PART I

LEARNING OBJECTIVES, SUMMARY OVERVIEW, AND PROBLEMS
LEARNING OUTCOMES

After completing this chapter, you will be able to do the following:

• interpret interest rates as required rates of return, discount rates, or opportunity costs;
• explain an interest rate as the sum of a real risk-free rate and premiums that compensate investors for bearing distinct types of risk;
• calculate and interpret the effective annual rate, given the stated annual interest rate and the frequency of compounding;
• solve time value of money problems for different frequencies of compounding;
• calculate and interpret the future value (FV) and present value (PV) of a single sum of money, an ordinary annuity, an annuity due, a perpetuity (PV only), and a series of unequal cash flows;
• demonstrate the use of a time line in modeling and solving time value of money problems.

SUMMARY OVERVIEW

In this reading, we have explored a foundation topic in investment mathematics, the time value of money. We have developed and reviewed the following concepts for use in financial applications:

• The interest rate, $r$, is the required rate of return; $r$ is also called the discount rate or opportunity cost.
• An interest rate can be viewed as the sum of the real risk-free interest rate and a set of premiums that compensate lenders for risk: an inflation premium, a default risk premium, a liquidity premium, and a maturity premium.
• The future value, FV, is the present value, PV, times the future value factor, $(1 + r)^N$.
• The interest rate, $r$, makes current and future currency amounts equivalent based on their time value.
The stated annual interest rate is a quoted interest rate that does not account for compounding within the year.

The periodic rate is the quoted interest rate per period; it equals the stated annual interest rate divided by the number of compounding periods per year.

The effective annual rate is the amount by which a unit of currency will grow in a year with interest on interest included.

An annuity is a finite set of level sequential cash flows.

There are two types of annuities, the annuity due and the ordinary annuity. The annuity due has a first cash flow that occurs immediately; the ordinary annuity has a first cash flow that occurs one period from the present (indexed at $t = 1$).

On a time line, we can index the present as 0 and then display equally spaced hash marks to represent a number of periods into the future. This representation allows us to index how many periods away each cash flow will be paid.

Annuities may be handled in a similar fashion as single payments if we use annuity factors instead of single-payment factors.

The present value, $P_v$, is the future value, $F_v$, times the present value factor, $(1 + r)^{-N}$.

The present value of a perpetuity is $A/r$, where $A$ is the periodic payment to be received forever.

It is possible to calculate an unknown variable, given the other relevant variables in time value of money problems.

The cash flow additivity principle can be used to solve problems with uneven cash flows by combining single payments and annuities.

### Problems


1. The table below gives current information on the interest rates for two two-year and two eight-year maturity investments. The table also gives the maturity, liquidity, and default risk characteristics of a new investment possibility (Investment 3). All investments promise only a single payment (a payment at maturity). Assume that premiums relating to inflation, liquidity, and default risk are constant across all time horizons.

<table>
<thead>
<tr>
<th>Investment</th>
<th>Maturity (in Years)</th>
<th>Liquidity</th>
<th>Default Risk</th>
<th>Interest Rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>High</td>
<td>Low</td>
<td>2.0</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>Low</td>
<td>Low</td>
<td>2.5</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>Low</td>
<td>Low</td>
<td>$r_3$</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>High</td>
<td>Low</td>
<td>4.0</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>Low</td>
<td>High</td>
<td>6.5</td>
</tr>
</tbody>
</table>

Based on the information in the above table, address the following:
A. Explain the difference between the interest rates on Investment 1 and Investment 2.
B. Estimate the default risk premium.
C. Calculate upper and lower limits for the interest rate on Investment 3, $r_3$. 
2. A client has a $5 million portfolio and invests 5 percent of it in a money market fund projected to earn 3 percent annually. Estimate the value of this portion of his portfolio after seven years.

3. A client invests $500,000 in a bond fund projected to earn 7 percent annually. Estimate the value of her investment after 10 years.

4. For liquidity purposes, a client keeps $100,000 in a bank account. The bank quotes a stated annual interest rate of 7 percent. The bank's service representative explains that the stated rate is the rate one would earn if one were to cash out rather than invest the interest payments. How much will your client have in his account at the end of one year, assuming no additions or withdrawals, using the following types of compounding?
   A. Quarterly.
   B. Monthly.
   C. Continuous.

5. A bank quotes a rate of 5.89 percent with an effective annual rate of 6.05 percent. Does the bank use annual, quarterly, or monthly compounding?

6. A bank pays a stated annual interest rate of 8 percent. What is the effective annual rate using the following types of compounding?
   A. Quarterly.
   B. Monthly.
   C. Continuous.

7. A couple plans to set aside $20,000 per year in a conservative portfolio projected to earn 7 percent a year. If they make their first savings contribution one year from now, how much will they have at the end of 20 years?

8. Two years from now, a client will receive the first of three annual payments of $20,000 from a small business project. If she can earn 9 percent annually on her investments and plans to retire in six years, how much will the three business project payments be worth at the time of her retirement?

9. To cover the first year’s total college tuition payments for his two children, a father will make a $75,000 payment five years from now. How much will he need to invest today to meet his first tuition goal if the investment earns 6 percent annually?

10. A client has agreed to invest €100,000 one year from now in a business planning to expand, and she has decided to set aside the funds today in a bank account that pays 7 percent compounded quarterly. How much does she need to set aside?

11. A client can choose between receiving 10 annual $100,000 retirement payments, starting one year from today, or receiving a lump sum today. Knowing that he can invest at a rate of 5 percent annually, he has decided to take the lump sum. What lump sum today will be equivalent to the future annual payments?

12. A perpetual preferred stock position pays quarterly dividends of $1,000 indefinitely (forever). If an investor has a required rate of return of 12 percent per year compounded quarterly on this type of investment, how much should he be willing to pay for this dividend stream?

13. At retirement, a client has two payment options: a 20-year annuity at €50,000 per year starting after one year or a lump sum of €500,000 today. If the client's required rate of return on retirement fund investments is 6 percent per year, which plan has the higher present value and by how much?

14. You are considering investing in two different instruments. The first instrument will pay nothing for three years, but then it will pay $20,000 per year for four years. The second instrument will pay $20,000 for three years and $30,000 in the fourth year. All payments
are made at year-end. If your required rate of return on these investments is 8 percent annually, what should you be willing to pay for:

A. The first instrument?
B. The second instrument (use the formula for a four-year annuity)?

15. Suppose you plan to send your daughter to college in three years. You expect her to earn two-thirds of her tuition payment in scholarship money, so you estimate that your payments will be $10,000 a year for four years. To estimate whether you have set aside enough money, you ignore possible inflation in tuition payments and assume that you can earn 8 percent annually on your investments. How much should you set aside now to cover these payments?

16. A client is confused about two terms on some certificate-of-deposit rates quoted at his bank in the United States. You explain that the stated annual interest rate is an annual rate that does not take into account compounding within a year. The rate his bank calls APY (annual percentage yield) is the effective annual rate taking into account compounding. The bank's customer service representative mentioned monthly compounding, with $1,000 becoming $1,061.68 at the end of a year. To prepare to explain the terms to your client, calculate the stated annual interest rate that the bank must be quoting.

17. A client seeking liquidity sets aside €35,000 in a bank account today. The account pays 5 percent compounded monthly. Because the client is concerned about the fact that deposit insurance covers the account for only up to €100,000, calculate how many months it will take to reach that amount.

18. A client plans to send a child to college for four years starting 18 years from now. Having set aside money for tuition, she decides to plan for room and board also. She estimates these costs at $20,000 per year, payable at the beginning of each year, by the time her child goes to college. If she starts next year and makes 17 payments into a savings account paying 5 percent annually, what annual payments must she make?

19. A couple plans to pay their child’s college tuition for 4 years starting 18 years from now. The current annual cost of college is C$7,000, and they expect this cost to rise at an annual rate of 5 percent. In their planning, they assume that they can earn 6 percent annually. How much must they put aside each year, starting next year, if they plan to make 17 equal payments?

20. You are analyzing the last five years of earnings per share data for a company. The figures are $4.00, $4.50, $5.00, $6.00, and $7.00. At what compound annual rate did EPS grow during these years?

21. An analyst expects that a company’s net sales will double and the company’s net income will triple over the next five-year period starting now. Based on the analyst’s expectations, which of the following best describes the expected compound annual growth?
   A. Net sales will grow 15% annually and net income will grow 25% annually.
   B. Net sales will grow 20% annually and net income will grow 40% annually.
   C. Net sales will grow 25% annually and net income will grow 50% annually.