# CONTENTS

Preface xiii

## 1 Basics of Linear Algebra 1

1.1 Notation and Terminology 1
1.2 Vector and Matrix Norms 4
1.3 Dot Product and Orthogonality 8
1.4 Special Matrices 9
  1.4.1 Diagonal and triangular matrices 9
  1.4.2 Hessenberg matrices 10
  1.4.3 Nonsingular and inverse matrices 11
  1.4.4 Symmetric and positive definite matrices 12
  1.4.5 Matrix exponential 14
  1.4.6 Permutation matrices 15
  1.4.7 Orthogonal matrices 17
1.5 Vector Spaces 21
1.6 Linear Independence and Basis 24
1.7 Orthogonalization and Direct Sums 31
1.8 Column Space, Row Space, and Null Space 34
  1.8.1 Linear transformations 40
1.9 Orthogonal Projections 43
1.10 Eigenvalues and Eigenvectors 47

vii
<table>
<thead>
<tr>
<th>Section Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.11 Similarity</td>
<td>56</td>
</tr>
<tr>
<td>1.12 Bezier Curves and Postscript Fonts</td>
<td>59</td>
</tr>
<tr>
<td>1.12.1 Properties of Bezier curves</td>
<td>61</td>
</tr>
<tr>
<td>1.12.2 Composite Bezier curves</td>
<td>66</td>
</tr>
<tr>
<td>1.13 Final Remarks and Further Reading</td>
<td>68</td>
</tr>
<tr>
<td>Exercises</td>
<td>69</td>
</tr>
<tr>
<td>2 Ranking Web Pages</td>
<td>79</td>
</tr>
<tr>
<td>2.1 The Power Method</td>
<td>80</td>
</tr>
<tr>
<td>2.2 Stochastic, Irreducible, and Primitive Matrices</td>
<td>84</td>
</tr>
<tr>
<td>2.3 Google's PageRank Algorithm</td>
<td>92</td>
</tr>
<tr>
<td>2.3.1 The personalization vector</td>
<td>99</td>
</tr>
<tr>
<td>2.3.2 Speed of convergence and sparsity</td>
<td>100</td>
</tr>
<tr>
<td>2.3.3 Power method and reordering</td>
<td>105</td>
</tr>
<tr>
<td>2.4 Alternatives to the Power Method</td>
<td>106</td>
</tr>
<tr>
<td>2.4.1 Linear system formulation</td>
<td>107</td>
</tr>
<tr>
<td>2.4.2 Iterative aggregation/disaggregation (IAD)</td>
<td>111</td>
</tr>
<tr>
<td>2.4.3 IAD and linear systems</td>
<td>117</td>
</tr>
<tr>
<td>2.5 Final Remarks and Further Reading</td>
<td>120</td>
</tr>
<tr>
<td>Exercises</td>
<td>121</td>
</tr>
<tr>
<td>3 Matrix Factorizations</td>
<td>131</td>
</tr>
<tr>
<td>3.1 LU Factorization</td>
<td>132</td>
</tr>
<tr>
<td>3.1.1 The complex case</td>
<td>137</td>
</tr>
<tr>
<td>3.1.2 Solving several systems</td>
<td>137</td>
</tr>
<tr>
<td>3.1.3 The $PA = LU$ factorization</td>
<td>139</td>
</tr>
<tr>
<td>3.2 QR Factorization</td>
<td>142</td>
</tr>
<tr>
<td>3.2.1 QR and Gram–Schmidt</td>
<td>143</td>
</tr>
<tr>
<td>3.2.2 The complex case</td>
<td>147</td>
</tr>
<tr>
<td>3.2.3 QR and similarity</td>
<td>148</td>
</tr>
<tr>
<td>3.2.4 The QR algorithm</td>
<td>149</td>
</tr>
<tr>
<td>3.2.5 $QR$ and $LU$</td>
<td>151</td>
</tr>
<tr>
<td>3.3 Singular Value Decomposition (SVD)</td>
<td>155</td>
</tr>
<tr>
<td>3.3.1 The complex case</td>
<td>160</td>
</tr>
<tr>
<td>3.3.2 Low-rank approximations</td>
<td>161</td>
</tr>
<tr>
<td>3.3.3 $SVD$ and spectral norm</td>
<td>164</td>
</tr>
<tr>
<td>3.4 Schur Factorization</td>
<td>166</td>
</tr>
<tr>
<td>3.4.1 The complex case</td>
<td>171</td>
</tr>
<tr>
<td>3.4.2 Schur factorization and invariant subspaces</td>
<td>172</td>
</tr>
<tr>
<td>3.4.3 Exchanging eigenblocks</td>
<td>177</td>
</tr>
<tr>
<td>3.4.4 Block diagonalization</td>
<td>180</td>
</tr>
</tbody>
</table>
3.5 Information Retrieval 186
  3.5.1 Query matching 187
  3.5.2 Low-rank query matching 190
  3.5.3 Term–term comparison 192
3.6 Partition of Simple Substitution Cryptograms 194
  3.6.1 Rank-1 approximation 197
  3.6.2 Rank-2 approximation 199
3.7 Final Remarks and Further Reading 203
  Exercises 205

4 Least Squares 215
  4.1 Projections and Normal Equations 215
  4.2 Least Squares and QR Factorization 224
  4.3 Lagrange Multipliers 228
  4.4 Final Remarks and Further Reading 231
    Exercises 231

5 Image Compression 235
  5.1 Compressing with Discrete Cosine Transform 236
    5.1.1 1-D discrete cosine transform 236
    5.1.2 2-D discrete cosine transform 242
    5.1.3 Image compression and the human visual system 245
    5.1.4 Basis functions and images 247
    5.1.5 Low-pass filtering 250
    5.1.6 Quantization 254
    5.1.7 Compression of color images 257
  5.2 Huffman Coding 260
    5.2.1 Huffman coding and JPEG 262
  5.3 Compression with SVD 267
    5.3.1 Compressing grayscale images 268
    5.3.2 Compressing color images 268
  5.4 Final Remarks and Further Reading 269
    Exercises 271

6 Ordinary Differential Equations 277
  6.1 One-Dimensional Differential Equations 278
    6.1.1 Existence and uniqueness 278
    6.1.2 A simple population model 284
    6.1.3 Emigration 285
    6.1.4 Time-varying emigration 285
    6.1.5 Competition 286
7 Dynamical Systems

7.1 Linear Dynamical Systems
7.1.1 Dynamics in two dimensions
7.1.2 Trace-determinant analysis
7.1.3 Stable, unstable, and center subspaces

7.2 Nonlinear Dynamical Systems
7.2.1 Linearization around an equilibrium point
7.2.2 Linearization around a periodic orbit
7.2.3 Connecting orbits
7.2.4 Chaos
7.2.5 Bifurcations

7.3 Predator–prey Models with Harvesting
7.3.1 Boundedness of solutions
7.3.2 Equilibrium point analysis
7.3.3 Bifurcations
7.3.4 Connecting orbits
7.3.5 Other models

7.4 Final Remarks and Further Reading
Exercises

8 Mathematical Models

8.1 Optimization of a Waste Management System
8.1.1 Background
8.1.2 Description of the system
8.1.3 Development of the mathematical model
8.1.4 Building the objective function
8.1.5 Building the constraints
8.1.6 Numerical experiments

8.2 Grouping Problem in Networks
## Contents

8.2.1 Background 405  
8.2.2 The $N$-median approach 406  
8.2.3 The probabilistic approach 407  
8.2.4 Numerical experiments 409  

### 8.3 American Cutaneous Leishmaniasis 410  
8.3.1 Background 410  
8.3.2 Development of the mathematical model 413  
8.3.3 Equilibria and periodic orbits 414  
8.3.4 Stability properties 416  
8.3.5 Numerical computations 418  

### 8.4 Variable Population Interactions 420  
8.4.1 Model formulation 420  
8.4.2 Local stability of equilibria 421  
8.4.3 Bifurcations 425  

References 431  

Index 435