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Introduction

1.1 Stability and Control of a Dynamic System

In engineering, a system is understood to be a set of physical elements acting together and realizing a common goal. An important role in the analysis of the system is played by its mathematical model. It is created using the system structure and fundamental physical laws governing the system elements. In the case of complicated systems, mathematical models usually do not have a universal character but rather reflect some characteristic phenomena which are of interest. Because of mathematical complications, practically used system models are usually a compromise between a required accuracy of modeling and a degree of complication.

When formulating a system model, important terms are the system state and the state variables. The system state describes the system’s operating conditions. The state variables are the minimum set of variables $x_1, x_2, ..., x_n$ uniquely defining the system state. State variables written as a vector $\mathbf{x} = [x_1, x_2, ..., x_n]^T$ are referred to as the state vector. A normalized space of coordinates corresponding to the state variables is referred to as the state space. In the state space, each system state corresponds to a point defined by the state vector. Hence, a term “system state” often refers also to a point in the state space.

A system may be static when its state variables $x_1, x_2, ..., x_n$ are time invariant or dynamic when they are functions of time, that is $x_1(t), x_2(t), ..., x_n(t)$.

This book is devoted to the analysis of dynamic systems modeled by ordinary differential equations of the form

$$\dot{\mathbf{x}} = F(\mathbf{x}) \quad \text{or} \quad \mathbf{x} = A\mathbf{x} \quad (1.1)$$

where the first of the equations above describes a nonlinear system and the second describes a linear system. $F(\mathbf{x})$ is a vector of nonlinear functions and $A$ is a square matrix.

A curve $\mathbf{x}(t)$ in the state space containing system states (points) in consecutive time instants is referred to as the system trajectory. A trivial one-point trajectory $\mathbf{x}(t) = \dot{\mathbf{x}} = \text{const}$ is referred to as the equilibrium point (state), if in that point all the partial derivatives are zero (no movement), that is $\dot{\mathbf{x}} = 0$. According to Eq. (1.1), the coordinates of the point satisfy the following equations

$$F(\dot{\mathbf{x}}) = 0 \quad \text{or} \quad A\dot{\mathbf{x}} = 0 \quad (1.2)$$

A nonlinear system may have more than one equilibrium point because nonlinear equations may have generally more than one solution. In the case of linear systems, according to Cramér’s theorem concerning linear equations, there exists only one uniquely specified equilibrium point $\dot{\mathbf{x}} = 0$ if and only if the matrix $A$ is nonsingular (det $A \neq 0$).

All the states of a dynamic system, apart from equilibrium states, are dynamic states because the derivatives $\dot{\mathbf{x}} \neq 0$ for those states are nonzero, which means a movement. Disturbance means a random (usually unintentional) event affecting the system. Disturbances affecting dynamic systems are modeled by changes in their coefficients (parameters) or by nonzero initial conditions of differential equations.

Let $x_1(t)$ be a trajectory of a dynamic system (Figure 1.1a) corresponding to some initial conditions. The system is considered stable in the Lyapunov sense if for any $t_0$ it is possible to choose a number $\eta$ such that for all the other initial conditions satisfying the constraint $|x_2(t_0) - x_1(t_0)| < \eta$ there holds $|x_2(t) - x_1(t)| < \varepsilon$ for $t_0 \leq t < \infty$. In other words stability means that if the trajectory $x_2(t)$ starts close enough (as defined by $\eta$) to the trajectory $x_1(t)$ then it remains close to it (number $\varepsilon$). Moreover, if the trajectory $x_2(t)$ tends with time toward the trajectory $x_1(t)$, i.e. $\lim_{t \to \infty} |x_2(t) - x_1(t)| = 0$, then the dynamic system is asymptotically stable.

The above definition concerns any trajectory of a dynamic system. Hence it must also be valid for a trivial trajectory such as the equilibrium point $x$. In this particular case (Figure 1.1b), the trajectory $x_1(t)$ is a point $\hat{x}$ and the initial condition $x_2(t_0)$ of trajectory $x_2(t)$ lies in the vicinity of the point defined by $\eta$. The dynamic system is stable in the equilibrium point $\hat{x}$ if for $t_0 \leq t < \infty$ the trajectory $x_2(t)$ does not leave an area defined by the number $\varepsilon$. Moreover, if the trajectory $x_2(t)$ tends with time toward the equilibrium point $\hat{x}$, i.e. $\lim_{t \to \infty} |x_2(t) - \hat{x}| = 0$, then the system is said to be asymptotically stable at the equilibrium point $\hat{x}$. On the other hand, if the trajectory $x_2(t)$ tends with time to leave the area defined by $\varepsilon$, then the dynamic system is said to be unstable at the equilibrium point $\hat{x}$.

It can be shown that the stability of a linear system does not depend on the size of a disturbance. Hence if a linear system is stable for a small disturbance then it is also globally stable for any large disturbance.

The situation is different with nonlinear systems as their stability generally depends on the size of a disturbance. A nonlinear system may be stable for a small disturbance but unstable for a large disturbance. The largest disturbance for which a nonlinear system is still stable is referred to as a critical disturbance.

Dynamic systems are designed and constructed with a particular task in mind and assuming that they will behave in a particular way following a disturbance. A purposeful action affecting a dynamic system which aims to achieve a particular behavior is referred to as a control. The definition of control is illustrated in Figure 1.2. The following signals have been defined:

- $u(t)$ – a control signal which affects the system to achieve a desired behavior
- $y(t)$ – an output signal which serves to assess whether or not the control achieved the desired goal
- $x(t)$ – system state variables
- $z(t)$ – disturbances.

Control can be open loop or closed loop. In the case of open-loop control (Figure 1.2a), control signals are created by a control device which tries to achieve a desired system behavior without obtaining any information about the output signals. Such control makes sense only when it is possible to predict the shape of output signals from the...
control signals. However, if there are additional disturbances which are not a part of the control, then their action may lead to the control objective not being achieved.

In the case of closed-loop control (Figure 1.2b), control signals are chosen based on the control task and knowledge of the system output signals describing whether the control task has been achieved. Hence the control is a function of its effects and acts until the control task has been achieved.

Closed-loop control is referred to as feedback control or regulation. The control device is then called a regulator and the path connecting the output signals with the control device (regulator) is called the feedback loop.

A nonlinear dynamic system with its control can be generally described by the following set of algebraic and differential equations

\[ \dot{x} = F(x, u) \quad \text{and} \quad y = G(x, u) \]  \hspace{1cm} (1.3)

while a linear dynamic system model is

\[ \dot{x} = Ax + Bu \quad \text{and} \quad y = Cx + Du \]  \hspace{1cm} (1.4)

It is easy to show that, for small changes in state variables and output and control signals, Eqs. (1.4) are linear approximations of nonlinear Eqs. (1.3). In other words linearization of (1.3) leads to equations

\[ \Delta \dot{x} = A\Delta x + B\Delta u \quad \text{and} \quad \Delta y = C\Delta x + D\Delta u \]  \hspace{1cm} (1.5)

where \( A, B, C, \) and \( D \) are the matrices of derivatives of functions \( F \) and \( G \) with respect to \( x \) and \( u \).

### 1.2 Classification of Power System Dynamics

An electrical power system (EPS) consists of many individual elements connected together to form a large complex and dynamic system capable of generating, transmitting, and distributing electrical energy over a large geographical area. Because of this interconnection of elements, a large variety of dynamic interactions are possible, some of which will only affect some of the elements, others fragments of the system, while others may affect the system as a whole. As each dynamic effect displays certain unique features, power system dynamics can be conveniently divided into groups characterized by their cause, consequence, timeframe, physical character, or the place in the system where they occur.

Of prime concern is the way the EPS will respond to both a changing power demand and to various types of disturbance, the two main causes of power system dynamics. A changing power demand introduces a wide spectrum of dynamic changes into the system, each of which occurs on a different timescale. In this context the fastest dynamics are due to sudden changes in demand and are associated with the transfer of energy between the rotating masses in the generators and the loads. Slightly slower are the voltage and frequency control actions needed to maintain system operating conditions until finally the very slow dynamics corresponding to the way in which the generation is adjusted to meet the slow daily demand variations take effect. Similarly, the way in which the system responds to disturbances also covers a wide spectrum of dynamics and associated timeframes. In this case the fastest dynamics are those associated with the very fast wave phenomena that occur in high-voltage transmission lines. These are followed by fast electromagnetic changes in the electrical machines themselves before the
relatively slow electromechanical rotor oscillations occur. Finally, the very slow prime mover and automatic generation control actions take effect.

Based on their physical character, the different power system dynamics may be divided into four groups defined as: wave, electromagnetic, electromechanical, and thermodynamic. This classification also corresponds to the timeframe involved and is shown in Figure 1.3. Although this broad classification is convenient, it is by no means absolute, with some of the dynamics belonging to two or more groups while others lie on the boundary between groups. Figure 1.3 shows the fastest dynamics to be the wave effects, or surges, in high-voltage transmission lines and correspond to the propagation of electromagnetic waves caused by lightning strikes or switching operations. The timeframe of these dynamics is from microseconds to milliseconds. Much slower are the electromagnetic dynamics that take place in the machine windings following a disturbance, operation of the protection system, or the interaction between the electrical machines and the network. Their timeframe is from milliseconds to a second. Slower still are the electromechanical dynamics due to the oscillation of the rotating masses of the generators and motors that occur following a disturbance, operation of the protection system, and voltage and prime mover control. The timeframe of these dynamics is from seconds to several seconds. The slowest dynamics are the thermodynamic changes which result from boiler control action in steam power plants as the demands of the automatic generation control are implemented.

Careful inspection of Figure 1.3 shows the classification of power system dynamics with respect to timeframe to be closely related to where the dynamics occur within the system. For example, moving from the left to right along the timescale in Figure 1.1 corresponds to moving through the EPS from the electrical RLC circuits of the transmission network, through the generator armature windings to the field and damper winding, then along the generator rotor to the turbine until finally the boiler is reached.

The fast wave phenomena, caused by lightning and switching overvoltages, occur almost exclusively in the network and basically do not propagate beyond the transformer windings. The electromagnetic phenomena mainly involve the generator armature and damper windings and partly the network. These electromechanical phenomena, namely the rotor oscillations and accompanying network power swings, mainly involve the rotor field and damper windings and the rotor inertia. As the power system network connects the generators together this enables interactions between swinging generator rotors to take place. An important role is played here by the automatic voltage control and the prime mover control. Slightly slower than the electromechanical phenomena are the frequency oscillations, in which the rotor dynamics still play an important part, but are influenced to a much greater extent by the action of the turbine governing systems and the automatic generation control. Automatic generation control also influences the thermodynamic changes through the boiler control action in steam power plants.

The fact that the timeframe of the dynamic phenomena is closely related to where it occurs within the EPS has important consequences for the modeling of the system elements. In particular, moving from left to right along
Figure 1.3 corresponds to a reduction in the accuracy required in the models used to represent the network elements, but an increase in the accuracy in the models used first to represent the electrical components of the generating unit and then, further to the right, the mechanical and thermal parts of the unit. This important fact is taken into account in the general structure of this book when later chapters describe the different power system dynamic phenomena.

1.3 Two Pairs of Important Quantities

This book is devoted to the analysis of electromechanical phenomena and control processes in power systems. The main elements of electrical power networks are transmission lines and transformers which are usually modeled by four-terminal (two-port) RLC elements. Those models are connected together according to the network configuration to form a network diagram.

For further use in this book, some general relationships will be derived below for a two-port $\pi$-equivalent circuit in which the series branch consists of only an inductance and the shunt branch is completely neglected. The equivalent circuit and the phasor diagram of such an element are shown in Figure 1.4a. The voltages $V$ and $E$ are phase voltages, while $P$ and $Q$ are single-phase powers. The phasor $E$ has been obtained by adding voltage drop $jXL$ perpendicular to $I$, to the voltage $V$. The triangles OAD and BAC are similar. Analyzing triangles BAC and OBC gives

$$|BC| = XI \cos \varphi = E \sin \delta \quad \text{hence} \quad I \cos \varphi = \frac{E}{X} \sin \delta$$

(1.6)

$$|AC| = XI \sin \varphi = E \cos \delta - V \quad \text{hence} \quad I \sin \varphi = \frac{E}{X} \cos \delta - \frac{V}{X}$$

(1.7)

Active (real) power leaving the element is expressed as $P = VI \cos \varphi$. Substituting (1.6) into that equation gives

$$P = \frac{EV}{X} \sin \delta$$

(1.8)

This equation shows that active power $P$ depends on the product of phase voltages and the sine of the angle $\delta$ between their phasors. In power networks, node voltages must be within a small percentage of their nominal values. Hence such small variations cannot influence the value of active power. The conclusion is that large changes of active power, from negative to positive values, correspond to changes in the sine of the angle $\delta$. The characteristic $P(\delta)$ is referred to as the power-angle characteristic, while the angle $\delta$ is referred to as the power angle or the load angle. Because of stability considerations discussed in Chapter 5, the system can operate only in that part of the characteristic which is shown by a solid line in Figure 1.4b. The smaller the reactance $X$, the higher the amplitude of the characteristic.
The per-phase reactive power leaving the element is expressed as \( Q = VI \sin \phi \). Substituting (1.7) into that equation gives

\[
Q = \frac{EV}{X} \cos \delta - \frac{V^2}{X} \tag{1.9}
\]

The term \( \cos \delta \) is determined by the value of active power because the relationship between the sine and cosine is \( \cos \delta = \sqrt{1 - \sin^2 \delta} \). Using that equation and (1.8) gives

\[
Q = \sqrt{\left(\frac{EV}{X}\right)^2 - P^2} - \frac{V^2}{X} \tag{1.10}
\]

The characteristic \( Q(V) \) corresponds to an inverted parabola (Figure 1.4b). Because of the stability considerations discussed in Chapter 8, the system can operate only in that part of the characteristic which is shown by a solid line. The smaller the reactance \( X \), the steeper the parabola, and even small changes in \( V \) cause large changes in reactive power. Obviously the inverse relationship also takes place: a change in reactive power causes a change in voltage.

The above analysis points out that \( Q, V \) and \( P, \delta \) form two pairs of strongly connected variables. Hence one should always remember that voltage control strongly influences reactive power flows and vice versa. Similarly, when talking about active power \( P \) one should remember that it is connected with angle \( \delta \). That angle is also strongly connected with system frequency \( f \), as discussed later in the book. Hence the pair \( P, f \) is also strongly connected and important for understanding power system operation.

### 1.4 Stability of a Power System

Power system stability is understood as the ability to regain an equilibrium state after being subjected to a physical disturbance. Section 1.3 showed that three quantities are important for power system operation: (i) angles of nodal voltages \( \delta \) also called power or load angles; (ii) frequency \( f \); and (iii) nodal voltage magnitudes \( V \). Those quantities are especially important from the point of view of defining and classifying power system stability. Hence power system stability can be divided (Figure 1.5) into: (i) rotor (or power) angle stability; (ii) frequency stability; and (iii) voltage stability.

![Figure 1.5](image)

**Figure 1.5** Classification of power system stability (based on CIGRE Report No. 325). Source: Reproduced by permission of CIGRE.
As power systems are nonlinear, their stability depends on both the initial conditions and the size of a disturbance. Consequently, angle and voltage stability can be divided into small- and large-disturbance stability. Small disturbance angle stability is also referred to as the steady-state stability.

Power system stability is mainly connected with electromechanical phenomena (Figure 1.3). However, it is also affected by fast electromagnetic phenomena and slow thermodynamic phenomena. Hence, depending on the type of phenomena, one can refer to short-term stability and long-term stability. All of them are discussed in detail in this book.

1.5 Security of a Power System

A set of imminent disturbances is referred to as contingencies. Power system security is understood as the ability of the EPS to survive plausible contingencies without interruption to customer service. Power system security and power system stability are related terms. Stability is an important factor of power system security, but security is a wider term than stability. Security not only includes stability but also encompasses the integrity of an EPS and assessment of the equilibrium state from the point of view of overloads, under- or overvoltages and underfrequency.

From the point of view of power system security, the operating states may be classified as in Figure 1.6. Most authors credit Dy Liacco (1968) for defining and classifying these states.

In the normal state, an EPS satisfies the power demand of all the customers, all the quantities important for power system operation assume values within their technical constraints, and the system is able to withstand any plausible contingencies.

The alert state arises when some quantities that are important for power system operation (e.g. line currents or nodal voltages) exceed their technical constraints due to an unexpected rise in demand or a severe contingency, but the EPS is still intact and supplies its customers. In that state a further increase in demand or another contingency may threaten power system operation and preventive actions must be undertaken to restore the system to its normal state.

In the emergency state the EPS is still intact and supplies its customers but the violation of constraints is more severe. The emergency state usually follows the alert state when preventive actions have not been undertaken or have not been successful. A power system may assume the emergency state directly from the normal state following unusually severe contingencies like multiple faults. When a system is in the emergency state, it is necessary to undertake effective corrective actions leading first to the alert state and then to the normal state.

A power system can transpose to the in extremis state from the emergency state if no corrective actions have been undertaken and the system is already not intact due to a reduction of power supply following load shedding or when generators were tripped because of a lack of synchronism. The extreme variant of that state is a partial or complete blackout.

To return an EPS from an in extremis state to an alert or normal state, a restorative state is necessary in which power system operators perform control actions in order to reconnect all the facilities and restore all system loads.

Assessment of power system security can be divided into static and dynamic security. Static security assessment (SSA) includes the following computational methods:

- for the pre-contingency states, determine the available transfer capability of transmission links and identify network congestion;
- for the post-contingency states, verify the bus voltages and line power flow limits.

Figure 1.6 Classification of power system operating states (based on CIGRE Report No. 325). Source: Reproduced by permission of CIGRE.
Those tasks of SSA have always been the subject of great interest for power dispatching centers. However, when
the industry was still vertically integrated (see Chapter 2), security management was relatively easy to execute
because any decisions affecting the outputs or control settings of power plants could be implemented internally
within a utility controlling both generation and transmission. Security management is not that easy to execute in
the unbundled industry structure when the system operator has no direct control of generation. Any decisions
affecting outputs or control settings of power plants have to be implemented using commercial agreements with
power plants or enforced through the Grid Code. Especially, the analysis of available transfer capacity and con-
gestion management have important implications for power plants as they directly affect their outputs, and there-
fore revenues.

SSA methods assume that every transition from the pre- to post-contingency state takes place without suffering
any instability phenomena. Dynamic security assessment (DSA) includes methods to evaluate stability and quality
of the transition from the pre- to post-contingency state. Typical criteria for DSA include:

1) rotor (power) angle stability, voltage stability, frequency stability;
2) frequency excursion during the dynamic state (dip or rise) beyond specified threshold levels;
3) voltage excursion during the dynamic state (dip or rise) beyond specified threshold levels;
4) damping of power swings inside subsystems and between subsystems on an interconnected network.

Criteria (1) and (2) are assessed using computer programs executing transient security assessment (TSA). Crite-
rion (3) is assessed by programs executing voltage security assessment (VSA), and criterion (4) is assessed using
programs executing small-signal stability assessment (SSSA).

Recent years have seen a number of total and partial blackouts in many countries of the world. These events have
spurred a renewed interest among system operators in the tools for SSA and DSA. There are a variety of online DSA
architectures. Figure 1.7 shows an example of the DSA architecture. The main components are denoted by boxes
drawn with dashed lines.

![Diagram of DSA Architecture](image)

**Figure 1.7** Components of DSA according to CIGRE Report No. 325. Source: Reproduced by permission of CIGRE.
The task of the component “measurement” is online data acquisition and taking a snapshot of power system conditions. Supervisory control and data acquisition (SCADA) systems usually collect measurements of real and reactive power in network branches, busbar voltages, frequency at a few locations in the system, status of switchgear, and the position of tap changers of transformers. As is shown in Section 2.7, new SCADA systems are often augmented by phasor measurement units (PMUs) collecting synchronized voltage phasor measurements.

The “modeling” component uses online data from the “measurement” component and augments them with offline data, obtained from a database, describing the parameters of power system elements and contingencies to be analyzed. The task of the “modeling” component is to create an online power system model using the identification of the power system configuration and state estimation. That component may also contain computer programs for the creation of equivalent models of neighboring systems. Contingencies vary according to the type of security being examined and in general need to be able to cater for a variety of events like short-circuits at any location, opening any line or transformer, loss of the largest generating unit or largest load in a region, multiple faults (when considered to be credible), and so on.

The next important component is “computation.” Its task is system model validation and security assessment. The accuracy of the security assessment depends on the quality of the system model. Offline data delivered to the “modeling” component are validated through field testing of devices. Online data of the network configuration and system state obtained from the “measuring” component are validated using bad measurement data identification and removal which is made possible by redundancy of measurements. The best methodology for power system model validation is via a comparison of simulation results of the dynamic response of the EPS with recorded responses following some disturbances. To achieve this, the “measurement” component sends data from disturbance recorders to the “computation” component. The tools for the security assessment consist of a number of computer programs executing voltage stability analysis, small-signal stability analysis, transient stability analysis by hybrid methods combining system simulation, and the direct Lyapunov method described in the textbook by Pavella et al. (2000). Intelligent systems are also used employing learning from the situations previously seen.

The “reporting and visualization” component is very important for a system operator employing the described architecture. Computer programs of the “computation” component process a huge amount of data and analyze a large number of variants. On the other hand, the operator must receive a minimum number of results displayed in the most synthetic, preferably graphic, way. Some DSA displays have been shown in CIGRE Report No. 325. If the EPS is in a normal state, the synthetic results should report how close the system is to an insecure state to give the operator an idea of what might happen. If the system moves to an alert state or to an emergency state, the displayed result should also contain information about preventive or corrective action. This information is passed on to the “control” component.

The “control” component assists the operator in preventive and corrective actions executed to improve the power system operation. Some information produced by security assessment programs may be used to produce remedial control actions, which can be automatically executed by real-time control. The description of the current state of the art in DSA can be found in CIGRE Report No. 325.