# Contents

**Series Editor’s Foreword** xiii

**Preface** xv

**Acknowledgments** xix

**List of Abbreviations** xxi

**About the Companion Website** xxiii

## 1 Polarization of Monochromatic Waves. Background of the Jones Matrix Methods. The Jones Calculus 1

1.1 Homogeneous Waves in Isotropic Media 1

1.1.1 Plane Waves 1

1.1.2 Polarization. Jones Vectors 3

1.1.3 Coordinate Transformation Rules for Jones Vectors. Orthogonal Polarizations. Decomposition of a Wave into Two Orthogonally Polarized Waves 9

1.2 Interface Optics for Isotropic Media 14

1.2.1 Fresnel’s Formulas. Snell’s Law 14

1.2.2 Reflection and Transmission Jones Matrices for a Plane Interface between Isotropic Media 20

1.3 Wave Propagation in Anisotropic Media 23

1.3.1 Wave Equations 23

1.3.2 Waves in a Uniaxial Layer 25

1.3.3 A Simple Birefringent Layer and Its Principal Axes 30

1.3.4 Transmission Jones Matrices of a Simple Birefringent Layer at Normal Incidence 32

1.3.5 Linear Retarders 36

1.3.6 Jones Matrices of Absorptive Polarizers. Ideal Polarizer 38

1.4 Jones Calculus 41

1.4.1 Basic Principles of the Jones Calculus 42

1.4.2 Three Useful Theorems for Transmissive Systems 46

1.4.3 Reciprocity Relations. Jones’s Reversibility Theorem 50

1.4.4 Theorem of Polarization Reversibility for Systems Without Diattenuation 53

1.4.5 Particular Variants of Application of the Jones Calculus. Cartesian Jones Vectors for Wave Fields in Anisotropic Media 55

**References** 57
2 The Jones Calculus: Solutions for Ideal Twisted Structures and Their Applications in LCD Optics 59
2.1 Jones Matrix and Eigenmodes of a Liquid Crystal Layer with an Ideal Twisted Structure 59
2.2 LCD Optics and the Gooch–Tarry Formulas 64
2.3 Interactive Simulation 67
2.4 Parameter Space 69
References 73

3 Optical Equivalence Theorem 75
3.1 General Optical Equivalence Theorem 75
3.2 Optical Equivalence for the Twisted Nematic Liquid Crystal Cell 77
3.3 Polarization Conserving Modes 77
3.3.1 LP1 Modes 78
3.3.2 LP2 Modes 79
3.3.3 LP3 Modes 80
3.3.4 CP Modes 81
3.4 Application to Nematic Bistable LCDs 82
3.4.1 2π Bistable TN Displays 82
3.4.2 π Bistable TN Displays 83
3.5 Application to Reflective Displays 84
3.6 Measurement of Characteristic Parameters of an LC Cell 86
3.6.1 Characteristic Angle Ω 86
3.6.2 Characteristic Phase Γ 87
References 87

4 Electro-optical Modes: Practical Examples of LCD Modeling and Optimization 91
4.1 Optimization of LCD Performance in Various Electro-optical Modes 91
4.1.1 Electrically Controlled Birefringence 91
4.1.2 Twist Effect 101
4.1.3 Supertwist Effect 109
4.1.4 Optimization of Optical Performance of Reflective LCDs 116
4.2 Transreflective LCDs 119
4.2.1 Dual-Mode Single-Cell-Gap Approach 119
4.2.2 Single-Mode Single-Cell-Gap Approach 122
4.3 Total Internal Reflection Mode 124
4.4 Ferroelectric LCDs 131
4.4.1 Basic Physical Properties 131
4.4.2 Electro-optical Effects in FLC Cells 135
4.5 Birefringent Color Generation in Dichromatic Reflective FLCDs 145
References 149

5 Necessary Mathematics, Radiometric Terms, Conventions, Various Stokes and Jones Vectors 153
5.1 Some Definitions and Relations from Matrix Algebra 153
5.1.1 General Definitions 153
5.1.2 Some Important Properties of Matrix Products 160
5.1.3 Unitary Matrices, Unimodular Unitary 2 × 2 Matrices, STU Matrices 160
5.1.4 Norms of Vectors and Matrices 163
5.1.5 Kronecker Product of Matrices 166
5.1.6 Approximations 167
## Contents

8.3 Berreman’s Method  
8.3.1 Transfer Matrices  
8.3.2 Transfer Matrix of a Homogeneous Layer  
8.3.3 Transfer Matrix of a Smoothly Inhomogeneous Layer, Staircase Approximation  
8.3.4 Coordinate Systems  
8.4 Simplifications, Useful Relations, and Advanced Techniques  
8.4.1 Orthogonality Relations and Other Useful Relations for Eigenwave Bases  
8.4.2 Simple General Formulas for Transmission Operators of Interfaces  
8.4.3 Calculation of Transmission and Reflection Operators of Layered Systems by Using the Adding Technique  
8.5 Transmissivities and Reflectivities  
8.6 Mathematical Properties of Transfer Matrices and Transmission and Reflection EW Jones Matrices of Lossless Media and Reciprocal Media  
8.6.1 Properties of Matrix Operators for Nonabsorbing Regions  
8.6.2 Properties of Matrix Operators for Reciprocal Regions  
8.7 Calculation of EW $4 \times 4$ Transfer Matrices for LC Layers  
8.8 Transformation of the Elements of EW Jones Vectors and EW Jones Matrices Under Changes of Eigenwave Bases  
8.8.1 Coordinates of the EW Jones Vector of a Wave Field in Different Eigenwave Bases  
8.8.2 EW Jones Operators in Different Eigenwave Bases  
References  

9 Choice of Eigenwave Bases for Isotropic, Uniaxial, and Biaxial Media  
9.1 General Aspects of EWB Specification. EWB-generating routines  
9.2 Isotropic Media  
9.3 Uniaxial Media  
9.4 Biaxial Media  
References  

10 Efficient Methods for Calculating Optical Characteristics of Layered Systems for Quasimonochromatic Incident Light. Main Routines of LMOPTICS Library  
10.1 EW Stokes Vectors and EW Mueller Matrices  
10.2 Calculation of the EW Mueller Matrices of the Overall Transmission and Reflection of a System Consisting of “Thin” and “Thick” Layers  
10.3 Main Routines of LMOPTICS  
10.3.1 Routines for Computing $4 \times 4$ Transfer Matrices and EW Jones Matrices  
10.3.2 Routines for Computing EW Mueller Matrices  
10.3.3 Other Useful Routines  
References  

11 Calculation of Transmission Characteristics of Inhomogeneous Liquid Crystal Layers with Negligible Bulk Reflection  
11.1 Application of Jones Matrix Methods to Inhomogeneous LC Layers  
11.1.1 Calculation of Transmission Jones Matrices of LC Layers Using the Classical Jones Calculus  
11.1.2 Extended Jones Matrix Methods  
11.2 NBRA. Basic Differential Equations  
11.3 NBRA. Numerical Methods  
11.3.1 Approximating Multilayer Method  
References
A.4 Transflective LCD
  A.4.1 Vertical Aligned Nematic Cell 525
A.5 Switchable Viewing Angle LCD 535
A.6 Optimal e-paper Configurations 535
A.7 Color Filter Optimization 536
  References 536

B Some Derivations and Examples 537
B.1 Conservation Law for Energy Flux 537
B.2 Lorentz’s Lemma 538
B.3 Nonexponential Waves 538
B.4 To the Power Series Method (Section 11.3.3) 540
B.5 One of the Ways to Obtain the Explicit Expressions for Transmission Jones Matrices of an Ideal Twisted LC Layer 541
  Reference 543

Index 545