INDEX

$C^1$ norm, 365
$LU$ factorization, 132
and permutations, 139
existence and uniqueness, 136
$N$-median problem, 405
$QR$ factorization, 142, 153
and least squares, 224
and linear systems, 154
Householder, 153
thin, 144
uniqueness, 147
$SVD$ compression, 267
$A$-invariant, 166
AC coefficients, 262, 264
Adjacency matrix, 86
Adjoint of a matrix, 3
Aggregated matrix, 112, 129
Aggregation/disaggregation
  exact, 113
Aggregation/disaggregation
  approximate, 115
Algebraic multiplicity, 54
American cutaneous leishmaniasis, 410
Approximate aggregation matrix, 114
Approximate eigenvector, 75
Asymptotically stable equilibrium, 328
Backward substitution, 133
Banach space, 356
Baseline JPEG, 257
Basis, 27
canonical, 27, 28
orthonormal, 175
Beads, 295
Bernstein polynomials, 29, 59
Best teacher, 197
Bezier curve, 61, 76
composite, 66
midpoint property, 64
Bezier spline, 66
Bifurcation
  saddle-node, 430
Bifurcation, 365, 379
cusp, 393
fold, 369, 380
heteroclinic, 394
homoclinic, 393
Hopf, 372, 380, 384, 430
period doubling, 363, 392
pitchfork, 370
saddle-node, 369, 392, 426
saddle-node of periodic orbits, 393
singularity, 373
transition, 367, 384, 425, 430
Bifurcation diagram, 367, 380
Bit size, 203
Block diagonalization, 180
Block Schur factorization, 170, 175, 359
Block triangular matrix, 10, 167, 169
eigenvalues of, 51
Boundedness of solutions, 376

Canonical basis, 27, 28
Cauchy sequence, 281
Cauchy–Schwarz inequality, 69
Center, 329, 332, 335
Center subspace, 338
Chaos, 360
Characteristic equation, 48, 288
Choleski, 13
Choleski factorization, 206
Chrominance matrix, 258
Chua’s circuit, 363
Cipher, 195
Circuits, 302
Clustering, 192
Codomain, 42
Column space, 34
orthonormal basis, 158
orthonormal basis of, 143
Competition, 286, 421
Complementary spaces, 32
Compression of color images, 257
Compression with SVD, 267
Condition number, 18, 73, 154
Connecting orbits, 352, 380, 418
Consistent norm, 8
Contraction, 280, 341
Control points, 61
Convergent sequence, 281
Convex hull, 64
Cooperative systems, 415
Cosine of an angle, 187
Cryptogram, 194
Cusp bifurcation, 393

Damped equations, 298
Damping, 288
Damping constant, 288
Dangling node, 95, 109, 118, 128
reordering, 109
DC coefficient, 262
DCT, 236
1-dimensional, 242
1-dimensional, 236
DCT coefficients, 252
DCT interpolation theorem, 237
DCT least squares approximation, 240, 245
Defect of an eigenvalue, 311
Defective eigenvalue, 54, 310
Degenerate equilibrium point, 334
Dependence on initial conditions, 340
Determinant, 11, 58
Diagonal matrix, 9
Diagonalizable matrix, 57, 76
Differentiable manifold, 343
Dirichlet frequency matrix, 195
Dimension formula, 39, 42
Dimension of a vector space, 29

Direct sum, 32, 338
basis of, 75
Discrete cosine transform, 236, 242
Domain, 42
Dominant eigenspace, 124
Dominant eigenvector, 81
Dot product, 8
Double-scroll attractor, 363
Dynamical system, 327

Eigenbasis, 177
Eigenblock, 177
Eigenpair, 177
Eigenspace, 54
dominant, 124
generalized, 387
Eigenvalue, 48, 307
complex, 309
defective, 54, 310
distinct, 308
of a symmetric matrix, 52, 75
of transpose, 50
repeated, 309
simple, 54

Eigenvector, 48, 307
approximate, 75
dominant, 81
generalized, 54, 310, 311, 322
left, 179

Electrical resonance, 306
Elementary row operations, 133
Emigration, 284
time-varying, 285

Entropy, 260
Equilibrium equations, 229
Equilibrium point, 327, 341
asymptotically stable, 328
stable, 328
unstable, 328
Exchanging eigenblocks, 177
Existence and uniqueness, 278, 283, 340

Fixed point, 280
Fixed point iteration, 281
Flame fronts, 391
Floquet exponent, 350
Floquet multiplier, 349
Floquet’s theorem, 388
Flow of the system, 342
Focus, 329, 331, 335
Fold bifurcation, 369, 380
Frobenius norm, 6, 7, 69, 70, 209
Full rank matrix, 143
Fundamental matrix solution, 311
Fundamental theorem of linear algebra, 39

Gaussian elimination, 133
Generalized eigenspace, 387
Generalized eigenvector, 54, 310, 311, 322
INDEX 437

Geometric multiplicity, 54
Gerchgorin circle theorem, 75, 211
Golubev, 92
Golubev matrix, 97
Gram–Schmidt, 31
Gram-Schmidt, 72, 143
Graph of a matrix, 86
Gronwall's inequality, 388
Grouping in networks, 404

Haar matrix, 273
Hadamard matrix, 273
Hartman–Grobman theorem, 347
Harvesting, 375, 430
Hermite matrix, 13
Hessenberg matrix, 10, 151
properly, 211
Heteroclinic bifurcation, 394
Heteroclinic orbit, 352
Humeomorphism, 343, 389
Homoclinic bifurcation, 393
Homoclinic orbit, 352
Hooke's law, 288
Hopf bifurcation, 372, 380, 384, 430
Host-parasite, 421
Householder
QR factorization, 153
Householder matrix, 19, 151, 152
Hue, 246
Huffman coding, 260
Huffman coding tree, 261
Human visual system, 246
Hyperbolic equilibrium, 342
Hyperbolic periodic orbit, 349

IAD algorithm, 115
Identity matrix, 11
Image, 35, 42
Impedance, 306
Induced norm, 7, 69
Information retrieval, 186
Initial value problem, 278
Inner product, 8
Invariant set, 338
Invariant subspace, 48, 124, 166, 172, 173
Inverse discrete cosine transform, 238, 242
Inverse matrix, 11
computation of, 138
Invertible matrix, 11
Irreducible matrix, 86, 96
Isolated equilibrium, 334
Iterative aggregation/disaggregation, 111, 115

JPEG, 257
JPEG standard, 254

Kernel, 42
Kirchhoff's law, 304
Krylov subspace, 211

Kuramoto–Sivashinsky equations, 391
Lagrange multipliers, 228
Lastgrad, 405
Leading principal submatrix, 14, 136
Least squares, 218
and QR, 224
rank deficient, 226
Left eigenvector, 55, 74, 76, 179
Left inverse matrix, 72
Level shifting, 251
Linear combination, 24
Linear independence, 26
Linear operator, 41
Linear regression, 219
Linear system, 40, 48, 73, 106
overdetermined, 216
Linear systems, 132, 137, 141
Linear transformation, 40
Linearization, 342
around a periodic orbit, 348
around an equilibrium, 342
Link matrix, 86
Lipschitz function, 279, 341
Locally Lipschitz, 341, 389
Logistic equation, 286
Lorenz system, 358, 361
Lossless compression, 246
Lossy compression, 246
Lotka–Volterra model, 375
Low-pass filtering, 250
Low-rank approximation, 161, 163, 197, 199
Lower triangular matrix, 9
Luminance quantization matrix, 254

M-matrix, 127
Manifold, 343
Markov chain, 111
Matrix, 1
addition, 2
adjacency, 86
adjoint, 3
aggregated, 112, 129
block triangular, 10, 167
chrominance, 258
diagonal, 9
diagonalizable, 57, 76
digram frequency, 195
exponential, 14
full rank, 143
Haar, 273
Hadamard, 273
Hermitean, 13
Hessenberg, 10, 151
Householder, 19, 151, 152
identity, 11
inverse, 11
invertible, 11
irreducible, 86, 96
null space, 38
Nullity, 42

Ordered block Schur factorization, 170, 175
Orthogonal complement, 33, 73
Orthogonal matrix, 17, 72, 73
Orthogonal projection, 44, 46, 72, 234
Orthogonal vectors, 9
Orthogonality, 8
Orthogonalization, 31
Orthogonally invariant, 72
Orthonormal basis, 31, 46, 175
Orthonormal extension, 20, 178
Orthonormal vectors, 9
Orthonormalization, 144
Overdetermined system, 216

PageRank vector, 93
Particular solution, 293
Peixoto's theorem, 366
Period doubling bifurcation, 363, 392
Periodic orbit, 349
Periodic solution, 348
Permutation matrix, 15
Perp, 33
Perron–Frobenius theorem, 89
Personalization vector, 97, 99
Phase angle, 294
Phase condition, 353
Phase portrait, 327
Phase space, 327
Pitchfork bifurcation, 370
Pixel, 246
Population model, 283
Positive definite matrix, 12, 14
Positive matrix, 86
Postscript fonts, 59, 67
Power method, 80, 103
rate of convergence, 82
Practical resonance, 301
Predator–prey models, 374, 390, 392, 393
Primitive matrix, 91
Principal submatrix, 14
Probabilistic vector, 90
Projection, 32, 72
Projection boundary conditions, 353, 356
Projection matrix, 43, 76, 221, 233
Pseudoinverse, 45, 209

QR algorithm, 149
Quadratic form, 216
Quantization, 254
Query matching, 187

Rössler attractor, 392
Radio tuning, 506
Range, 35, 42
Rank, 35, 42, 76
Rank deficient least squares, 226

link, 86
lower triangular, 70
M-, 127
nilpotent, 212
nonnegative, 84
nonsingular, 11
orthogonal, 17, 72, 73
orthogonal projection, 44
permutation, 15
positive, 86
positive definite, 12, 14
primitive, 91
product, 2
projection, 43, 76, 221, 233
pseudo-inverse, 45, 209
rank-1, 159
reducible, 86
skew Hermitian, 34
skew symmetric, 34, 75
square root, 210
stochastic, 85, 95
strictly diagonal dominant, 71
symmetric, 12
symmetric positive definite, 13
term document, 186
transition, 111
transpose, 3
trapezoidal, 10
triangular, 9, 70
tridiagonal, 70
unitary, 17
Matrix exponential, 72, 326
Matrix norm, 5, 59
spectral, 165
submultiplicative, 70
Monodromy matrix, 349
Morse–Smale system, 366
Multiplicity, 54
Multipliers, 134
Mutualism, 421

Natural frequency, 293
Network grouping, 404
Newton’s law, 288
Nilpotent matrix, 212
Node, 329, 330, 335, 336
Nonhomogeneous equations, 292
Nonhomogeneous systems, 314
Nonnegative matrix, 84
Nonsingular matrix, 11
Nonwandering set, 366
Norm
$C^1$, 365
Euclidean, 5
Frobenius, 6, 7, 209
induced, 7, 69
of a matrix, 5
of a vector, 4
Normal equations, 44, 83, 218

PageRank vector, 93
Particular solution, 293
Peixoto’s theorem, 366
Period doubling bifurcation, 363, 392
Periodic orbit, 349
Periodic solution, 348
Permutation matrix, 15
Perp, 33
Perron–Frobenius theorem, 89
Personalization vector, 97, 99
Phase angle, 294
Phase condition, 353
Phase portrait, 327
Phase space, 327
Pitchfork bifurcation, 370
Pixel, 246
Population model, 283
Positive definite matrix, 12, 14
Positive matrix, 86
Postscript fonts, 59, 67
Power method, 80, 103
rate of convergence, 82
Practical resonance, 301
Predator–prey models, 374, 390, 392, 393
Primitive matrix, 91
Principal submatrix, 14
Probabilistic vector, 90
Projection, 32, 72
Projection boundary conditions, 353, 356
Projection matrix, 43, 76, 221, 233
Pseudoinverse, 45, 209

QR algorithm, 149
Quadratic form, 216
Quantization, 254
Query matching, 187

Rössler attractor, 392
Radio tuning, 506
Range, 35, 42
Rank, 35, 42, 76
Rank deficient least squares, 226

link, 86
lower triangular, 70
M-, 127
nilpotent, 212
nonnegative, 84
nonsingular, 11
orthogonal, 17, 72, 73
orthogonal projection, 44
permutation, 15
positive, 86
positive definite, 12, 14
primitive, 91
product, 2
projection, 43, 76, 221, 233
pseudo-inverse, 45, 209
rank-1, 159
reducible, 86
skew Hermitian, 34
skew symmetric, 34, 75
square root, 210
stochastic, 85, 95
strictly diagonal dominant, 71
symmetric, 12
symmetric positive definite, 13
term document, 186
transition, 111
transpose, 3
trapezoidal, 10
triangular, 9, 70
tridiagonal, 70
unitary, 17
Matrix exponential, 72, 326
Matrix norm, 5, 59
spectral, 165
submultiplicative, 70
Monodromy matrix, 349
Morse–Smale system, 366
Multiplicity, 54
Multipliers, 134
Mutualism, 421

Natural frequency, 293
Network grouping, 404
Newton’s law, 288
Nilpotent matrix, 212
Node, 329, 330, 335, 336
Nonhomogeneous equations, 292
Nonhomogeneous systems, 314
Nonnegative matrix, 84
Nonsingular matrix, 11
Nonwandering set, 366
Norm
$C^1$, 365
Euclidean, 5
Frobenius, 6, 7, 209
induced, 7, 69
of a matrix, 5
of a vector, 4
Normal equations, 44, 83, 218
INDEX

Rank-1 approximation, 197
Rank-1 matrix, 159
modification, 108
Rank-2 approximation, 199
Rayleigh quotient, 121
Reducible matrix, 86
Reordering, 109, 128
Residual, 184
Resonance, 297
electrical, 306
practical, 301
Resonance frequency, 306
RGB, 257
RLC circuits, 302
Row space, 36
orthonormal basis, 158
Saddle, 328, 329, 335
Saddle-node bifurcation, 369, 392
of periodic orbits, 393
Saturation, 246
Schur factorization, 166, 174, 354
block, 170, 175, 359
complex, 171
ordered, 176, 180, 354
real, 168
uniqueness, 170
Separatrices, 352
Sherman–Morrison formula, 108, 128
Similarity, 56, 148, 327
invariant properties, 58
Simple eigenvalue, 54
Singular value decomposition, 155, 163
uniqueness, 158
Singular values, 155, 160
Singular vectors, 156
Singularity bifurcation, 393
Singularity bifurcations, 373
Skew hermitian matrix, 34
Skew symmetric matrix, 34, 75
Span, 25
Spd, 13
Spectral radius, 70, 123
Spectrum of a matrix, 51
Spectral norm, 165
Spring systems, 287
Square root of a matrix, 210
Stable equilibrium, 328
Stable manifold theorem, 343
for periodic orbits, 350
Stable subspace, 338
Stationary distribution, 111
Steady solution, 300
Stochastic complement, 112
Stochastic matrix, 85, 95
Strange attractor
Chua, 363
Lorenz, 362
Rössler, 392
Strictly diagonal dominant, 71
Strongly connected graph, 86
Structural stability, 365
Subdominant eigenvalue, 82
Submatrix, 14
leading principal, 14, 136
principal, 14
Submultiplicative inequality, 70
Subspace, 22, 73
invariant, 166, 172, 173
Successive approximations, 341, 344
Sum of subspaces, 24
dimension of, 30
Superposition principle, 313
Sylvester equation, 180
Symmetric matrix, 12
eigenvalues, eigenvectors of, 75
inverse of, 75
product of, 75
Symmetric positive definite, 13
Teaching, 197
Term–term comparison, 192
Term-document matrix, 186
Threshold condition, 416
Threshold policy harvesting, 375
Trace, 3, 69
Trace-determinant analysis, 335, 377, 417
Transcritical bifurcation, 367, 384
Transient solution, 299
Transition matrix, 111
Transpose, 3, 69
Transversal intersection, 354, 366
Triangular matrix, 10
Traveling waves, 390
Triangular matrix, 9, 70
eigenvalues of, 49
Tridiagonal matrix, 70
Tuning instruments, 296
Undamped equations, 292
Unit ball, 5
Unit lower triangular, 134
Unit vector, 9
Unitary matrix, 17
Unstable equilibrium, 328
Unstable subspace, 338
Upper triangular matrix, 9
Van der Pol oscillators, 360
Variable population interactions, 420
Variation of constants formula, 323
Vector, 4
orthonormal, 9
probabilistic, 90
unit, 9
Vector norm, 4
Vector space, 21
Vector subspace, 22, 73
Vfc rule, 195, 196
Waste water management, 395
Weakly diagonal dominant, 71
Wronskian, 319
Zigzag pattern, 265