \mathbf{\hat{x}} + A_y \mathbf{\hat{y}} \).

**Resolving a Vector into Its Components**

If the magnitude and direction of a vector are known, it is possible to find the components of the vector. The process of finding the components is called “resolving the vector into its components.” As Example 8 illustrates, this process can be carried out with the aid of trigonometry, because the two perpendicular vector components and the original vector form a right triangle.
Since the vector components and the original vector form a right triangle, the Pythagorean theorem can be applied to check the validity of calculations such as those in Example 8. Thus, with the components obtained in Example 8, the theorem can be used to verify that the magnitude of the original vector is indeed 175 m, as given initially:

\[ r = \sqrt{(112 \text{ m})^2 + (134 \text{ m})^2} = 175 \text{ m} \]

It is possible for one of the components of a vector to be zero. This does not mean that the vector itself is zero, however. For a vector to be zero, every vector component must individually be zero. Thus, in two dimensions, saying that \( \vec{A} = 0 \) is equivalent to saying that \( A_x = 0 \) and \( A_y = 0 \). Or, stated in terms of scalar components, if \( \vec{A} = 0 \), then \( A_x = 0 \) and \( A_y = 0 \).

**Problem-Solving Insight**
You can check to see whether the components of a vector are correct by substituting them into the Pythagorean theorem in order to calculate the magnitude of the original vector.

**Problem-Solving Insight**
Either acute angle of a right triangle can be used to determine the components of a vector. The choice of angle is a matter of convenience. For instance, instead of the 50.0° angle, it is also possible to use the angle \( \alpha \) in Figure 1.20. Since \( \alpha + 50.0° = 90.0° \), it follows that \( \alpha = 40.0° \). The solution using \( \alpha \) yields the same answers as the solution using the 50.0° angle:

\[ \cos \alpha = \frac{y}{r} \]
\[ y = r \cos \alpha = (175 \text{ m})(\cos 40.0°) = 134 \text{ m} \]
\[ \sin \alpha = \frac{x}{r} \]
\[ x = r \sin \alpha = (175 \text{ m})(\sin 40.0°) = 112 \text{ m} \]

Since the vector components and the original vector form a right triangle, the Pythagorean theorem can be applied to check the validity of calculations such as those in Example 8. Thus, with the components obtained in Example 8, the theorem can be used to verify that the magnitude of the original vector is indeed 175 m, as given initially:

\[ r = \sqrt{(112 \text{ m})^2 + (134 \text{ m})^2} = 175 \text{ m} \]

**Math Skills**

**Check Your Understanding**
(The answers are given at the end of the book.)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Magnitude</th>
<th>Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \vec{A} )</td>
<td>100 m</td>
<td>30° north of east</td>
</tr>
<tr>
<td>( \vec{B} )</td>
<td>100 m</td>
<td>30° south of west</td>
</tr>
<tr>
<td>( \vec{C} )</td>
<td>50 m</td>
<td>30° south of west</td>
</tr>
<tr>
<td>( \vec{D} )</td>
<td>100 m</td>
<td>60° east of north</td>
</tr>
</tbody>
</table>
13. Two vectors, \( \mathbf{A} \) and \( \mathbf{B} \), are shown in the drawing.

(a) What are the signs (+ or −) of the scalar components, \( A_x \) and \( A_y \), of the vector \( \mathbf{A} \)?

(b) What are the signs of the scalar components, \( B_x \) and \( B_y \), of the vector \( \mathbf{B} \)?

(c) What are the signs of the scalar components, \( R_x \) and \( R_y \), of the vector \( \mathbf{R} \), where \( \mathbf{R} = \mathbf{A} + \mathbf{B} \)?

14. Are two vectors with the same magnitude necessarily equal?

15. The magnitude of a vector has doubled, its direction remaining the same. Can you conclude that the magnitude of each component of the vector has doubled?

16. The tail of a vector is fixed to the origin of an \( x, y \) axis system. Originally the vector points along the \( +x \) axis and has a magnitude of 12 units. As time passes, the vector rotates counterclockwise. What are the sizes of the \( x \) and \( y \) components of the vector for the following rotational angles? 

(a) \( 90^\circ \)  
(b) \( 180^\circ \)  
(c) \( 270^\circ \)  
(d) \( 360^\circ \)

17. A vector has a component of zero along the \( x \) axis of a certain axis system. Does this vector necessarily have a component of zero along the \( x \) axis of another (rotated) axis system?

### 1.8 Addition of Vectors by Means of Components

The components of a vector provide the most convenient and accurate way of adding (or subtracting) any number of vectors. For example, suppose that vector \( \mathbf{A} \) is added to vector \( \mathbf{B} \). The resultant vector is \( \mathbf{C} \), where \( \mathbf{C} = \mathbf{A} + \mathbf{B} \). Figure 1.21 illustrates this vector addition, along with the \( x \) and \( y \) vector components of \( \mathbf{A} \) and \( \mathbf{B} \). In part (b) of the drawing, the vectors \( \mathbf{A} \) and \( \mathbf{B} \) have been removed, because we can use the vector components of these vectors in place of them. The vector component \( \mathbf{B}_y \) has been shifted downward and arranged tail to head with vector component \( \mathbf{A}_x \). Similarly, the vector component \( \mathbf{A}_y \) has been shifted to the right and arranged tail to head with the vector component \( \mathbf{B}_x \). The \( x \) components are colinear and add together to give the \( x \) component of the resultant vector \( \mathbf{C} \). In like fashion, the \( y \) components are colinear and add together to give the \( y \) component of \( \mathbf{C} \). In terms of scalar components, we can write

\[
C_x = A_x + B_x \quad \text{and} \quad C_y = A_y + B_y
\]

The vector components \( \mathbf{C}_x \) and \( \mathbf{C}_y \) of the resultant vector form the sides of the right triangle shown in Figure 1.21c. Thus, we can find the magnitude of \( \mathbf{C} \) by using the Pythagorean theorem:

\[
C = \sqrt{C_x^2 + C_y^2}
\]

The angle \( \theta \) that \( \mathbf{C} \) makes with the \( x \) axis is given by \( \theta = \tan^{-1}(C_y/C_x) \). Example 9 illustrates how to add several vectors using the component method.

Figure 1.21 (a) The vectors \( \mathbf{A} \) and \( \mathbf{B} \) add together to give the resultant vector \( \mathbf{C} \). The \( x \) and \( y \) components of \( \mathbf{A} \) and \( \mathbf{B} \) are also shown. (b) The drawing illustrates that \( \mathbf{C}_x = \mathbf{A}_x + \mathbf{B}_x \) and \( \mathbf{C}_y = \mathbf{A}_y + \mathbf{B}_y \). (c) Vector \( \mathbf{C} \) and its components form a right triangle.
EXAMPLE 9 | The Component Method of Vector Addition

A jogger runs 145 m in a direction 20.0° east of north (displacement vector \( \vec{A} \)) and then 105 m in a direction 35.0° south of east (displacement vector \( \vec{B} \)). Using components, determine the magnitude and direction of the resultant vector \( \vec{C} \) for these two displacements.

**Reasoning** Figure 1.22 shows the vectors \( \vec{A} \) and \( \vec{B} \), assuming that the y axis corresponds to the direction due north. The vectors are arranged in a tail-to-head fashion, with the resultant vector \( \vec{C} \) drawn from the tail of \( \vec{A} \) to the head of \( \vec{B} \). The components of the vectors are also shown in the figure. Since \( \vec{C} \) and its components form a right triangle (red in the drawing), we will use the Pythagorean theorem and trigonometry to express the magnitude and directional angle \( \theta \) for \( \vec{C} \) in terms of its components. The components of \( \vec{C} \) will then be obtained from the components of \( \vec{A} \) and \( \vec{B} \) and the data given for these two vectors.

**Knowns and Unknowns** The data for this problem are listed in the table that follows:

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnitude of vector ( \vec{A} )</td>
<td></td>
<td>145 m</td>
<td></td>
</tr>
<tr>
<td>Direction of vector ( \vec{A} )</td>
<td></td>
<td>20.0° east of north</td>
<td>See Figure 1.22.</td>
</tr>
<tr>
<td>Magnitude of vector ( \vec{B} )</td>
<td></td>
<td>105 m</td>
<td></td>
</tr>
<tr>
<td>Direction of vector ( \vec{B} )</td>
<td></td>
<td>35.0° south of east</td>
<td>See Figure 1.22.</td>
</tr>
</tbody>
</table>

**Unknown Variables**
- Magnitude of resultant vector \( C \)
- Direction of resultant vector \( \theta \)

**Modeling the Problem**

**STEP 1** **Magnitude and Direction of \( \vec{C} \)** In Figure 1.22 the vector \( \vec{C} \) and its components \( C_x \) and \( C_y \) form a right triangle, as the red arrows show. Applying the Pythagorean theorem to this right triangle shows that the magnitude of \( \vec{C} \) is given by Equation 1a at the right. From the red triangle it also follows that the directional angle \( \theta \) for the vector \( \vec{C} \) is given by Equation 1b at the right.

\[
C = \sqrt{C_x^2 + C_y^2} \quad \text{(1a)}
\]
\[
\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) \quad \text{(1b)}
\]

**STEP 2** **Components of \( \vec{C} \)** Since vector \( \vec{C} \) is the resultant of vectors \( \vec{A} \) and \( \vec{B} \), we have \( \vec{C} = \vec{A} + \vec{B} \) and can write the scalar components of \( \vec{C} \) as the sum of the scalar components of \( \vec{A} \) and \( \vec{B} \):

\[
C_x = A_x + B_x \quad \text{and} \quad C_y = A_y + B_y
\]

These expressions can be substituted into Equations 1a and 1b for the magnitude and direction of \( \vec{C} \), as shown at the right.

**Solution** Algebraically combining the results of each step, we find that

**STEP 1**

\[
C = \sqrt{C_x^2 + C_y^2} = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}
\]
\[
\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{A_y + B_y}{A_x + B_x}\right)
\]

**STEP 2**

To use these results we need values for the individual components of \( \vec{A} \) and \( \vec{B} \).
Referring to Figure 1.22, we find these values to be

\[ A_x = (145 \text{ m}) \sin 20.0^\circ = 49.6 \text{ m} \quad \text{and} \quad A_y = (145 \text{ m}) \cos 20.0^\circ = 136 \text{ m} \]
\[ B_x = (105 \text{ m}) \cos 35.0^\circ = 86.0 \text{ m} \quad \text{and} \quad B_y = -(105 \text{ m}) \sin 35.0^\circ = -60.2 \text{ m} \]

Note that the component \( B_y \) is negative, because \( \vec{B} \) points downward, in the negative \( y \) direction in the drawing.

Substituting these values into the results for \( C \) and \( \theta \) gives

\[
C = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2} = \sqrt{(49.6 \text{ m} + 86.0 \text{ m})^2 + (136 \text{ m} - 60.2 \text{ m})^2} = 155 \text{ m} \\
\theta = \tan^{-1} \left( \frac{A_y + B_y}{A_x + B_x} \right) = \tan^{-1} \left( \frac{136 \text{ m} - 60.2 \text{ m}}{49.6 \text{ m} + 86.0 \text{ m}} \right) = 29^\circ 
\]

**Related Homework:** Problems 45, 47, 50, 54
CONCEPT SUMMARY

1.2 Units The SI system of units includes the meter (m), the kilogram (kg), and the second (s) as the base units for length, mass, and time, respectively. One meter is the distance that light travels in a vacuum in a time of 1/299 792 458 second. One kilogram is the mass of a standard cylinder of platinum–iridium alloy kept at the International Bureau of Weights and Measures. One second is the time for a certain type of electromagnetic wave emitted by cesium-133 atoms to undergo 9 192 631 770 wave cycles.

1.3 The Role of Units in Problem Solving To convert a number from one unit to another, multiply the number by the ratio of the two units. For instance, to convert 979 meters to feet, multiply 979 meters by the factor (3.281 foot/1 meter).

The dimension of a quantity represents its physical nature and the type of unit used to specify it. Three such dimensions are length [L], mass [M], time [T]. Dimensional analysis is a method for checking mathematical relations for the consistency of their dimensions.

\[
\sin \theta = \frac{h_o}{h} \quad \text{(1.1)}
\]

\[
\cos \theta = \frac{h_o}{h} \quad \text{(1.2)}
\]

\[
\tan \theta = \frac{h_o}{h} \quad \text{(1.3)}
\]

\[
\theta = \sin^{-1} \left( \frac{h_o}{h} \right) \quad \text{(1.4)}
\]

\[
\theta = \cos^{-1} \left( \frac{h_o}{h} \right) \quad \text{(1.5)}
\]

\[
\theta = \tan^{-1} \left( \frac{h_o}{h} \right) \quad \text{(1.6)}
\]

\[
h^2 = h_o^2 + h^2 \quad \text{(1.7)}
\]

1.4 Trigonometry The sine, cosine, and tangent functions of an angle \( \theta \) are defined in terms of a right triangle that contains \( \theta \), as in Equations 1.1–1.3, where \( h_o \) and \( h \) are, respectively, the lengths of the sides opposite and adjacent to the angle \( \theta \), and \( h \) is the length of the hypotenuse.

The inverse sine, inverse cosine, and inverse tangent functions are given in Equations 1.4–1.6.

The Pythagorean theorem states that the square of the length of the hypotenuse of a right triangle is equal to the sum of the squares of the lengths of the other two sides, according to Equation 1.7.

1.5 Scalars and Vectors A scalar quantity is described by its size, which is also called its magnitude. A vector quantity has both a magnitude and a direction. Vectors are often represented by arrows, the length of the arrow being proportional to the magnitude of the vector and the direction of the arrow indicating the direction of the vector.

1.6 Vector Addition and Subtraction One procedure for adding vectors utilizes a graphical technique, in which the vectors to be added are arranged in a tail-to-head fashion. The resultant vector is drawn from the tail of the first vector to the head of the last vector. The subtraction of a vector is treated as the addition of a vector that has been multiplied by a scalar factor of \(-1\). Multiplying a vector by \(-1\) reverses the direction of the vector.

1.7 The Components of a Vector In two dimensions, the vector components of a vector \( \vec{A} \) are two perpendicular vector \( \vec{A}_x \) and \( \vec{A}_y \) that are parallel to the \( x \) and \( y \) axes, respectively, and that add together vectorially so that \( \vec{A} = \vec{A}_x + \vec{A}_y \). The scalar component \( A_x \) has a magnitude that is equal to that of \( \vec{A}_x \) and is given a positive sign if \( \vec{A}_x \) points along the \( +x \) axis and a negative sign if \( \vec{A}_x \) points along the \(-x \) axis. The scalar component \( A_y \) is defined in a similar manner.

Two vectors are equal if, and only if, they have the same magnitude and direction. Alternatively, two vectors are equal in two dimensions if the \( x \) vector components of each are equal and the \( y \) vector components of each are equal. A vector is zero if, and only if, each of its vector components is zero.

1.8 Addition of Vectors by Means of Components If two vectors \( \vec{A} \) and \( \vec{B} \) are added to give a resultant \( \vec{C} \) such that \( \vec{C} = \vec{A} + \vec{B} \), then \( C_x = A_x + B_x \) and \( C_y = A_y + B_y \), where \( C_x, A_x, \) and \( B_x \) are the scalar components of the vectors along the \( x \) direction, and \( C_y, A_y, \) and \( B_y \) are the scalar components of the vectors along the \( y \) direction.

FOCUS ON CONCEPTS

Note to Instructors: The numbering of the questions shown here reflects the fact that they are only a representative subset of the total number that are available online. However, all of the questions are available for assignment via an online homework management program such as WileyPLUS or WebAssign.

Section 1.6 Vector Addition and Subtraction

1. During a relay race, runner A runs a certain distance due north and then hands off the baton to runner B, who runs for the same distance in a direction south of east. The two displacement vectors \( \vec{A} \) and \( \vec{B} \) can be added together to give a resultant vector \( \vec{R} \). Which drawing correctly shows the resultant vector? (a) 1  (b) 2  (c) 3  (d) 4
2. How is the magnitude $R$ of the resultant vector $\mathbf{R}$ in the drawing related to the magnitudes $A$ and $B$ of the vectors $\mathbf{A}$ and $\mathbf{B}$? (a) The magnitude of the resultant vector $\mathbf{R}$ is equal to the sum of the magnitudes of $\mathbf{A}$ and $\mathbf{B}$, or $R = A + B$. (b) The magnitude of the resultant vector $\mathbf{R}$ is greater than the sum of the magnitudes of $\mathbf{A}$ and $\mathbf{B}$, or $R > A + B$. (c) The magnitude of the resultant vector $\mathbf{R}$ is less than the sum of the magnitudes of $\mathbf{A}$ and $\mathbf{B}$, or $R < A + B$.

5. The first drawing shows three displacement vectors, $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$, which are added in a tail-to-head fashion. The resultant vector is labeled $\mathbf{R}$. Which of the following drawings shows the correct resultant vector for $\mathbf{A} + \mathbf{B} - \mathbf{C}$? (a) 1 (b) 2 (c) 3

6. The first drawing shows the sum of three displacement vectors, $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$. The resultant vector is labeled $\mathbf{R}$. Which of the following drawings shows the correct resultant vector for $\mathbf{A} - \mathbf{B} - \mathbf{C}$? (a) 1 (b) 2 (c) 3

Section 1.7 The Components of a Vector

8. A person is jogging along a straight line, and her displacement is denoted by the vector $\mathbf{A}$ in the drawings. Which drawing represents the correct vector components, $\mathbf{A}_x$ and $\mathbf{A}_y$, for the vector $\mathbf{A}$? (a) 1 (b) 2 (c) 3 (d) 4

11. A person drives a car for a distance of 450.0 m. The displacement $\mathbf{A}$ of the car is illustrated in the drawing. What are the scalar components of this displacement vector?
(a) $A_x = 0$ m and $A_y = +450.0$ m
(b) $A_x = 0$ m and $A_y = -450.0$ m
(c) $A_x = +450.0$ m and $A_y = +450.0$ m
(d) $A_x = -450.0$ m and $A_y = 0$ m
(e) $A_x = -450.0$ m and $A_y = +450.0$ m

12. Drawing $a$ shows a displacement vector $\mathbf{A}$ (450.0 m along the $-y$ axis). In this $x$, $y$ coordinate system the scalar components are $A_x = 0$ m and $A_y = -450.0$ m. Suppose that the coordinate system is rotated counterclockwise by 35.0°, but the magnitude (450.0 m) and direction of vector $\mathbf{A}$ remain unchanged, as in drawing $b$. What are the scalar components, $A_1$ and $A_2$, of the vector $\mathbf{A}$ in the rotated $x'$, $y'$ coordinate system?
(a) $A_1 = -369$ m and $A_2 = -258$ m
(b) $A_1 = +369$ m and $A_2 = -258$ m
(c) $A_1 = +258$ m and $A_2 = +369$ m
(d) $A_1 = +258$ m and $A_2 = -369$ m
(e) $A_1 = -258$ m and $A_2 = -369$ m

15. Suppose the vectors $\mathbf{A}$ and $\mathbf{B}$ in the drawing have magnitudes of 6.0 m and are directed as shown. What are $A_x$ and $B_x$, the scalar components of $\mathbf{A}$ and $\mathbf{B}$ along the $x$ axis?

<table>
<thead>
<tr>
<th>$A_x$</th>
<th>$B_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(+6.0 \text{ m}) \cos 35^\circ = +4.9 \text{ m}$</td>
<td>$-(6.0 \text{ m}) \cos 35^\circ = -4.9 \text{ m}$</td>
</tr>
<tr>
<td>$(+6.0 \text{ m}) \sin 35^\circ = +3.4 \text{ m}$</td>
<td>$-(6.0 \text{ m}) \sin 35^\circ = -3.4 \text{ m}$</td>
</tr>
<tr>
<td>$-(6.0 \text{ m}) \cos 35^\circ = -4.9 \text{ m}$</td>
<td>$(+6.0 \text{ m}) \sin 35^\circ = +3.4 \text{ m}$</td>
</tr>
<tr>
<td>$-(6.0 \text{ m}) \sin 35^\circ = -3.4 \text{ m}$</td>
<td>$(+6.0 \text{ m}) \sin 35^\circ = +3.4 \text{ m}$</td>
</tr>
</tbody>
</table>

Section 1.8 Addition of Vectors by Means of Components

17. Drawing $a$ shows two vectors $\mathbf{A}$ and $\mathbf{B}$, and drawing $b$ shows their components. The scalar components of these vectors are as follows:

$A_x = -4.9$ m  $A_y = +3.4$ m
$B_x = +4.9$ m  $B_y = +3.4$ m
When the vectors \( \vec{A} \) and \( \vec{B} \) are added, the resultant vector is \( \vec{R} \), so that \( \vec{R} = \vec{A} + \vec{B} \). What are the values of \( R \) and \( R_y \), the \( x \) and \( y \) components of \( \vec{R} \)?

**PROBLEMS**

Problems that are not marked with a star are considered the easiest to solve. Problems that are marked with a single star (*) are more difficult, while those marked with a double star (**) are the most difficult.

**Note to Instructors:** Most of the homework problems in this chapter are available for assignment via an online homework management program such as WileyPLUS or WebAssign, and those marked with the icons \( \text{ssm} \) and \( \text{mmh} \) are presented in WileyPLUS using a guided tutorial format that provides enhanced interactivity. See the Preface for additional details.

\( \text{ssm} \) Solution is in the Student Solutions Manual.

\( \text{mmh} \) Problem-solving help is available online at www.wiley.com/college/cutnell.

**Section 1.2** Units

**Section 1.3** The Role of Units in Problem Solving

1. \( \text{ssm} \) A student sees a newspaper ad for an apartment that has 1330 square feet (ft\(^2\)) of floor space. How many square meters of area are there?

2. Bicyclists in the Tour de France reach speeds of 34.0 miles per hour (mi/h) on flat sections of the road. What is this speed in (a) kilometers per hour (km/h) and (b) meters per second (m/s)?

3. \( \text{ssm} \) Vesna Vulovic survived the longest fall on record without a parachute when her plane exploded and she fell 6 miles, 551 yards. What is this distance in meters?

4. Suppose a man’s scalp hair grows at a rate of 0.35 mm per day. What is this growth rate in feet per century?

5. Given the quantities \( a = 9.7 \) m, \( b = 4.2 \) s, \( c = 69 \) m/s, what is the value of the quantity \( d = a b^2 c^3 \)?

6. Consider the equation \( v = \frac{1}{2} z x t^2 \). The dimensions of the variables \( v \), \( x \), and \( t \) are [L]/[T], [L], and [T], respectively. The numerical factor 3 is dimensionless. What must be the dimensions of the variable \( z \), such that both sides of the equation have the same dimensions? Show how you determined your answer.

7. \( \text{ssm} \) A bottle of wine known as a magnum contains a volume of 1.5 liters. A bottle known as a jeroboam contains 0.792 U.S. gallons. How many magnums are there in one jeroboam?

8. The CGS unit for measuring the viscosity of a liquid is the poise (P): 1 P = 1 g/(s \cdot cm). The SI unit for viscosity is the kg/(s \cdot m). The viscosity of water at 0 °C is 1.78 \times 10^{-3} \text{ kg/(s \cdot m)}. Express this viscosity in poise.

9. Azelastine hydrochloride is an antihistamine nasal spray. A standard-size container holds one fluid ounce (oz) of the liquid. You are searching for this medication in a European drugstore and are asked how many milliliters (mL) there are in one fluid ounce. Using the following conversion factors, determine the number of milliliters in a volume of one fluid ounce: 1 gallon (gal) = 128 oz, 3.785 \times 10^{-3} \text{ cubic meters (m}^3\) = 1 gal, and 1 mL = 10^{-3} \text{ m}^3.

10. A partly full paint can has 0.67 U.S. gallons of paint left in it. (a) What is the volume of the paint in cubic meters? (b) If all the remaining paint is used to coat a wall evenly (wall area = 13 m\(^2\)), how thick is the layer of wet paint? Give your answer in meters.

11. \( \text{ssm} \) A spring is hanging down from the ceiling, and an object of mass \( m \) is attached to the free end. The object is pulled down, thereby stretching the spring, and then released. The object oscillates up and down, and the time \( T \) required for one complete up-and-down oscillation is given by the equation \( T = 2\pi \sqrt{m/k} \), where \( k \) is known as the spring constant. What must be the dimension of \( k \) for this equation to be dimensionally correct?

**Section 1.4** Trigonometry

12. You are driving into St. Louis, Missouri, and in the distance you see the famous Gateway to the West arch. This monument rises to a height of 192 m. You estimate your line of sight with the top of the arch to be 2.0° above the horizontal. Approximately how far (in kilometers) are you from the base of the arch?

13. \( \text{ssm} \) A highway is to be built between two towns, one of which lies 35.0 km south and 72.0 km west of the other. What is the shortest length of highway that can be built between the two towns, and at what angle would this highway be directed with respect to due west?

14. \( \text{mmh} \) A hill that has a 12.0% grade is one that rises 12.0 m vertically for every 100.0 m of distance in the horizontal direction. At what angle is such a hill inclined above the horizontal?

15. \( \text{mmh} \) The corners of a square lie on a circle of diameter \( D = 0.35 \) m. Each side of the square has a length \( L \). Find \( L \).

16. \( \text{mmh} \) The drawing shows a person looking at a building on top of which an antenna is mounted. The horizontal distance between the person’s eyes and the building is 85.0 m. In part a the person is looking at
the base of the antenna, and his line of sight makes an angle of 35.0° with the horizontal. In part b the person is looking at the top of the antenna, and his line of sight makes an angle of 38.0° with the horizontal. How tall is the antenna?

17. The two hot-air balloons in the drawing are 48.2 and 61.0 m above the ground. A person in the left balloon observes that the right balloon is 13.3° above the horizontal. What is the horizontal distance \( x \) between the two balloons?

18. What is the value of each of the angles of a triangle whose sides are 95, 150, and 190 cm in length? (Hint: Consider using the law of cosines given in Appendix E.)

19. The drawing shows sodium and chloride ions positioned at the corners of a cube that is part of the crystal structure of sodium chloride (common table salt). The edges of the cube are each 0.281 nm (1 nm = 1 nanometer = \( 10^{-9} \) m) in length. What is the value of the angle \( \theta \) in the drawing?

20. A person is standing at the edge of the water and looking out at the ocean (see the drawing). The height of the person’s eyes above the water is \( h = 1.6 \) m, and the radius of the earth is \( R = 6.38 \times 10^6 \) m. (a) How far is it to the horizon? In other words, what is the distance \( d \) from the person’s eyes to the horizon? (Note: At the horizon the angle between the line of sight and the radius of the earth is 90°.) (b) Express this distance in miles.

21. Three deer, A, B, and C, are grazing in a field. Deer B is located 62 m from deer A at an angle of 51° north of west. Deer C is located 77° north of east relative to deer A. The distance between deer B and C is 95 m. What is the distance between deer A and C? (Hint: Consider the law of cosines given in Appendix E.)

Section 1.6 Vector Addition and Subtraction

23. Two workers are trying to move a heavy crate. One pushes on the crate with a force \( \vec{A} \), which has a magnitude of 445 newtons and is directed due west. The other pushes with a force \( \vec{B} \), which has a magnitude of 325 newtons and is directed due north. What are the magnitude and direction of the resultant force \( \vec{A} + \vec{B} \) applied to the crate? (b) Suppose that the second worker applies a force \( -\vec{B} \) instead of \( \vec{B} \). What then are the magnitude and direction of the resultant force \( \vec{A} - \vec{B} \) applied to the crate? In both cases express the direction relative to due west.

24. A force vector \( \vec{F}_2 \) points due east and has a magnitude of 200 newtons. A second force \( \vec{F}_4 \) is added to \( \vec{F}_2 \). The resultant of the two vectors has a magnitude of 400 newtons and points along the east/west line. Find the magnitude and direction of \( \vec{F}_4 \). Note that there are two answers.

25. Consider the following four force vectors:

\[
\begin{align*}
\vec{F}_1 &= 50.0 \text{ newtons, due east} \\
\vec{F}_2 &= 10.0 \text{ newtons, due east} \\
\vec{F}_3 &= 40.0 \text{ newtons, due west} \\
\vec{F}_4 &= 30.0 \text{ newtons, due west}
\end{align*}
\]

Which two vectors add together to give a resultant with the smallest magnitude, and which two vectors add to give a resultant with the largest magnitude? In each case specify the magnitude and direction of the resultant.

26. Vector \( \vec{A} \) has a magnitude of 63 units and points due west, while vector \( \vec{B} \) has the same magnitude and points due south. Find the magnitude and direction of \( (a) \vec{A} + \vec{B} \) and \( (b) \vec{A} - \vec{B} \). Specify the directions relative to due west.

27. Two bicyclists, starting at the same place, are riding toward the same campground by two different routes. One cyclist rides 1080 m due east
and then turns due north and travels another 1430 m before reaching the
campground. The second cyclist starts out by heading due north for 1950 m
and then turns and heads directly toward the campground. (a) At the turn-
ing point, how far is the second cyclist from the campground? (b) In what
direction (measured relative to due east) must the second cyclist head
during the last part of the trip?

28. The drawing shows a triple jump on a checkerboard, starting at
the center of square A and ending on the center of square B. Each side of
a square measures 4.0 cm. What is the magnitude of the displacement of
the colored checker during the triple jump?

29. Given the vectors \( \vec{F} \) and \( \vec{Q} \) shown on the
grid, sketch and calculate the magnitudes of the vectors (a) \( \vec{M} = \vec{F} + \vec{Q} \) and (b) \( \vec{K} = 2\vec{F} - \vec{Q} \).
Use the tail-to-head method and express the magnitudes in centimeters with the aid of the
grid scale shown in the drawing.

30. Vector \( \vec{A} \) has a magnitude of 12.3 units and points due west.
Vector \( \vec{B} \) points due north. (a) What is the magnitude of \( \vec{A} + \vec{B} \) if \( \vec{A} + \vec{B} \) has
a magnitude of 15.0 units? (b) What is the direction of \( \vec{A} + \vec{B} \) relative
to due west? (c) What is the magnitude of \( \vec{A} - \vec{B} \) if \( \vec{A} - \vec{B} \) has a magnitude
of 15.0 units? (d) What is the direction of \( \vec{A} - \vec{B} \) relative to due west?

31. A car is being pulled out of the mud by two forces that are applied
by the two ropes shown in the drawing. The dashed line in the drawing
bisects the 30.0° angle. The magnitude of the force applied by each
rope is 2900 newtons. Arrange the force vectors tail to head and use the
graphical technique to answer the following questions. (a) How much
force would a single rope need to apply to accomplish the same effect as
the two forces added together? (b) How would the single rope be directed
relative to the dashed line?

32. A jogger travels a route that has two parts. The first is a dis-
placement \( \vec{A} \) of 2.50 km due south, and the second involves a displace-
ment \( \vec{B} \) that points due east. (a) The resultant displacement \( \vec{A} + \vec{B} \) has a
magnitude of 3.75 km. What is the magnitude of \( \vec{B} \), and what is the
direction of \( \vec{A} + \vec{B} \) relative to due south? (b) Suppose that \( \vec{A} - \vec{B} \) had a
magnitude of 3.75 km. What then would be the magnitude of \( \vec{B} \), and what
is the direction of \( \vec{A} - \vec{B} \) relative to due south?

33. At a picnic, there is a contest in which hoses are used to shoot water
at a beach ball from three directions. As a result, three forces act on the
ball, \( \vec{F}_1 \), \( \vec{F}_2 \), and \( \vec{F}_3 \) (see the drawing). The magnitudes of \( \vec{F}_1 \) and \( \vec{F}_2 \) are
\( F_1 = 50.0 \) newtons and \( F_2 = 90.0 \) newtons. Using a scale drawing and the
graphical technique, determine (a) the magnitude of \( \vec{F}_3 \) and (b) the angle \( \theta \)
such that the resultant force acting on the ball is zero.

Section 1.7 The Components of a Vector

34. A force vector has a magnitude of 575 newtons and points at an
angle of 36.0° below the positive x axis. What are (a) the x scalar com-
ponent and (b) the y scalar component of the vector?

35. Vector \( \vec{A} \) points along the +y axis and has a magnitude of 100.0
units. Vector \( \vec{B} \) points at an angle of 60.0° above the +x axis and has a
magnitude of 200.0 units. Vector \( \vec{C} \) points along the +x axis and has a
magnitude of 150.0 units. Which vec-
tor has (a) the largest x component and
(b) the largest y component?

36. Soccer player #1 is 8.6 m from
the goal (see the drawing). If she
kicks the ball directly into the net, the
ball has a displacement labeled
\( \vec{X} \). If, on the other hand, she first kicks it
to player #2, who then kicks it into the
net, the ball undergoes two successive
displacements, \( \vec{X}_1 \) and \( \vec{X}_2 \). What are the
magnitudes and directions of \( \vec{X}_1 \)
and \( \vec{X}_2 \)?

37. The components of vector \( \vec{A} \) are \( A_x \) and \( A_y \) (both positive), and
the angle that it makes with respect to the positive x axis is \( \theta \). Find the angle \( \theta \)
if the components of the displacement vector \( \vec{A} \) are (a) \( A_x = 12 \) m
and \( A_y = 12 \) m, (b) \( A_x = 17 \) m and \( A_y = 12 \) m, and (c) \( A_x = 12 \) m
and \( A_y = 17 \) m.

38. During takeoff, an airplane climbs with a speed of 180 m/s at an
angle of 34° above the horizontal. The speed and direction of the airplane
constitute a vector quantity known as the velocity. The sun is shining
directly overhead. How fast is the shadow of the plane moving along the
ground? (That is, what is the magnitude of the horizontal component of
the plane’s velocity?)

39. The x vector component of a displacement vector \( \vec{F} \) has a mag-
nitude of 125 m and points along the negative x axis. The y vector compo-
nent has a magnitude of 184 m and points along the negative y axis. Find
the magnitude and direction of \( \vec{F} \). Specify the direction with respect to
the negative x axis.

40. Your friend has slipped and fallen. To help her up, you pull with a force
\( \vec{F} \), as the drawing shows. The vertical component of this force is 130 newtons, and the horizontal
component is 150 newtons. Find (a) the
magnitude of \( \vec{F} \) and (b) the angle \( \theta \).
41. The displacement vector $\vec{A}$ has scalar components of $A_x = 80.0 \text{ m}$ and $A_y = 60.0 \text{ m}$. The displacement vector $\vec{B}$ has a scalar component of $B_x = 60.0 \text{ m}$ and a magnitude of $B = 75.0 \text{ m}$. The displacement vector $\vec{C}$ has a magnitude of $C = 100.0 \text{ m}$ and is directed at an angle of $36.9^\circ$ above the $+x$ axis. Two of these vectors are equal. Determine which two, and support your choice with a calculation.

42. Two racing boats set out from the same dock and speed away at the same constant speed of $101 \text{ km/h}$ for half an hour ($0.500 \text{ h}$), the blue boat headed $25.0^\circ$ south of west, and the green boat headed $37.0^\circ$ south of west. During this half hour (a) how much farther west does the blue boat travel, compared to the green boat, and (b) how much farther south does the green boat travel, compared to the blue boat? Express your answers in km.

43. The magnitude of the force vector $\vec{F}$ is 82.3 newtons. The $x$ component of this vector is directed along the $+x$ axis and has a magnitude of 74.6 newtons. The $y$ component points along the $+y$ axis. (a) Find the direction of the $\vec{F}$ relative to the $+x$ axis. (b) Find the component of $\vec{F}$ along the $+y$ axis.

44. The drawing shows a force vector that has a magnitude of 475 newtons. Find the (a) $x$, (b) $y$, and (c) $z$ components of the vector.

45. Consult Multiple-Concept Example 9 in preparation for this problem. A golfer, putting on a green, requires three strokes to “hole the ball.” During the first putt, the ball rolls 5.0 m due east. For the second putt, the ball travels 2.1 m at an angle of $20.0^\circ$ north of east. The third putt is 0.50 m 50.0$^\circ$ south of east. What displacement (magnitude and direction relative to due east) would have been needed to “hole the ball” on the very first putt?

46. The three displacement vectors in the drawing have magnitudes of $A = 5.00 \text{ m}$, $B = 5.00 \text{ m}$, and $C = 4.00 \text{ m}$. Find the resultant (magnitude and directional angle) of the three vectors by means of the component method. Express the directional angle as an angle above the positive or negative x axis.

47. Multiple-Concept Example 9 reviews the concepts that play a role in this problem. Two forces are applied to a tree stump to pull it out of the ground. Force $\vec{F}_A$ has a magnitude of 2240 newtons and points $34.0^\circ$ south of east, while force $\vec{F}_B$ has a magnitude of 3160 newtons and points due south. Using the component method, find the magnitude and direction of the resultant force $\vec{F}_A + \vec{F}_B$ that is applied to the stump. Specify the direction with respect to due east.

48. A baby elephant is stuck in a mud hole. To help pull it out, game keepers use a rope to apply a force $\vec{F}_A$, as part of the drawing shows. By itself, however, force $\vec{F}_A$ is insufficient. Therefore, two additional forces $\vec{F}_B$ and $\vec{F}_C$ are applied, as in part $b$ of the drawing. Each of these additional forces has the same magnitude $F$. The magnitude of the resultant force acting on the elephant in part $b$ of the drawing is $k$ times larger than that in part $a$. Find the ratio $F/\vec{F}_A$ when $k = 2.00$.

49. Displacement vector $\vec{A}$ points due east and has a magnitude of 2.00 km. Displacement vector $\vec{B}$ points due north and has a magnitude of 3.75 km. Displacement vector $\vec{C}$ points due west and has a magnitude of 2.50 km. Displacement vector $\vec{D}$ points due south and has a magnitude of 3.00 km. Find the magnitude and direction (relative to due west) of the resultant vector $\vec{A} + \vec{B} + \vec{C} + \vec{D}$.

50. Multiple-Concept Example 9 provides background pertinent to this problem. The magnitudes of the four displacement vectors shown in the drawing are $A = 16.0 \text{ m}$, $B = 11.0 \text{ m}$, $C = 12.0 \text{ m}$, and $D = 26.0 \text{ m}$. Determine the magnitude and directional angle for the resultant that occurs when these vectors are added together.

51. On a safari, a team of naturalists sets out toward a research station located 4.8 km away in a direction $42^\circ$ north of east. After traveling in a straight line for 2.4 km, they stop and discover that they have been traveling $22^\circ$ north of east, because their guide misread his compass. What are (a) the magnitude and (b) the direction (relative to due east) of the displacement vector now required to bring the team to the research station?

52. Two geological field teams are working in a remote area. A global positioning system (GPS) tracker at their base camp shows the location of the first team as 38 km away, $19^\circ$ north of west, and the second team as 29 km away, $35^\circ$ east of north. When the first team uses its GPS to check the position of the second team, what does the GPS give for the second team’s (a) distance from them and (b) direction, measured from due east?

53. A sailboat race course consists of four legs, defined by the displacement vectors $\vec{A}$, $\vec{B}$, $\vec{C}$, and $\vec{D}$, as the drawing indicates. The magnitudes of the first three vectors are $A = 3.20 \text{ km}$, $B = 5.10 \text{ km}$, and $C = 4.80 \text{ km}$. The finish line of the course coincides with the starting line. Using the data in the drawing, find the distance of the fourth leg and the angle $\theta$.

54. Multiple-Concept Example 9 deals with the concepts that are important in this problem. A grasshopper makes four jumps. The displacement vectors are (1) $27.0 \text{ cm}$, due west; (2) $23.0 \text{ cm}$, $35.0^\circ$ south of west; (3) $28.0 \text{ cm}$, $55.0^\circ$ south of east; and (4) $35.0 \text{ cm}$, $63.0^\circ$ north of east. Find the magnitude and direction of the resultant displacement. Express the direction with respect to due west.
*55. mnh Vector \( \mathbf{A} \) has a magnitude of 145 units and points 35.0° north of west. Vector \( \mathbf{B} \) points 65.0° east of north. Vector \( \mathbf{C} \) points 15.0° west of south. These three vectors add to give a resultant vector that is zero. Using components, find the magnitudes of (a) vector \( \mathbf{B} \) and (b) vector \( \mathbf{C} \).

*56. The route followed by a hiker consists of three displacement vectors \( \mathbf{A} \), \( \mathbf{B} \), and \( \mathbf{C} \). Vector \( \mathbf{A} \) is along a measured trail and is 1550 m in a direction 25.0° north of east. Vector \( \mathbf{B} \) is not along a measured trail, but the hiker uses a compass and knows that the direction is 41.0° east of south. Similarly, the direction of vector \( \mathbf{C} \) is 35.0° north of west. The hiker ends up back where she started. Therefore, it follows that the resultant displacement vector is zero, or \( \mathbf{A} + \mathbf{B} + \mathbf{C} = 0 \). Find the magnitudes of (a) vector \( \mathbf{B} \) and (b) vector \( \mathbf{C} \).

### ADDITIONAL PROBLEMS

57. A chimpanzee sitting against his favorite tree gets up and walks 51 m due east and 39 m due south to reach a termite mound, where he eats lunch. (a) What is the shortest distance between the tree and the termite mound? (b) What angle does the shortest distance make with respect to due east?

58. A monkey is chained to a stake in the ground. The stake is 3.00 m from a vertical pole, and the chain is 3.40 m long. How high can the monkey climb up the pole?

59. ssm The speed of an object and the direction in which it moves constitute a vector quantity known as the velocity. An ostrich is running at a speed of 17.0 m/s in a direction of 68.0° north of west. What is the magnitude of the ostrich's velocity component that is directed (a) due north and (b) due west?

60. The volume of liquid flowing per second is called the volume flow rate \( Q \) and has the dimensions of [L]/[T]. The flow rate of a liquid through a hypodermic needle during an injection can be estimated with the following equation:

\[
Q = \frac{\pi R^2 (\bar{P}_2 - \bar{P}_1)}{8\eta L}
\]

The length and radius of the needle are \( L \) and \( R \), respectively, both of which have the dimension [L]. The pressures at opposite ends of the needle are \( \bar{P}_2 \) and \( \bar{P}_1 \), both of which have the dimensions of [M]/([L][T]^2]. The symbol \( \eta \) represents the viscosity of the liquid and has the dimensions of [M]/([L][T]). The symbol \( \pi \) stands for pi and, like the number 8 and the exponent \( n \), has no dimensions. Using dimensional analysis, determine the value of \( n \) in the expression for \( Q \).

61. An ocean liner leaves New York City and travels 18.0° north of east for 155 km. How far east and how far north has it gone? In other words, what are the magnitudes of the components of the ship’s displacement vector in the directions (a) due east and (b) due north?

62. A pilot flies her route in two straight-line segments. The displacement vector \( \mathbf{A} \) for the first segment has a magnitude of 244 km and a direction 30.0° north of east. The displacement vector \( \mathbf{B} \) for the second segment has a magnitude of 175 km and a direction due west. The resultant displacement vector is \( \mathbf{R} = \mathbf{A} + \mathbf{B} \) and makes an angle \( \theta \) with the direction due east. Using the component method, find the magnitude of \( \mathbf{R} \) and the directional angle \( \theta \).

63. A circus performer begins his act by walking out along a nearly horizontal high wire. He slips and falls to the safety net, 25.0 ft below. The magnitude of his displacement from the beginning of the walk to the net is 26.7 ft. (a) How far out along the high wire did he walk? (b) Find the angle that his displacement vector makes below the horizontal.

64. A force vector points at an angle of 52° above the +x axis. It has a y component of +290 newtons. Find (a) the magnitude and (b) the x component of the force vector.

65. ssm Vector \( \mathbf{A} \) has a magnitude of 6.00 units and points due east. Vector \( \mathbf{B} \) points due north. (a) What is the magnitude of \( \mathbf{B} \), if the vector \( \mathbf{A} + \mathbf{B} \) points 60.0° north of east? (b) Find the magnitude of \( \mathbf{A} + \mathbf{B} \).

66. Three forces act on an object, as indicated in the drawing. Force \( \mathbf{F}_1 \) has a magnitude of 21.0 newtons (21.0 N) and is directed 30.0° to the left of the +y axis. Force \( \mathbf{F}_2 \) has a magnitude of 15.0 N and points along the +x axis. What must be the magnitude and direction (specified by the angle \( \theta \) in the drawing) of the third force \( \mathbf{F}_3 \) such that the vector sum of the three forces is 0 N?

67. Before starting this problem, review Conceptual Example 7. The force vector \( \mathbf{F}_A \) has a magnitude of 90.0 newtons and points due east. The force vector \( \mathbf{F}_B \) has a magnitude of 135 newtons and points 75° north of east. Use the graphical method and find the magnitude and direction of (a) \( \mathbf{F}_A - \mathbf{F}_B \) (give the direction with respect to due east) and (b) \( \mathbf{F}_B - \mathbf{F}_A \) (give the direction with respect to due west).

68. You live in the building on the left in the drawing, and a friend lives in the other building. The two of you are having a discussion about the heights of the buildings, and your friend claims that the height of his building is more than 1.50 times the height of yours. To resolve the issue you climb to the roof of your building and estimate that your line of sight to the top edge of the other building makes an angle of 21° above the horizontal, whereas your line of sight to the base of the other building makes an angle of 52° below the horizontal. Determine the ratio of the height of the taller building to the height of the shorter building. State whether your friend is right or wrong.

69. What are the \( x \) and \( y \) components of the vector that must be added to the following three vectors, so that the sum of the four vectors is zero? Due east is the +x direction, and due north is the +y direction.

- \( \mathbf{A} = 113 \) units, 60.0° south of west
- \( \mathbf{B} = 222 \) units, 35.0° south of east
- \( \mathbf{C} = 177 \) units, 23.0° north of east
This chapter has presented an introduction to the mathematics of trigonometry and vectors, which will be used throughout the text. In this section we apply some of the important features of this mathematics, and review some concepts that can help in anticipating some of the characteristics of the numerical answers.

**70.** The figure shows two displacement vectors $\mathbf{A}$ and $\mathbf{B}$. Vector $\mathbf{A}$ points at an angle of 22° above the $x$ axis and has an unknown magnitude. Vector $\mathbf{B}$ has an $x$ component $B_x = 35.0$ m and has an unknown $y$ component $B_y$. These two vectors are equal. **Concepts:** (i) What does the condition that vector $\mathbf{A}$ equals $\mathbf{B}$ imply about the magnitudes and directions of the vectors? (ii) What does the condition that vector $\mathbf{A}$ equals $\mathbf{B}$ imply about the $x$ and $y$ components of the vectors? **Calculations:** Find the magnitude of $\mathbf{A}$ and the value of $B_y$.

**71.** The figure shows three displacement vectors $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$. These vectors are arranged in tail-to-head fashion, and they add together to give a resultant displacement $\mathbf{R}$, which lies along the $x$ axis. Note that the vector $\mathbf{B}$ is parallel to the $x$ axis. **Concepts:** (i) How is the magnitude of $\mathbf{A}$ related to its scalar components $A_x$ and $A_y$? (ii) Do any of the vectors in the figure have a zero value for either their $x$ or $y$ components? (If so, which ones?) (iii) What does the fact that $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ add together to give $\mathbf{R}$ tell you about the components of these vectors? **Calculations:** What is the magnitude of the vector $\mathbf{A}$ and its directional angle $\theta$?