Preface

Traditionally, insurance has been built on the assumption of independence, and the law of large numbers has governed the determination of premiums. But these days, the increasing complexity of insurance and reinsurance products has led to increased actuarial interest in the modelling of dependent risks.

In many situations, insured risks tend to behave alike. For instance, in group life insurance the remaining lifetimes of husband and wife can be shown to possess a certain degree of ‘positive dependence’. The emergence of catastrophes and the interplay between insurance and finance also offer good examples in which dependence plays an important role in pricing and reserving.

Several concepts of bivariate and multivariate positive dependence have appeared in the mathematical literature. Undoubtedly, the most commonly encountered dependence property is actually ‘lack of dependence’, in other words mutual independence. Actuaries have so far mostly been interested in positive dependence properties expressing the notion that ‘large’ (or ‘small’) values of the random variables tend to occur together. Negative dependence properties express the notion that ‘large’ values of one variable tend to occur together with small values of the others. Instances of this phenomenon naturally arise in life insurance (think, for instance, of the death and survival benefits after year \( k \) in an endowment insurance, which are mutually exclusive and hence negatively correlated), or for the purpose of competitive pricing. Note that, in general, a negative dependence results in more predictable losses for the insurance company than mutual independence. The independence assumption is thus conservative in such a case. Moreover, assuming independence is mathematically convenient, and also obviates the need for elaborate models to be devised and statistics to be kept on mutual dependence of claims.

There is only one way for risks to be independent, but there are of course infinitely many ways for them to be correlated. For efficient risk management, actuaries need to be able to answer the fundamental question: is the correlation structure dangerous? And if it is, how dangerous is the situation? Therefore, tools to quantify, to compare and to model the strength of dependence between different risks have now become essential.

The purpose of this book is to provide its readership with methods to:

- measure risk
- compare risks
- measure the strength of dependence
- compare dependence structures
- model the dependence structure.

To illustrate the theoretical concepts, we will give many applications in actuarial science.
This book is innovative in many respects. It integrates the theory of stochastic orders, one of the methodological cornerstones of risk theory, the theory of risk measures, the very foundation of risk management, and the theory of stochastic dependence, which has become increasingly important as new types of risks emerge.

More specifically, risk measures will be used to generate stochastic orderings, by identifying pairs of risks about which a class of risk measures agree. Stochastic orderings are then used to define positive dependence relationships.

The copula concept is examined in detail. Apart from the well-known correlation coefficient, other measures of dependence are presented, as well as multivariate stochastic orderings, to evaluate the strength of dependence between risks. We also emphasize the numerous connections existing between multivariate and univariate stochastic orders.

In the third part of the book, we discuss some applications in actuarial mathematics. We first review credibility models. In these models, past claims history not only of the risk itself, but also of related risks, is used to determine the future premium. This method is based on the serial correlation among the annual claim characteristics (frequencies or severities) induced by their sharing a common random effect, and on the correlation between ‘related’ risks caused by a similar effect. We describe the kind of dependence induced by credibility models, and establish numerous stochastic inequalities showing that the classical credibility construction pioneered by Bühlmann produces very intuitive results.

Secondly, we will derive bounds on actuarial quantities involving correlated risks whose joint distribution is (partially) unknown or too cumbersome to work with. Our focus will be on stop-loss premiums and Value-at-Risk.

Next, we will present probabilistic distances, and show the close connection between this theoretical tool and stochastic orderings. In particular, the relevance of probabilistic distances for the analysis of dependent risks will be demonstrated.

This book complements our Modern Actuarial Risk Theory (Kaas et al. 2001), which only scratches the surface of the material found here. Since the traditional actuarial risk theory assumes independence between the different random variables of interest, the present book may be thought of as an advanced course on risk theory dropping this hypothesis.

The target audience of this book consists of academics and practitioners who are eager to master modern modelling tools for dependent risks. The inclusion of many exercises also makes the book suitable as the basis for advanced courses on risk management in incomplete markets, as a complement to Kaas et al. (2001).

Sometimes, we will give proofs only under simplifying assumptions, in order to help the reader understand the underlying reasoning, bringing out the main ideas without obscuring them with mathematical technicalities. Some proofs are omitted. Appropriate references to the literature will guide the readers interested in a more thorough mathematical treatment of the topic.

Insurance markets are prominent examples of incomplete markets, since the products sold by insurance companies cannot be replicated by some financial trading strategy. We firmly believe that this book should be of interest not only to actuaries but more generally to traders aware that perfect hedges do not exist in reality. The main effect of accounting for market incompleteness has indeed been to bring utility theory back into pricing. More generally, it should bridge quantitative finance and actuarial science.

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