CHAPTER 1

CONCEPTS OF FERROMAGNETISM

The quality of magnetism first apparent to the ancient world, and to us today, is the tractive force that exists between two bodies such as lodestone or iron. When a magnetized body is dipped into iron filings they cling to it, especially in certain places called poles usually located near the ends of the magnet (Fig. 1-1). The concept of poles is useful in defining the two quantities basic to magnetism, magnetic field strength and intensity of magnetization.

This chapter is to recall to the reader these and various other concepts associated with ferromagnetism. It is not intended to comprise definitions. Some definitions have been formulated by the American Society for Testing Materials [49A2] and the American Standards Association [42A2]. In this book the cgs and practical systems are used.

Magnetic Field.—A magnet will attract a piece of iron even though the two are not in contact, and this action-at-a-distance is said to be caused by the magnetic field, or field of force. This field may be explored by sprinkling iron filings around a magnet, whereupon they form in lines that converge on the poles and indicate also the direction a small compass needle would take if placed at any point (Fig. 1–2).

Poles exert forces on each other: north and south poles attract each other and like poles repel with a force that varies inversely as the square of the distance between them. A unit pole is a convenient concept defined so that two unit poles of like kind, one centimeter apart in vacuum, would repel each other with a force of one dyne. The strength of the field of force, the magnetic field strength, or magnetizing force \( H \), may be defined in terms of magnetic poles: one centimeter from a unit pole the field strength is one
A magnetic field may be produced by a current of electricity as well as by a magnet, and the unit of field strength can also be defined in terms of current. (In the rationalized MKS system, not used in this book, the unit of field strength is one ampere-turn/meter, and is \( \frac{4\pi}{1000} \) or 0.01256 oersteds.)

**Fig. 1–2.** Iron filings indicate the directions of the magnetic field near a group of magnets and show the lines of force emanating from S poles and converging on N poles.

A magnetic field has direction as well as strength; the direction is that in which a north pole, subjected to it, tends to move, or that indicated by the north-seeking end of a small compass needle placed at the point.

**Intensity of Magnetization and Magnetic Induction.**—In order to describe the magnetic properties of materials, one must have a quantitative measure of magnetization. Such a measure is the intensity of magnetization, defined in terms of the number of unit poles in a piece of given cross-sectional area. Suppose that a uniformly magnetized bar, of length \( l \) and cross-sectional area \( a \), has \( m \) unit north poles at one end and \( m \) unit south poles at the other (Fig. 1–3). The intensity of magnetization is then \( \frac{m}{a} \) and is represented by the symbol \( I \).

It may be shown that \( I \) is also the magnetic moment per unit volume; for the magnetic moment is pole strength times interpolar distance \( (M = ml) \), and \( I = \frac{M}{v} \), if \( v \) is the volume \( al \).

Faraday showed that some of the properties of magnetism may be likened
to a flow and conceived endless *lines of induction* that represent the direction and, by their concentration, the flow at any point. The lines pass from a magnetized material into the air at a north pole, enter again at a south pole, and pass through the material from the south pole back to the north to form a closed loop.

The total number of lines crossing a given area at right angles is the *flux* in that area. The flux per unit area is the *flux density*, or *magnetic induction*, and is represented by the symbol \( B \). Both \( H \) and \( I \) contribute to the lines of induction, but in magnetic materials the contribution of \( I \) is generally the larger. The magnetic induction is defined by the relation:

\[
B = H + 4\pi I, 
\]

the addition being vectorial when \( H \) and \( I \) differ in direction. The occurrence of the factor \( 4\pi \) is caused by the fact that a unit pole gives rise to a unit field everywhere on the surface of a sphere of unit radius enclosing the pole, and this sphere has an area of \( 4\pi \). The lines of induction may be visualized with the aid of Fig. 1–2 showing the pattern obtained with iron filings. The cgs unit of induction is the *gauss*. (The rationalized MKS unit, the weber/(meter)\(^2\), is equal to \( 10^4 \) gauss.)

Alternatively, \( B \) can be defined in terms of the electromotive force created by the relative movement of electric circuit elements and lines of induction, and \( I \) can be derived from the definitions of \( B \) and \( H \).

**Magnetization and Permeability Curves.**—When a piece of unmagnetized iron is brought near a magnet or subjected to the magnetic field of an electric current, the magnetization induced in the iron by the field is described by a magnetization curve obtained by plotting the intensity of magnetization \( I \) or the magnetic induction \( B \) against the field strength \( H \). Such curves are of fundamental importance for describing the magnetic properties of materials, and many of them are shown on the following pages. A magnetization curve for iron is shown as the solid line of Fig. 1–4.

The behavior of a material is also described by its permeability curve and hysteresis loop. The ratio \( B/H \) is called the permeability \( \mu \), and this represents the relative increase in flux caused by the presence of the magnetic material.* It is quite useful when \( B \) is considered to be due to \( H \). The permeability curve is obtained by plotting the permeability \( \mu \) against either \( H \), \( B \) (Fig. 1–5), \( B - H \), or \( I \). In any case the curve rises from a point on the \( \mu \)-axis above the origin (the initial permeability is non-zero) to a maximum (the maximum permeability) and falls off rapidly and then more slowly toward a value of one (not zero). The quantity \( B - H = 4\pi I \) attains a ceiling value, known as *saturation induction* and represented by the symbol \( B_s \); when it is used as abscissa, the axis of

* In the rationalized MKS system the permeability of free space is \( 4\pi \cdot 10^{-7} \) (weber/meter\(^2\)) / (ampere-turn/meter).
absissa ends at a finite distance from the origin, and so does the curve of \( \mu \) vs \( B - H \).

**Fig. 1-4.** Magnetization curve (solid) and hysteresis loop (dotted). Some important magnetic quantities are illustrated.

**Fig. 1-5.** Permeability curves of iron, with \( \mu \) plotted against \( H \) and \( B \). \( I \) and \( B - H \) are also used as abscissae.

**Hysteresis Loop.**—If the field strength is first increased from zero to a high value and then decreased again, as indicated by the arrows of Fig. 1–4, it is observed that the original curve is not retraced; the induction "lags
behind” the field and follows a characteristic curve, shown by the broken line. This phenomenon was named *hysteresis* by Ewing, and the characteristic curve is called a hysteresis loop. On a loop symmetrical about the origin, the value of $H$ for which $B = 0$ is called the *coercive force* $H_C$, and this is often used as a measure of quality of the material. The value of induction for $H = 0$ is the *residual induction* $B_r$. When the field strength has been sufficient to magnetize the material practically to saturation, the coercive force and residual induction become the *coercivity* and *retentivity*. The values of $H$ and $B$ at the tips of a loop are usually called $H_m$ and $B_m$.

**Ferromagnetism, Paramagnetism, and Diamagnetism.**—Materials which have magnetic properties similar to iron (e.g., nickel and cobalt and many alloys of these three elements) are ferromagnetic. In another class of materials, more numerous, the permeabilities are only slightly greater than one, usually between 1.000 and 1.001 (except near 0°K when they may be much larger). As a rule these materials do not show hysteresis, and their permeabilities are independent of field strength and are either independent of temperature or decrease with increasing temperature. Such materials are *paramagnetic*. Among the paramagnetic substances are many of the salts of the iron and the rare earth families and the platinum and palladium metals, the elements sodium, potassium and oxygen, and the ferromagnetic metals above the Curie points. They may be solids, liquids, or gases. *Diamagnetic* substances the magnetization is directed oppositely to the field, i.e., they have permeabilities somewhat less than one. They are, therefore, repelled from the poles of an electromagnet and tend to move toward a weaker field. Many of the metals and most of the nonmetals are diamagnetic.

Paramagnetic and diamagnetic substances are described more conveniently by their susceptibilities than by their permeabilities. The *susceptibility* is a measure of the increase in magnetic moment caused by the application of a field, and is defined as

$$\kappa = I/H,$$

or the equivalent,

$$\kappa = (\mu - 1)/4\pi.$$

For diamagnetic materials the susceptibility is negative, and for bismuth it has a value of $-0.000013$. For substances like iron the susceptibility may be 1000 or more; values as high as 10 000 have been recorded. Other kinds of susceptibility are referred to in Chap. 11.

It is sometimes difficult to draw the line separating ferromagnetic from paramagnetic substances. The important attributes of a ferromagnetic substance are dependence of permeability on the field strength and on the previous magnetic history (hysteresis), approach of the magnetization to a finite limit as the field strength is indefinitely increased (saturation), the
presence of small, magnetized regions containing many atoms and having magnet moments comparable with the saturation moment of the material even when the material is unmagnetized (spontaneous magnetization), and disappearance of the characteristics already mentioned when the temperature is raised to a certain temperature, the **Curie point.** (See Figs. 2–5 and 7.) From a practical point of view one may say arbitrarily that a material is ferromagnetic if it has a permeability greater than 1.1. From an atomic point of view the atomic moments of a ferromagnetic material align themselves parallel to each other, against the forces of thermal agitation. In some materials the atomic forces align neighboring atoms antiparallel—this phenomenon is **antiferromagnetism,** and it is characterized by hysteresis and a Curie point. On account of their small permeabilities these materials are classed as paramagnetic.

In the broad sense of the word, as used by Faraday, paramagnetism includes ferromagnetism ($\mu > 1$). More often, and in this book, ferromagnetism is considered a separate classification. Ferromagnetic materials are usually designated “magnetic”; all materials are either ferromagnetic, paramagnetic, or diamagnetic.

**Kinds of Permeability.**—The **normal permeability,** often referred to simply as the permeability, is $\mu = B/H$, measured when the specimen is in the “cyclic magnetic state.” Under these conditions the material responds equally when the field is applied in either of the two opposite directions. Before such a measurement the specimen is ordinarily demagnetized by applying an alternating field with amplitude high enough to cause the induction to approach saturation, then slowly reducing the amplitude to zero. The material may be demagnetized also by heating the material above the Curie point and cooling in zero field; in this case the magnetization curve is referred to as the “virgin” curve. In either case the curve represents the values of $B$ (or $B - H$ or $I$) and $H$ measured as $H$ is increased from zero, or it is (preferably) the locus of the tips of hysteresis loops taken with increasing amplitudes of $H$, the field having been reversed several times at each amplitude. The latter is sometimes called the “commutation” curve.

The **initial permeability** $\mu_0$ is the limit approached by the normal permeability as $B$ and $H$ are decreased toward zero. Some permeability curves in weak fields are shown in Fig. 1–6; extrapolation is linear when $H$ is sufficiently small.

The **maximum permeability** $\mu_m$ is the largest value of normal permeability obtained by varying the amplitude of $H$ (see Figs. 1–4 and 5).

The **incremental permeability** $\mu_\Delta$ refers to the permeability measured with superposed fields. Let one (“biasing”) field $H_b$ be applied and held constant, and another field $H_\Delta$ be applied and alternated cyclically, causing an alternating induction of amplitude $B_\Delta$. Then $\mu_\Delta = B_\Delta/H_\Delta$ (Fig. 1–4).
When $H_\Delta$ approaches zero, $\mu_\Delta$ approaches a limiting value $\mu_r$, the reversible permeability. When the material is demagnetized and $H_b = 0$, $\mu_r = \mu_0$. Both $\mu_\Delta$ and $\mu_r$ are dependent on the value of $H_b$ and on the previous magnetic history of the specimen; $\mu_\Delta$ is also dependent on the magnitude of $H_\Delta$.

Occasionally the term "differential permeability" is used—it is simply the slope of the $B$ vs $H$ curve, $dB/dH$.

**Saturation.**—As $H$ is increased indefinitely the intensity of magnetization $I$ and the *intrinsic induction* $B - H$ of a ferromagnetic material approach finite limits, commonly referred to as "saturation." The induction $B_s$ on the contrary, increases indefinitely. This is shown for a permanent magnet material, Alnico 5, in Fig. 1–7. The limit of $B - H$ at saturation is designated simply by $B_s$, and the limit of $I$ is $I_s$. Thus $B_s = 4\pi I_s$. The saturation at the absolute zero of temperature is represented by the symbol $I_0$.

The magnetic moment per gram $\sigma$ is equal to the intensity of magnetization divided by the density $d$:

$$\sigma = I/d.$$
The saturation per gram is \( \sigma_s = I_s/d \). The magnetic moment per gram atom \( \sigma_A \) is
\[
\sigma_A = AI/d,
\]
\( A \) being the atomic weight.

There is an advantage in using \( \sigma \) when measurements of magnetic moment are made at various temperatures, for then one merely divides the magnetic moment by the known weight, whereas, to calculate \( I \), the density at each temperature must also be known.

The magnetic moment per atom is
\[
\mu_A = \frac{AI_0}{N_0d} = \frac{A\sigma_0}{N_0},
\]
\( N_0 \) being the number of atoms per gram atom \( (6.025 \times 10^{23}) \). The value of \( \mu_A \) is \( 2.06 \times 10^{-20} \) for Fe. It is more usual to express the atomic moment in terms of the Bohr magneton \( \beta \), the moment which arises from the motion of a single electron moving in its smallest orbit. The number of Bohr magnetons per atom is
\[
n_0 = \mu_A/\beta,
\]
\( \beta \) being \( 9.27 \times 10^{-21} \) erg/gauss. The values of \( n_0 \) for Fe, Co, and Ni are 2.22, 1.71, and 0.60, respectively.

**Ideal Magnetization.**—This refers to the magnetization remaining after applying a constant field \( H \), superposing on it a field varying continually from \(+H_A\) to \(-H_A\), large enough in amplitude to cause practical saturation in each direction, then reducing the amplitude of \( H_A \) slowly to zero. The resulting \( I \) or \( B \) is plotted against \( H \), and the ideal or anhysteretic magnetization curve so obtained has the characteristic shape shown in Fig. 1–8, with no point of inflection. The ideal permeability, the ratio of the \( B \) so obtained to the constant \( H \), has a very high finite maximum at \( H = 0 \), of about the same magnitude as the maximum value of \( dB/dH \) for the hysteresis loop. The course of the ideal curve is near the curve defined by the midpoints of horizontal chords in the maximum loop.

**Demagnetization Curve.**—This is the portion of the hysteresis loop that lies in the second quadrant, between the points marked \( B_r \) and \( H_e \) in Fig. 1–4, and is shown for an important permanent magnet material on the left side of Fig. 1–9. This curve is especially important in the evaluation of materials used for permanent magnets, for in the use of such materials they are subjected to fields that tend to reduce the magnetization they originally possessed. A more specific criterion is the so-called energy product, obtained by multiplying together the magnitudes of \( B \) and \( H \) for a given point on the demagnetization curve. The energy product for all points on the demagnetization curve of Fig. 1–9 is plotted against \( B \) in the right-hand side of the same figure. The maximum value \((BH)_m\) of
this curve is the best single criterion for a material for use as a permanent magnet. More detailed consideration of the demagnetization curve and the energy product are given in Chap. 9.

![Demagnetization Curve](image1)

**Fig. 1-8.** Normal and ideal magnetization curves of typical shape. Material is iron.

![Energy Product Curve](image2)

**Fig. 1-9.** Demagnetization curve (left) and energy product curve (right) of Alnico 5.

**Demagnetizing Field. Air Gaps.**—When a rod is magnetized by an applied field $H_a$, its ends carry magnetic poles which themselves cause magnetic fields to be present in all parts of the rod. Normally these
fields are directed in the opposite direction to the field $H_a$, as shown in Fig. 1-10, and are therefore called demagnetizing fields. The true field acting on a given section of the bar, e.g., its middle, is then the resultant of the applied field and the demagnetizing field, $\Delta H$:

$$H = H_a - \Delta H.$$ 

The demagnetizing field is approximately proportional to the intensity of magnetization

$$\Delta H = NI = \frac{N}{4\pi} (B - H).$$

The proportionality factor $N$ is called the demagnetizing factor and depends primarily on the shape of the test body. Sometimes $N/4\pi$ is represented by the symbol $D_B$. It is zero in a ring, and in a rod when the ratio of length to diameter is very large. A more complete discussion is given in Chap. 19.

When $B-H$ is plotted against $H_a$, the curve is lower than the $B-H$ vs $H$ curve, as illustrated in Fig. 1-10, and is said to be "sheared." The horizontal distance between them is proportional to $B-H$, as indicated.

Similarly a ring specimen containing an air gap has a sheared magnetization curve, and a demagnetizing factor may be associated with an air gap.
the length of which is a given fraction of the length of the magnetic circuit.

**Magnetostriction.**—When a body is magnetized, its dimensions are changed slightly, by not more than a few parts per million, and such changes are referred to as magnetostriction. The change in length in the direction parallel to the magnetization is that most often measured, and this change \( \Delta l \) divided by the original length \( l \) is the *Joule magnetostriction*.

The symbol \( \lambda \) is usually used for \( \Delta l / l \). Unless otherwise specified, "magnetostriction" refers to this Joule magnetostriction. The magnetostriction of nickel and 68 Permalloy, as dependent on field strength, is shown in Fig. 1–11. Changes of volume are measurable, but the relative change in volume, \( \Delta V / V \), is usually much smaller than the relative change in length, \( \Delta l / l \).

**Magnetic Anisotropy.**—In single crystals of iron (and of other substances) the magnetic properties depend on the direction in which they are measured. Although some of these crystals are cubic and have some isotropic properties, such as their interaction with light, they are magnetically anisotropic in their response to magnetic fields of any considerable magnitude. Crystals of non-cubic symmetry are anisotropic with respect to light and to magnetic fields of any magnitude.

In some polycrystalline materials the various crystals are oriented more or less at random, and the properties in different directions are not greatly

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**Fig. 1–11.** The magnetostrictive change in length of 68 Permalloy and nickel.
different. In many materials, however, as in rolled metal sheets, the process of fabrication produces some regularity in the distribution of orientations, and the magnetic properties are markedly anisotropic. The anisotropy usually persists even after the material is annealed.

Figure 1–12 shows a $B$ vs $H$ curve obtained by applying $H$, and measuring $B$, in the direction of a crystal axis ([100] direction) in iron containing 3% silicon, and another curve for which $H$ and $B$ are equally inclined to the three cubic axes ([111] direction). Curves are also shown for a sheet that has been rolled and then annealed, as measured in directions (1) parallel to the direction in which it was rolled (rolling direction, $RD$) and (2) at right angles to this direction (cross direction, $CD$), respectively.

**Spontaneous Magnetization.**—A ferromagnetic substance has long been regarded as an assemblage of small permanent magnets. When the material is unmagnetized, the magnets are arranged with haphazard orientations; when it is magnetized, they are lined up with their axes
approximately parallel. The nature of this small magnet has been the subject of much consideration over a period of years. It is now known that all ferromagnetic materials are composed of many small magnets or *domains*, each of which consists of many atoms. Within a domain all of the atoms are aligned parallel and the domain is thus saturated, even when no field is applied. The material is therefore said to be "spontaneously magnetized." When the magnetization of the material is changed, the atoms turn together in groups (each atomic magnet about its own axis), the atoms in each group remaining parallel to each other so that they are aligned more nearly with the magnetic field applied to the material. The domain theory is discussed in more detail later.