“People aren’t friends till they have said all they can say, and are able to sit together, at work or rest, hour-long without speaking . . .”
The practice of finance brings with it risk exposures that have to be understood and managed, for all participants. Banks are (or should be) at the forefront of this discipline. In fact the art of banking is the art of managing risk on both sides of the balance sheet. In this chapter, we introduce the “universe” of risk for banks, and develop the individual segments in subsequent chapters. In addition, there is a primer on the value-at-risk (VaR) risk measurement tool. Many regulators request the banks they supervise to employ the VaR technique when calculating risk exposure and thereby their minimum capital requirement, and as such it is worthwhile being aware of the methodology, the assumptions behind it, and its weaknesses.

THE RISK MANAGEMENT UNIVERSE FOR BANKS

It is sensible to group balance sheet related risks in banking separate from other risks. The former includes the main one for banks, credit risk, as well as market risk, non-traded market risk, and liquidity risk. The latter would include technology risk, operational risk, and conduct risk.

Introduction

Banks are by their nature risk-taking institutions and as far as possible seek to meet the precise needs of their customers. This “business as usual” gives rise to balance sheet risk exposure which must be managed. Essentially banks offer the lending terms, maturities, rate options, currency, optionality, and contingencies demanded by their clients, and take on the range of risks associated with providing these services. Because banks have a wide range of clients with varying borrowing and deposit requirements, exposures may to some degree offset each other, but will not match completely in terms of timing, amount, and currency.

1 This section was co-written with Ed Bace and Peter Eisenhardt.
Banks must have the scale, systems, expertise and staff to concentrate on financial risk. Throughout history, banks have generally charged their clients sufficiently for taking on the range of risks and consequently earned profits. Banks can match fund loans to maturity immediately to offset interest-rate risks, which sometimes negates profitability. Banks can also choose to hedge through a range of mechanisms. Finally, balance sheet positions can be left open with risk retained if the view is taken that markets will move in a direction favourable to the exposure, and be reduced or offset in the course of new business.

In this chapter, we take a high-level view of risk management before detailing the risks in subsequent chapters. Specifically, we will focus on the aggregation of risk, concentration and correlation, determination of risk tolerance and its translation into limits, and means of risk mitigation.

**Risk Management: High Level**

A bank is defined by its risk management approach and framework. It is integral to strategy, and must be determined at the Board and senior management level. The regulatory framework merely sets out minimum standards. Management must use all its experience and knowledge to establish risk policies that fit the bank’s markets and capabilities, by means of:

- Defining a business model based on competitive strengths in markets and products;
- Identifying and understanding the risks and returns of the model, and reviewing and adjusting constantly as conditions change and markets evolve;
- Quantifying the risks and developing and maintaining risk management monitoring capabilities supported by robust IT systems;
- Constructing a broad array of possible techniques for mitigating risks;
- Instilling a strong culture of risk management and ensuring objectives and the risk strategy are understood at all levels of the bank.

Risk oversight is the responsibility of the Board and stands apart from risk management, which is the responsibility of management. Internal audit teams are important in risk management, focusing on protecting against fraud and the preparation of sound financial reports. However, the Board is accountable ultimately for all aspects of risk management at every level. Good practice dictates that the Board defines risk management issues that will always require direct elevation for its attention. In other words, the Board must own every aspect of risk tolerance and risk appetite for the bank, from credit risk on origination through to market risk and operational risk as a business-as-usual (BAU) function. This culminates in a formal risk appetite and guidance statement that every member of the Board must sign up to and must maintain responsibility for.
The reward system of the bank must encourage employees to seek out business and realise returns without taking excessive or disproportionate risks. It is crucial that banks have a strong independent risk function to ensure “producers” (lending officers, sales teams, and traders) keep revenues and new business in balance with risk. This risk function must be backed up and fully supported by senior management. A key challenge is to ensure consistency across all areas, so that perceived banking “stars” or over-hyped “businesses of the future” are held to the same risk–reward standards.

Figure 5.1 is a stylised view of the central risk management function in a bank.

**FIGURE 5.1** Overview of a risk management framework
*Source: The Principles of Banking (2012).*

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**Aggregation of Risk**

Banks must place sufficient emphasis on their processes to identify, monitor, and aggregate risk. Action cannot be taken to dispose of or mitigate risk and limit losses without a complete picture. It is inefficient to take aggressive steps to deal with risk in individual parts of the bank when these risks may actually have some offset in others, so that the need for selling or hedging is limited. This includes lending, trading, funding, interest-rate risk exposure, and off-balance sheet businesses.

In order to understand both the diversification and correlation of the portfolio, and likely outcomes in various environments, the bank requires
aggregate data. In times of economic stress, regulators and other banks cannot help, and potential merger partners with fresh capital cannot be found, without comprehensive management information systems to assess the situation in a timely manner.

The Financial Stability Board issued a report in 2011 noting that aggregate data reporting at global systemically important banks was inadequate, and set a deadline of 2016 to meet supervisory expectations. Complex group structures and businesses ranging across legal entities and regions must not hinder risk data aggregation.

Risk aggregation models include:

- **Summation**: add together individual capital components;
- **Constant diversification**: subtract fixed risk percentage from summation to reflect diversification;
- **Variance-Covariance**: components weighted based on bilateral correlation;
- **Copulas**: use of multi-variate probability theory to evaluate marginal risks;
- **Full modelling**: simulate impact of multiple risk factors on all components.

Banks must consider clarity and simplicity, IT cost, non-linearity of risks, and over-reliance on assumptions in assessing the most appropriate of these models.

The simplest and most traditional approach in a credit risk setting is to consider just two scenarios, default and no default. A binomial tree is built over a number of time periods until maturity. A more advanced approach was, however, developed, where the observable market prices, data for bonds, indices for credit rating and sector categories were used to compute a volatility value for the credit portfolio.

Variance-Covariance value-at-risk (VaR) was introduced in the mid-1990s and quickly became popular, revolutionising credit portfolio risk management. The model joined previous market risk methodologies to estimate portfolio VaR to include upgrades and downgrades, as well as defaults. Credit risk monitoring shifted to a more mark-to-market approach.

Risk management models must be effective not only in normal conditions, but also in extreme situations. However, in the past a VaR confidence level was generally calibrated to only two standard deviations (95%). This was not effective in the extreme markets of the late 1990s with financial crises in Asia and Russia. Tail risk (three standard deviations beyond the mean) must also be captured and monitored. Risks with disproportionately high probabilities of extreme outcomes are referred to as having “fat tails”.

Nasim Taleb’s book, *The Black Swan: The Impact of the Highly Improbable*, is a widely discussed and popular study of the extreme impact of unpredictable outlier events. That said, the book offers very little if anything in the
way of practical guidance as to how the impact of such tail events can be mitigated in practice. For this reason, the most important aspect of the risk management culture at a bank must be that every bank must know its risk and understand the nature of its exposure to economic fluctuations to a sufficient degree of familiarity that it can mitigate the exposures in a satisfactory manner.

Any model is dependent on assumptions, which are generally based on historical experience. The key assumption in using historical experience is that this is representative of future experience. Numbers generated by a model therefore cannot be taken as the final assessment of risk. One of the worst assumptions in banking history was that, because there had never been a nation-wide housing slump, a portfolio of US retail mortgages could be made safe by diversifying across regions. This helped lead to the creation of high volumes of mortgage-backed securities that suffered large financial losses.

Banks may choose to compare and benchmark their risk management against peers and competitors to validate models. While helpful, adherence to the same methodologies and assumptions can lead to systemic (as opposed to idiosyncratic) risk. Homogeneity in credit risk assumptions is in part addressed by the BIS by allowing banks to assign their own internal ratings to individual credits.

Management judgement is also an important element of risk management, as not all risks can be quantified using mathematical and statistical techniques. Some risks, such as reputational risk, are largely unquantifiable.

Regulatory capital requirements address diversification, concentration, and correlation risk to some extent; however, sound judgements by experienced management can be used to provide greater insights into these risks.

**Diversification**

The concept of diversification is intuitive and well known to all, as evidenced by the popular saying “don’t put all your eggs in one basket”. Diversification helps protect against losses arising from idiosyncratic risk. It does not help with systemic risk, which is what would lead to catastrophic losses or total failure of the financial system. A well-diversified portfolio will perform in line with overall markets and the economy, and have more predictable volatility and manageability.

Diversification can be a problem for specialised and regional banks. Examples in the US include specialised mortgage banks and regional Savings and Loans Associations. Prior to a relaxation of interstate banking laws, many US banks were forced to lend within their states, some of which were dominated by just a few industries.

Diversification is not a solution to all the risks faced by banks. Banks that diversify by lending to sectors and regions where they have limited
expertise, insufficient ability to monitor transactions, and competitive disadvantages may find that there are other risks which arise as a result. This issue would be greater for smaller banks with limited resources. In short, banks must focus on both individual risks and the overall portfolio.

Diversification is not a substitute for quality lending. Although banks have put in place mechanisms, such as credit provisions and risk capital, which allow them to absorb losses up to a certain point, and only those within their estimation of their risk exposure, badly considered diversification can lead to diminished returns or even large losses. Diversification of a portfolio for the purpose of investment returns must be viewed very differently from diversification for credit risk management. Managing the portfolio in terms of investment returns will be more focused on the “upside”, while credit risk management is concerned with the “downside” risk. Default correlation increases significantly in deteriorating market conditions, while asset return correlation is less sensitive.

Diversification analysis will review industry sectors, size, products, and regions. In doing so, methodologies to quantify expected losses given varying degrees of diversification can be a useful tool in decision-making.

**Concentration**

Excessive concentration of credit risk has been identified by global regulators as one of the key causes of bank crises and failures. Traditionally, concentration risk was viewed as the risk of over-exposure to single borrowers, but the emphasis is now on excessive correlation across the entire portfolio. BIS studies have concluded that sector concentrations have had a bigger impact on bank credit portfolios than single name exposures. While name concentration is straightforward, correlation risk management is challenging in terms of both setting appropriate assumptions and performing analytics.

Basel II is based on the Asymptotic Single Risk Factor model, where each asset class (for example, corporate, bank, sovereign, retail) is assigned a single correlation to the overall performance of the economy. No form of concentration risk was considered in the previous calculation of capital requirements, and thus did not fully incentivise diversification. As banks realised the need to more fully analyse correlations in the portfolio, more sophisticated methodologies have been developed.

Historically, many regulatory regimes across the world set a large exposure limit to a single entity or related entities of 25% of a bank’s regulatory capital.

In April 2014, the BIS issued a new standard addressing concentration risk, considering exposure to both instrument-based asset classes (for
example, asset-backed securities and collateralised debt obligations) and systemically important financial institutions (SIFIs) and individual companies. The revised framework will help ensure a common minimum standard for measuring, aggregating, and controlling concentration risk across jurisdictions.

**Correlation**

Even if single name exposures are limited, and transactions are evaluated and structured on a sound basis, management must make correlation assessments on the aggregate portfolio. If different borrowers are sensitive to the same changes in economic, business, regulatory, financial market, and political conditions, a much larger exposure akin to a single large borrower is created, which can result in heavy losses.

The US Comptroller of the Currency has identified as highly correlated credit exposures to borrowers, those:

- Related through group structure;
- Dependent on the same guarantor;
- Dependent on the selling of the same manufacturer’s product;
- In the same industry or economic sector;
- In the financial sector;
- Within a geographic area dominated by few business enterprises;
- Owned by a foreign government;
- Secured by a common debt or equity instrument.

Product areas where correlation risk has been identified by global regulators as having created large concentration risk include retail products (for example, credit cards, home equity), leveraged loans, CDOs, and commercial real estate.

Once pools of correlated risks are identified, it is necessary to examine whether there is further correlation across those pools. As one example, there could be a correlation between transportation and hotel sector exposures.

For maximum diversification, banks would construct a loan portfolio with the lowest possible correlations. The lower the correlation, the fewer the number of loans needed for a diverse portfolio. New loans might increase diversification. But again, this must be balanced against expertise and competitive position in markets and clients.

It should also be noted that correlation is difficult to measure as a result of data constraints. For example, in order to determine correlation between asset classes and default rates for an asset class and the economy as accurately as possible, one would need data points for all possible scenarios. This
would include periods of low, medium, and high economic stress, as well as corresponding default rate data. In reality, this data is difficult to obtain and validate, and often does not contain sufficient information around extreme scenarios to allow for accurate calibration of correlation.

Risk Management Tolerance and Parameters

The risk appetite of a bank is driven to a large extent by the size of its free or available capital, and to a material extent by its funding model and its ability to raise sufficient levels of long-term (or stable) liquidity. Banks must maintain minimum regulatory capital ratios. Any strategy must be based on having enough capital to meet these requirements in even the most challenging environments. However, banks must not stop at maintaining minimum regulatory capital, but should also perform a thorough analysis of their business model and capital needs under various scenarios. The amount of capital required as assessed by a bank is known as risk capital or economic capital.

A risk appetite statement must be coordinated by the risk management team and include senior management before being approved by the Board. Shareholders, regulators, and rating agencies will be part of the process. Parameters are likely to include:

- Desired External Risk Rating;
- Risk-adjusted Return on Capital (RAROC) hurdle rate for new business;
- Weighted Average Portfolio Rating;
- Relative internal capital allocation to Credit, Interest Rate, Market and Operational Risk;
- Concentration Thresholds (Obligor, Sector, Country);
- Asset–Liability Management mismatch;
- Liquidity Thresholds and metrics;
- Connection to Business Strategy;
- Acceptable earnings volatility;
- Controlled or undesirable exposures;
- Targeted levels of Risk Asset Ratios or leverage.

The statement will anticipate scenarios of market stress and plan possible responses. Mechanisms for early identification and escalation of limits breaches also need to be constructed.

Allocation of Risk Based Capital

Allocation of risk based capital is not only about risk, but must be linked also to potential rewards. In order to increase returns, the bank will need to take on additional risk, and it is necessary to
optimise risk capital allocation to the portfolio in seeking higher profitability. Banker and trader pay must be linked not only to returns, but also to the amount of risk based capital used in its generation, as higher risk positions attract higher regulatory and economic capital. In assessing returns, banks must consider potential “ancillary” business (for example, investment banking and custody fees) which might be realised as the client relationship is strengthened through the extension of credit.

Capital must be allocated with a view towards sustainability. Business plans must anticipate ongoing capital needs, costs, and availability, without which growth and continued competitiveness will not be attainable. Consideration must be given to the time value of money, in that businesses generate cash flows and returns in differing time frames. Management must consider tax consequences and varying rates when assessing returns.

In allocating capital, banks can perform a comparison between the economic capital and regulatory capital charges applicable to different areas of their business. In doing so, they can identify those areas of their business which attract high amounts of regulatory capital, but low amounts of economic capital. Management would then need to understand why the regulator believes this area of business to be significantly more risky than the bank does. Similarly, the bank can identify those areas of their business which attract low amounts of regulatory capital, but high amounts of economic capital. Performing this type of analysis will help the bank with strategic business planning and in setting growth and performance targets for the future.

**Setting Loan Sanctioning Criteria**

New loans must be sanctioned only within the context of the risk appetite statement and risk weighted capital criteria. For corporate and wholesale banking, a request to provide credit should involve the completion of a form to be submitted to a separate entity from the relationship bankers, whether it be a credit committee, or (in the case of smaller banks) executive management and ALCO. The request will contain all the information needed for the committee to discuss the risk and return of the loan, terms, and the client, so a determination can be made as to whether it fits into the broader strategy. For retail banking, new loans can be granted via automated applications, which will be approved or referred by loan origination officers.

“Know Your Customer” (KYC) is a key element of the loan origination process. For corporate and wholesale banking, relationship bankers should have an extensive dialogue with clients to have a thorough understanding of their business, strategy, and risks. The extension of credit will be only part of an ongoing process. Any entity is likely to undergo some difficulties
over the term of a loan, and good communication will enable the bank to work with the borrower. For retail banking, the bank will maintain behavioural scorecards and delinquency information, which will enable them to monitor loan performance and take action should the loan be at risk of defaulting.

Beyond the credit risk of the borrower and the rate charged, the committee must also evaluate the specifics of the loan, these specifics being:

- Legal framework and jurisdiction risks;
- Collateral: acceptable forms and enforceability;
- Disclosure requirements;
- Ability to terminate: material adverse change clauses and covenants;
- Transferability: can the loan be sold?;
- Currencies;
- Fixed or floating rate;
- Pre-payment options.

The Loan Market Association (LMA) strives to improve liquidity, efficiency, and transparency in the primary and secondary syndicated loan markets in Europe, the Middle East, and Africa by issuing recommended standard market practices and documentation.

Survival Models for Credit Risk Management

Just as survival analysis is used in insurance to predict mortality rates, at the same time, duration analysis can be used to predict frequencies of default throughout the economic cycle. An example of survival analysis being used to calculate the Loss Given Default (LGD) parameter is presented below:

1. The probability of an account being written off, given that it has “survived” to a certain duration in default ($P(W|D)$), is calculated for each duration in default. This is calculated as the exposure which is written off at the end of the period divided by the exposure at risk at the start of the period;
2. The next step is to calculate the loss given write-off at that duration in default (LGW). This is calculated as the loss at write-off (which is the difference between the outstanding balance and any recoveries made) divided by the outstanding balance at write-off.

In this way the two parameters, probability of write-off and loss, can be combined to provide an estimate of loss given default that is dependent on the duration spent in default. This curve is usually upward sloping, with loans suffering a higher loss the longer they have remained in default.
Unlike with mortality rates, recurring or repeated event models are required in banking, i.e. “default” is a non-absorbing state from which cure is possible.

**Asset–Liability Modelling for Balance Sheet Management** Actuaries have traditionally been involved in performing asset–liability modelling for life insurers and pension funds. Actuaries determine funding status, cash requirements, and balance sheet positions, and make projections under varying future economic and capital market environments. Projections are deterministic (scenario based) or stochastic (using Monte Carlo techniques). These same skills and techniques are applicable to banks in their Asset–Liability Management process.

**Cash Flow Models for Budgeting and Balance Sheet Management** Cash flow is an important measure of sustainability of a bank or business. Banks must be able to estimate cash flows (timing and severity) in the most extreme scenarios so as to be able to devise a strategy to ensure adequate funding. Understanding cash flows under stressed economic conditions also forms part of a bank’s regulatory reporting requirements.

**Risk Management and Mitigation**

Before focusing on technical and complicated risk management matters, management should always know and be thinking about a few basic questions: what are the bank’s largest individual, sector, product, and regional exposures? In what circumstances should these exposures be reduced or grown? And, very importantly, what are the means and options for reducing these exposures under a range of circumstances? The bank would also want to understand how to grow certain exposures to meet their strategic and business objectives, without taking on excessive risk.

**Positions** The risk portfolio of a bank can be adjusted by monitoring maturing business and adjusting policies for new assets accordingly. New business for sectors can be frozen, reduced, or subjected to stricter standards (for example, increased spread and fees, more collateral, tighter covenants). Emphasis can be placed on booking business in sectors that can be expected to perform in a manner counter to the concentration risk. These sectors may outperform in different phases of the economic cycle (for example, consumer staples are usually relatively strong in a downturn).
A more direct approach is the outright sale of loan and bond positions. Selling has the advantage of immediacy and avoids the risk that a hedging instrument does not perform as expected (basis risk). Liquidity varies based on factors including market size and standardisation, complexity and terms of the debt instrument, and borrower credit rating. In stressed markets, liquidity can be expected to diminish when it is needed most.

**Credit Hedging** Historically, best-practice credit risk management discipline dictated that banks apply a sound credit policy at origination. In other words, prevention is better than cure. This reflects the fact that credit risk is, for practical purposes, difficult to hedge. This remains true in the era of credit derivatives, which (despite being in use since 1994) are used by only a small minority of the world’s banks to mitigate credit risk. It remains the case that the optimum approach to credit risk for most banks involves diversification of the loan book, avoiding concentration with one borrower or sector and the operation of conservative origination principles.

For a more proactive approach, one means of hedging credit risk was to short the bonds of the borrower or similar borrowers. However, it is not always possible to borrow the bonds in sufficient size (if at all), and the repo rate can be prohibitively expensive. Basis risk can again be a challenge. This is not a feasible hedge approach for the vast majority of the world’s banks.

Since the early 1990s, the management and transfer of credit risk has been transformed by the emergence of credit derivatives, which are contracts and instruments that separate and transfer credit or default risk from the lender/noteholder to another party. Credit Default Swaps (CDS) spreads are watched closely by issuers, investors, and banks, and are key to issuance, investment and pricing decisions.

Credit derivatives have been a source of discussion and debate. In the early years, growing pains were experienced in coping with processing of increasing volumes and there were controversies in agreeing exactly when credit events were triggered. Many buyers of protection were dismayed when there was no payout on Greece because default was accepted “voluntarily”. As with all derivatives, greater scrutiny has been brought to bear since the crisis given the size and complexity of the market.

In essence then, managing credit risk is all about sound origination policies, sticking to one’s knitting, and avoiding concentration. Prevention, to reiterate, is better than cure.
The risk management department was one of the fastest growing areas in investment and commercial banks during the 1990s, and again after the crash of 2008. A string of high-profile banking losses and failures, typified by the fall of Barings Bank in 1995, highlighted the importance of risk management to bank managers and shareholders alike. In response to the volatile and complex nature of risks that they were exposed to, banks set up specialist risk management departments, whose functions included both measuring and managing risk. As a value-added function, risk management can assist banks not only in managing risk, but also in understanding the nature of their profit and loss, and so help increase return on capital. It is now accepted that senior directors of banks need to be thoroughly familiar with the concept of risk management. One of the primary tools of the risk manager is value-at-risk (VaR), which is a quantitative measure of the risk exposure of an institution. For a while VaR was regarded as somewhat inaccessible, and only the preserve of mathematicians and quantitative analysts. Although VaR is indeed based on statistical techniques that may be difficult to grasp for the layman, its basic premise can, and should, be explained in straightforward fashion, in a way that enables non-academics to become comfortable with the concept. The problem with VaR is that while it was only ever a measure, based on some strong assumptions, of approximate market risk exposure (it is unsuited to measuring risk exposure in the banking book), it suffers in the eyes of its critics in having the cachet of science. This makes it arcane and inaccessible, while paradoxically being expected to be much more accurate than it was ever claimed to be. Losses suffered by banks during the crash of 2007–08 were much larger than any of their VaR values, which is where the measure comes in for criticism. But we can leave that aside for now, and concentrate just on introducing the technicalities.

Later in the book we describe and explain the calculation and application of VaR. We begin here with a discussion of risk.

DEFINING RISK

Any transaction or undertaking with an element of uncertainty as to its future outcome carries an element of risk: risk can be thought of as uncertainty. To associate particular assets such as equities, bonds or
corporate cash flows with types of risk, we need to define ‘risk’ itself. It is useful to define risk in terms of a risk horizon, the point at which an asset will be realised, or turned into cash. All market participants, including speculators, have an horizon, which may be as short as a half-day. Essentially then, the horizon is the time period relating to the risk being considered.

Once we have established a notion of horizon, a working definition of risk is the uncertainty of the future total cash value of an investment on the investor’s horizon date. This uncertainty arises from many sources. For participants in the financial markets risk is essentially a measure of the volatility of asset returns, although it has a broader definition as being any type of uncertainty as to future outcomes. The types of risk that a bank or securities house is exposed to as part of its operations in the bond and capital markets are characterised below.

THE ELEMENTS OF RISK: CHARACTERISING RISK

Banks and other financial institutions are exposed to a number of risks during the course of normal operations. The different types of risk are broadly characterised as follows:

- **Market risk** – risk arising from movements in prices in financial markets. Examples include foreign exchange (FX) risk, interest rate risk and basis risk. In essence market risk applies to ‘tradeable instruments, ones that are marked-to-market in a trading book, as opposed to assets that are held to maturity, and never formally repriced, in a banking book.
- **Credit risk** – something called issuer risk refers to risk that a customer will default. Examples include sovereign risk, marginal risk and force majeure risk.
- **Liquidity risk** – this refers to two different but related issues: for a Treasury or money markets’ person, it is the risk that a bank has insufficient funding to meet commitments as they arise. That is, the risk that funds cannot be raised in the market as and when required. For a securities or derivatives trader, it is the risk that the market for assets becomes too thin to enable fair and efficient trading to take place. This is the risk that assets cannot be sold or bought as and when required. We should differentiate therefore between funding liquidity and trading liquidity whenever using the expression *liquidity*. 
Operational risk – risk of loss associated with non-financial matters such as fraud, system failure, accidents and ethics. Table 1.1 assigns sources of risk for a range of fixed interest, FX, interest rate derivative and equity products. The classification has assumed a 1-year horizon, but the concepts apply to any time horizon.

Forms of market risk

Market risk reflects the uncertainty as to an asset’s price when it is sold. Market risk is the risk arising from movements in financial market prices. Specific market risks will differ according to the type of asset under consideration:

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**Table 1.1** Characterising risk.

- **Operational risk** – risk of loss associated with non-financial matters such as fraud, system failure, accidents and ethics. Table 1.1 assigns sources of risk for a range of fixed interest, FX, interest rate derivative and equity products. The classification has assumed a 1-year horizon, but the concepts apply to any time horizon.

**Forms of market risk**

Market risk reflects the uncertainty as to an asset’s price when it is sold. Market risk is the risk arising from movements in financial market prices. Specific market risks will differ according to the type of asset under consideration:
• Currency risk – this arises from exposure to movements in FX rates. A version of currency risk is transaction risk, where currency fluctuations affect the proceeds from day-to-day transactions.

• Interest rate risk – this arises from the impact of fluctuating interest rates and will directly affect any entity borrowing or investing funds. The most common exposure is simply to the level of interest rates but some institutions run positions that are exposed to changes in the shape of the yield curve. The basic risk arises from revaluation of the asset after a change in rates.

• Equity risk – this affects anyone holding a portfolio of shares, which will rise and fall with the level of individual share prices and the level of the stock market.

• Other market risk – there are residual market risks which fall in this category. Among these are volatility risk, which affects option traders, and basis risk, which has a wider impact. Basis risk arises whenever one kind of risk exposure is hedged with an instrument that behaves in a similar, but not necessarily identical manner. One example would be a company using 3-month interest rate futures to hedge its commercial paper (CP) programme. Although eurocurrency rates, to which futures prices respond, are well correlated with CP rates, they do not invariably move in lock step. If CP rates moved up by 50 basis points but futures prices dropped by only 35 basis points, the 15-bps gap would be the basis risk in this case.

Other risks

• Liquidity risk – in banking, this refers to the risk that a bank cannot raise funds to refinance loans as the original borrowing becomes past due. It is sometimes also referred to as rollover risk. In other words, it refers to the risk of an inability to continue to raise funds to replace maturing liabilities. There is also another (related) liquidity risk, which refers to trading liquidity. This is the risk that an asset on the balance sheet cannot be sold at a previously perceived fair value, or cannot be sold at all, and hence experiences illiquidity.

• Credit risk – the risk that an obligor (the entity that has borrowed funds from you) defaults on the loan repayments.

• Counterparty risk – all transactions involve one or both parties in counterparty risk, the potential loss that can arise if one party were to default on its obligations. Counterparty risk is
most relevant in the derivatives market, where every contract is
marked-to-market daily and so a positive MTM is taken to the
profit & loss (P&L) account. If the counterparty defaults before
the contract has expired, there is risk that the actual P&L will not
be realized. In the credit derivatives market, a counterparty that
has sold protection on the third-party reference name on the credit
derivative contract and which subsequently defaults will mean the
other side to the trade is no longer protected against the default of
that third party.

• **Reinvestment risk** – if an asset makes any payments before the
  investor’s horizon, whether it matures or not, the cash flows will
  have to be reinvested until the horizon date. Since the reinvestment
  rate is unknown when the asset is purchased, the final cash flow is
  uncertain.

• **Sovereign risk** – this is a type of credit risk specific to a government
  bond. Post 2008, there is material risk of default by an industrial-
  ised country. A developing country may default on its obligation (or
  declare a debt ‘moratorium’) if debt payments relative to domestic
  product reach unsustainable levels.

• **Prepayment risk** – this is specific to mortgage-backed and asset-
  backed bonds. For example, mortgage lenders allow the homeowner
  to repay outstanding debt before the stated maturity. If interest rates
  fall prepayment will occur, which forces reinvestment at rates lower
  than the initial yield.

• **Model risk** – some financial instruments are heavily dependent on
  complex mathematical models for pricing and hedging. If the model
  is incorrectly specified, is based on questionable assumptions or
  does not accurately reflect the true behaviour of the market, banks
  trading these instruments could suffer extensive losses.

**RISK MANAGEMENT**

The risk management function grew steadily in size and importance within
commercial and investment banks during the 1990s. Risk management
departments exist not to eliminate the possibility of all risk, should such
action indeed be feasible or desirable; rather, to control the frequency, extent
and size of such losses in such a way as to provide the minimum surprise to
senior management and shareholders.

Risk exists in all competitive business although the balance between
financial risks of the type described above and general and management
risk varies with the type of business engaged in. The key objective of the
risk management function within a financial institution is to allow for a
clear understanding of the risks and exposures the firm is engaged in, such that monetary loss is deemed acceptable by the firm. The acceptability of any loss should be on the basis that such (occasional) loss is to be expected as a result of the firm being engaged in a particular business activity. If the bank’s risk management function is effective, there will be no over-reaction to any unexpected losses, which may increase eventual costs to many times the original loss amount.

The risk management function

While there is no one agreed organisation structure for the risk management function, the following may be taken as being reflective of the typical bank set-up:

- an independent, ‘middle office’ department responsible for drawing up and explicitly stating the bank’s approach to risk, and defining trading limits and the areas of the market that the firm can have exposure to;
- the head of the risk function reporting to an independent senior manager, who is a member of the executive board;
- monitoring the separation of duties between front, middle and back office, often in conjunction with an internal audit function;
- reporting to senior management, including firm’s overall exposure and adherence of the front office to the firm’s overall risk strategy;
- communication of risks and risk strategy to shareholders;
- where leading edge systems are in use, employment of the risk management function to generate competitive advantage in the market as well as control.

The risk management function is more likely to deliver effective results when there are clear lines of responsibility and accountability. It is also imperative that the department interacts closely with other areas of the front and back office.

In addition to the above the following are often accepted as ingredients of a risk management framework in an institution engaged in investment banking and trading activity:

- proactive management involvement in risk issues;
- daily overview of risk exposure profile and profit & loss (P&L) reports;
- VaR as a common measure of risk exposure, in addition to other measures including ‘jump risk’ to allow for market corrections;
Banks and Risk Management

- defined escalation procedures to deal with rising levels of trading loss, as well as internal ‘stop-loss’ limits;
- independent daily monitoring of risk utilisation by middle-office risk management function;
- independent production of daily P&L, and independent review of front-office closing prices on a daily basis;
- independent validation of market pricing, and pricing and VaR models.

These guidelines, adopted universally in the investment banking community, should assist in the development of an influential and effective risk management function for all financial institutions. We say ‘should’, but of course the experience of JPMorgan, Soc Gen and UBS in the 21st century, shows that the existence of large and seemingly sophisticated risk management infrastructures does not preclude multi-billion dollar trading losses.

Managing risk

The different stakeholders in a bank or financial institution will have slightly different perspectives on risk and its management. If we were to generalise, shareholders will wish for stable earnings as well as the highest possible return on capital. From the point of view of business managers though, the perspective may be slightly different and possibly shorter term. For them, risk management often takes the following route:

- create as diversified a set of business lines as possible, and within each business line diversify portfolios to maximum extent;
- establish procedures to enable some measure of forecasting of market prices;
- hedge the portfolio to minimise losses when market forecasts suggest that losses are to be expected.

The VaR measurement tool falls into the second and third areas of this strategy. It is used to give an idea of risk exposure (generally, to market and credit risk only) so that banks can stay within trading limits, and to feed into the hedge calculation.
Value-at-Risk (VaR) is essentially a measure of volatility, specifically how volatile a bank’s assets are. Assets that exhibit high volatility present higher risk. VaR also takes into account the correlation between different sets of assets in the overall portfolio. If the market price performance of assets is closely positively correlated, this also presents higher risk. So, before we begin the discussion of VaR we need to be familiar with these two concepts. Readers who have an investor’s understanding of elementary statistics may skip this chapter and move straight to Chapter 3.

STATISTICAL CONCEPTS

The statistics used in VaR calculations are based on well-established concepts. There are standard formulae for calculating the mean and standard deviation of a set of values. If we assume that $X$ is a random variable with particular values $x$, we can apply the basic formula to calculate the mean and standard deviation. Remember that the mean is the average of the set of values or observations, while the standard deviation is a measure of the dispersion away from the mean of the range of values. In fact, the standard deviation is the square root of the variance, but the variance, being the sum of squared deviations of each value from the mean divided by the number of observations, has less practical value for us.

Arithmetic mean

We say that the random variable is $X$, so the mean is $E(X)$. In a time series of observations of historical data, the probability values are the frequencies of the observed values. The mean is:

$$E(X) = \frac{\sum x_i}{n} \quad (2.1)$$

where $1/n = \text{Assigned probability to a single value among } n$; and $n = \text{Number of observations}$.

The standard deviation of the set of values is:

$$\sigma(X) = \frac{1}{n} \sqrt{\sum (x_i - E(X))^2} \quad (2.2)$$
The probability assigned to a set of values is given by the type of distribution and, in fact, from a distribution we can determine the mean and standard deviation depending on the probabilities $p_i$ assigned to each value $x_i$ of the random variable $X$. The sum of all probabilities must be 100%. From probability values then, the mean is given by:

$$ E(X) = \frac{\sum p_i x_i}{n} \quad (2.3) $$

The variance is the average weighted by the probabilities of the squared deviations from the mean; so, of course, the standard deviation – which we now call the volatility – is the square root of this value. The volatility is given by:

$$ \sigma(X) = \sqrt{\sum p_i \left[ x_i - E(X) \right]^2} \quad (2.4) $$

In the example in Table 2.1 we show the calculation of the mean, the variance and the standard deviation as calculated from an Excel spreadsheet. The expectation is the mean of all the observations, while the variance is, as we noted earlier, the sum of squared deviations from the mean. The standard deviation is the square root of the variance.

<table>
<thead>
<tr>
<th>Dates</th>
<th>Observations</th>
<th>Deviations from mean</th>
<th>Squared deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>22</td>
<td>4.83</td>
<td>23.36</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
<td>-2.17</td>
<td>4.69</td>
</tr>
<tr>
<td>3</td>
<td>13</td>
<td>-4.17</td>
<td>17.36</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
<td>-3.17</td>
<td>10.03</td>
</tr>
<tr>
<td>5</td>
<td>16</td>
<td>-1.17</td>
<td>1.36</td>
</tr>
<tr>
<td>6</td>
<td>17</td>
<td>-0.17</td>
<td>0.03</td>
</tr>
<tr>
<td>7</td>
<td>16</td>
<td>-1.17</td>
<td>1.36</td>
</tr>
<tr>
<td>8</td>
<td>19</td>
<td>1.83</td>
<td>3.36</td>
</tr>
<tr>
<td>9</td>
<td>21</td>
<td>3.83</td>
<td>14.69</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
<td>2.83</td>
<td>8.03</td>
</tr>
<tr>
<td>11</td>
<td>17</td>
<td>-0.17</td>
<td>0.03</td>
</tr>
<tr>
<td>12</td>
<td>16</td>
<td>-1.17</td>
<td>1.36</td>
</tr>
<tr>
<td>Sum</td>
<td>206</td>
<td>Sum</td>
<td>85.66</td>
</tr>
<tr>
<td>Mean</td>
<td>17.17</td>
<td>Variance</td>
<td>7.788</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Standard deviation</td>
<td>2.791</td>
</tr>
</tbody>
</table>

**Table 2.1** Calculation of standard deviation.
What happens when we have observations that can assume any value within a range, rather than the discrete values we have seen in our example? When there is a probability that a variable can have a value of any measure between a range of specified values, we have a continuous distribution.

**Probability distributions**

A probability distribution is a model for an actual or empirical distribution. If we are engaged in an experiment in which a coin is tossed a number of times, the number of heads recorded will be a discrete value of 0, 1, 2, 3, 4, or so on, depending on the number of times we toss the coin. The result is called a ‘discrete’ random variable. Of course, we know that the probability of throwing a head is 50%, because there are only two outcomes in a coin-toss experiment, heads or tails. We may throw the coin three times and get three heads (it is unlikely but by no means exceptional); however, performing the experiment a great number of times should produce something approaching our 50% result. So, an experiment with a large number of trials would produce an empirical distribution which would be close to the theoretical distribution as the number of tosses increases.

This example illustrates a discrete set of outcomes (0, 1, 2, 3); in other words, a discrete probability distribution. It is equally possible to have a continuous probability distribution: for example, the probability that the return on a portfolio lies between 3% and 7% is associated with a continuous probability distribution because the final return value can assume any value between those two parameters.

**The normal distribution**

A very commonly used theoretical distribution is the normal distribution, which is plotted as a bell-shaped curve and is familiar to most practitioners in business. The theoretical distribution actually looks like many observed distributions such as the height of people, shoe sizes, and so on. The distribution is completely described by the mean and the standard deviation. The normal distribution $N(\mu, \sigma)$ has mean $\mu$ and standard deviation $\sigma$. The probability function is given by:

$$P(X = x) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left[-\frac{(x - \mu)^2}{2\sigma^2}\right]$$

The distribution is standardised as $N(0, 1)$ with a mean of 0 and a standard deviation of 1. It is possible to obtain probability values for any part of the distribution by using the standardised curve and converting variables to
this standardised distribution; thus, the variable $Z = (X - \mu)/\sigma$ follows the
standardised normal distribution $N(0, 1)$ with probability:

$$P(Z = Z) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{Z^2}{2\sigma^2}\right) \quad (2.6)$$

The Central Limit Theorem (known also as the law of large numbers) is
the basis for the importance of the normal distribution in statistical theory,
and in real life a large number of distributions tend towards the normal, pro-
vided that there are a sufficient number of observations. This explains the
importance of the normal distribution in statistics. If we have large numbers
of observations – for example, the change in stock prices, or closing prices
in government bonds – it makes calculations straightforward if we assume
that they are normally distributed.

For both option pricing theory and VaR, it is assumed that the returns
from holding an asset are normally distributed. It is often convenient to
define the return in logarithmic form as:

$$\ln\left(\frac{P_t}{P_{t-1}}\right)$$

where $P_t = $ Price today;
$P_{t-1} = $ Previous price.

If this is assumed to be normally distributed, then the underlying price
will have a log-normal distribution. The log-normal distribution never goes
to a negative value, unlike the normal distribution, and hence is intuitively
more suitable for asset prices. The distribution is illustrated as Figure 2.1.
The normal distribution is assumed to apply to the returns associated with stock prices, and indeed all financial time series observations. However, it is not strictly accurate, as it implies extreme negative values that are not observed in practice. For this reason the log-normal distribution is used instead, in which case the logarithm of the returns is used instead of the return values themselves; this also removes the probability of negative stock prices. In the log-normal distribution, the logarithm of the random variable follows a normal distribution. The log-normal distribution is asymmetric, unlike the normal curve, because it does not have negatives at the extreme values.

**Confidence intervals**

Assume an estimate \( \bar{x} \) of the average of a given statistical population where the true mean of the population is \( \mu \). Suppose that we believe that on average \( \bar{x} \) is an unbiased estimator of \( \mu \). Although this means that on average \( \bar{x} \) is accurate, the specific sample that we observe will almost certainly be above or below the true level. Accordingly, if we want to be reasonably confident that our inference is correct, we cannot claim that \( \mu \) is precisely equal to the observed \( \bar{x} \).

Instead, we must construct an interval estimate or confidence interval of the following form:

\[
\mu = \bar{x} \pm \text{Sampling error}
\]

The crucial question is: How wide must this confidence interval level be? The answer, of course, will depend on how much \( \bar{x} \) fluctuates. We first set our requirements for level of confidence; that is, how certain we wish to be statistically. If we wish to be incorrect only 1 day in 20 – that is, we wish to be right 19 days each month (a month is assumed to have 20 working days) – that would equate to a 95% confidence interval that our estimate is accurate. We also assume that our observations are normally distributed. In that case we would expect that the population would be distributed along the lines portrayed in Figure 2.2.

In the normal distribution, 2.5% of the outcomes are expected to fall more than 1.96 standard deviations from the mean. So, that means 95% of the outcomes would be expected to fall within ±1.96 standard deviations. That is, there is a 95% chance that the random variable will fall between −1.96 standard deviations and +1.96 standard deviations. This would be referred to as a ‘two-sided’ (or ‘two-tailed’) confidence interval. It gives the probability of a move upwards or downwards by the random variable outside the limits we are expecting.

In the financial markets, we do not however expect negative prices, so that values below 0 are not really our concern. In this scenario, it makes sense to consider a one-sided test if we are concerned with the risk of loss: a move upward into profit is of less concern (certainly to a risk manager...
anyway!). From the statistical tables associated with the normal distribution we know that 5% of the outcomes are expected to fall more than 1.645 (rounded to 1.65) standard deviations from the mean. This would be referred to as a one-sided confidence interval.

**VOLATILITY**

In financial market terms, volatility is a measure of how much the price of an asset moves each day (or week or month, and so on). Speaking generally, higher volatility equates to higher profit or loss risk. Bankers must be familiar with volatility, as assets that exhibit higher volatility must be priced such that their returns incorporate a ‘risk premium’ to compensate the holder for the added risk exposure.

**Example 2.1**

We demonstrate volatility from first principles here. Table 2.2 shows two portfolios, outwardly quite similar. They have virtually identical means from an observation of portfolio returns over ten observation periods. However, the standard deviation shows a different picture, and we see that Portfolio B exhibits much greater volatility than Portfolio A. Its future performance is much harder to predict with any reasonable confidence. Portfolio B carries higher risk and so would carry higher VaR. We see also from Table 2.2 that the standard deviation is a measure of the dispersion away from the mean of all the observations. To be comfortable that the statistical measures are as accurate as possible, we need the greatest number of observations.
Volatility is important for both VaR measurement and in the valuation of options. It is a method of measuring current asset price against the distribution of the asset’s future price. Statistically, volatility is defined as the fluctuation in the underlying asset price over a certain period of time. Fluctuation is derived from the change in price between one day’s closing price and the next day’s closing price. Where the asset price is stable it will exhibit low volatility, and the opposite when price movements are large and/or unstable.

We saw from Table 2.2 that the average values for low- and high-volatility portfolios were similar; however, the distribution of the recordings differ. The low-volatility portfolio showed low variability in the distribution. High-volatility assets show a wider variability around the mean.

Market practitioners wish to obtain a volatility value that approximates around the normal distribution. This is done by recording a sufficiently large volume of data and reducing the price change intervals to as small an amount as possible; this means that the price changes can be described statistically by the normal distribution curve. We saw earlier in the chapter that the normal distribution curve has two numerical properties known as the mean and the standard deviation. The mean is the average reading taken at the centre of the curve, and the standard deviation is a value which represents the dispersion around the mean. We demonstrate some examples at Figures 2.3 and 2.4.

Figure 2.3  Differing standard deviations.
## Table 2.2

<table>
<thead>
<tr>
<th>Excel row</th>
<th>Observations</th>
<th>Portfolio A</th>
<th>Portfolio B</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>1</td>
<td>5.08%</td>
<td>3.50%</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>5.00%</td>
<td>5.00%</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>5.05%</td>
<td>6.25%</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
<td>5.00%</td>
<td>7.10%</td>
</tr>
<tr>
<td>10</td>
<td>5</td>
<td>5.05%</td>
<td>3.75%</td>
</tr>
<tr>
<td>11</td>
<td>6</td>
<td>5.00%</td>
<td>5.75%</td>
</tr>
<tr>
<td>12</td>
<td>7</td>
<td>5.01%</td>
<td>2.50%</td>
</tr>
<tr>
<td>13</td>
<td>8</td>
<td>5.20%</td>
<td>4.75%</td>
</tr>
<tr>
<td>14</td>
<td>9</td>
<td>5.06%</td>
<td>5.25%</td>
</tr>
<tr>
<td>15</td>
<td>10</td>
<td>5.00%</td>
<td>6.75%</td>
</tr>
<tr>
<td>16</td>
<td>Mean</td>
<td>5.05%</td>
<td>5.06%</td>
</tr>
<tr>
<td>17</td>
<td>Standard deviation</td>
<td>0.000 622272</td>
<td>0.000 622272</td>
</tr>
<tr>
<td>18</td>
<td>Excel formula</td>
<td>=AVERAGE(C7:C16)</td>
<td>=AVERAGE(D7:D16)</td>
</tr>
<tr>
<td>19</td>
<td></td>
<td>=STDEV(C7:C16)</td>
<td>=STDEV(D7:D16)</td>
</tr>
</tbody>
</table>

**Table 2.2**

Standard deviation.
In Figure 2.5, the standard deviation is shown correlated with dispersion. The curve can be divided into segments which represent specific percentages within each band.

We see from Figure 2.5 that 68.3% of data fall within ±1 standard deviation, 95.5% of data fall within ±2 standard deviations and 99.7% fall within ±3 standard deviations.

The normal distribution curve can also be used to predict future daily share fluctuation over a measured period of time. Future price distribution uses volatility expressed as a 1 standard deviation price change at the end of 1 year.

This can be expressed as a percentage:

\[
\text{1 standard deviation price change } (p) = \text{Volatility (\%)} \times \text{Current asset price (p)}
\]
Although the value of an option relies upon estimated future volatility, volatility is shown also as historical volatility and implied volatility. Historical volatility is the actual price fluctuation in a given time period. The value will depend on the length of the observation period and when the value was observed. This naturally smooths out day-to-day fluctuations – a moving average of historical volatility can be shown graphically in a similar way as conventional share prices. Figure 2.6 shows a 5-day historical volatility chart reversing through a specified time period.

Although historical volatility can show trends over a greater period of time – for example, 4 years – it can also make distinctly significant and highly variable changes. Therefore, there can be no certainty that a past trend is in any way indicative of a share’s future performance.

Implied volatility is a necessary tool to obtain the predicted value of an option which has been obtained from the present value of that option by entering different levels of volatility into an option pricing model, until the current market price is reached. This iterative process effectively reduces the margin of error. In a working situation, most option pricing models allow the calculation of implied volatility by entering the present market price for an option.

Future volatility is the predicted or expected price fluctuation of a period of time until the option has expired. Evidently, this will be affected not only by the calculated implied volatility but also by the expectation of the share’s price trend.

**THE NORMAL DISTRIBUTION AND VaR**

As we will see from Chapter 3 there is more than one way to calculate VaR for an asset portfolio. Many VaR models use the normal curve to calculate the
Table 2.4  Normal distribution illustrated for portfolio return.

<table>
<thead>
<tr>
<th>Observation</th>
<th>Mean return</th>
<th>Target return</th>
<th>Standard deviation of return</th>
<th>Number of standard deviations</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5%</td>
<td>4%</td>
<td>1.63%</td>
<td>−0.612369</td>
<td>27.01%</td>
</tr>
<tr>
<td>2</td>
<td>5%</td>
<td>5%</td>
<td>1.63%</td>
<td>0.000</td>
<td>50.00%</td>
</tr>
<tr>
<td>3</td>
<td>5%</td>
<td>6%</td>
<td>1.63%</td>
<td>0.612369</td>
<td>72.99%</td>
</tr>
<tr>
<td>4</td>
<td>5%</td>
<td>7%</td>
<td>1.63%</td>
<td>1.224739</td>
<td>88.97%</td>
</tr>
<tr>
<td>5</td>
<td>5%</td>
<td>8%</td>
<td>1.63%</td>
<td>1.837109</td>
<td>96.69%</td>
</tr>
<tr>
<td>6</td>
<td>5%</td>
<td>8%</td>
<td>1.63%</td>
<td>1.837109</td>
<td>96.69%</td>
</tr>
</tbody>
</table>

Excel formula
Number of standard deviations = (D9-D8)/D10
Probability = NORMSDIST(D11)
estimation of losses over a specified time period. Normal distribution curve tables, which can be looked up in any number of statistics textbooks or on the Internet, show the probability of an observation moving a specific distance away from the recorded mean. Some specific probabilities are given in Table 2.3.

Table 2.3 shows that 95% of all observations in a normal distribution lie within ±1.65 standard deviations of the mean. A 95% percentile is often used in VaR calculations. Let us take this further. Consider a gilt portfolio of £20 million with a mean return of 5% per annum. If the standard deviation of the returns is 1.63 what probability is there that returns will fall to 4% within the holding period of 1 year? We require the area of the normal curve at which the 4% value is marked – that is, 27% of the area to the left of the mean. The results are shown at Table 2.4, which also shows the Excel formula.

We should note that, although the markets assume a normal distribution of asset (equity and bond) prices, from observation we know that prices follow a more skewed distribution. In practice, asset prices exhibit what is termed ‘leptokurtosis’, also known as ‘fat tails’, which is a normal distribution with fatter tails than the theoretical. In other words, extreme price movements such as stock market corrections occur more frequently than the normal distribution would suggest.

Options traders need to correct for this more than others. The standard option pricing model, the Black–Scholes formula, which we look at in Chapter 5, uses the normal distribution to calculate the delta of the option – the $N(d_1)$ part of the formula – and the probability that the option will be exercised, the $N(d_2)$ part. In practice, the implied volatility of an option is higher if it is deeply in-the-money or out-of-the-money. This is known as the option ‘smile’, and reflects market understanding that the normal distribution is not a completely accurate description of market price behaviour.

CORRELATION

The correlation between different assets and classes of assets is an important measure for risk managers because of the role diversification plays in risk reduction. Correlation is a measure of how much the price of one asset moves in relation to the price of another asset. In a portfolio comprised of only two assets, the VaR of this portfolio is reduced if the correlation between the two assets is weak or negative.

The simplest measure of correlation is the correlation coefficient. This is a value between −1 and +1, with a perfect positive correlation indicated by 1, while a perfect negative correlation is given by −1. Note that this assumes a linear (straight line) relationship between the two assets. A correlation of 0 suggests that there is no linear relationship.
We illustrate these values at Table 2.5, which is a hypothetical set of observations showing the volatilities of four different government benchmark bonds. Note also the Excel formula so that readers can reproduce their own analyses. We assume these bonds are different sovereign names. Bonds 1, 3 and 4 have very similar average returns, but the relationship between Bond 3 and Bond 1 is negatively closely correlated, whereas Bond 4 is positively closely correlated with Bond 1. Bond 2 has a very low positive correlation with Bond 1, and we conclude that there is very little relationship in the price movement of these two bonds.

What are we to make of these four different sovereign names with regard to portfolio diversification? On first glance, Bonds 1 and 3 would appear to offer perfect diversification because they are strongly negatively correlated. However, calculating a diversified VaR for such a portfolio would underestimate risk exposure in times of market correction – which is, after all, when managers most want to know what their risk is. This is because, even though the bonds are negatively related, they can both be expected to fall in value when the market overall is dropping. Bond 2 is no good for risk mitigation, it is strongly positively correlated. Bond 2 has essentially no relationship with Bond 1; however, it is also the most risky security in the portfolio.

We will apply what we have learned here in Chapter 3.

---

**Value-At-Risk**

**Chapter 3: VALUE-AT-RISK**

The advent of value-at-risk (VaR) as an accepted methodology for quantifying market risk and its adoption by bank regulators are part of the development of risk management. The application of VaR has been extended from its initial use in securities houses to commercial banks and corporates, following its introduction in October 1994 when JPMorgan launched RiskMetrics free over the Internet.

In this chapter we look at the different methodologies employed to calculate VaR, and also illustrate its application to simple portfolios. We look first at the variance–covariance method, which is arguably the most popular estimation technique.

**WHAT IS VaR?**

VaR is an estimate of an amount of exposure cash value. It is based on probabilities, so cannot be relied on with certainty, but reflects rather a level of confidence which is selected by the user in advance. VaR measures the
<table>
<thead>
<tr>
<th>Cell</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Observation</td>
<td>Government bond 1</td>
<td>Government bond 2</td>
<td>Government bond 3</td>
<td>Government bond 4</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>5.35%</td>
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<td>7.15%</td>
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</tr>
<tr>
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<td>6.00%</td>
<td>9.00%</td>
<td>7.30%</td>
<td>6.00%</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>5.50%</td>
<td>9.60%</td>
<td>6.90%</td>
<td>5.80%</td>
</tr>
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<td>9</td>
<td>4</td>
<td>6.00%</td>
<td>13.70%</td>
<td>7.20%</td>
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<td>5.90%</td>
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<td>5.90%</td>
</tr>
<tr>
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<td>6</td>
<td>6.50%</td>
<td>10.80%</td>
<td>6.00%</td>
<td>6.05%</td>
</tr>
<tr>
<td>12</td>
<td>7</td>
<td>7.15%</td>
<td>10.10%</td>
<td>6.10%</td>
<td>7.00%</td>
</tr>
<tr>
<td>13</td>
<td>8</td>
<td>6.80%</td>
<td>12.40%</td>
<td>5.60%</td>
<td>6.80%</td>
</tr>
<tr>
<td>14</td>
<td>9</td>
<td>6.75%</td>
<td>14.70%</td>
<td>5.40%</td>
<td>6.70%</td>
</tr>
<tr>
<td>15</td>
<td>10</td>
<td>7.00%</td>
<td>13.50%</td>
<td>5.45%</td>
<td>7.20%</td>
</tr>
<tr>
<td>16</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>Mean return</td>
<td>6.30%</td>
<td>11.68%</td>
<td>6.30%</td>
<td>6.30%</td>
</tr>
<tr>
<td>19</td>
<td>Volatility</td>
<td>0.006 31</td>
<td>0.018 97</td>
<td>0.007 60</td>
<td>0.006 22</td>
</tr>
<tr>
<td>20</td>
<td>Correlation with bond 1</td>
<td>0.357 617 936</td>
<td>–0.758 492 885</td>
<td>0.933 620 205</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Excel formula</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean return</td>
<td>= AVERAGE(E6:E15)</td>
</tr>
<tr>
<td>Volatility</td>
<td>= STDEV(E6:E15)</td>
</tr>
<tr>
<td>Correlation with bond 1</td>
<td>= CORREL(E6:E15,D6:D15)</td>
</tr>
</tbody>
</table>

Table 2.5 Correlation.
volatility of a company’s asset prices, and so the greater the volatility, the higher the probability of loss.

## Definition

Essentially VaR is a measure of the volatility of a bank trading book. It is the characteristics of volatility that traders, risk managers and others wish to become acquainted with when assessing a bank’s risk exposure. The mathematics behind measuring and estimating volatility is slightly involved, and we do not go into it here. However, by making use of a volatility estimate, a trader or senior manager can gain some idea of the risk exposure of the trading book, using the VaR measure.

**VaR is defined as follows:**

**VaR is a measure of market risk. It is the maximum loss which can occur with X% confidence over a holding period of t days.**

VaR is the expected loss of a portfolio over a specified time period for a set level of probability. So, for example, if a daily VaR is stated as £100,000 to a 95% level of confidence, this means that during the day there is a only a 5% chance that the loss will be **greater** than £100,000. VaR measures the potential loss in market value of a portfolio using estimated volatility and correlations. It is measured within a given confidence interval, typically 95% or 99%. The technique seeks to measure possible losses from a position or portfolio under ‘normal’ circumstances. The definition of normality is critical to the estimation of VaR and is a statistical concept; its importance varies according to the VaR calculation methodology that is being used.

Broadly speaking, the calculation of a VaR estimate follows four steps:

1. **Determine the time horizon over which one wishes to estimate a potential loss** – this horizon is set by the user. In practice, time horizons of 1 day to 1 year have been used. For instance, bank front-office traders are often interested in calculating the amount they might lose in a 1-day period. Regulators and participants in illiquid markets may want to estimate exposures to market risk over a longer period. In any case a time horizon must be specified by the decision-maker.

2. **Select the degree of certainty required, which is the confidence level that applies to the VaR estimate** – knowing the largest likely loss a bank will suffer 95 times out of 100, or in fact on 1 day out of 20 (i.e., a 95% degree of confidence in this estimate, or confidence interval) may be sufficient. For regulatory requirements a 99% confidence interval may be more appropriate. Senior management and shareholders are often interested in the potential loss arising from catastrophe situations, such as a stock market crash, so for them a 99% confidence level is more appropriate.
3. Create a probability distribution of likely returns for the instrument or portfolio under consideration – several methods may be used. The easiest to understand is the distribution of recent historical returns for the asset or portfolio which often looks like the curve associated with the normal distribution. After determining a time horizon and confidence interval for the estimate, and then collating the history of market price changes in a probability distribution, we can apply the laws of statistics to estimate VaR.

4. Calculate the VaR estimate – this is done by observing the loss amount associated with that area beneath the normal curve at the critical confidence interval value that is statistically associated with the probability chosen for the VaR estimate in Step 2.

These four steps will in theory allow us to calculate a VaR estimate ‘longhand’, although in practice mathematical models exist that will do this for us. Bearing these steps in mind, we can arrive at a practical definition of VaR not much removed from our first one:

**VaR is the largest likely loss from market risk (expressed in currency units) that an asset or portfolio will suffer over a time interval and with a degree of certainty selected by the user.**

We stress, of course, that this would be under ‘normal’, that is, unstressed conditions. There are a number of methods for calculating VaR, all logically sustainable but nevertheless reliant on some strong assumptions, and estimates prepared using the different methodologies can vary dramatically. At this point it is worthwhile reminding ourselves what VaR is not. It is not a unified method for measuring risk, as the different calculation methodologies each produce different VaR values. In addition, as it is a quantitative statistical technique, VaR only captures risks that can be quantified. Therefore, it does not measure (nor does it seek to measure) other risks that a bank or securities house will be exposed to, such as liquidity risk or operational risk. Most importantly, VaR is not ‘risk management’. This term refers to the complete range of duties and disciplines that are involved in minimising and managing bank risk exposure. VaR is but one ingredient of risk management, a measurement tool for market risk exposure. So the mean and standard deviation parameters of the statistical distribution are key to the VaR estimate.

**METHODOLOGY**

**Centralised database**

To implement VaR, all of a firm’s positions data must be gathered into one centralised database. Once this is complete the overall risk has to be
calculated by aggregating the risks from individual instruments across the entire portfolio. The potential move in each instrument (i.e., each risk factor) has to be inferred from past daily price movements over a given observation period. For regulatory purposes this period is at least 1 year. Hence, the data on which VaR estimates are based should capture all relevant daily market moves over the previous year. The main assumption underpinning VaR – and which in turn may be seen as its major weakness – is that the distribution of future price and rate changes will follow past variations. Therefore, the potential portfolio loss calculations for VaR are worked out using distributions from historic price data in the observation period.

Correlation assumptions

VaR requires that the user decide which exposures are allowed to offset each other and by how much. For example, is the Japanese yen correlated to movements in the euro or the Mexican peso? Consider also the price of crude oil to movements in the price of natural gas: if there is a correlation, to what extent is the degree of correlation? VaR requires that the user determine correlations within markets as well as across markets. The mapping procedures used as part of the VaR process also have embedded correlation assumptions. For example, mapping individual stocks into the S&P 500 or fixed interest securities into the swap curve translate into the assumption that individual financial instruments move as the market overall. This is reasonable for diversified portfolios but may fall down for undiversified or illiquid portfolios.

To calculate the VaR for a single security, we would calculate the standard deviation of its price returns. This can be done using historical data, but also using the implied volatility contained in exchange-traded option prices. We would then select a confidence interval and apply this to the standard deviation, which would be our VaR measure. This is considered in more detail later.

There are three main methods for calculating VaR. As with all statistical models, they depend on certain assumptions. They are:

- the correlation method (or variance/covariance method);
- historical simulation;
- Monte Carlo simulation.

Correlation method

This is also known as the variance–covariance, parametric or analytic method. This method assumes the returns on risk factors are normally distributed, the
correlations between risk factors are constant and the delta (or price sensitivity to changes in a risk factor) of each portfolio constituent is constant. Using the correlation method, the volatility of each risk factor is extracted from the historical observation period. Historical data on investment returns are therefore required. The potential effect of each component of the portfolio on the overall portfolio value is then worked out from the component’s delta (with respect to a particular risk factor) and that risk factor’s volatility.

There are different methods of calculating relevant risk factor volatilities and correlations. We consider two alternatives:

(i) Simple historic volatility (correlation) – this is the most straightforward method but the effects of a large one-off market move can significantly distort volatilities (correlations) over the required forecasting period. For example, if using 30-day historic volatility, a market shock will stay in the volatility figure for 30 days until it drops out of the sample range and, correspondingly, causes a sharp drop in (historic) volatility 30 days after the event. This is because each past observation is equally weighted in the volatility calculation.

(ii) A more sophisticated approach is to weight past observations unequally. This is done to give more weight to recent observations so that large jumps in volatility are not caused by events that occurred some time ago. Two methods for unequal weighting are the generalised autoregressive conditional heteroscedasticity (GARCH) models and exponentially weighted moving averages. GARCH models are fine-tuned to each risk factor time series, while exponentially weighted averages can be computed with little more complication than simple historic volatility. Both methods rely on the assumption that future volatilities can be predicted from historic price movements.

**Historical simulation method**

The historical simulation method for calculating VaR is the simplest and avoids some of the pitfalls of the correlation method. Specifically, the three main assumptions behind correlation (normally distributed returns, constant correlations, constant deltas) are not needed in this case. For historical simulation the model calculates potential losses using actual historical returns in the risk factors and so captures the non-normal distribution of risk factor returns. This means rare events and crashes can be included in the results. As the risk factor returns used for revaluing the portfolio are actual past movements, the correlations in the calculation are also actual past correlations. They capture the dynamic nature of correlations as well as scenarios when the usual correlation relationships break down.
Monte Carlo simulation method

The third method, Monte Carlo simulation, is more flexible than the previous two. As with historical simulation, Monte Carlo simulation allows the risk manager to use actual historical distributions for risk factor returns rather than having to assume normal returns. A large number of randomly generated simulations are run forward in time using volatility and correlation estimates chosen by the risk manager. Each simulation will be different, but in total the simulations will aggregate to the chosen statistical parameters (i.e., historical distributions and volatility and correlation estimates). This method is more realistic than the previous two models and, therefore, is more likely to estimate VaR more accurately. However, its implementation requires powerful computers and there is also a trade-off in that the time to perform calculations is longer.

Validity of the volatility–correlation VaR estimate

The level of confidence in the VaR estimation process is selected by the number of standard deviations of variance applied to the probability distribution. A standard deviation selection of 1.645 provides a 95% confidence level (in a one-tailed test) that the potential estimated price movement will not be more than a given amount based on the correlation of market factors to the position’s price sensitivity. This confidence level is advocated by the RiskMetrics version of volatility–correlation VaR.

HOW TO CALCULATE VaR

A conceptual illustration of the normal distribution being applied for VaR is given at Figure 3.1.

A market risk estimate can be calculated by following these steps:

1. Value the current portfolio using today’s prices, the components of which are ‘market factors’. For example, the market factors that affect the value of a bond denominated in a foreign currency are the term structure of that currency’s interest rate (either the zero-coupon curve or the par yield curve) and the exchange rate.
2. Revalue the portfolio using alternative prices based on changed market factors and calculate the change in the portfolio value that would result.
3. Revaluing the portfolio using a number of alternative prices gives a distribution of changes in value. Given this, a portfolio VaR can be specified in terms of confidence levels.
4. The risk manager can calculate the maximum the firm can lose over a specified time horizon at a specified probability level.
In implementing VaR the main problem is finding a way to obtain a series of vectors of different market factors. We will see how the various methodologies try to resolve this issue for each of the three methods that can be used to calculate VaR.

**Historical method**

Values of the market factors for a particular historical period are collected and changes in these values over the time horizon are observed for use in the calculation. For instance, if a 1-day VaR is required using the past 100 trading days, each of the market factors will have a vector of observed changes that will be made up of the 99 changes in value of the market factor. A vector of alternative values is created for each of the market factors by adding the current value of the market factor to each of the values in the vector of observed changes.

The portfolio value is found using the current and alternative values for the market factors. The changes in portfolio value between the current value and the alternative values are then calculated. The final step is to sort the changes in portfolio value from the lowest value to highest value and determine the VaR based on the desired confidence interval. For a 1-day, 95% confidence level VaR using the past 100 trading days, the VaR would be the 95th most adverse change in portfolio value.

**Simulation method**

The first step is to define the parameters of the distributions for the changes in market factors, including correlations among these factors. Normal and log-normal distributions are usually used to estimate changes in market
factors, while historical data are most often used to define correlations among market factors. The distributions are then used in a Monte Carlo simulation to obtain simulated changes in the market factors over the time horizon to be used in the VaR calculation.

A vector of alternative values is created for each of the market factors by adding the current value of the market factor to each of the values in the vector of simulated changes. Once this vector of alternative values of the market factors is obtained, the current and alternative values for the portfolio, the changes in portfolio value and the VaR are calculated exactly as in the historical method.

Variance–covariance, analytic or parametric method

This is similar to the historical method in that historical values of market factors are collected in a database. The next steps are then to:

(i) decompose the instruments in the portfolio into the cash-equivalent positions in more basic instruments;
(ii) specify the exact distributions for the market factors (or ‘returns’); and
(iii) calculate the portfolio variance and VaR using standard statistical methods.

We now look at these steps in greater detail.

Decompose financial instruments

The analytic method assumes that financial instruments can be decomposed or ‘mapped’ into a set of simpler instruments that are exposed to only one market factor. For example, a 2-year UK gilt can be mapped into a set of zero-coupon bonds representing each cash flow. Each of these zero-coupon bonds is exposed to only one market factor – a specific UK zero-coupon interest rate. Similarly, a foreign currency bond can be mapped into a set of zero-coupon bonds and a cash foreign exchange amount subject to movement in the spot foreign exchange (FX) rate.

Specify distributions

The analytic method makes assumptions about the distributions of market factors. For example, the most widely used analytic method, JPMorgan’s RiskMetrics, assumes that the underlying distributions are normal. With normal distributions all the historical information is summarised in the mean and standard deviation of the returns (market factors), so users do not need to keep all the historical data.
Calculate portfolio variance and VaR

If all the market factors are assumed to be normally distributed, the portfolio, which is the sum of the individual instruments, can also be assumed to be normally distributed. This means that portfolio variance can be calculated using standard statistical methods (similar to modern portfolio theory), given by:

\[
\sigma_p = \sqrt{\alpha_j^2 \sigma_j^2 + \alpha_k^2 \sigma_k^2 + 2 \alpha_j \alpha_k \rho_{jk} \sigma_j \sigma_k}
\]  

(3.1)

where

- \( \alpha_j \) = Home currency present value of the position in market factor \( j \);
- \( \sigma_j^2 \) = Variance of market factor \( j \);
- \( \rho_{jk} \) = Correlation coefficient between market factors \( j \) and \( k \).

The portfolio VaR is then a selected number of portfolio standard deviations; for example, 1.645 standard deviations will isolate 5% of the area of the distribution in the lower tail of the normal curve, providing 95% confidence in the estimate. Consider an example where, using historical data, the portfolio variance for a package of UK gilts is £348.57. The standard deviation of the portfolio would be \( \sqrt{348.57} \), which is £18.67. A 95% 1-day VaR would be 1.645 \times £18.67, which is £30.71.

Of course, a bank’s trading book will contain many hundreds of different assets, and the method employed above, useful for a two-asset portfolio, will become unwieldy. Therefore, matrices are used to calculate the VaR of a portfolio where many correlation coefficients are used. This is considered below.

Matrix calculation of variance–covariance VaR

Consider the following hypothetical portfolio of £10,000,000.00 invested in two assets, as shown in Table 3.1(i). The standard deviation of each asset has been calculated on historical observation of asset returns. Note that returns are returns of asset prices, rather than the prices themselves; they are calculated from the actual prices by taking the ratio of closing prices. The returns are then calculated as the logarithm of price relatives. The mean and standard deviation of the returns are then calculated using standard statistical formulae. This would then give the standard deviation of daily price relatives, which is converted to an annual figure by multiplying it by the square root of the number of days in a year, usually taken to be 250.

We wish to calculate the portfolio VaR at the 95% level. The Excel formulae are shown at Table 3.1(ii).

The standard equation is used to calculate the variance of the portfolio, using the individual asset standard deviations and the asset weightings; the VaR of the book is the square root of the variance. Multiplying this figure
by the current value of the portfolio gives us the portfolio VaR, which is £2,113,300.72.

Using historical volatility means that we must define the horizon of the time period of observations, as well as the frequency of observations. Typically, a daily measure is used due to the ease of collating information, with the result that we need to use the ‘square root of time’ rule when moving to another time period. This applies when there are no bounds to returns data. This was illustrated above when we referred to the square root for the number of working days in a year. As an example, if we assume a 2% daily volatility, the 1-year volatility then becomes:

\[
\sigma_{\text{1 year}} = \sigma_{\text{1 day}} \sqrt{250} \\
= 2\% \times 15.811 \\
= 31.622\%
\]

<table>
<thead>
<tr>
<th>Asset</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bond 1</td>
<td>8</td>
<td>Standard deviation</td>
<td>11.83%</td>
<td>17.65%</td>
<td></td>
</tr>
<tr>
<td>Bond 2</td>
<td>9</td>
<td>Portfolio weighting</td>
<td>60%</td>
<td>40%</td>
<td></td>
</tr>
<tr>
<td>Correlation coefficient</td>
<td>10</td>
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<td>0.647</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Portfolio value</td>
<td>11</td>
<td></td>
<td>£10,000,000.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Confidence level</td>
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<td>95%</td>
<td></td>
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<tr>
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<td></td>
</tr>
<tr>
<td>Standard deviation</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>95% c.i. standard deviations</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>Value-at-Risk</td>
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<td>0.211330072</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Value-at-Risk £</td>
<td>17</td>
<td></td>
<td>£2,113,300.72</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.1(i) Two-asset portfolio VaR.
<table>
<thead>
<tr>
<th>D</th>
<th>E</th>
<th>F</th>
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<th>H</th>
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<tbody>
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<td></td>
<td>Asset</td>
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<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>Asset</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>Bond 1</td>
<td>Bond 2</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>Standard deviation</td>
<td>11.83%</td>
<td>17.65%</td>
</tr>
<tr>
<td>11</td>
<td></td>
<td>Portfolio weighting</td>
<td>60%</td>
<td>40%</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td>Correlation coefficient</td>
<td></td>
<td>0.647</td>
</tr>
<tr>
<td>13</td>
<td></td>
<td>Portfolio value</td>
<td></td>
<td>£10,000,000.00</td>
</tr>
<tr>
<td>14</td>
<td></td>
<td>Confidence level</td>
<td></td>
<td>95%</td>
</tr>
<tr>
<td>15</td>
<td></td>
<td>Portfolio variance</td>
<td>=F9^2<em>F10^2+G9^2</em>G10^2+2<em>F9</em>F10<em>G9</em>G10</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
<td>Standard deviation</td>
<td>=H15^0.5</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td></td>
<td>95% c.i. standard deviations</td>
<td>=NORMSINV(H13)</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td></td>
<td>Value-at-Risk</td>
<td>=H18*H16</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td></td>
<td>Value-at-Risk £</td>
<td>=H20*H12</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 3.1(ii)** Spreadsheet formulae for Table 3.1(i).
Using this rule we can convert values for market volatility over any period of time.

The RiskMetrics VaR methodology uses matrices to obtain the same results that we have shown here. This is because, once a portfolio starts to contain many assets, the method we described above becomes unwieldy. Matrices allow us to calculate VaR for a portfolio containing many hundreds of assets, which would require assessment of the volatility of each asset and correlations of each asset to all the others in the portfolio. We can demonstrate how the parametric methodology uses variance and correlation matrices to calculate the variance, and hence the standard deviation, of a portfolio. The matrices are shown at Figure 3.2. Note that multiplication of matrices carries with it some unique rules; readers who are unfamiliar with matrices should refer to a standard mathematics text.

As shown at Figure 3.2, using the same two-asset portfolio described, we can set a $2 \times 2$ matrix with the individual standard deviations inside; this is labelled the ‘variance’ matrix. The standard deviations are placed on the horizontal axis of the matrix, and a ‘0’ entered in the other cells. The second matrix is the correlation matrix, and the correlation of the two assets is placed in cells corresponding to the other asset; that is why a ‘1’ is placed in the other cells, as an asset is said to have a correlation of 1 with itself. The two matrices are then multiplied to produce another matrix, labelled ‘VC’ in Figure 3.2.¹

---

**Figure 3.2** Matrix variance–covariance calculation for the two-asset portfolio shown in Table 3.1.

¹ A spreadsheet calculator such as Microsoft Excel has a function for multiplying matrices which may be used for any type of matrix. The function is ‘=MMULT(’ typed in all the cells of the product matrix.
The VC matrix is then multiplied by the V matrix to obtain the variance–covariance matrix or VCV matrix. This shows the variance of each asset; for Bond 1 this is 0.01399, which is expected as that is the square of its standard deviation, which we were given at the start. The matrix also tells us that Bond 1 has a covariance of 0.0135 with Bond 2. We then set up a matrix of the portfolio weighting of the two assets, and this is multiplied by the VCV matrix. This produces a $1 \times 2$ matrix, which we need to change to a single number; so, this is multiplied by the W matrix, reset as a $2 \times 1$ matrix, which produces the portfolio variance. This is 0.016507. The standard deviation is the square root of the variance, and is 0.1284795 or 12.848%, which is what we obtained before. In our illustration it is important to note the order in which the matrices were multiplied, as this will obviously affect the result. The volatility matrix contains the standard deviations along the diagonal, and ‘0’ s are entered in all the other cells. So, if the portfolio we were calculating has 50 assets in it, we would require a $50 \times 50$ matrix and enter the standard deviations for each asset along the diagonal line. All the other cells would have a ‘0’ in them. Similarly, for the weighting matrix this is always one row, and all the weights are entered along the row. To take the example just given the result would be a $1 \times 50$ weighting matrix.

The correlation matrix in the simple example above is set up as shown in Table 3.2.

The correlation matrix at Table 3.2 shows that Asset 1 has a correlation of 0.647 with Asset 2. All correlation tables always have unity along the diagonal because an asset will have a correlation of 1 with itself. So, a three-asset portfolio of the following correlations

\[
\begin{align*}
\text{Correlation 1, 2} & = 0.647 \\
\text{Correlation 1, 3} & = 0.455 \\
\text{Correlation 2, 3} & = 0.723
\end{align*}
\]

would look like Table 3.3.

The matrix method for calculating the standard deviation is more effective than the first method we described, because it can be used for a portfolio containing a large number of assets. In fact, this is exactly the methodology used by RiskMetrics, and the computer model used for the calculation will

<table>
<thead>
<tr>
<th></th>
<th>Asset 1</th>
<th>Asset 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asset 1</td>
<td>1</td>
<td>0.647</td>
</tr>
<tr>
<td>Asset 2</td>
<td>0.647</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3.2 Asset correlation.
be set up with matrices containing the data for hundreds, if not thousands, of different assets.

The variance–covariance method captures the diversification benefits of a multi-product portfolio because the correlation coefficient matrix is used in the calculation. For instance, if the two bonds in our hypothetical portfolio had a negative correlation the VaR number produced would be lower. It was also the first methodology introduced by JPMorgan in 1994. To apply it, a bank would require data on volatility and correlation for the assets in its portfolio. These data are actually available from the RiskMetrics website (and other sources), so a bank does not necessarily need its own data. It may wish to use its own datasets, however, should it have them, to tailor the application to its own use. The advantages of the variance–covariance methodology are that:

- it is simple to apply and fairly straightforward to explain;
- datasets for its use are immediately available.

The drawbacks of the variance–covariance method are that it assumes stable correlations and measures only linear risk; it also places excessive reliance on the normal distribution, and returns in the market are widely believed to have ‘fatter tails’ than a true to normal distribution. This phenomenon is known as leptokurtosis; that is, the non-normal distribution of outcomes. Another disadvantage is that the process requires mapping. To construct a weighting portfolio for the RiskMetrics tool, cash flows from financial instruments are mapped into precise maturity points, known as grid points. We will review this later in the chapter; however, in most cases assets do not fit into neat grid points, and complex instruments cannot be broken down accurately into cash flows. The mapping process makes assumptions that frequently do not hold in practice.

Nevertheless, the variance–covariance method is still popular in the market, and is frequently the first VaR method installed at a bank.

### Confidence intervals

Many models estimate VaR at a given confidence interval, under normal market conditions. This assumes that market returns generally follow a

<table>
<thead>
<tr>
<th>Asset 1</th>
<th>Asset 2</th>
<th>Asset 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asset 1</td>
<td>1</td>
<td>0.647</td>
</tr>
<tr>
<td>Asset 2</td>
<td>0.647</td>
<td>1</td>
</tr>
<tr>
<td>Asset 3</td>
<td>0.455</td>
<td>0.723</td>
</tr>
</tbody>
</table>

Table 3.3 Correlation matrix: three-asset portfolio.
random pattern but one that approximates over time to a normal distribution. The level of confidence at which the VaR is calculated will depend on the nature of the trading book’s activity and what the VaR number is being used for. The original amendment to the Basel Capital Accord stipulated a 99% confidence interval and a 10-day holding period if the VaR measure is to be used to calculate the regulatory capital requirement. However, certain banks prefer to use other confidence levels and holding periods; the decision on which level to use is a function of asset types in the portfolio, quality of market data available and the accuracy of the model itself, which will have been tested over time by the bank.

For example, a bank may view a 99% confidence interval as providing no useful information, as it implies that there should only be two or three breaches of the VaR measure over the course of 1 year; that would leave no opportunity to test the accuracy of the model until a relatively long period of time had elapsed, in the meantime the bank would be unaware if the model was generating inaccurate numbers. A 95% confidence level implies the VaR level being exceeded around 1 day each month, if a year is assumed to contain 250 days. If a VaR calculation is made using 95% confidence, and a 99% confidence level is required for, say, regulatory purposes, we need to adjust the measure to take account of the change in standard deviations required. For example, a 99% confidence interval corresponds to 2.32 standard deviations, while a 95% level is equivalent to 1.645 standard deviations. Thus, to convert from 95% confidence to 99% confidence, the VaR figure is divided by 1.645 and multiplied by 2.32.

In the same way there may be occasions when a firm will wish to calculate VaR over a different holding period from that recommended by the Basel Committee. The holding period of a portfolio’s VaR calculation should represent the period of time required to unwind the portfolio; that is, sell off the assets on the book. A 10-day holding period is recommended but would be unnecessary for a highly liquid portfolio; for example, a market-making book holding government bonds.

To adjust the VaR number to fit it to a new holding period we simply scale it upwards or downwards by the square root of the time period required. For example, a VaR calculation measured for a 10-day holding period will be $\sqrt{10}$ times larger than the corresponding 1-day measure.

**COMPARISON BETWEEN METHODS**

The three methods produce different VaR estimates and these are more marked with portfolios that contain options. The analytic method usually estimates the market risk of option positions based on delta (or delta and gamma). This results in inaccurate risk estimates for large changes in the
price of the underlying; it also ignores the potential effect of changes in
the volatility of the underlying. The historic and simulation methods can
account for changes in all the market factors that affect an option price, and
the revaluation process allows the market risk of options to be more accu-
rately measured for larger changes in market factors.

A comparison of the three methodologies is presented at Table 3.7,
summarised from Risk in November 1997.

Choosing between methods

The composition of a bank’s portfolio is a prime factor in deciding which
method to implement. For portfolios with no options the analytic method
may be most suitable because it does not require pricing models. Publicly
available software and data (e.g., RiskMetrics) makes installation simpler.

Historical or simulation methods are more appropriate for portfolios
with option positions. The historical method is conceptually simple and the
required pricing models are often available as add-ins for spreadsheet pack-
ages. The main obstacle to using the simulation method is the complex task
of doing Monte Carlo simulations; although the software is available the
process is time-consuming.

To calculate a single position VaR example in simple fashion, consider
Example 3.2.
EXAMPLE 3.2 VaR calculation

| VaR | Amount of position * Volatility of instrument; |
| Volatility | % of value which may be lost with a certain possibility (e.g., 95%); |
| Position | A bond trader is long of $40 million US 10-year Treasury benchmark; |
| Market risk | US 10-year volatility is 0.932%; |
| VaR = | $40 million * 0.932% = $372,800. |

For a two-position VaR example the portfolio now needs to consider correlations and the following expression is applied:

$$VaR = \sqrt{VaR_1^2 + VaR_2^2 + 2 \rho VaR_1 VaR_2}$$

where

- $VaR_1$ = Value-at-risk for Instrument 1;
- $VaR_2$ = Value-at-risk for Instrument 2;
- $\rho$ = Correlation between the price movements of Instrument 1 and 2.

The individual VaRs are calculated as before.

Essentially, RiskMetrics follows the procedure detailed in the previous section for analytic method VaR estimates.

The core of RiskMetrics is:

- a method mapping position, forecasting volatilities and correlations, and risk estimation;
- a daily updated set of estimated volatilities and correlations of rates and prices.

The DEaR and VaR are the maximum estimated loss in market value of a given position that can be expected to be incurred with 95% certainty until the position can be neutralised or reassessed.

Assessment

The key technical assumptions made by RiskMetrics are:

- conditional multivariate normality of returns and assets;
- exponentially weighted moving average forecasts of volatility (as against GARCH or stochastic models);
- variance–covariance method of calculation (as against historical simulation).
The key limitations are:

- limited applicability to options and non-linear positions generally;
- simplicity of its mapping process, assuming cash flows on standardised grid points on the time line;
- like any VaR model, no coverage of liquidity risk, funding risk, credit risk, or operational risk.

Comparison with the historical approach

The historical approach is preferred in some firms because of its simplicity. It differs from RiskMetrics in three respects:

- it makes no explicit assumption about the variances of portfolio assets and the correlations between them;
- it makes no assumptions about the shape of the distribution of asset returns, including no assumption of normality;
- it requires no simplification or mapping of cash flows.

To calculate VaR using this approach all that is required is a historical record of the daily profit and loss (P&L) of the portfolio under consideration. Hence, a major strength of the historical approach is the minimal analytical capability required. An additional benefit is the lack of cash flow mapping. The simplification process can create substantial risk distortion, particularly if there are options in the portfolio. Under RiskMetrics, options are converted into their delta equivalents.

The main drawback of the historical approach is that since it is based strictly on the past it is not useful for scenario analysis. With RiskMetrics we can alter the assumed variances and correlations to see how the VaR would be affected. This is not possible under the historical approach.

COMPARING VaR CALCULATION FOR DIFFERENT METHODOLOGIES

The different approaches to calculating VaR produce a wide range of results. As we illustrate here, this difference occurs even with the simplest portfolio (in this case, a holding of just one vanilla fixed income instrument). If this variation is so marked for just one asset holding, how much more dispersion must there be for complex portfolios that include exotic instruments? The point here is that one must be aware that care needs to be taken when interpreting and using VaR numbers. The actual loss that might occur in practice can bear no relation to the previous day’s VaR calculation. This being the case, it suggests that banks should be ultra conservative when setting VaR.
limits for credit and market risk, on the grounds that the true risk exposure might be many times what the bank’s model is suggesting. Relying excessively on VaR model output is not recommended.

Figure 3.3 shows Bloomberg screen PORT, which needs to be set up by the user to hold the desired securities (in other words, the portfolio is bespoke to the user’s needs). We see that the portfolio consists of one bond, the Aston Martin Capital Ltd $9\frac{1}{4}$% 2018, a £304 million issue from 2011 and quoted at £100.30 as at 18 December 2012. At that price the holding of £1 million had a market value of £1,030,563.

This Bloomberg screen has a VaR functionality built in, which allows you to compare VaR numbers by methodology. We see this at Figure 3.4. From this screen we observe the following values, for the 95% confidence interval:

- Monte Carlo VaR: 9,579
- 1-year historical VaR: 1,778
- 2-year historical VaR: 4,101
- 3-year historical VaR: 3,050
- Parametric VaR: 11,570

![Figure 3.3 PORT screen showing holding of GBP1 million Aston Martin Capital Ltd $9\frac{1}{4}$% 2018 sterling corporate bond.](www.bloomberg.com)
This variation is so great as to render the results almost unusable. The investor can take the entire range or simply select an average or the worst-case scenario. It is at this point that using VaR itself becomes slightly subjective. The other interesting observation is the 2-year historical VaR exceeding the 3-year historical VaR. This forces the question, what time horizon is most appropriate if selecting historical VaR as the preferred methodology? The answer is not clear-cut, and of course there is no one right answer. Banks generally tend to obtain a feel for their preferred approach based on a number of market and operational factors.

Finally, Figure 3.5 is the distribution of VaR results by confidence interval selected. As expected we observe reducing estimates the closer we decrease towards 95% c.i., and this trend continuing the further below 95% c.i. the user falls. It also shows the resulting P&L distribution.
POLICY TEMPLATES

For practitioners we have placed the following policy document templates on the book’s website, contained in the folder for Chapter 5:

- Trading Book policy;
- FX hedging policy.

SELECTED BIBLIOGRAPHY AND REFERENCES


