CHAPTER 1
Measurement

1-1 MEASURING THINGS, INCLUDING LENGTHS

Learning Objectives

After reading this module, you should be able to . . .

1.01 Identify the base quantities in the SI system.
1.02 Name the most frequently used prefixes for SI units.
1.03 Change units (here for length, area, and volume) by using chain-link conversions.
1.04 Explain that the meter is defined in terms of the speed of light in vacuum.

Key Ideas

● Physics is based on measurement of physical quantities. Certain physical quantities have been chosen as base quantities (such as length, time, and mass); each has been defined in terms of a standard and given a unit of measure (such as meter, second, and kilogram). Other physical quantities are defined in terms of the base quantities and their standards and units.
● The unit system emphasized in this book is the International System of Units (SI). The three physical quantities displayed in Table 1-1 are used in the early chapters. Standards, which must be both accessible and invariable, have been established for these base quantities by international agreement. These standards are used in all physical measurement, for both the base quantities and the quantities derived from them. Scientific notation and the prefixes of Table 1-2 are used to simplify measurement notation.
● Conversion of units may be performed by using chain-link conversions in which the original data are multiplied successively by conversion factors written as unity and the units are manipulated like algebraic quantities until only the desired units remain.
● The meter is defined as the distance traveled by light during a precisely specified time interval.

What Is Physics?

Science and engineering are based on measurements and comparisons. Thus, we need rules about how things are measured and compared, and we need experiments to establish the units for those measurements and comparisons. One purpose of physics (and engineering) is to design and conduct those experiments.

For example, physicists strive to develop clocks of extreme accuracy so that any time or time interval can be precisely determined and compared. You may wonder whether such accuracy is actually needed or worth the effort. Here is one example of the worth: Without clocks of extreme accuracy, the Global Positioning System (GPS) that is now vital to worldwide navigation would be useless.

Measuring Things

We discover physics by learning how to measure the quantities involved in physics. Among these quantities are length, time, mass, temperature, pressure, and electric current.

We measure each physical quantity in its own units, by comparison with a standard. The unit is a unique name we assign to measures of that quantity—for example, meter (m) for the quantity length. The standard corresponds to exactly 1.0 unit of the quantity. As you will see, the standard for length, which corresponds
to exactly 1.0 m, is the distance traveled by light in a vacuum during a certain fraction of a second. We can define a unit and its standard in any way we care to. However, the important thing is to do so in such a way that scientists around the world will agree that our definitions are both sensible and practical.

Once we have set up a standard—say, for length—we must work out procedures by which any length whatever, be it the radius of a hydrogen atom, the wheelbase of a skateboard, or the distance to a star, can be expressed in terms of the standard. Rulers, which approximate our length standard, give us one such procedure for measuring length. However, many of our comparisons must be indirect. You cannot use a ruler, for example, to measure the radius of an atom or the distance to a star.

**Base Quantities.** There are so many physical quantities that it is a problem to organize them. Fortunately, they are not all independent; for example, speed is the ratio of a length to a time. Thus, what we do is pick out—by international agreement—a small number of physical quantities, such as length and time, and assign standards to them alone. We then define all other physical quantities in terms of these base quantities and their standards (called base standards). Speed, for example, is defined in terms of the base quantities length and time and their base standards.

Base standards must be both accessible and invariable. If we define the length standard as the distance between one’s nose and the index finger on an outstretched arm, we certainly have an accessible standard—but it will, of course, vary from person to person. The demand for precision in science and engineering pushes us to aim first for invariability. We then exert great effort to make duplicates of the base standards that are accessible to those who need them.

**The International System of Units**

In 1971, the 14th General Conference on Weights and Measures picked seven quantities as base quantities, thereby forming the basis of the International System of Units, abbreviated SI from its French name and popularly known as the metric system. Table 1-1 shows the units for the three base quantities—length, mass, and time—that we use in the early chapters of this book. These units were defined to be on a “human scale.”

Many SI derived units are defined in terms of these base units. For example, the SI unit for power, called the watt (W), is defined in terms of the base units for mass, length, and time. Thus, as you will see in Chapter 7,

\[
1 \text{ watt} = 1 \text{ W} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^3, \tag{1-1}
\]

where the last collection of unit symbols is read as kilogram-meter squared per second cubed.

To express the very large and very small quantities we often run into in physics, we use scientific notation, which employs powers of 10. In this notation,

\[
3 \, 560 \, 000 \, 000 \, \text{m} = 3.56 \times 10^9 \, \text{m}, \tag{1-2}
\]

and

\[
0.000 \, 000 \, 492 \, \text{s} = 4.92 \times 10^{-7} \, \text{s}. \tag{1-3}
\]

Scientific notation on computers sometimes takes on an even briefer look, as in 3.56 E9 and 4.92 E–7, where E stands for “exponent of ten.” It is briefer still on some calculators, where E is replaced with an empty space.

As a further convenience when dealing with very large or very small measurements, we use the prefixes listed in Table 1-2. As you can see, each prefix represents a certain power of 10, to be used as a multiplication factor. Attaching a prefix to an SI unit has the effect of multiplying by the associated factor. Thus, we can express a particular electric power as

\[
1.27 \times 10^9 \, \text{watts} = 1.27 \, \text{gigawatts} = 1.27 \, \text{GW} \tag{1-4}
\]
or a particular time interval as

\[ 2.35 \times 10^{-9} \text{s} = 2.35 \text{ nanoseconds} = 2.35 \text{ ns}. \quad (1-5) \]

Some prefixes, as used in milliliter, centimeter, kilogram, and megabyte, are probably familiar to you.

**Changing Units**

We often need to change the units in which a physical quantity is expressed. We do so by a method called *chain-link conversion*. In this method, we multiply the original measurement by a *conversion factor* (a ratio of units that is equal to unity). For example, because 1 min and 60 s are identical time intervals, we have

\[ \frac{1 \text{ min}}{60 \text{ s}} = 1 \quad \text{and} \quad \frac{60 \text{ s}}{1 \text{ min}} = 1. \]

Thus, the ratios \( \frac{1 \text{ min}}{60 \text{ s}} \) and \( \frac{60 \text{ s}}{1 \text{ min}} \) can be used as conversion factors. This is *not* the same as writing \( \frac{1}{60} = 1 \) or \( 60 = 1 \); each *number* and its *unit* must be treated together.

Because multiplying any quantity by unity leaves the quantity unchanged, we can introduce conversion factors wherever we find them useful. In chain-link conversion, we use the factors to cancel unwanted units. For example, to convert 2 min to seconds, we have

\[ 2 \text{ min} = (2 \text{ min})(1) = (2 \text{ min}) \left( \frac{60 \text{ s}}{1 \text{ min}} \right) = 120 \text{ s}. \quad (1-6) \]

If you introduce a conversion factor in such a way that unwanted units do *not* cancel, invert the factor and try again. In conversions, the units obey the same algebraic rules as variables and numbers.

Appendix D gives conversion factors between SI and other systems of units, including non-SI units still used in the United States. However, the conversion factors are written in the style of “1 min = 60 s” rather than as a ratio. So, you need to decide on the numerator and denominator in any needed ratio.

**Length**

In 1792, the newborn Republic of France established a new system of weights and measures. Its cornerstone was the meter, defined to be one ten-millionth of the distance from the north pole to the equator. Later, for practical reasons, this Earth standard was abandoned and the meter came to be defined as the distance between two fine lines engraved near the ends of a platinum–iridium bar, the *standard meter bar*, which was kept at the International Bureau of Weights and Measures near Paris. Accurate copies of the bar were sent to standardizing laboratories throughout the world. These *secondary standards* were used to produce other, still more accessible standards, so that ultimately every measuring device derived its authority from the standard meter bar through a complicated chain of comparisons.

Eventually, a standard more precise than the distance between two fine scratches on a metal bar was required. In 1960, a new standard for the meter, based on the wavelength of light, was adopted. Specifically, the standard for the meter was redefined to be 1 650 763.73 wavelengths of a particular orange-red light emitted by atoms of krypton-86 (a particular isotope, or type, of krypton) in a gas discharge tube that can be set up anywhere in the world. This awkward number of wavelengths was chosen so that the new standard would be close to the old meter-bar standard.
By 1983, however, the demand for higher precision had reached such a point that even the krypton-86 standard could not meet it, and in that year a bold step was taken. The meter was redefined as the distance traveled by light in a specified time interval. In the words of the 17th General Conference on Weights and Measures:

The meter is the length of the path traveled by light in a vacuum during a time interval of 1/299 792 458 of a second.

This time interval was chosen so that the speed of light \( c \) is exactly

\[ c = 299 792 458 \text{ m/s}. \]

Measurements of the speed of light had become extremely precise, so it made sense to adopt the speed of light as a defined quantity and to use it to redefine the meter.

Table 1-3 shows a wide range of lengths, from that of the universe (top line) to those of some very small objects.

## Significant Figures and Decimal Places

Suppose that you work out a problem in which each value consists of two digits. Those digits are called **significant figures** and they set the number of digits that you can use in reporting your final answer. With data given in two significant figures, your final answer should have only two significant figures. However, depending on the mode setting of your calculator, many more digits might be displayed. Those extra digits are meaningless.

In this book, final results of calculations are often rounded to match the least number of significant figures in the given data. (However, sometimes an extra significant figure is kept.) When the leftmost of the digits to be discarded is 5 or more, the last remaining digit is rounded up; otherwise it is retained as is. For example, 11.3516 is rounded to three significant figures as 11.4 and 11.3279 is rounded to three significant figures as 11.3. (The answers to sample problems in this book are usually presented with the symbol \( \approx \) instead of \( \approx \) even if rounding is involved.)

When a number such as 3.15 or \( 3.15 \times 10^3 \) is provided in a problem, the number of significant figures is apparent, but how about the number 3000? Is it known to only one significant figure (\( 3 \times 10^3 \))? Or is it known to as many as four significant figures (\( 3.000 \times 10^3 \))? In this book, we assume that all the zeros in such given numbers as 3000 are significant, but you had better not make that assumption elsewhere.

Don’t confuse **significant figures** with **decimal places**. Consider the lengths 35.6 mm, 3.56 m, and 0.00356 m. They all have three significant figures but they have one, two, and five decimal places, respectively.

### Sample Problem 1.01  Estimating order of magnitude, ball of string

The world’s largest ball of string is about 2 m in radius. To the nearest order of magnitude, what is the total length \( L \) of the string in the ball?

**KEY IDEA**

We could, of course, take the ball apart and measure the total length \( L \), but that would take great effort and make the ball’s builder most unhappy. Instead, because we want only the nearest order of magnitude, we can estimate any quantities required in the calculation.

**Calculations:** Let us assume the ball is spherical with radius \( R = 2 \) m. The string in the ball is not closely packed (there are uncountable gaps between adjacent sections of string). To allow for these gaps, let us somewhat overestimate...
the cross-sectional area of the string by assuming the cross section is square, with an edge length \( d = 4 \text{ mm} \). Then, with a cross-sectional area of \( d^2 \) and a length \( L \), the string occupies a total volume of
\[
V = (\text{cross-sectional area})(\text{length}) = d^2L.
\]
This is approximately equal to the volume of the ball, given by \( \frac{4}{3}\pi R^3 \), which is about \( 4R^3 \) because \( \pi \) is about 3. Thus, we have the following
\[
d^2L = 4R^3,
\]
or
\[
L = \frac{4R^3}{d^2} = \frac{4(2 \text{ m})^3}{(4 \times 10^{-3} \text{ m})^2} = 2 \times 10^6 \text{ m} \approx 10^6 \text{ m} = 10^3 \text{ km}. \quad \text{(Answer)}
\]
(Note that you do not need a calculator for such a simplified calculation.) To the nearest order of magnitude, the ball contains about 1000 km of string!

**1-2 TIME**

Learning Objectives

After reading this module, you should be able to . . .

1.05 Change units for time by using chain-link conversions.

1.06 Use various measures of time, such as for motion or as determined on different clocks.

Key Idea

- The second is defined in terms of the oscillations of light emitted by an atomic (cesium-133) source. Accurate time signals are sent worldwide by radio signals keyed to atomic clocks in standardizing laboratories.

**Time**

Time has two aspects. For civil and some scientific purposes, we want to know the time of day so that we can order events in sequence. In much scientific work, we want to know how long an event lasts. Thus, any time standard must be able to answer two questions: “When did it happen?” and “What is its duration?” Table 1-4 shows some time intervals.

Any phenomenon that repeats itself is a possible time standard. Earth’s rotation, which determines the length of the day, has been used in this way for centuries; Fig. 1-1 shows one novel example of a watch based on that rotation. A quartz clock, in which a quartz ring is made to vibrate continuously, can be calibrated against Earth’s rotation via astronomical observations and used to measure time intervals in the laboratory. However, the calibration cannot be carried out with the accuracy called for by modern scientific and engineering technology.

**Table 1-4** Some Approximate Time Intervals

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Time Interval in Seconds</th>
<th>Measurement</th>
<th>Time Interval in Seconds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lifetime of the proton (predicted)</td>
<td>( 3 \times 10^{40} )</td>
<td>Time between human heartbeats</td>
<td>( 8 \times 10^{-1} )</td>
</tr>
<tr>
<td>Age of the universe</td>
<td>( 5 \times 10^{17} )</td>
<td>Lifetime of the muon</td>
<td>( 2 \times 10^{-6} )</td>
</tr>
<tr>
<td>Age of the pyramid of Cheops</td>
<td>( 1 \times 10^{11} )</td>
<td>Shortest lab light pulse</td>
<td>( 1 \times 10^{-16} )</td>
</tr>
<tr>
<td>Human life expectancy</td>
<td>( 2 \times 10^9 )</td>
<td>Lifetime of the most unstable particle</td>
<td>( 1 \times 10^{-23} )</td>
</tr>
<tr>
<td>Length of a day</td>
<td>( 9 \times 10^4 )</td>
<td>The Planck time(^a)</td>
<td>( 1 \times 10^{-43} )</td>
</tr>
</tbody>
</table>

\(^a\)This is the earliest time after the big bang at which the laws of physics as we know them can be applied.

**Figure 1-1** When the metric system was proposed in 1792, the hour was redefined to provide a 10-hour day. The idea did not catch on. The maker of this 10-hour watch wisely provided a small dial that kept conventional 12-hour time. Do the two dials indicate the same time?
Atomic clocks are so consistent that, in principle, two cesium clocks would have to run for 6000 years before their readings would differ by more than 1 second. Even such accuracy pales in comparison with that of clocks currently being developed; their precision may be 1 part in $10^{18}$—that is, 1 second in $1 \times 10^{18}$ seconds (which is about 3 x $10^{10}$ years).

To meet the need for a better time standard, atomic clocks have been developed. An atomic clock at the National Institute of Standards and Technology (NIST) in Boulder, Colorado, is the standard for Coordinated Universal Time (UTC) in the United States. Its time signals are available by shortwave radio (stations WWV and WWVH) and by telephone (303-499-7111). Time signals (and related information) are also available from the United States Naval Observatory at website http://tycho.usno.navy.mil/time.html. (To set a clock extremely accurately at your particular location, you would have to account for the travel time required for these signals to reach you.)

Figure 1-2 shows variations in the length of one day on Earth over a 4-year period, as determined by comparison with a cesium (atomic) clock. Because the variation displayed by Fig. 1-2 is seasonal and repetitious, we suspect the rotating Earth when there is a difference between Earth and atom as timekeepers. The variation is due to tidal effects caused by the Moon and to large-scale winds.

The 13th General Conference on Weights and Measures in 1967 adopted a standard second based on the cesium clock:

One second is the time taken by 9 192 631 770 oscillations of the light (of a specified wavelength) emitted by a cesium-133 atom.

Atomic clocks are so consistent that, in principle, two cesium clocks would have to run for 6000 years before their readings would differ by more than 1 second. Even such accuracy pales in comparison with that of clocks currently being developed; their precision may be 1 part in $10^{18}$—that is, 1 second in $1 \times 10^{18}$ seconds (which is about 3 x $10^{10}$ years).

1-3 MASS

Learning Objectives

After reading this module, you should be able to . . .

1.07 Change units for mass by using chain-link conversions.

1.08 Relate density to mass and volume when the mass is uniformly distributed.

Key Ideas

- The kilogram is defined in terms of a platinum–iridium standard mass kept near Paris. For measurements on an atomic scale, the atomic mass unit, defined in terms of the atom carbon-12, is usually used.

- The density $\rho$ of a material is the mass per unit volume:

$$\rho = \frac{m}{V}.$$

Mass

The Standard Kilogram

The SI standard of mass is a cylinder of platinum and iridium (Fig. 1-3) that is kept at the International Bureau of Weights and Measures near Paris and assigned, by

Figure 1-3 The international 1 kg standard of mass, a platinum–iridium cylinder 3.9 cm in height and in diameter.
international agreement, a mass of 1 kilogram. Accurate copies have been sent to standardizing laboratories in other countries, and the masses of other bodies can be determined by balancing them against a copy. Table 1-5 shows some masses expressed in kilograms, ranging over about 83 orders of magnitude.

The U.S. copy of the standard kilogram is housed in a vault at NIST. It is removed, no more than once a year, for the purpose of checking duplicate copies that are used elsewhere. Since 1889, it has been taken to France twice for recomparison with the primary standard.

### A Second Mass Standard

The masses of atoms can be compared with one another more precisely than they can be compared with the standard kilogram. For this reason, we have a second mass standard. It is the carbon-12 atom, which, by international agreement, has been assigned a mass of 12 atomic mass units (u). The relation between the two units is

\[ 1 \text{ u} = 1.660 \, 538 \, 86 \times 10^{-27} \text{ kg}, \quad (1-7) \]

with an uncertainty of ±10 in the last two decimal places. Scientists can, with reasonable precision, experimentally determine the masses of other atoms relative to the mass of carbon-12. What we presently lack is a reliable means of extending that precision to more common units of mass, such as a kilogram.

### Density

As we shall discuss further in Chapter 14, density \( \rho \) (lowercase Greek letter rho) is the mass per unit volume:

\[ \rho = \frac{m}{V} \quad (1-8) \]

Densities are typically listed in kilograms per cubic meter or grams per cubic centimeter. The density of water (1.00 gram per cubic centimeter) is often used as a comparison. Fresh snow has about 10% of that density; platinum has a density that is about 21 times that of water.

### Sample Problem 1.02  Density and liquefaction

A heavy object can sink into the ground during an earthquake if the shaking causes the ground to undergo liquefaction, in which the soil grains experience little friction as they slide over one another. The ground is then effectively quicksand. The possibility of liquefaction in sandy ground can be predicted in terms of the void ratio \( e \) for a sample of the ground:

\[ e = \frac{V_{\text{voids}}}{V_{\text{grains}}} \quad (1-9) \]

Here, \( V_{\text{grains}} \) is the total volume of the sand grains in the sample and \( V_{\text{voids}} \) is the total volume between the grains (in the voids). If \( e \) exceeds a critical value of 0.80, liquefaction can occur during an earthquake. What is the corresponding sand density \( \rho_{\text{sand}} \)? Solid silicon dioxide (the primary component of sand) has a density of \( \rho_{\text{SiO}_2} = 2.600 \times 10^3 \text{ kg/m}^3 \).

### Table 1-5  Some Approximate Masses

<table>
<thead>
<tr>
<th>Object</th>
<th>Mass in Kilograms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Known universe</td>
<td>( 1 \times 10^{53} )</td>
</tr>
<tr>
<td>Our galaxy</td>
<td>( 2 \times 10^{41} )</td>
</tr>
<tr>
<td>Sun</td>
<td>( 2 \times 10^{30} )</td>
</tr>
<tr>
<td>Moon</td>
<td>( 7 \times 10^{22} )</td>
</tr>
<tr>
<td>Asteroid Eros</td>
<td>( 5 \times 10^{15} )</td>
</tr>
<tr>
<td>Small mountain</td>
<td>( 1 \times 10^{12} )</td>
</tr>
<tr>
<td>Ocean liner</td>
<td>( 7 \times 10^{7} )</td>
</tr>
<tr>
<td>Elephant</td>
<td>( 5 \times 10^{3} )</td>
</tr>
<tr>
<td>Grape</td>
<td>( 3 \times 10^{-3} )</td>
</tr>
<tr>
<td>Speck of dust</td>
<td>( 7 \times 10^{-10} )</td>
</tr>
<tr>
<td>Penicillin molecule</td>
<td>( 5 \times 10^{-17} )</td>
</tr>
<tr>
<td>Uranium atom</td>
<td>( 4 \times 10^{-25} )</td>
</tr>
<tr>
<td>Proton</td>
<td>( 2 \times 10^{-27} )</td>
</tr>
<tr>
<td>Electron</td>
<td>( 9 \times 10^{-31} )</td>
</tr>
</tbody>
</table>

### KEY IDEA

The density of the sand \( \rho_{\text{sand}} \) in a sample is the mass per unit volume — that is, the ratio of the total mass \( m_{\text{sand}} \) of the sand grains to the total volume \( V_{\text{total}} \) of the sample:

\[ \rho_{\text{sand}} = \frac{m_{\text{sand}}}{V_{\text{total}}} \quad (1-10) \]

**Calculations:** The total volume \( V_{\text{total}} \) of a sample is

\[ V_{\text{total}} = V_{\text{grains}} + V_{\text{voids}} \]

Substituting for \( V_{\text{voids}} \) from Eq. 1-9 and solving for \( V_{\text{grains}} \) lead to

\[ V_{\text{grains}} = \frac{V_{\text{total}}}{1 + e} \quad (1-11) \]
From Eq. 1-8, the total mass $m_{\text{sand}}$ of the sand grains is the product of the density of silicon dioxide and the total volume of the sand grains:

$$m_{\text{sand}} = \rho_{\text{SiO}_2} V_{\text{grains}}, \quad (1-12)$$

Substituting this expression into Eq. 1-10 and then substituting for $V_{\text{grains}}$ from Eq. 1-11 lead to

$$\rho_{\text{sand}} = \frac{\rho_{\text{SiO}_2} V_{\text{total}}}{V_{\text{total}} + e} = \frac{\rho_{\text{SiO}_2}}{1 + e}. \quad (1-13)$$

Substituting $\rho_{\text{SiO}_2} = 2.600 \times 10^3 \text{ kg/m}^3$ and the critical value of $e = 0.80$, we find that liquefaction occurs when the sand density is less than

$$\rho_{\text{sand}} = \frac{2.600 \times 10^3 \text{ kg/m}^3}{1.80} = 1.4 \times 10^3 \text{ kg/m}^3. \quad (\text{Answer})$$

A building can sink several meters in such liquefaction.

**Measurement in Physics**

Physics is based on measurement of physical quantities. Certain physical quantities have been chosen as **base quantities** (such as length, time, and mass); each has been defined in terms of a **standard** and given a **unit** of measure (such as meter, second, and kilogram). Other physical quantities are defined in terms of the base quantities and their standards and units.

**SI Units**

The unit system emphasized in this book is the International System of Units (SI). The three physical quantities displayed in Table 1-1 are used in the early chapters. Standards, which must be both accessible and invariable, have been established for these base quantities by international agreement. These standards are used in all physical measurement, for both the base quantities and the quantities derived from them. Scientific notation and the prefixes of Table 1-2 are used to simplify measurement notation.

**Changing Units**

Conversion of units may be performed by using **chain-link conversions** in which the original data are multiplied successively by conversion factors written as unity and the units are manipulated like algebraic quantities until only the desired units remain.

**Length**

The meter is defined as the distance traveled by light during a precisely specified time interval.

**Time**

The second is defined in terms of the oscillations of light emitted by an atomic (cesium-133) source. Accurate time signals are sent worldwide by radio signals keyed to atomic clocks in standardizing laboratories.

**Mass**

The kilogram is defined in terms of a platinum–iridium standard mass kept near Paris. For measurements on an atomic scale, the atomic mass unit, defined in terms of the atom carbon-12, is usually used.

**Density**

The density $\rho$ of a material is the mass per unit volume:

$$\rho = \frac{m}{V}. \quad (1-8)$$
You can easily convert common units and measures electronically, but you still should be able to use a conversion table, such as those in Appendix D. Table 1-6 is part of a conversion table for a system of volume measures once common in Spain: a volume of 1 fanega is equivalent to 55.501 dm³ (cubic decimeters). To complete the table, what numbers (to three significant figures) should be entered in (a) the cahiz column, (b) the fanega column, (c) the cuartilla column, and (d) the almude column, starting with the top blank? Express 7.00 almudes in (e) medios, (f) cahizes, and (g) cubic centimeters (cm³).

Table 1-6 Problem 6

<table>
<thead>
<tr>
<th></th>
<th>cahiz</th>
<th>fanega</th>
<th>cuartilla</th>
<th>almude</th>
<th>medio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>12</td>
<td>48</td>
<td>144</td>
<td>288</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>4</td>
<td>12</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>3</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Hydraulic engineers in the United States often use, as a unit of volume of water, the acre-foot, defined as the volume of water that will cover 1 acre of land to a depth of 1 ft. A severe thunderstorm dumped 2.0 in. of rain in 30 min on a town of area 26 km². What volume of water, in acre-feet, fell on the town?

Harvard Bridge, which connects MIT with its fraternities across the Charles River, has a length of 364.4 Smoots plus one ear. The unit of one Smoot is based on the length of Oliver Reed Smoot, Jr., class of 1962, who was carried or dragged 1-Smoot lengths along the bridge. The marks have been repainted biannually by fraternity pledges since the initial measurement, usually during times of traffic congestion so that the police cannot easily interfere. (Presumably, the police were originally upset because they on (a) clock A, (b) clock B, and (c) clock C, (d) when clock A reads 400 s, what does clock B read? (d) When clock C reads 15.0 s, what does clock B read? (Assume negative readings for prezero times.)

Problem 9. Antarctica is roughly semicircular, with a radius of 2000 km (Fig. 1-5). The average thickness of its ice cover is 3000 m. How many cubic centimeters of ice does Antarctica contain? (Ignore the curvature of Earth.)

Module 1-2 Time

Until 1883, every city and town in the United States kept its own local time. Today, travelers reset their watches only when the time change equals 1.0 h. How far, on the average, must you travel in degrees of longitude between the time-zone boundaries at which your watch must be reset by 1.0 h? (Hint: Earth rotates 360° in about 24 h.)

For about 10 years after the French Revolution, the French government attempted to base measures of time on multiples of ten: One week consisted of 10 days, one day consisted of 10 hours, one hour consisted of 100 minutes, and one minute consisted of 100 seconds. What are the ratios of (a) the French decimal week to the standard week and (b) the French decimal second to the standard second?

The fastest growing plant on record is a Hesperoyucca whipplei that grew 3.7 m in 14 days. What was its growth rate in micrometers per second?

Three digital clocks A, B, and C run at different rates and do not have simultaneous readings of zero. Figure 1-6 shows simultaneous readings on pairs of the clocks for four occasions. (At the earliest occasion, for example, B reads 25.0 s and C reads 92.0 s.) If two events are 600 s apart on clock A, how far apart are they on (a) clock B and (b) clock C? (c) When clock A reads 400 s, what does clock B read? (d) When clock C reads 15.0 s, what does clock B read? (Assume negative readings for prezero times.)

A lecture period (50 min) is close to 1 microcentury. (a) How long is a microcentury in minutes? (b) Using

\[
\text{percentage difference} = \left(1 - \frac{\text{approximation}}{\text{actual}}\right) \times 100,
\]

find the percentage difference from the approximation.

A fortnight is a charming English measure of time equal to 2.0 weeks (the word is a contraction of “fourteen nights”). That is a nice amount of time in pleasant company but perhaps a painful string of microseconds in unpleasant company. How many microseconds are in a fortnight?

Time standards are now based on atomic clocks. A promising second standard is based on pulsars, which are rotating neutron stars (highly compact stars consisting of neutrons). Some rotate at a rate that is highly stable, sending out a radio beacon that sweeps briefly across Earth once with each rotation, like a lighthouse beacon. Pulsar PSR 1937 + 21 is an example; it rotates once every 1.557 806 448 872 75 ± 3 ms, where the trailing ±3 indicates the uncertainty in the last decimal place (it does not mean ±3 ms). (a) How many rotations does PSR 1937 + 21 make in 7.00 days? (b) How much time does the pulsar take to rotate exactly one million times and (c) what is the associated uncertainty?
Five clocks are being tested in a laboratory. Exactly at noon, as determined by the WWV time signal, on successive days of a week the clocks read as in the following table. Rank the five clocks according to their relative value as good timekeepers, best to worst. Justify your choice.

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<thead>
<tr>
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<tbody>
<tr>
<td>B</td>
<td>11:59:59</td>
<td>12:00:02</td>
<td>11:59:57</td>
<td>12:00:07</td>
<td>12:00:02</td>
<td>11:59:56</td>
<td>12:00:03</td>
</tr>
<tr>
<td>E</td>
<td>12:03:59</td>
<td>12:02:49</td>
<td>12:01:54</td>
<td>12:01:52</td>
<td>12:01:32</td>
<td>12:01:22</td>
<td>12:01:12</td>
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Because Earth's rotation is gradually slowing, the length of each day increases: The day at the end of 1.0 century is 1.0 s longer than the day at the start of the century. In 20 centuries, what is the total of the daily increases in time?

Suppose that, while lying on a beach near the equator watching the Sun set over a calm ocean, you start a stopwatch just as the top of the Sun disappears. You then stand, elevating your eyes by a height $H = 1.70 \text{ m}$, and stop the watch when the top of the Sun again disappears. If the elapsed time is $t = 11.1 \text{ s}$, what is the radius $r$ of Earth?

**Module 1-3 Mass**

The record for the largest glass bottle was set in 1992 by a team in Millville, New Jersey—they blew a bottle with a volume of 193 U.S. fluid gallons. (a) How much short of 1.0 million cubic centimeters is that? (b) If the bottle were filled with water at the leisurely rate of 1.8 g/min, how long would the filling take? Water has a density of 1000 kg/m$^3$.

Earth has a mass of $5.98 \times 10^{24}$ kg. The average mass of the atoms that make up Earth is 40 u. How many atoms are there in Earth?

Gold, which has a density of 19.32 g/cm$^3$, is the most ductile metal and can be pressed into a thin leaf or drawn out into a long fiber. (a) If a sample of gold, with a mass of 27.63 g, is pressed into a leaf of 1.000 $\mu$m thickness, what is the area of the leaf? (b) If, instead, the gold is drawn out into a cylindrical fiber of radius 2.500 $\mu$m, what is the length of the fiber?

(a) Assuming that water has a density of exactly 1 g/cm$^3$, find the mass of one cubic meter of water in kilograms. (b) Suppose that it takes 10.0 h to drain a container of 5700 m$^3$ of water. What is the “mass flow rate,” in kilograms per second, of water from the container?

Grains of fine California beach sand are approximately spheres with an average radius of 50 $\mu$m and are made of silicon dioxide, which has a density of 2600 kg/m$^3$. What mass of sand grains would have a total surface area (the total area of all the individual spheres) equal to the surface area of a cube 1.00 m on an edge?

During heavy rain, a section of a mountainside measuring 2.5 km horizontally, 0.80 km up along the slope, and 2.0 m deep slips into a valley in a mud slide. Assume that the mud ends up uniformly distributed over a surface area of the valley measuring 0.40 km $\times$ 0.40 km and that mud has a density of 1900 kg/m$^3$. What is the mass of the mud sitting above a 4.0 m$^2$ area of the valley floor?

One cubic centimeter of a typical cumulus cloud contains 50 to 500 water drops, which have a typical radius of 10 $\mu$m. For that range, give the lower value and the higher value, respectively, for the following. (a) How many cubic meters of water are in a cylindrical cumulus cloud of height 3.0 km and radius 1.0 km? (b) How many 1-liter pop bottles would that water fill? (c) Water has a density of 1000 kg/m$^3$. How much mass does the water in the cloud have?

Iron has a density of 7.87 g/cm$^3$, and the mass of an iron atom is $9.27 \times 10^{-26}$ kg. If the atoms are spherical and tightly packed, (a) what is the volume of an iron atom and (b) what is the distance between the centers of adjacent atoms?

A mole of atoms is $6.02 \times 10^{23}$ atoms. To the nearest order of magnitude, how many moles of atoms are in a large domestic cat? The masses of a hydrogen atom, an oxygen atom, and a carbon atom are 1.0 u, 16 u, and 12 u, respectively. (Hint: Cats are sometimes known to kill a mole.)

On a spending spree in Malaysia, you buy an ox with a weight of 28.9 piculs in the local unit of weights: 1 picul = 100 gins, 1 gin = 16 tahils, 1 tahil = 10 chees, and 1 chee = 10 hoons. The weight of 1 hoon corresponds to a mass of 0.3779 g. When you arrange to ship the ox home to your astonished family, how much mass in kilograms must you declare on the shipping manifest? (Hint: Set up multiple chain-link conversions.)

Water is poured into a container that has a small leak. The mass $m$ of the water is given as a function of time $t$ by $m = 5.00t^{0.8} - 3.00t + 20.00$, with $t \geq 0$, $m$ in grams, and $t$ in seconds. (a) At what time is the water mass greatest, and (b) what is that greatest mass? In kilograms per minute, what is the rate of mass change at (c) $t = 2.00$ s and (d) $t = 5.00$ s?

A vertical container with base area measuring 14.0 cm$^2$ by 17.0 cm is being filled with identical pieces of candy, each with a volume of 50.0 mm$^3$ and a mass of 0.0200 g. Assume that the volume of the empty spaces between the candies is negligible. If the height of the candies in the container increases at the rate of 0.250 cm/s, at what rate (kilograms per minute) does the mass of the candies in the container increase?

**Additional Problems**

In the United States, a doll house has the scale of 1:12 of a real house (that is, each length of the doll house is $\frac{1}{12}$ that of the real house) and a miniature house (a doll house to fit within a doll house) has the scale of 1:144 of a real house. Suppose a real house (that is, each length of the doll house is $\frac{1}{144}$ of the real house) and a miniature house (a doll house to fit within a doll house) has a height of 3.0 m.

**Figure 1-7 Problem 32.**
A ton is a measure of volume frequently used in shipping, but that use requires some care because there are at least three types of tons: A displacement ton is equal to 7 barrels bulk, a freight ton is equal to 8 barrels bulk, and a register ton is equal to 20 barrels bulk. A barrel bulk is another measure of volume: 1 barrel bulk = 0.1415 m³. Suppose you spot a shipping order for “73 tons” of M&M candies, and you are certain that the client who sent the order intended “ton” to refer to volume (instead of weight or mass, as discussed in Chapter 5). If the client actually meant displacement tons, how many extra U.S. bushels of the candies will he receive? (a) The mistaken tourist believes she needs and (b) the car actually requires?

A cord is a volume of cut wood equal to a stack 8 ft long, 4 ft wide, and 4 ft high. How many cords are in 1.0 m³?

One molecule of water (H₂O) contains two atoms of hydrogen and one atom of oxygen. A hydrogen atom has a mass of 1.0 u and an atom of oxygen has a mass of 16 u, approximately. (a) What is the mass in kilograms of one molecule of water? (b) How many molecules of water are in the world’s oceans, which have an estimated total mass of 1.4 × 10²¹ kg?

A person on a diet might lose 2.3 kg per week. Express the mass loss rate in milligrams per second, as if the dieter could sense the second-by-second loss.

What mass of water fell on the town in Problem 7? Water has a density of 1.0 × 10³ kg/m³.

(a) A unit of time sometimes used in microscopic physics is the shake. One shake equals 10⁻⁸ s. Are there more shakes in a second than there are seconds in a year? (b) Humans have existed for about 10⁸ years, whereas the universe is about 10¹⁰ years old. If the age of the universe is defined as 1 “universe day,” where a universe day consists of “universe seconds” as a normal day consists of normal seconds, how many universe seconds have humans existed?

A unit of area often used in measuring land areas is the hectare, defined as 10⁴ m². An open-pit coal mine consumes 75 hectares of land, down to a depth of 26 m, each year. What volume of earth, in cubic kilometers, is removed in this time?

An astronomical unit (AU) is the average distance between Earth and the Sun, approximately 1.50 × 10¹³ km. The speed of light is about 3.0 × 10⁸ m/s. Express the speed of light in astronomical units per minute.

The common Eastern mole, a mammal, typically has a mass of 75 g, which corresponds to about 7.5 moles of atoms. (A mole of atoms is 6.02 × 10²³ atoms.) In atomic mass units (u), what is the average mass of the atoms in the common Eastern mole?

A traditional unit of length in Japan is the ken (1 ken = 1.97 m). What are the ratios of (a) square kens to square meters and (b) cubic kens to cubic meters? What is the volume of a cylindrical water tank of height 5.50 kens and radius 3.00 kens in (c) cubic kens and (d) cubic meters?

You receive orders to sail due east for 24.5 mi to put your salvage ship directly over a sunken pirate ship. However, when your divers probe the ocean floor at that location and find no evidence of a ship, you radio back to your source of information, only to discover that the sailing distance was supposed to be 245 nautical miles, not regular miles. Use the Length table in Appendix D to calculate how far horizontally you are from the pirate ship in kilometers.
51 The cubit is an ancient unit of length based on the distance between the elbow and the tip of the middle finger of the measurer. Assume that the distance ranged from 43 to 53 cm, and suppose that ancient drawings indicate that a cylindrical pillar was to have a length of 9 cubits and a diameter of 2 cubits. For the stated range, what are the lower value and the upper value, respectively, for (a) the cylinder’s length in meters, (b) the cylinder’s length in millimeters, and (c) the cylinder’s volume in cubic meters?

52 As a contrast between the old and the modern and between the large and the small, consider the following: In old rural England 1 hide (between 100 and 120 acres) was the area of land needed to sustain one family with a single plough for one year. (An area of 1 acre is equal to 4047 m².) Also, 1 wapentake was the area of land needed by 100 such families. In quantum physics, the cross-sectional area of a nucleus (defined in terms of the chance of a particle hitting and being absorbed by it) is measured in units of barns, where 1 barn is $1 \times 10^{-28}$ m². (In nuclear physics jargon, if a nucleus is “large,” then shooting a particle at it is like shooting a bullet at a barn door, which can hardly be missed.) What is the ratio of 25 wapentakes to 11 barns?

53 SSM An astronomical unit (AU) is equal to the average distance from Earth to the Sun, about $92.9 \times 10^6$ mi. A parsec (pc) is the distance at which a length of 1 AU would subtend an angle of exactly 1 second of arc (Fig. 1-8). A light-year (ly) is the distance that light, traveling through a vacuum with a speed of 186 000 mi/s, would cover in 1.0 year. Express the Earth–Sun distance in (a) parsecs and (b) light-years.

54 The description for a certain brand of house paint claims a coverage of 460 ft²/gal. (a) Express this quantity in square meters per liter. (b) Express this quantity in an SI unit (see Appendices A and D). (c) What is the inverse of the original quantity, and (d) what is its physical significance?

55 Strangely, the wine for a large wedding reception is to be served in a stunning cut-glass receptacle with the interior dimensions of 40 cm x 40 cm x 30 cm (height). The receptacle is to be initially filled to the top. The wine can be purchased in bottles of the sizes given in the following table. Purchasing a larger bottle instead of multiple smaller bottles decreases the overall cost of the wine. To minimize the cost, (a) which bottle sizes should be purchased and how many of each should be purchased and, once the receptacle is filled, how much wine is left over in terms of (b) standard bottles and (c) liters?

1 standard bottle
1 magnum = 2 standard bottles
1 jeroboam = 4 standard bottles
1 rehoboam = 6 standard bottles
1 methuselah = 8 standard bottles
1 salmanazar = 12 standard bottles
1 balthazar = 16 standard bottles = 11.356 L
1 nebuchadnezzar = 20 standard bottles

56 The corn–hog ratio is a financial term used in the pig market and presumably is related to the cost of feeding a pig until it is large enough for market. It is defined as the ratio of the market price of a pig with a mass of 3.108 slugs to the market price of a U.S. bushel of corn. (The word “slug” is derived from an old German word that means “to hit”; we have the same meaning for “slug” as a verb in modern English.) A U.S. bushel is equal to 35.238 L. If the corn–hogs ratio is listed as 5.7 on the market exchange, what is it in the metric units of

$$\frac{\text{price of 1 kilogram of pig}}{\text{price of 1 liter of corn}} ?$$

(Hint: See the Mass table in Appendix D.)

57 You are to fix dinners for 400 people at a convention of Mexican food fans. Your recipe calls for 2 jalapeño peppers per serving (one serving per person). However, you have only habanero peppers on hand. The spiciness of peppers is measured in terms of the scoville heat unit (SHU). On average, one jalapeño pepper has a spiciness of 4000 SHU and one habanero pepper has a spiciness of 300 000 SHU. To get the desired spiciness, how many habanero peppers should you substitute for the jalapeño peppers in the recipe for the 400 dinners?

58 A standard interior staircase has steps each with a rise (height) of 19 cm and a run (horizontal depth) of 23 cm. Research suggests that the stairs would be safer for descent if the run were, instead, 28 cm. For a particular staircase of total height 4.57 m, how much farther into the room would the staircase extend if this change in run were made?

59 In purchasing food for a political rally, you erroneously order shucked medium-size Pacific oysters (which come 8 to 12 per U.S. pint) instead of shucked medium-size Atlantic oysters (which come 26 to 38 per U.S. pint). The filled oyster container shipped to you has the interior measure of 1.0 m × 12 cm × 20 cm, and a U.S. pint is equivalent to 0.4732 liter. By how many oysters does the order short of your anticipated count?

60 An old English cookbook carries this recipe for cream of nettle soup: “Boil stock of the following amount: 1 breakfastcup plus 1 teacup plus 6 tablespoons plus 1 dessertspoon. Using gloves, separate nettle tops until you have 0.5 quart; add the tops to the boiling stock. Add 1 tablespoon of cooked rice and 1 saltspoon of salt. Simmer for 15 min.” The following table gives some of the conversions among old (premetric) British measures and among common (still premetric) U.S. measures. (These measures just scream for metrication.) For liquid measures, 1 British teaspoon = 1 U.S. teaspoon. For dry measures, 1 British teaspoon = 2 U.S. teaspoons and 1 British quart = 1 U.S. quart. In U.S. measures, how much (a) stock, (b) nettle tops, (c) rice, and (d) salt are required in the recipe?

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<thead>
<tr>
<th>Old British Measures</th>
<th>U.S. Measures</th>
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<tbody>
<tr>
<td>teaspoon = 2 saltspoons</td>
<td>tablespoon = 3 teaspoons</td>
</tr>
<tr>
<td>dessertspoon = 2 teaspoons</td>
<td>half cup = 8 tablespoons</td>
</tr>
<tr>
<td>tablespoon = 2 dessertspoons</td>
<td>cup = 2 half cups</td>
</tr>
<tr>
<td>teacup = 8 tablespoons</td>
<td>breakfastcup = 2 teacups</td>
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</tbody>
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