Chapter 1

INTRODUCTION TO BONDS
Bonds are the basic ingredient of the world’s debt-capital markets, which in turn are the cornerstone of the world’s economy. Consider how many television news programmes contain a slot during which the newscaster informs viewers where the main stock market indexes closed that day and where key foreign exchange rates ended up. More usefully, the financial sections of most newspapers also indicate at what yield the government long bond closed. This coverage reflects the fact that bond prices are affected directly by economic and political events, and yield levels on certain government bonds are fundamental economic indicators. The yield level on the US Treasury long bond, for instance, mirrors the market’s view on US interest rates, inflation, public-sector debt, and economic growth.

The media report the bond yield level because it is so important to the country’s economy – as important as the level of the equity market and more relevant as an indicator of the health and direction of the economy. Because of the size and crucial nature of the debt markets, a large number of market participants, ranging from bond issuers to bond investors and associated intermediaries are interested in analysing them. This chapter introduces the building blocks of the analysis.

Bonds are debt instruments that represent cash flows payable during a specified time period. They are a form of debt, much like how a bank loan is a form of debt. The cash flows they represent are the interest payments on the loan and the loan redemption. Unlike commercial bank loans, however, bonds are tradeable in a secondary market. Bonds are commonly referred to as fixed-income instruments. This term goes back to a time when bonds paid fixed coupons each year. That is no longer necessarily the case. Asset-backed bonds, for instance, are issued in a number of tranches – related securities from the same issuer – each of which pays a different fixed or floating coupon. Nevertheless, this is still commonly referred to as the fixed-income market.

In the first edition of this book I wrote:

*Unlike bank loans however bonds can be traded in a market.*

Actually, the first part of this statement cannot really be said to be accurate anymore. There is a thriving secondary market, certainly for US dollar and pound sterling loans, in bank loans these days. However, it is viewed as a separate market, and is not as liquid as the
A bond is a debt capital market instrument issued by a borrower, who is then required to repay to the lender/investor the amount borrowed plus interest, over a specified period of time. Usually, bonds are considered to be those debt securities with terms to maturity of over 1 year. Debt issued with a maturity of less than 1 year is considered to be money market debt. There are many different types of bonds that can be issued. The most common bond is the conventional (or plain vanilla or bullet) bond. This is a bond paying a regular (annual or semiannual) fixed interest rate over a fixed period to maturity or redemption, with the return of principal (the par or nominal value of the bond) on the maturity date. All other bonds will be variations on this.

There is a wide range of parties involved in the bond markets. We can group them broadly into borrowers and investors, plus the institutions and individuals who are part of the business of bond trading. Borrowers access the bond markets as part of their financing requirements; hence, borrowers can include sovereign governments, local authorities, public-sector organisations and corporates. Virtually all businesses operate with a financing structure that is a mixture of debt and equity finance. The debt finance almost invariably contains a form of bond finance, so it is easy to see what an important part of the global economy the bond markets are. As we shall see in the following chapters, there is a range of types of debt that can be raised to meet the needs of individual borrowers, from short-term paper issued as part of a company’s cash flow requirements, to very long-dated bonds that form part of the financing of key projects. An example of the latter was the issue in the summer of 2005 of 50-year bonds by the UK government. The other main category of market participant are investors, those who lend money to borrowers by buying their bonds. Investors range from private individuals to fund managers such as those who manage pensions funds. Often an institution will be active in the markets as both a borrower and an

1 The secondary market is the market in which bonds and loans are traded after they have been struck between borrower and lender. The bonds are traded between third parties who generally would not have been party to the initial primary market transaction. Liquidity refers to the ease with which bonds can be bought and sold by market participants.
investor. The banks and securities houses that facilitate trading in bonds in both the *primary* and *secondary* markets are also often observed to be both borrowers and investors in bonds. The bond markets in developed countries are large and *liquid*, a term used to describe the ease with which it is possible to buy and sell bonds. In emerging markets a debt market usually develops ahead of an equity market, led by trading in government *bills* and bonds. This reflects the fact that, as in developed economies, government debt is usually the largest in the domestic market and the highest quality paper available.

We look first at some important features of bonds. This is followed by a detailed look at pricing and yield. We conclude this introductory chapter with some spreadsheet illustrations.

**DESCRIPTION**

Bonds are identified by just one or two key features.

**Type of issuer**  A key feature of a bond is the nature of the issuer. There are four issuers of bonds: sovereign governments and their agencies, local government authorities, supranational bodies such as the World Bank, and corporations. Within the corporate bond market there is a wide range of issuers, each with differing abilities to satisfy their contractual obligations to investors. An issuer’s ability to make these payments is identified by its *credit rating*.

**Term to maturity**  The *term to maturity* of a bond is the number of years after which the issuer will repay the obligation. During the term the issuer will also make periodic interest payments on the debt. The *maturity* of a bond refers to the date that the debt will cease to exist, at which time the issuer will redeem the bond by paying the principal. The practice in the market is often to refer simply to a bond’s ‘term’ or ‘maturity’. The provisions under which a bond is issued may allow either the issuer or investor to alter a bond’s term to maturity. The term to maturity is an important consideration in the make-up of a bond. It indicates the time period over which the bondholder can expect to receive the coupon payments and the number of years before the principal will be paid in full. The bond’s *yield* also depends on the term to maturity. Finally, the price of a bond will fluctuate over its life as yields in the market change and as it approaches maturity. As we will discover later, the *volatility* of a
bond’s price is dependent on its maturity; assuming other factors constant, the longer a bond’s maturity the greater the price volatility resulting from a change in market yields.

**Principal and coupon rate** The principal of a bond is the amount that the issuer agrees to repay the bondholder on the maturity date. This amount is also referred to as the redemption value, maturity value, par value or face amount. The coupon rate or nominal rate is the interest rate that the issuer agrees to pay each year. The annual amount of the interest payment made is called the coupon. The coupon rate multiplied by the principal of the bond provides the cash amount of the coupon. For example, a bond with a 7% coupon rate and a principal of £1,000,000 will pay annual interest of £70,000. In the UK, US and Japan the usual practice is for the issuer to pay the coupon in two semi-annual instalments. For bonds issued in European markets and the Eurobond market, coupon payments are made annually. Sometimes one will encounter bonds that pay interest on a quarterly basis.

All bonds make periodic interest payments except for zero-coupon bonds. These bonds allow a holder to realise interest by being sold substantially below their principal value. The bonds are redeemed at par, with the interest amount then being the difference between the principal value and the price at which the bond was sold. We will explore zero-coupon bonds in greater detail later.

Another type of bond makes floating-rate interest payments. Such bonds are known as floating-rate notes and their coupon rates are reset periodically in line with a predetermined benchmark, such as an interest-rate index.

**Embedded options** Some bonds include a provision in their offer particulars that gives either the bondholder and/or the issuer an option to enforce early redemption of the bond. The most common type of option embedded in a bond is a call feature. A call provision grants the issuer the right to redeem all or part of the debt before the specified maturity date. An issuing company may wish to include such a feature as it allows it to replace an old bond issue with a lower coupon rate issue if interest rates in the market have declined. As a call feature allows the issuer to change the maturity date of a bond it is considered harmful to the bondholder’s interests; therefore, the market price of the bond will reflect this. A call option is included in all asset-backed securities based on mortgages, for obvious reasons (asset-backed bonds are considered in Chapter 10). A bond issue may
also include a provision that allows the investor to change the maturity of the bond. This is known as a put feature and gives the bondholder the right to sell the bond back to the issuer at par on specified dates. The advantage to the bondholder is that if interest rates rise after the issue date, thus depressing the bond's value, the investor can realise par value by putting the bond back to the issuer.

A convertible bond is an issue giving the bondholder the right to exchange the bond for a specified number of shares (equity) in the issuing company. This feature allows the investor to take advantage of favourable movements in the price of the issuer's shares.

The presence of embedded options in a bond makes valuation more complex compared with plain vanilla bonds, and will be considered separately.

**OUTLINE OF MARKET PARTICIPANTS**

There is a large variety of players in the bond markets, each trading some or all of the different instruments available to suit their own purposes. We can group the main types of players according to the time horizon of their investment activity.

- **Short-term institutional investors** – these include banks and building societies, money market fund managers, central banks and the treasury desks of some types of corporates. Such bodies are driven by short-term investment views, often subject to close guidelines, and will be driven by the total return available on their investments. Banks will have an additional requirement to maintain liquidity, often in fulfilment of regulatory authority rules, by holding a proportion of their assets in the form of easily tradeable short-term instruments.

- **Long-term institutional investors** – typically these types of investors include pension funds and life assurance companies. Their investment horizon is long-term, reflecting the nature of their liabilities; often they will seek to match these liabilities by holding long-dated bonds.

- **Mixed horizon institutional investors** – this is possibly the largest category of investors and will include general insurance companies and most corporate bodies. Like banks and financial sector companies, they are also very active in the primary market, issuing bonds to finance their operations.

- **Market professionals** – this category includes firms that one would not automatically classify as ‘investors’ although they
will also have an investment objective. Their time horizon will range from 1 day to the very long term. They include the proprietary trading desks of investment banks, as well as bond market makers in securities houses and banks who are providing a service to their customers. Proprietary traders will actively position themselves in the market in order to gain trading profit – for example, in response to their view on where they think interest-rate levels are headed. These participants will trade direct with other market professionals and investors, or via brokers.

Figure 1.1 shows a screen from the Bloomberg news and analytics system, widely used by capital market participants such as investment banks and hedge funds. It is screen DES, which is the description page that can be pulled up for virtually every bond in existence. Our example shows a bond issued by Ford Motor Company, the 2.25% of 2007. We see that all the key identifying features of the bond, such as coupon and maturity date, are listed, together with a confirmation of the bond’s credit rating of Baa3 and BB+. Of course, this bond has since expired; the credit rating for Ford, after the credit crunch, was a much lower B3 as at March 2010.
BOND ANALYSIS

In the past, bond analysis was frequently limited to calculating gross redemption yield, or yield to maturity. Today, basic bond mathematics involves different concepts and calculations. The level of understanding required to master bond pricing is quite high, and beyond the scope of this book. We concentrate instead on the essential elements required for a basic understanding.

In the analysis that follows, bonds are assumed to be default-free. This means there is no possibility that the interest payments and principal repayment will not be made. Such an assumption is entirely reasonable for government bonds such as US Treasuries and UK gilt-edged securities. It is less so when you are dealing with the debt of corporate and lower-rated sovereign borrowers. The valuation and analysis of bonds carrying default risk, however, are based on those of default-free government securities. Essentially, the yield investors demand from borrowers whose credit standing is not risk-free is the yield on government securities plus some credit risk premium.

Financial arithmetic: The time value of money

Bond prices are expressed ‘per 100 nominal’ – that is, as a percentage of the bond’s face value. (The convention in certain markets is to quote a price per 1,000 nominal, but this is rare.) For example, if the price of a US dollar-denominated bond is quoted as 98.00, this means that for every $100 of the bond’s face value, a buyer would pay $98. The principles of pricing in the bond market are the same as those in other financial markets: the price of a financial instrument is equal to the sum of the present values of all the future cash flows from the instrument. The interest rate used to derive the present values of the cash flows, known as the discount rate, is key, since it reflects where the bond is trading and how its return is perceived by the market. All the factors that identify the bond – including the nature of the issuer, the maturity date, the coupon and the currency in which it was issued – influence the bond’s discount rate. Comparable bonds have similar discount rates. The following sections explain the traditional approach to bond pricing for plain-vanilla instruments, making certain assumptions to keep the analysis simple.
Present value and discounting

Since fixed-income instruments are essentially collections of cash flows, it is useful to begin by reviewing two key concepts of cash flow analysis: discounting and present value. Understanding these concepts is essential. In the following discussion, the interest rates cited are assumed to be the market-determined rates.

Financial arithmetic demonstrates that the value of $1 received today is not the same as that of $1 received in the future. Assuming an interest rate of 10% a year, a choice between receiving $1 in a year and receiving the same amount today is really a choice between having $1 a year from now and having $1 plus $0.10 – the interest on $1 for 1 year at 10% per annum.

The notion that money has a time value is basic to the analysis of financial instruments. Money has time value because of the opportunity to invest it at a rate of interest. A loan that makes one interest payment at maturity is accruing simple interest. Short-term instruments are usually such loans. Hence, the lenders receive simple interest when the instrument expires. The formula for deriving terminal, or future, value of an investment with simple interest is shown as (1.1):

\[ FV = PV(1 + r) \]

where \( FV \) = Future value of the instrument;
\( PV \) = Initial investment, or the present value, of the instrument;
\( r \) = Interest rate.

The market convention is to quote annualised interest rates: the rate corresponding to the amount of interest that would be earned if the investment term were 1 year. Consider a 3-month deposit of $100 in a bank earning a rate of 6% a year. The annual interest gain would be $6. The interest earned for the 90 days of the deposit is proportional to that gain, as calculated below:

\[ I_{90} = 6.00 \times \frac{90}{365} \]
\[ = 6.00 \times 0.2465 \]
\[ = 1.479 \]

The investor will receive $1.479 in interest at the end of the term. The total value of the deposit after the 3 months is therefore $100
plus $1.479. To calculate the terminal value of a short-term investment – that is, one with a term of less than a year – accruing simple interest, equation (1.1) is modified as follows:

\[ FV = PV \left(1 + r \left(\frac{\text{Days}}{\text{Year}}\right)\right) \]  

(1.2)

where \( r \) = Annualised rate of interest;  
\( \text{Days} \) = Term of the investment;  
\( \text{Year} \) = Number of days in the year.

Note that, in the sterling markets, the number of days in the year is taken to be 365, but most other markets – including the dollar and euro markets – use a 360-day year. (These conventions are discussed more fully below.)

Now, consider an investment of $100, again at a fixed rate of 6% a year, but this time for 3 years. At the end of the first year, the investor will be credited with interest of $6. Therefore, for the second year the interest rate of 6% will be accruing on a principal sum of $106. Accordingly, at the end of year 2, the interest credited will be $6.36. This illustrates the principle of compounding: earning interest on interest. Equation (1.3) computes the future value for a sum deposited at a compounding rate of interest:

\[ FV = PV(1 + r)^n \]  

(1.3)

where \( r \) = Periodic rate of interest (expressed as a decimal);  
\( n \) = Number of periods for which the sum is invested.

This computation assumes that the interest payments made during the investment term are reinvested at an interest rate equal to the first year’s rate. That is why the example above stated that the 6% rate was fixed for 3 years. Compounding obviously results in higher returns than those earned with simple interest.

Now, consider a deposit of $100 for 1 year, still at a rate of 6% but compounded quarterly. Again assuming that the interest payments will be reinvested at the initial interest rate of 6%, the total return at the end of the year will be:

\[ 100 \times \left( (1 + 0.015) \times (1 + 0.015) \times (1 + 0.015) \times (1 + 0.015) \right) \]

\[ = 100 \times [(1 + 0.015)^4] \]

\[ = 100 \times 1.6136 \]

\[ = $106.136 \]
The terminal value for quarterly compounding is thus about 13 cents more than that for annual compounded interest.

In general, if compounding takes place $m$ times per year, then at the end of $n$ years, $mn$ interest payments will have been made, and the future value of the principal is computed using the formula (1.4):

$$FV = PV \left(1 + \frac{r}{m}\right)^{mn}$$

(1.4)

As the example above illustrates, more frequent compounding results in higher total returns. In Box 1.1 we show the interest rate factors corresponding to different frequencies of compounding on a base rate of 6% a year:

$$\text{Interest rate factor} = \left(1 + \frac{r}{m}\right)^m$$

<table>
<thead>
<tr>
<th>Compounding frequency</th>
<th>Interest rate factor for 6%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual</td>
<td>$(1 + r) = 1.060,000$</td>
</tr>
<tr>
<td>Semiannual</td>
<td>$\left(1 + \frac{r}{2}\right)^2 = 1.060,900$</td>
</tr>
<tr>
<td>Quarterly</td>
<td>$\left(1 + \frac{r}{4}\right)^4 = 1.061,364$</td>
</tr>
<tr>
<td>Monthly</td>
<td>$\left(1 + \frac{r}{12}\right)^{12} = 1.061,678$</td>
</tr>
<tr>
<td>Daily</td>
<td>$\left(1 + \frac{r}{365}\right)^{365} = 1.061,831$</td>
</tr>
</tbody>
</table>

This shows that the more frequent the compounding, the higher the annualised interest rate. The entire progression indicates that a limit can be defined for continuous compounding – i.e., where $m = \text{Infinity}$. To define the limit, it is useful to rewrite equation
(1.4) as (1.5):

\[ FV = PV \left[ \left( 1 + \frac{r}{m} \right)^{m/r} \right]^m \]

\[ = PV \left[ \left( 1 + \frac{1}{m/r} \right)^{m/r} \right]^m \]

\[ = PV \left[ \left( 1 + \frac{1}{w} \right)^w \right]^m \]  \hspace{1cm} (1.5)

where \( w = \frac{m}{r} \).

As compounding becomes continuous and \( m \) and hence \( w \) approach infinity, the expression in the square brackets in (1.5) approaches the mathematical constant \( e \) (the base of natural logarithmic functions), which is equal to approximately 2.718281.

Substituting \( e \) into (1.5) gives us:

\[ FV = PV e^{rn} \]  \hspace{1cm} (1.6)

In (1.6) \( e^{rn} \) is the exponential function of \( rn \). It represents the continuously compounded interest-rate factor. To compute this factor for an interest rate of 6% over a term of 1 year, set \( r \) to 6% and \( n \) to 1, giving:

\[ e^{rn} = e^{0.06 \times 1} = (2.718281)^{0.06} = 1.061837 \]

The convention in both wholesale and personal, or retail, markets is to quote an annual interest rate, whatever the term of the investment, whether it be overnight or 10 years. Lenders wishing to earn interest at the rate quoted have to place their funds on deposit for 1 year. For example, if you open a bank account that pays 3.5% interest and close it after 6 months, the interest you actually earn will be equal to 1.75% of your deposit. The actual return on a 3-year building society bond that pays a 6.75% fixed rate compounded annually is 21.65%. The quoted rate is the annual 1-year equivalent. An overnight deposit in the wholesale, or interbank, market is still quoted as an annual rate, even though interest is earned for only 1 day.

Quoting annualised rates allows deposits and loans of different maturities and involving different instruments to be compared. Be careful when comparing interest rates for products that have different payment frequencies. As shown in the earlier examples, the actual interest earned on a deposit paying 6% semiannually will be greater than on one paying 6% annually. The convention in
the money markets is to quote the applicable interest rate taking into account payment frequency.

The discussion thus far has involved calculating future value given a known present value and rate of interest. For example, $100 invested today for 1 year at a simple interest rate of 6% will generate $100 \times (1 + 0.06) = $106 at the end of the year. The future value of $100 in this case is $106. Conversely, $100 is the present value of $106, given the same term and interest rate. This relationship can be stated formally by rearranging equation (1.3) – i.e., \( FV = PV(1 + r)^n \) – as shown in (1.7):

\[
P V = \frac{F V}{(1 + r)^n}
\]  

Equation (1.7) applies to investments earning annual interest payments, giving the present value of a known future sum.

To calculate the present value of an investment, you must prorate the interest that would be earned for a whole year over the number of days in the investment period, as was done in (1.2). The result is equation (1.8):

\[
P V = \frac{F V}{\left(1 + \frac{r}{\text{Days/Year}}\right)^{\frac{\text{Days}}{\text{Year}}}}
\]

When interest is compounded more than once a year, the formula for calculating present value is modified, as it was in (1.4). The result is shown in equation (1.9):

\[
P V = \frac{F V}{\left(1 + \frac{r}{m}\right)^{mn}}
\]

For example, the present value of $100 to be received at the end of 5 years, assuming an interest rate of 5%, with quarterly compounding is:

\[
P V = \frac{100}{\left(1 + \frac{0.05}{4}\right)^{(4)(5)}} = $78.00
\]

Interest rates in the money markets are always quoted for standard maturities, such as overnight, ‘tom next’ (the overnight interest rate starting tomorrow, or ‘tomorrow to the next’), ‘spot next’ (the overnight rate starting 2 days forward), 1 week, 1 month, 2 months and so on, up to 1 year. If a bank or corporate customer wishes to borrow for
a nonstandard period, or ‘odd date’, an interbank desk will calculate
the rate chargeable, by interpolating between two standard-period
interest rates. Assuming a steady, uniform increase between stan-
dard periods, the required rate can be calculated using the formula
for straight line interpolation, which apportions the difference
equally among the stated intervals. This formula is shown as (1.10):

\[ r = r_1 + (r_2 - r_1) \times \frac{n - n_1}{n_2 - n_1} \quad (1.10) \]

where

- \( r \) = Required odd-date rate for \( n \) days;
- \( r_1 \) = Quoted rate for \( n_1 \) days;
- \( r_2 \) = Quoted rate for \( n_2 \) days.

Say the 1-month (30-day) interest rate is 5.25% and the 2-month
(60-day) rate is 5.75%. If a customer wishes to borrow money for 40
days, the bank can calculate the required rate using straight line
interpolation as follows: the difference between 30 and 40 is one-
third that between 30 and 60, so the increase from the 30-day to the
40-day rate is assumed to be one-third the increase between the
30-day and the 60-day rates, giving the following computation:

\[ 5.25\% + \left( \frac{5.75\% - 5.25\%}{3} \right) = 5.4167\% \]

What about the interest rate for a period that is shorter or longer than
the two whose rates are known, rather than lying between them?
What if the customer in the example above wished to borrow money
for 64 days? In this case, the interbank desk would extrapolate from
the relationship between the known 1-month and 2-month rates,
again assuming a uniform rate of change in the interest rates along
the maturity spectrum. So, given the 1-month rate of 5.25% and the
2-month rate of 5.75%, the 64-day rate would be:

\[ 5.25 + \left( 5.75 - 5.25 \right) \times \frac{34}{30} = 5.8167\% \]

Just as future and present value can be derived from one another,
given an investment period and interest rate, so can the interest rate
for a period be calculated given a present and a future value. The basic
equation is merely rearranged again to solve for \( r \). This, as will be
discussed below, is known as the investment’s yield.
Discount factors and boot-strapping the discount function

An $n$-period discount factor is the present value of one unit of currency that is payable at the end of period $n$. Essentially, it is the present value relationship expressed in terms of $1$. A discount factor for any term is given by (1.11):

$$d_n = \frac{1}{(1 + r)^n}$$  \hspace{1cm} (1.11)

where $n =$ Period of discount.

For instance, the 5-year discount factor for a rate of 6% compounded annually is:

$$d_5 = \frac{1}{(1 + 0.06)^5} = 0.747258$$

The set of discount factors for every period from 1 day to 30 years and longer is termed the discount function. Since the following discussion is in terms of $PV$, discount factors may be used to value any financial instrument that generates future cash flows. For example, the present value of an instrument generating a cash flow of $103.50 payable at the end of 6 months would be determined as follows, given a 6-month discount factor of 0.98756:

$$PV = \frac{FV}{(1 + r)^n} = FV \times d_n = $103.50 \times 0.98756 = $102.212$$

Discount factors can also be used to calculate the future value of a present investment by inverting the formula. In the example above, the 6-month discount factor of 0.98756 signifies that $1 receivable in 6 months has a present value of $0.98756. By the same reasoning, $1 today would in 6 months be worth:

$$\frac{1}{d_{0.5}} = \frac{1}{0.98756} = $1.0126$$

It is possible to derive discount factors from current bond prices. This process can be illustrated using the set of hypothetical bonds, all
assumed to have semiannual coupons, that are shown in Table 1.1, together with their prices.

The first bond in Table 1.1 matures in precisely 6 months. Its final cash flow will be $103.50, comprising the final coupon payment of $3.50 and the redemption payment of $100. The price, or present value, of this bond is $101.65. Using this, the 6-month discount factor may be calculated as follows:

\[
d_{0.5} = \frac{101.65}{103.50} = 0.98213
\]

Using this 6-month discount factor, the 1-year factor can be derived from the second bond in Table 1.1, the 8% due 2001. This bond pays a coupon of $4 in 6 months, and in 1 year makes a payment of $104, consisting of another $4 coupon payment plus $100 return of principal.

The price of the 1-year bond is $101.89. As with the 6-month bond, the price is also its present value, equal to the sum of the present values of its total cash flows. This relationship can be expressed in the following equation:

\[
101.89 = 4 \times d_{0.5} + 104 \times d_1
\]

The value of \(d_{0.5}\) is known to be 0.98213. That leaves \(d_1\) as the only unknown in the equation, which may be rearranged to solve for it:

\[
d_1 = \left[\frac{101.89 - 4(0.98213)}{104}\right] = \frac{97.96148}{104} = 0.94194
\]

The same procedure can be repeated for the remaining two bonds, using the discount factors derived in the previous steps to derive the set of discount factors in Table 1.2. These factors may also be graphed as a continuous function, as shown in Figure 1.2.

<table>
<thead>
<tr>
<th>Coupon (%)</th>
<th>Maturity date</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>07-Jun-2001</td>
<td>101.65</td>
</tr>
<tr>
<td>8</td>
<td>07-Dec-2001</td>
<td>101.89</td>
</tr>
<tr>
<td>6</td>
<td>07-Jun-2002</td>
<td>100.75</td>
</tr>
<tr>
<td>6.50</td>
<td>07-Dec-2002</td>
<td>100.37</td>
</tr>
</tbody>
</table>
This technique of calculating discount factors, known as ‘bootstrapping’, is conceptually neat, but may not work so well in practice. Problems arise when you do not have a set of bonds that mature at precise 6-month intervals. Liquidity issues connected with individual bonds can also cause complications. This is true because the price of the bond, which is still the sum of the present values of the cash flows, may reflect liquidity considerations (e.g., hard to buy or sell the bond, difficult to find) that do not reflect the market as a whole but peculiarities of that specific bond. The approach, however, is still worth knowing.

Note that the discount factors in Figure 1.2 decrease as the bond’s maturity increases. This makes intuitive sense, since the present value of something to be received in the future diminishes the farther in the future the date of receipt lies.

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Table 1.2  Discount factors calculated using the bootstrapping technique.

<table>
<thead>
<tr>
<th>Coupon (%)</th>
<th>Maturity date</th>
<th>Term (years)</th>
<th>Price</th>
<th>(d(n))</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>07-Jun-2001</td>
<td>0.5</td>
<td>101.65</td>
<td>0.98213</td>
</tr>
<tr>
<td>8</td>
<td>07-Dec-2001</td>
<td>1.0</td>
<td>101.89</td>
<td>0.94194</td>
</tr>
<tr>
<td>6</td>
<td>07-Jun-2002</td>
<td>1.5</td>
<td>100.75</td>
<td>0.92211</td>
</tr>
<tr>
<td>6.50</td>
<td>07-Dec-2002</td>
<td>2.0</td>
<td>100.37</td>
<td>0.88252</td>
</tr>
</tbody>
</table>

Figure 1.2  Hypothetical discount function.
BOND PRICING AND YIELD: THE TRADITIONAL APPROACH

The discount rate used to derive the present value of a bond’s cash flows is the interest rate that the bondholders require as compensation for the risk of lending their money to the issuer. The yield investors require on a bond depends on a number of political and economic factors, including what other bonds in the same class are yielding. Yield is always quoted as an annualised interest rate. This means that the rate used to discount the cash flows of a bond paying semiannual coupons is exactly half the bond’s yield.

Bond pricing

The fair price of a bond is the sum of the present values of all its cash flows, including both the coupon payments and the redemption payment. The price of a conventional bond that pays annual coupons can therefore be represented by formula (1.12):

\[
P = \frac{C}{(1 + r)} + \frac{C}{(1 + r)^2} + \frac{C}{(1 + r)^3} + \cdots + \frac{C}{(1 + r)^N} + \frac{M}{(1 + r)^N} = \sum_{n=1}^{N} \frac{C}{(1 + r)^n} + \frac{M}{(1 + r)^N}
\]

(1.12)

where

- \( P \) = Bond’s fair price;
- \( C \) = Annual coupon payment;
- \( r \) = Discount rate, or required yield;
- \( N \) = Number of years to maturity, and so the number of interest periods for a bond paying an annual coupon;
- \( M \) = Maturity payment, or par value, which is usually 100% of face value.

Bonds in the US domestic market – as opposed to international securities denominated in US dollars, such as USD Eurobonds – usually pay semiannual coupons. Such bonds may be priced using the expression in (1.13), which is a modification of (1.12) allowing for
twice-yearly discounting:

\[
P = \frac{C/2}{(1 + \frac{1}{2}r)} + \frac{C/2}{(1 + \frac{1}{2}r)^2} + \frac{C/2}{(1 + \frac{1}{2}r)^3} + \cdots + \frac{C/2}{(1 + \frac{1}{2}r)^{2N}} + \frac{M}{(1 + \frac{1}{2}r)^{2N}}
\]

\[
= \sum_{n=1}^{2N} \frac{C/2}{(1 + \frac{1}{2}r)^n} + \frac{M}{(1 + \frac{1}{2}r)^{2N}}
\]

\[
= \frac{C}{r} \left[ 1 - \frac{1}{(1 + \frac{1}{2}r)^{2N}} \right] + \frac{M}{(1 + \frac{1}{2}r)^{2N}}
\]  

(1.13)

Note that \(2N\) is now the power to which the discount factor is raised. This is because a bond that pays a semiannual coupon makes two interest payments a year. It might therefore be convenient to replace the number of years to maturity with the number of interest periods, which could be represented by the variable \(n\), resulting in formula (1.14):

\[
P = \frac{C}{r} \left[ 1 - \frac{1}{(1 + \frac{1}{2}r)^n} \right] + \frac{M}{(1 + \frac{1}{2}r)^n}
\]  

(1.14)

This formula calculates the fair price on a coupon payment date, so there is no accrued interest incorporated into the price. Accrued interest is an accounting convention that treats coupon interest as accruing every day a bond is held; this accrued amount is added to the discounted present value of the bond (the clean price) to obtain the market value of the bond, known as the dirty price. The price calculation is made as of the bond’s settlement date, the date on which it actually changes hands after being traded. For a new bond issue, the settlement date is the day when the investors take delivery of the bond and the issuer receives payment. The settlement date for a bond traded in the secondary market – the market where bonds are bought and sold after they are first issued – is the day the buyer transfers payment to the seller of the bond and the seller transfers the bond to the buyer.

Different markets have different settlement conventions. US Treasuries and UK gilts, for example, normally settle on \(T + 1\): one business day after the trade date, \(T\). Eurobonds, on the other hand, settle on \(T + 3\). The term value date is sometimes used in place of settlement date; however, the two terms are not strictly synonymous. A settlement date can fall only on a business day; a bond traded on a Friday, therefore, will settle on a Monday. A value
date, in contrast, can sometimes fall on a non-business day – when accrued interest is being calculated, for example.

Equation (1.14) assumes an even number of coupon payment dates remaining before maturity. If there are an odd number, the formula is modified as shown in (1.15):

\[
P = \frac{C}{r} \left[ 1 - \frac{1}{(1 + \frac{1}{2}r)^{2N+1}} \right] + \frac{M}{(1 + \frac{1}{2}r)^{2N+1}} \tag{1.15}
\]

Another assumption embodied in the standard formula is that the bond is traded for settlement on a day that is precisely one interest period before the next coupon payment. If the trade takes place between coupon dates, the formula is modified. This is done by adjusting the exponent for the discount factor using ratio \( i \), shown in (1.16):

\[
i = \frac{\text{Days from value date to next coupon date}}{\text{Days in the interest period}} \tag{1.16}
\]

The denominator of this ratio is the number of calendar days between the last coupon date and the next one. This figure depends on the day-count convention (see below) used for that particular bond. Using \( i \), the price formula is modified as (1.17) for annual coupon-paying bonds; for bonds with semiannual coupons, \( r/2 \) replaces \( r \):

\[
P = \frac{C}{(1 + r)^i} + \frac{C}{(1 + r)^{1+i}} + \frac{C}{(1 + r)^{2+i}} + \cdots + \frac{C}{(1 + r)^{n-1+i}} + \frac{M}{(1 + r)^{n-1+i}} \tag{1.17}
\]

where the variables \( C, M, n \) and \( r \) are as before.

**Box 1.2 Example: calculating consideration for a US Treasury bond.**

The consideration, or actual cash proceeds paid by a buyer for a bond, is the bond’s total cash value together with any costs such as commission. In this example, consideration refers only to the cash value of the bond.

What is the total consideration for £5 million nominal of a Eurobond, where the price is £114.50?
The price of the Eurobond is £114.50 per £100, so the consideration is:

\[
1.145 \times 5,000,000 = £5,725,000
\]

What consideration is payable for $5 million nominal of a US Treasury, quoted at a price of 99-16?

The US Treasury price is 99-16, which is equal to 99 and 16/32, or 99.50 per $100. The consideration is therefore:

\[
0.9950 \times 5,000,000 = $4,975,000
\]

If the price of a bond is below par, the total consideration is below the nominal amount; if it is priced above par, the consideration will be above the nominal amount.

As noted above, the bond market includes securities, known as zero-coupon bonds, or strips, that do not pay coupons. These are priced by setting \( C \) to 0 in the pricing equation. The only cash flow is the maturity payment, resulting in formula (1.18):

\[
P = \frac{M}{(1 + r)^N}
\]

where \( N \) = Number of years to maturity.

Note that, even though these bonds pay no actual coupons, their prices and yields must be calculated on the basis of quasi-coupon periods, which are based on the interest periods of bonds denominated in the same currency. A US dollar or a sterling 5-year zero-coupon bond, for example, would be assumed to cover ten quasi-coupon periods, and the price equation would accordingly be modified as (1.19):

\[
P = \frac{M}{(1 + \frac{1}{2}r)^n}
\]

**Box 1.3** Example: zero-coupon bond price.

(a) Calculate the price of a Treasury strip with a maturity of precisely 5 years corresponding to a required yield of 5.40%.

According to these terms, \( N = 5 \), so \( n = 10 \), and \( r = 0.054 \), so \( r/2 = 0.027 \). \( M = 100 \), as usual. Plugging these values into the
The pricing formula gives:

\[ P = \frac{100}{(1.027)^{10}} = \$76.611,782 \]

(b) Calculate the price of a French government zero-coupon bond with precisely 5 years to maturity, with the same required yield of 5.40%. Note that French government bonds pay coupon annually:

\[ P = \frac{100}{(1.054)^5} = €76,877,092 \]

It is clear from the bond price formula that a bond’s yield and its price are closely related. Specifically, the price moves in the opposite direction from the yield. This is because a bond’s price is the net present value of its cash flows; if the discount rate – that is, the yield required by investors – increases, the present values of the cash flows decrease. In the same way if the required yield decreases, the price of the bond rises. The stylised relationship between a bond’s price and any required yield level is illustrated by the graph in Figure 1.3, which plots the yield against the corresponding price to form a convex curve.

![Figure 1.3](image-url)  
*Figure 1.3* The price–yield relationship.
Box 1.4 Summary of the price–yield relationship.

- At issue, if a bond is priced at par, its coupon will equal the yield that the market requires, reflecting factors such as the bond’s term to maturity, the issuer’s credit rating and the yield on current bonds of comparable quality.
- If the required yield rises above the coupon rate, the bond price will decrease.
- If the required yield goes below the coupon rate, the bond price will increase.

Bond yield

The discussion so far has involved calculating the price of a bond given its yield. This procedure can be reversed to find a bond’s yield where its price is known. This is equivalent to calculating the bond’s internal rate of return (IRR), also known as its ‘yield to maturity’ or ‘gross redemption yield’ [also yield to workout]. These are among the various measures used in the markets to estimate the return generated from holding a bond.

In most markets, bonds are traded on the basis of their prices. Because different bonds can generate different and complicated cash-flow patterns, however, they are generally compared in terms of their yields. For example, market makers usually quote two-way prices at which they will buy or sell particular bonds, but it is the yield at which the bonds are trading that is important to the market maker’s customers. This is because a bond’s price does not tell buyers anything useful about what they are getting. Remember that in any market a number of bonds exist with different issuers, coupons and terms to maturity. It is their yields that are compared, not their prices.

The yield on any investment is the discount rate that will make the present value of its cash flows equal its initial cost or price. Mathematically, an investment’s yield, represented by \( r \), is the interest rate that satisfies the bond price equation, repeated here as (1.20):

\[
P = \sum_{n=1}^{N} \frac{C_n}{(1 + r)^n} + \frac{M}{(1 + r)^n} \quad (1.20)
\]
Other types of yield measure, however, are used in the market for different purposes. The simplest is the current yield, also known as the flat, interest or running yield. These are computed by formula (1.21):

\[
rc = \frac{C}{P} \times 100
\]  

(1.21)

where \(rc = \text{Current yield.}\)

In this equation the percentage for \(C\) is not expressed as a decimal. Current yield ignores any capital gain or loss that might arise from holding and trading a bond and does not consider the time value of money. It calculates the coupon income as a proportion of the price paid for the bond. For this to be an accurate representation of return, the bond would have to be more like an annuity than a fixed-term instrument.

Current yield is useful as a ‘rough and ready’ interest-rate calculation; it is often used to estimate the cost of or profit from holding a bond for a short-term. For example, if short-term interest rates, such as the 1-week or 3-month, are higher than the current yield, holding the bond is said to involve a running cost. This is also known as negative carry or negative funding. The concept is used by bond traders, market makers and leveraged investors, but it is useful for all market practitioners, since it represents the investor’s short-term cost of holding or funding a bond. The yield to maturity (\(YTM\)) – or, as it is known in sterling markets, gross redemption yield – is the most frequently used measure of bond return. Yield to maturity takes into account the pattern of coupon payments, the bond’s term to maturity and the capital gain (or loss) arising over the remaining life of the bond. The bond price formula shows the relationship between these elements and demonstrates their importance in determining price. The YTM calculation discounts the cash flows to maturity, employing the concept of the time value of money.

As noted above, the formula for calculating YTM is essentially that for calculating the price of a bond, repeated as (1.12). For the YTM of bonds with semiannual coupon, the formula must be modified, as in (1.13). Note, though, that this equation has two variables, the price \(P\) and yield \(r\). It cannot, therefore, be rearranged to solve for yield \(r\) explicitly. In fact, the only way to solve for the yield is to use numerical iteration. This involves estimating a value for \(r\) and
calculating the price associated with it. If the calculated price is higher than the bond’s current price, the estimate for \( r \) is lower than the actual yield, so it must be raised. This process of calculation and adjustment up or down is repeated until the estimates converge on a level that generates the bond’s current price.

To differentiate redemption yield from other yield and interest-rate measures described in this book, it will be referred to as \( \text{rm} \). Note that this section is concerned with the gross redemption yield, the yield that results from payment of coupons without deduction of any withholding tax. The \textit{net redemption yield} is what will be received if the bond is traded in a market where bonds pay coupon \textit{net}, without withholding tax. It is obtained by multiplying the coupon rate \( C \) by \([1 – \text{Marginal tax rate}]\). The net redemption yield is always lower than the gross redemption yield.

The key assumption behind the YTM calculation has already been discussed: that the redemption yield \( \text{rm} \) remains stable for the entire life of the bond, so that all coupons are reinvested at this same rate. The assumption is unrealistic, however. It can be predicted with virtual certainty that the interest rates paid by instruments with maturities equal to those of the bond at each coupon date will differ from \( \text{rm} \) at some point, at least, during the life of the bond. In practice, however, investors require a rate of return that is equivalent to the price that they are paying for a bond, and the redemption yield is as good a measurement as any.

A more accurate approach might be the one used to price interest-rate swaps: to calculate the present values of future cash flows using discount rates determined by the markets’ view on where interest rates will be at those points. These expected rates are known as \textit{forward} interest rates. Forward rates, however, are \textit{implied}, and a YTM derived using them is as speculative as one calculated using the conventional formula. This is because the real market interest rate at any time is invariably different from the one implied earlier in the forward markets. So, a YTM calculation made using forward rates would not equal the yield actually realised either. The zero-coupon rate, it will be demonstrated later, is the true interest rate for any term to maturity. Still, despite the limitations imposed by its underlying assumptions, the YTM is the main measure of return used in the markets.
Box 1.5 Example: yield to maturity for semiannual coupon bond.

A bond paying a semiannual coupon has a dirty price of $98.50, an annual coupon of 3% and exactly 1 year before maturity. The bond therefore has three remaining cash flows: two coupon payments of $1.50 each and a redemption payment of $100. Plugging these values into equation (1.13) gives:

\[ 98.50 = \frac{1.50}{(1 + \frac{1}{2} rm)} + \frac{101.50}{(1 + \frac{1}{2} rm)^2} \]

Note that the equation uses half of the YTM value \( rm \) because this is a semiannual paying bond. The expression above is a quadratic equation, which can be rearranged as:

\[ 98.50x^2 - 1.50x - 101.50 = 0, \quad \text{where } x = 1 + \frac{rm}{2} \]

The equation may now be solved using the standard solution for equations of the form \( ax^2 + bx + c = 0 \):

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

There are two solutions, only one of which gives a positive redemption yield. The positive solution is:

\[ \frac{rm}{2} = 0.022755 \quad \text{or} \quad rm = 4.551\% \]

YTM can also be calculated using mathematical iteration. Start with a trial value for \( rm \) of \( r_1 = 4\% \) and plug this into the right-hand side of equation (1.13). This gives a price \( P_1 \) of 99.050, which is higher than the dirty market price \( P_M \) of 98.50. The trial value for \( rm \) was therefore too low.

Next try \( r_2 = 5\% \). This generates a price \( P_2 \) of 98.114, which is lower than the market price. Because the two trial prices lie on either side of the market value, the correct value for \( rm \) must lie between 4 and 5%. Now use the formula for linear interpolation:

\[ rm = r_1 + (r_2 - r_1) \frac{P_1 - P_M}{P_1 - P_2} \]

Plugging in the appropriate values gives a linear approximation for the redemption yield of \( rm = 4.549\% \), which is near the solution obtained by solving the quadratic equation.
Calculating the redemption yield of bonds that pay semiannual coupons involves the semiannual discounting of those payments. This approach is appropriate for most US bonds and UK gilts. Government bonds in most of continental Europe and most Eurobonds, however, pay annual coupon payments. The appropriate method of calculating their redemption yields is to use annual discounting. The two yield measures are not directly comparable.

It is possible to make a Eurobond directly comparable with a UK gilt by using semiannual discounting of the former’s annual coupon payments or using annual discounting of the latter’s semiannual payments. The formulas for the semiannual and annual calculations appeared above as (1.13) and (1.12), respectively, and are repeated here as (1.22) and (1.23):

\[
P_d = \frac{C}{(1 + \frac{1}{2} rm)^2} + \frac{C}{(1 + \frac{1}{2} rm)^4} + \frac{C}{(1 + \frac{1}{2} rm)^6} + \cdots + \frac{C}{(1 + \frac{1}{2} rm)^{2N}} + \frac{M}{(1 + \frac{1}{2} rm)^{2N}} \tag{1.22}
\]

\[
P_d = \frac{C/2}{(1 + rm)^2} + \frac{C/2}{(1 + rm)^3} + \cdots + \frac{C/2}{(1 + rm)^N} + \frac{M}{(1 + rm)^N} \tag{1.23}
\]

Consider a bond with a dirty price – including the accrued interest the seller is entitled to receive – of $97.89, a coupon of 6% and 5 years to maturity. Table 1.3 shows the gross redemption yields this bond would have under the different yield-calculation conventions.

<table>
<thead>
<tr>
<th>Discounting</th>
<th>Payments</th>
<th>Yield to maturity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semiannual</td>
<td>Semiannual</td>
<td>6.500</td>
</tr>
<tr>
<td>Annual</td>
<td>Annual</td>
<td>6.508</td>
</tr>
<tr>
<td>Semiannual</td>
<td>Annual</td>
<td>6.428</td>
</tr>
<tr>
<td>Annual</td>
<td>Semiannual</td>
<td>6.605</td>
</tr>
</tbody>
</table>

These figures demonstrate the impact that the coupon-payment and discounting frequencies have on a bond’s redemption yield calculation. Specifically, increasing the frequency of discounting lowers the calculated yield, while increasing the frequency of payments raises it. When comparing yields for bonds that trade in markets
with different conventions, it is important to convert all the yields to the same calculation basis.

It might seem that doubling a semiannual yield figure would produce the annualised equivalent; the real result, however, is an underestimate of the true annualised yield. This is because of the multiplicative effects of discounting. The correct procedure for converting semiannual and quarterly yields into annualised ones is shown in (1.24).

a. General formula:

\[
rm_a = (1 + \text{Interest rate})^m - 1
\]

where \(m\) is the number of coupon payments per year.

b. Formulas for converting between semiannual and annual yields:

\[
rm_a = (1 + \frac{1}{2}rm_s)^2 - 1
\]
\[
rm_s = [(1 + rm_a)\frac{1}{2} - 1] \times 2
\]

c. Formulas for converting between quarterly and annual yields:

\[
rm_a = (1 + \frac{1}{4}rm_q)^4 - 1
\]
\[
rm_q = [(1 + rm_a)\frac{1}{4} - 1] \times 4
\]

where \(rm_q\), \(rm_s\) and \(rm_a\) are, respectively, the quarterly, semi-annually and annually discounted yields to maturity.

---

**Box 1.6  Example: comparing yields to maturity.**

A US Treasury paying semiannual coupons, with a maturity of 10 years, has a quoted yield of 4.89%. A Eurodollar government bond paying an annual coupon, with a similar maturity, is quoted at a yield of 4.96%. Which bond has the higher yield to maturity in practice?

The effective annual yield of the Treasury is:

\[
rm_a = (1 + \frac{1}{2} \times 0.0489)^2 - 1 = 4.9498\%
\]

Comparing the securities using the same calculation basis reveals that the European government bond does indeed have the higher yield.
The market convention is sometimes simply to double the semi-annual yield to obtain the annualised yields, despite the fact that this produces an inaccurate result. It is only acceptable to do this for rough calculations. An annualised yield obtained in this manner is known as a bond equivalent yield. It was noted earlier that the one disadvantage of the YTM measure is that its calculation incorporates the unrealistic assumption that each coupon payment, as it becomes due, is reinvested at the rate $r_m$. Another disadvantage is that it does not deal with the situation in which investors do not hold their bonds to maturity. In these cases, the redemption yield will not be as great. Investors might therefore be interested in other measures of return, such as the equivalent zero-coupon yield, which is considered a true yield.

Figure 1.4 shows Bloomberg page YA for the same Ford bond illustrated at Figure 1.1. It shows a number of yield measures for the bond; the principal one for our purposes – the yield to maturity – is the street yield shown as 8.285%.
To review, the redemption yield measure assumes that:

- the bond is held to maturity;
- all coupons during the bond’s life are reinvested at the same [redemption yield] rate.

Given these assumptions, the YTM can be viewed as an *expected* or *anticipated* yield. It is closest to reality when an investor buys a bond on first issue at par and holds it to maturity. Even then, however, the actual realised yield at maturity would be different from the YTM because of the unrealistic nature of the second assumption. It is clearly unlikely that all the coupons of any but the shortest-maturity bond will be reinvested at the same rate. As noted earlier, market interest rates are in a state of constant flux, and this would affect money reinvestment rates. Therefore, although YTM is the main market measure of bond levels, it is not a true interest rate. This is an important point.

Another problem with YTM is that it discounts a bond’s coupons at the yield specific to that bond. It thus cannot serve as an accurate basis for comparing bonds. Consider a 2-year and a 5-year bond. These securities will invariably have different YTMs. Accordingly, the coupon cash flows they generate in 2 years time will be discounted at different rates (assuming the yield curve is not flat). This is clearly not correct. The present value calculated today of a cash flow occurring in 2 years’ time should be the same whether that cash flow is generated by a short- or a long-dated bond.

**ACCRUED INTEREST**

All bonds except zero-coupon bonds accrue interest on a daily basis that is then paid out on the coupon date. As mentioned earlier, the formulas discussed so far calculate bonds’ prices as of a coupon payment date, so that no accrued interest is incorporated in the price. In all major bond markets, the convention is to quote this so-called clean price.

**Clean and dirty bond prices**

When investors buy a bond in the market, what they pay is the bond’s *all-in* price, also known as the dirty, or *gross price*, which is the clean price of a bond plus accrued interest.
Bonds trade either *ex-dividend* or *cum dividend*. The period between when a coupon is announced and when it is paid is the ex-dividend period. If the bond trades during this time, it is the seller, not the buyer, who receives the next coupon payment. Between the coupon payment date and the next ex-dividend date the bond trades cum dividend, so the buyer gets the next coupon payment.

Accrued interest compensates sellers for giving up all the next coupon payment even though they will have held their bonds for part of the period since the last coupon payment. A bond’s clean price moves with market interest rates. If the market rates are constant during a coupon period, the clean price will be constant as well. In contrast, the dirty price for the same bond will increase steadily as the coupon interest accrues from one coupon payment date until the next ex-dividend date, when it falls by the present value of the amount of the coupon payment. The dirty price at this point is below the clean price, reflecting the fact that accrued interest is now negative. This is because – if the bond is traded during the ex-dividend period – the seller, not the buyer, receives the next coupon, and the lower price is the buyer’s compensation for this loss. On the coupon date, the accrued interest is 0, so the clean and dirty prices are the same.

The net interest accrued since the last ex-dividend date is calculated using formula (1.25):

\[
AI = C \times \frac{N_{xt} - N_{xc}}{\text{Day base}}
\]

(1.25)

where

- \( AI \) = Next accrued interest;
- \( C \) = Bond coupon;
- \( N_{xc} \) = Number of days between the *ex-dividend* date and the coupon payment date;
- \( N_{xt} \) = Number of days between the *ex-dividend* date and the date for the calculation;
- Day base = Day-count base (see below).

When a bond is traded, accrued interest is calculated from and including the last coupon date up to and excluding the value date, usually the settlement date. Interest does not accrue on bonds whose issuer has defaulted.

As noted earlier, for bonds that are trading ex-dividend, the accrued coupon is negative and is subtracted from the clean price.
The negative accrued interest is calculated using formula (1.26):

\[
AI = -C \times \frac{\text{Days to next coupon}}{\text{Day base}}
\]

Certain classes of bonds – e.g., US Treasuries and Eurobonds – do not have ex-dividend periods and, therefore, trade cum dividend right up to the coupon date.

**Day-count conventions**

In calculating the accrued interest on a bond, the market uses the day-count convention appropriate to that bond. These conventions govern both the number of days assumed to be in a calendar year and how the days between two dates are figured. We show how the different conventions affect the accrual calculation in Box 1.7.

<table>
<thead>
<tr>
<th>Convention</th>
<th>Rules</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual/actual</td>
<td>The actual number of days between two dates is used. Leap years count for 366 days, non-leap years count for 365 days.</td>
</tr>
<tr>
<td>Actual/365 fixed</td>
<td>The actual number of days between two dates is used as the numerator. All years are assumed to have 365 days.</td>
</tr>
<tr>
<td>Actual/360</td>
<td>The actual number of days between two dates is used as the numerator. A year is assumed to have 12 months of 30 days each.</td>
</tr>
<tr>
<td>30/360</td>
<td>All months are assumed to have 30 days, resulting in a 360-day year. If the first date falls on the 31st, it is changed to the 30th. If the second date falls on the 31st, it is changed to the 30th, but only if the first date falls on the 30th or the 31st.</td>
</tr>
<tr>
<td>30E/360</td>
<td>All months are assumed to have 30 days, resulting in a 360-day year. If the first date falls on the 31st, it is changed to the 30th. If the second date falls on the 31st, it is changed to the 30th.</td>
</tr>
</tbody>
</table>
30E+/360 All months are assumed to have 30 days, resulting in a 360-day year. If the first date falls on the 31st, it is changed to the 30th. If the second date falls on the 31st, it is changed to the 1st and the month is increased by one.

In these conventions, the number of days between two dates includes the first date but not the second. Thus, using actual/365, there are 37 days between August 4 and September 10. The last two conventions assume 30 days in each month, no matter what the calendar says. So, for example, it is assumed that there are 30 days between 10 February and 10 March. Under the 30/360 convention, if the first date is the 31st, it is changed to the 30th; if the second date is the 31st and the first date is either the 30th or the 31st, the second date is changed to the 30th. The 30E/360 convention differs from this in that – if the second date is the 31st – it is changed to the 30th regardless of what the first date is.

**ILLUSTRATING BOND YIELD USING EXCEL SPREADSHEETS**

In this section we use Microsoft Excel to illustrate bond yield-to-maturity. We do this first by means of function references, then via a real-world illustration of how we can check yields quoted in the market using Excel.

Table 1.4 shows the spreadsheet used to calculate price, yield and duration for a hypothetical bond traded for settlement on 10 December 2005. It has a 5% coupon and matures in July 2012. Given the price we can calculate yield, and given yield we can calculate price and duration. We need to also set the coupon frequency, in this case semiannual, and the accrued interest day-count basis, in this case act/act, in order for the formulae to work.

Table 1.5 is the same spreadsheet but with the actual cell formulae shown.

We can apply the same logic with real-world bond prices and yields. Let us take a corporate bond first, the Ford 7\% of February 2007, traded for settlement on 6 January 2006. This bond accrues interest on a 30/360 basis.

As at the trade date this bond has three more coupons to pay plus its final maturity payment. The time to payment of each cash flow is
shown in column G of Table 1.6, which is a spreadsheet calculation of its yield. At our trade date, from Bloomberg page YA we see that the bond has a clean price of par and a redemption yield of 7.738%. This is shown at Figure 1.5. Can we check this on Excel?

Our confirmation is shown at Table 1.6. It is important to get the day-count fraction for the first coupon payment correct, and the confirm of this is shown at Figure 1.6, which is Bloomberg page 34.

**Table 1.4** Calculating bond price and yield using Microsoft Excel.

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Our confirmation is shown at Table 1.6. It is important to get the day-count fraction for the first coupon payment correct, and the confirm of this is shown at Figure 1.6, which is Bloomberg page 34.

**Table 1.5** Table 1.4 showing cell formulae.

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### Table 1.6  Bond yield calculation, Ford and US Treasury securities, 3 January 2006.

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**Notes:**
- **30/360-day basis**
- **Note numerator First-coupon day fraction 39/180**
- **Subsequent coupons are number of coupon periods plus some fraction**
- **act/act basis**
- **First coupon 32/184**
- **Subsequent coupon use coupon period plus some fraction**
Figure 1.5  Bloomberg page YA for Ford 7 3/8% 2007 bond, 3 Jan 2006. © 2006 Bloomberg Finance L.P. All rights reserved. Used with permission.

Figure 1.6  Bloomberg page DCX, 3 January 2006, settlement date for Ford bond. © 2006 Bloomberg Finance L.P. All rights reserved. Used with permission.
Table 1.7  Bond pricing from Table 1.6, showing cell formulae

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DCX with the relevant dates entered. We see that on the 30/360 basis the number of days accrued for the Ford bond for value 6 January 2006 is 39.

The spreadsheet cell formulae are shown at Table 1.7 (see previous page).

We perform a similar exercise for a US Treasury security, the 2\% February 2007 bond. This security settles on a T + 1 basis, so on our trade date of 3 January 2006 its yield is as at 4 January 2006. Also Treasury bonds accrue interest on an act/act basis. Its yield and price of 4.412% and 97.67, respectively, are given at Figure 1.7, the Bloomberg YA page. The confirm of these is shown at Table 1.6 again.

**BIBLIOGRAPHY**

INTRODUCTION TO BONDS
