Index

**a**

action, see load(s)
advanced finite element formulations
  assumed natural strain (ANS) 374, 379–380
  basic shell mathematical model (BSMM) 378
  continuum-based resultant shell FE (CBRS) 376
covariant shear strains interpolation (CSI) 378
discrete Kirchhoff element (DKT/DKQ) 332, 374–376
discrete Kirchhoff–Mindlin element (DKMT/DKMQ) 376
discrete strain gap (DSG) 379
enhanced degenerated FEs 373
hierarchic family of isogeometric shell FE 378–379
mixed interpolation of tensorial components (MIT) FE 377–378
non-uniform rational B-splines (NURBS) 378
triangular and quadrilateral flat shell FEs with 6 dofs per node 376
Airy’s stress function 79–80, 83, 88, 94, 258, 262–263
algebraic eigenvalue problem
  of linearized buckling in FEM 303
  of natural (free) vibrations 325–326, 328
algorithm
  for buckling analysis (FEM) 54
  for free vibrations analysis (FEM) 55
  incremental-iterative (Newton–Raphson method) 290, 300, 320, 321, 373
  for linear statics
    FDM 143
    FEM 54
assumed natural strains (ANS) 374, 379–380

**b**

base vectors 10, 11, 13, 64
Batdorf’s parameter 291, 311, 315, 316
benchmarks
  cantilever beam 88, 357
  conical shell 193
  covered tank 232
cylindrical shell 229, 238, 271, 281, 309, 335, 359, 360
  horizontal tube 246
  long 211
  pinched cylinder 360
  Scordelis–Lo roof 242, 359
  under self weight and hydrostatic pressure 197
  short 218
depth beam 94
hemisphere 231, 271, 360
  with asymmetric wind action 199
  with hydrostatic pressure 189, 191
  under self weight 186, 224
  with uniform normal pressure 187
membrane
  annular 105
circular 270
  rectangular 81, 83, 85, 94, 97, 293
plate
  annular 174
circular 166, 169, 171, 277, 306–309
benchmarks (contd.)
  rectangular 122, 127, 131, 135, 139, 144, 280, 301, 325
  shallow shell
    hyperbolic paraboloid 262
  square plate 359
  swept panel 357
  Bessel’s functions, see functions
  bifurcation of equilibrium states 291–293, 301
  bifurcation point type 290, 292, 299–300
  bi-Laplacian operator
    in Cartesian coordinate system 116
    in polar coordinate system 162
  boundary conditions 28–29, 45, 47
    for membrane 34, 77, 78, 103, 105
    for plate 35, 114, 116, 121, 161, 163
    for shell 33, 182, 184, 196, 206, 208, 223, 259, 261
  buckling problem 49–51, 55, 289–321
  FDM 53–54, 295–298
  FEM 54
    initial 303
    linearized 303
  C
    classification of FEs 56–70
    classification of shell structures 5, 21–24
    computational modeling 62
    constitutive equations, see equations
    continuity of approximation
      $C^0$ 64–66, 156, 345, 352
      $C^1$ 60, 64, 156, 345, 352
      $C^2$ 151
  convergence 168, 169, 312, 333, 343, 345, 355, 361, 376–377, 380
  coordinate (systems) 9–10
  critical load value, see buckling problem
  curvature
    Gaussian 12, 14
    mean 12, 14
  d
  degree(s) of freedom (dof)
    drilling rotation 63, 69, 374
    dynamic 324–325, 337
    mathematical 366, 369–370, 372–373, 379
  rotational 7, 63–64, 119, 375–376
  translational 64
  density 323–324
  discrete Kirchhoff constraints 332, 374–376
  displacement differential equation(s) 45–47
    for circular/annular membrane in axisymmetric state 104
    for circular/annular plate in axisymmetric state 163
    for circular/annular plate in general state 162
    for cylindrical shells in axisymmetric membrane-bending states 46, 209
    for cylindrical shells in general state 46
    for moderately thick rectangular plate 46, 122
    for rectangular membrane 46, 78
    for thin rectangular plate 46, 116
  displacement-force differential equations
    for shallow shell 258
  disturbance of membrane state, see local bending state
dof(s), see degree(s) of freedom
  e
  eigenproblem, see algebraic eigenvalue problem
  energy
    internal strain 29, 33–36, 78, 80, 103, 105, 114, 121, 162–163, 183–184, 196, 207–208, 223, 259, 261
    total potential 29, 33–36, 78, 80, 103, 105, 114, 121, 162–163, 183–184, 196, 207–208, 223, 259, 261
  enhanced assumed strains (EAS) 379–380
  equations
    constitutive 27, 29, 31, 34–36, 77, 81, 103, 105, 114, 121, 161, 163, 182, 184, 196, 206, 208–209, 223, 259, 261
    equilibrium 27, 29, 31, 34–36, 77, 81, 103, 105, 114, 121, 161, 163, 182, 184, 196, 206, 208–209, 223, 259, 261
    kinematic 27, 29, 31, 34–36, 77, 81, 103, 105, 114, 121, 161, 163, 182, 184, 196, 205, 208–209, 223, 259, 261
  equilibrium equations, see equations
equilibrium paths in stability analysis
primary/fundamental 289–290, 292, 294, 299
secondary/postbuckling 289–290, 292, 294, 299, 312–313

finite element (FE)
based on K–L thin shell theory 56–57, 64, 66, 68–69, 344
based on M–R moderately thick shell theory 57, 64–65, 344
curved 64
degenerated shell 65, 373
displacement-based 57, 62, 64, 66, 69, 80, 342, 347–349, 352, 361, 365, 367, 371, 374, 379
flat 58, 63, 376
four-node membrane
displacement-based 58
hybrid stress 369
rectangular 58, 369
four-node plate bending
conforming rectangular 60–61, 158, 304
generically one-dimensional shell 57, 66–67, 108, 318
Heterosis 305, 352
hybrid displacement 361, 366, 371
mixed (displacement-moment) 370
mixed (displacement-stress) 366, 368–369
nine-node thin plate and shell (mixed two-field model) 372
nonconforming flat three-and four-node FE for thin shell 63
shell 63–66, 373–374, 376
solid 65
three-dimensional solid for thick shell 65
triangular plate bending 370–371

triangular/quadrilateral membrane
(mixed displacement-stress) 368–369

finite element formulations 343, 365, 373, 378
hybrid 332, 362, 366–367, 369–373, 379
one-field 97, 342, 366–368, 373
three-field 366, 379
two-field 368–370, 372–373, 379–380

finite element method (FEM), see method of solution of BVP

force(s)
corner 115
damping 323
inertia 51, 323–325
reaction(s) 54, 109, 123–127, 131, 133, 150, 177, 181, 187, 193, 382
tangential 38, 79, 95, 266

formulation of BVP
local 29, 31–36, 76–77, 103, 105, 114, 121, 144, 161, 163, 182, 184, 196, 205–206, 208, 223, 259, 261

free vibrations, see vibration(s)
frequencies of natural/free vibrations, see vibration(s)
frequency spectrum 325
functional 395
complementary energy 366, 371–372
Hellinger–Reissner 366–368, 370, 372, 379
Hu–Washizu 366, 379
modified
complementary energy 369, 371
potential energy 371, 376, 379
Index

functional (contd.)

functions
  Bessel’s 306–308
  exponential-trigonometric 210–211, 250, 252
  shape
    bilinear 58–59, 344, 349–350, 368
    Hermitian 60–61, 67, 177
    Lagrange 328, 349, 352–353
    polynomial base 60
    Serendipity 349, 352–353

fundamental states
  in membrane
    pure in-plane bending 85–87, 350–351
    unidirectional tension/compression 81–83, 354
    uniform shear 83–85
  in plate under bending
    constant bending 356
    constant curvature 278–279, 355–356, 376
    constant transverse shear 356, 376
    constant twisting 356
    constant warping 354–356
  in shell
    pure bending 232, 352, 354

G
  Gaussian curvature, see curvature
  Gauss quadrature, see numerical integration
  generalized displacements 8, 18, 33, 37, 119, 366
  generalized strains 8, 21
  generalized stresses, see stress resultants
  geometrical parameters of middle surface 10, 14–15, 44
  global formulation, see formulation

H
  Heterosis, see finite element
  history of theory of plates and shells 6
  hp-adaptive FEM 69, 380

hyperbolic paraboloid, see benchmarks; shell shape
  hypothesis of Kirchhoff–Love
    kinematic 10, 16, 111
    static 18, 111

i
  imperfections 5, 289–291, 300, 313
  incremental-iterative algorithm, see algorithm
  inertia
    forces 51, 223–224
    moment 89, 91, 223
    rotational/rotatory 324–325, 328, 330–331, 334
  initial conditions 48, 323, 326
  instability, see structural stability
  isoparametric approach 57

k
  kinematic equations, see equations

l
  Lame parameters 8, 11, 14, 20–21, 38–42, 181, 205, 221
  Levy’s method, see method of solution of BVP

load(s)
  dead 289
  follower 289
  limit 290, 300
local bending state 210, 226, 233, 235, 249, 274, 283
local formulation, see formulation of BVP
locking
membrane 350, 352, 376, 379
shear 157, 350–352, 354, 376, 378–379
transverse shear 351–352, 378–379
loss of stability
buckling 289, 293, 298–301, 312–313
snap-through 289, 300, 319, 321

m
matrix of FE
  displacement stiffness 303
  linear stiffness 54, 59, 62, 66, 303, 347
  rank deficiency 349, 353
  singularity 305, 349, 353, 374
  stress stiffness 54, 303
mean curvature, see curvature
membrane
  annular 41, 102, 105
  circular 41, 102
forces
  normal 75
  principal 79
  radial 102
tangential 75, 102
rectangular 41, 75–101
membrane state in shells, see shell(s)
method of solution of BVP
Finite Difference Method (FDM) 51–54, 96, 143–150, 295–297
Levy’s method/single trigonometric series (STSM) 130–131
Navier’s method/double trigonometric series (DTSM) 127–130
Ritz method 151–153, 327
trigonometric series in polar coordinate system 164
metric tensor of middle surface 11–14
middle surface/plane 4, 9–12, 16, 19–22, 42–43, 76, 111, 119
Mindlin–Reissner theory 5, 18, 45, 57, 119–122, 155, 158, 328, 333, 355, 376, see also theory
moments
  bending 113, 120, 160, 207
  principal 115
twisting 113, 120, 160, 207
n
natural vibrations, see vibration(s)
Navier’s method, see method of solution of BVP
Newton–Raphson method, see algorithm
nonlinear analysis of stability 289, 300, 319
numerical integration/Gauss quadrature
  full 59, 94, 156, 305, 347
  reduced 59, 94, 156, 305, 347, 373
  selective 94, 305, 347, 373
o
overstiffness, see locking
p
patch test 355–356
plate
  annular 41, 160, 174–177
  with asymmetric load 171–174
  moderately thick 5, 18, 22, 119, 121, 155–159, 331–333, 342, 356
  with point load 157–159, 169–171, 356
  sinusoidally loaded 122–127
Index

plate (contd.)
  with variable thickness 153–55
  on Winkler foundation 118, 149
point on equilibrium path in stability analysis
  bifurcation 290, 299, 300, 313
  limit 290, 300, 313, 320
  snap-back 319
  snap–through 319
principle of minimum potential energy 30, 117, 151
Pucher’s equations 260–262
q
quadratic form of middle surface 11, 348
quality of FEs 347
r
radius of curvature 5, 12, 21, 38–42
reaction(s), see force(s)
reference load, see load(s)
relative error 333, 361
requirements/criteria for FEs 57, 343–345
resultant forces/moments, see stress resultants
rigid-body motions 231, 344, 348, 354, 378
Ritz method, see method of solution of BVP rotation
  around normal to middle surface/drilling 16, 17, 37, 63, 358, 374–376
  of normal to middle surface 16, 17, 37, 61, 111, 119
s
Sanders shell theory, see theory
Scordelis–Lo roof, see benchmarks
shape functions, see functions
shear correction coefficient 156, 342
shell(s), see also benchmarks; shell shape; stress state
curved 10, 22
flat (see membrane; plate)
in membrane-bending state 19, 36, 37, 205–228, 242–246, 256–260
in membrane state 37, 181–204, 209, 238–242, 260–266
moderately thick 5, 18, 22, 64, 342
shallow 5, 10, 22, 41, 244, 256–267, 299, 319
thick 5, 22, 65
thin 5, 16, 18, 22, 23, 31–33, 63, 205–208
shell shape
  with axisymmetric geometry 39, 185, 233
  barrel 318
  complex roof 267
  conical 40, 193, 195
  cylindrical 10, 14, 40, 44, 195, 205, 243, 248, 310, 318, 354, 360, 381
  hemispherical 13, 186–191, 194, 199–204, 221, 270–276, 360
  horizontal tube 248, 253–254
  hyperbolic paraboloid 15, 41, 262
  hyperboloid 318
  paraboloid 41, 257
  spherical 10, 13, 39, 221, 225, 382
sign convention
  for forces and moments 37, 85, 122, 161
  for generalized displacements 37
  for generalized resultant forces 19, 113, 161
  for kinematic and static boundary quantities 38, 77
  for membrane forces 76, 104, 257
  for stresses 28
  for surface loads 37, 113, 161
solution
  singular 170, 173, 307, 349
source of bending effects in shell, see local bending state
spectral analysis of stiffness matrix  344, 347–349
star of difference quotient coefficients 146–147
stiffness matrix, see matrix of FE
stiffness of cross-section
  bending   32, 35, 46, 114, 115, 342
  membrane  32, 34, 46, 77, 342
  transverse shear  46, 120, 121, 342
strain compatibility condition 79, 258
strains
  bending (curvatures and warping) 112, 120, 160, 207
  membrane (normal and shear) 75, 102, 182, 195, 207
  transverse shear 120
stress resultants
membrane forces
  circumferential 102, 186, 193, 195, 207
  meridional 185, 193, 195, 207
  normal 75
  radial 102
  tangential 75, 102
moments
  bending 113, 120, 160, 207
  twisting 113, 120, 160, 207
transverse shear forces 113, 120, 160, 207
stress state
  bending 113, 115
  membrane 75
  transverse shear 112
structural stability 289
surface
  conical (see shell shape)
  cylindrical (see shell shape)
  equidistant 12–13, 19–20, 112
  hemispherical (see shell shape)
  hyperbolic paraboloid (see shell shape)
  hyperboloid (see shell shape)
  middle (see middle surface/plane)
  spherical (see shell shape)

\text{t}

temperature change distribution along the thickness
  linear 268–270, 275, 277, 280–286

uniform 268–270
temperature
  for beams
    Bernoulli–Euler theory 90–92
    Timoshenko theory 90–92
  of elasticity 26, 76
geofometrically linear (small displacement and rotations) 5, 23, 31, 133
geofometrically nonlinear 5, 23, 290, 373
for membranes 34, 45, 75–80, 102–105, 292, 295–296
for plates
  classical plate theory (CPT) (see Kirchhoff–Love theory)
  first-order shear deformation theory (FSDT) (see Mindlin–Reissner theory)
  Mindlin–Reissner (M–R) theory 5, 18, 45, 57, 119–122, 155, 158, 328, 333, 355, 376
  moderately thick plate theory (see Mindlin–Reissner theory)
  one-parameter theory of plates (see Kirchhoff–Love theory)
  thin plate theory (see Kirchhoff–Love theory)
  third-order shear deformation theory (TSDT) 155
  three-parameter theory of plates (see Mindlin–Reissner theory)
  von Kármán theory of plates with moderately large deflections 5, 23, 159
for shells
  five-parameter theory of moderately thick shells 5, 8, 64, 119, 342
  Sanders/thin shell theory 21, 31–33, 38, 205, 221, 374
  three-parameter theory of thin shells in membrane-bending states 45
  two-parameter theory of thin shells in axisymmetric membrane-bending states 47
thermal load, see load(s)

transverse shear forces, see force(s)

\( U \)
uniform load, see load(s)

\( V \)
vector
of boundary generalized displacements 26, 33, 75, 102, 160, 182, 207, 221
of boundary loads 26, 33, 75, 102, 160, 182, 207, 221
of generalized displacements 17, 18, 26, 37, 75, 102, 113, 120, 160, 182, 207, 221
of generalized strains 26, 37, 75, 102, 113, 120, 160, 182, 207, 221
of generalized stresses 26, 37, 75, 102, 113, 120, 160, 182, 207, 222 (see also stress resultants)
of nodal substitutes of the boundary loads for FE 60, 62
of nodal substitutes of the surface loads for FE 60, 62

of rotations 17, 120
of surface loads 33, 37, 102, 160, 182, 207, 221
visualization of displacement field 203–204, 245
verification tests, see benchmarks; patch test
vibration(s)
free/natural
of cylindrical shells 333–337
of rectangular plates 51, 53, 325–333
frequencies 51, 53, 324
modes 50, 329, 336
virtual work principle 37, 365
von Kármán theory, see theory

\( W \)
Winkler foundation 118, 149
work of external loads 29–30, 33–36, 78, 80, 103, 105, 114, 121, 162, 163, 183, 184, 196, 207, 208, 223, 259

\( Z \)
zero eigenvalues of FE stiffness matrix 348–349
zero-energy deformation modes 349–350, 376