1

History of tidal and turbine science

1.1 Introduction

Renewable energy in general and tidal power in particular are receiving a great deal of attention at present, driven by issues including the security of supply and the availability of hydrocarbon energy sources. Tidal power has traditionally implied the construction of turbines in a barrage for the generation of electricity (cf. Section 3.2 below). This book, however, moves the focus forward to analyse the physics and deployment of tidal stream power devices that are fixed to the seabed and generate electricity directly from the ebbing and flooding of the currents.

The Analysis of Tidal Stream Power Jack Hardisty

This chapter introduces tides and tidal phenomena through consideration of the original work of some of the great innovators. The first sections deal with the early recognition of tidal processes and the rapid progress following the adoption of the heliocentric model for the solar system. Biographical summaries are used to introduce the fundamental work of Isaac Newton and the other luminaries. Attention then turns to the development of turbine science, and progress is reviewed from the earliest water wheels through the turbines of the late eighteenth century to modern turbines and to their application in low-head tidal streams.

The essence of the problem is to be able to understand and to predict the electrical power output from, and the economics of, a tidal stream device throughout periods from a tidal cycle to a year or more. The result is a complex flow that may accelerate from rest to speeds in excess of $3\,\mathrm{m\,s^{-1}}$ before peaking, reversing, and accelerating and decelerating in the opposite direction. Within this general, daily, or twice-daily pattern will be the effects of turbulence, which generates peak flows and outputs that are significantly larger than the mean. There are also variations from the seabed to the surface. The flow will peak just below the surface and will decelerate as the seabed is approached. The rate of deceleration depends both upon the overall flow velocity and upon the nature of the seabed. Further complications include the fact that the magnitude of the flows increases and decreases over the lunar cycle, peaking a few days after the new and the full Moons and increasing still further around the equinoxes in March and September. Consideration must then be given to the type of device, to its efficiency in converting flow energy into rotary motion and then into electrical power, and, overall, to the cost of the construction and operational maintenance of the device. This is a fascinating story written alongside 3000 years of cultural, religious, and scientific history.

A very good bibliography of tidal power developments is given by Charlier (2003). This chapter is based, in part, on the books by Defant (1961), Cartwright (1999) and Andrews and Jelley (2007). Cartwright offers a scholarly historical treatise on tidal science, but it is difficult to identify the significant developments within a plethora of players. Here, by sacrificing many of the named contributors (Cartwright lists more than 200, while only 10 are identified here), it is hoped that a clearer derivation of the important principles emerges. These principles are then taken forward in the later chapters. The development of tidal turbines is based upon the first chapter in Round (2004) and the references contained therein. The principles of flow-driven power machines are shown to emerge from the work of Archimedes, Vetruvius, and Poncelet and lead to the modern turbines of Francis, Kaplan, Pelton, and others. Again, these principles are taken forward in later chapters.

Part 1 Tidal science

1.2 Antiquity: Aristotle and Ptolemy

Antiquity forms the earliest period in the traditional division of European history into three 'ages': the classical civilization of Antiquity, the Middle Ages between about AD 500 and AD 1500, and then Modern Times. The early history of tidal

science is described by Pugh (1996) who reports that the first evidence of man's interaction with the rise and fall of the tide, implying some understanding of the processes, is the discovery by Indian archaeologists of a tidal dock near Ahmedabad, dating from 2000 BC (Pannikar & Srinivasan, 1971). The earliest known reference to the connection between the tides and the Moon is found in the Samaveda of the Indian Vedic period (2000–1400 BC).

1.2.1 Aristotle and cosmology

The development of tidal science began in Antiquity with the cosmology of Aristotle (Figure 1.1) who observed that 'ebbings and risings of the sea always come around with the Moon and upon certain fixed times'. Aristotle, working in the third century BC, used his books *On the Heavens* and *Physics* to put forward his notion of an ordered universe divided into two distinct parts, the earthly region and the heavens. The earthly region was made up of the four elements: earth, water, air, and fire. Earth was the heaviest, and its natural place was the centre of the cosmos, and for that reason, Aristotle maintained, the Earth was situated at the centre of the cosmos. The heavens, on the other hand, were made up of an entirely different substance, called the aether, and the heavenly bodies were part of spherical shells of aether from the Moon out to Mercury, Venus, the Sun, Mars, Jupiter, Saturn, and the fixed stars. Aristotle argued that the orbits of the heavenly bodies were circular and that they travelled at a constant speed.

Ingenious as this cosmology was, it turned out to be wholly unsatisfactory for astronomy. Heavenly bodies did not move with perfect circular motions: they accelerated, decelerated, and in the cases of the planets even stopped and reversed their motions. Although Aristotle and his contemporaries tried to account for these variations by splitting individual planetary spheres into components, these constructions were very complex and, ultimately, doomed to failure. Furthermore, no matter how complex a system of spheres for an individual planet became, these spheres were still

Figure 1.1 Aristotle (384–322 BC).

centred on the Earth. The distance of a planet from the Earth could therefore not be varied in this system, but planets varied in brightness. Since variations in intrinsic brightness were ruled out, and since spheres did not allow for a variation in planetary distances from the Earth, variations in brightness could not be accounted for in this system.

Other developments in tidal science at this time included those by Pytheas who travelled through the Strait of Gibraltar to the British Isles and reported the half-monthly variations in the range of the Atlantic Ocean tides, and that the greatest ranges (Spring tides) occurred near the new and the full Moons.

Many other aspects of the relationship between tides and the Moon are noted in Pliny the Elder's (AD 23–79) *Natural History*. Pliny described how the maximum tidal ranges occur a few days after the new or full Moon, and how the tides at the equinoxes in March and September have a larger range than those at the summer solstice in June and winter solstice in December.

1.2.2 Ptolemy's geometrical solar system

Mathematicians who wished to create geometrical models of the solar system in order to account for the actual motions of heavenly bodies began using different constructions within a century of Aristotle's death. Although these violated Aristotle's physical and cosmological principles somewhat, they were ultimately successful in accounting for the motions of heavenly bodies. We see the culmination of these efforts in the work of Claudius Ptolemy (Figure 1.2). In his great astronomical work *Almagest*, Ptolemy presented a complete system of mathematical constructions that accounted successfully for the observed motion of each heavenly body. Ptolemy used three basic constructions, the eccentric, the epicycle, and the equant, to describe the movements of the planets, the Sun, and the Moon. With such combinations of constructions, Ptolemy was able to account for the motions of heavenly bodies within the standards of observational accuracy of his day.

Figure 1.2 Claudius Ptolemy (AD 90–168).

Early explanations for the tides were curious; Aristotle is credited with the law that no animal dies except when the water is ebbing. This idea survived into popular culture. For example, as recently as 1595 in the North of England, the phase of the tide was recorded at the time of each person's death. Eastern cultures held the belief that the water was the blood of the Earth and that the tides were caused by the Earth breathing.

1.3 Middle Ages: Copernicus to Galileo

The Middle Ages was a period of great cultural, political, and economic change in Europe and is typically dated from around AD 500 to approximately AD 1500. During the early Middle Ages, for example, the Venerable Bede (673–735) was familiar with the tides along the coast of Northumbria in England, and was able to calculate the tides using the 19 year lunar cycle. By the early ninth century, tide tables and diagrams showing how Neap and Spring tides alternate were appearing in several manuscripts.

1.3.1 Copernicus's heliocentric solar system

Although Aristotelian cosmology and Ptolemaic astronomy were still dominant, it was the fundamental change brought about by Copernicus's heliocentric view that removed the barriers to progress and opened the way for the advancement of tidal science by Newton's deterministic analysis in the seventeenth century.

The Polish astronomer Copernicus (Figure 1.3) proposed that the planets have the Sun as the fixed point to which their motions are to be referred and that the Earth is a planet that, besides orbiting the Sun annually, also turns once daily on its own axis. He also recognized that the very slow long-term changes in the direction of this axis account for the precession of the equinoxes. This of the heavens is

Figure 1.3 Nicolaus Copernicus (1473–1543).

usually called the heliocentric, or 'Sun-centred', system – derived from the Greek *helios*, meaning 'Sun'. Copernicus wrote about these ideas in a manuscript called the *Commentariolus* ('Little Commentary') during the period 1508–1514. However, the work that contains the final version of his theory, *De revolutionibus orbium coelestium libri vi* ('Six Books Concerning the Revolutions of the Heavenly Orbs'), did not appear in print until 1543, the year of his death.

1.3.2 Tycho Brahe's observations

Tyge (latinized as Tycho) Brahe (Figure 1.4) was born in Skane, Sweden (The Galileo Project, 2007). He attended the Universities of Copenhagen and Leipzig, and then travelled through Germany, studying at Wittenberg, Rostock, and Basel. His interest in astronomy was aroused during this period, and he bought several astronomical instruments. Tycho Brahe lost part of his nose in a duel with another student, in Wittenberg in 1566, and for the rest of his life he wore a metal insert to cover the scar. He returned to Denmark in 1570.

Brahe accepted an offer from King Frederick II in the 1570s to fund an observatory. He was given the little island of Hven in the Sont near Copenhagen, and there he built Uraniburg, which became the finest observatory in Europe. Brahe designed and built new instruments, calibrated them, and instituted nightly observations. He also ran his own printing press. His observations were not published during his lifetime, but he employed Johannes Kepler as an assistant to calculate planetary orbits from the data. Brahe did not entirely abandon the Copernican, Earth-centred approach because, he argued, if the Earth were not at the centre of the universe, physics, as it was then known, was utterly undermined. Instead, he developed a system that combined the best of both worlds. He kept the Earth in the centre of the universe, so that he could retain Aristotelian physics. The Moon and Sun revolved

Figure 1.4 Tycho Brahe (1546–1601).

about the Earth, and the shell of the fixed stars was centred on the Earth. But Mercury, Venus, Mars, Jupiter, and Saturn revolved about the Sun. This Tychonic world system became popular early in the seventeenth century among those who felt forced to reject the Ptolemaic arrangement of the planets (in which the Earth was the centre of all motions) but who, for various reasons, could not accept the Copernican alternative.

1.3.3 Kepler's laws of planetary motion and the 'sphere of influence of the Moon' tidal theory

Brahe's student Johannes Kepler (Figure 1.5) used simple mathematics to describe how planets move. Kepler was able to use Tycho Brahe's data to develop three empirical laws that described the movement of the planets and that supported the heliocentric Copernican system (Drennon, 2007):

- **Kepler's first law** states that the orbit of a planet about the Sun is an ellipse with the Sun's centre of mass at one focus. Thus, the orbit of a planet (Figure 1.6) is given by

$$\frac{x^2}{a_{\mathrm{K}}^2} + \frac{y^2}{b_{\mathrm{K}}^2} = 1 \tag{1.1}$$

  where x and y are the Cartesian coordinates, and a_{K} and b_{K} are the lengths of the semi-minor and semi-major axes respectively. (Note that all symbols are defined

Figure 1.5 Johannes Kepler (1571–1630).

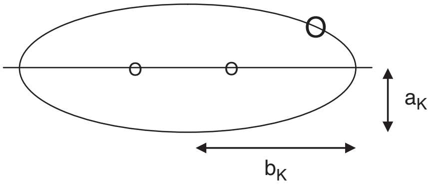

Figure 1.6 Johannes Kepler's first law of planetary motion.

in the Notation at the beginning of the book and will rarely be defined henceforth in the text.)

- **Kepler's second law** states that a line joining a planet and the Sun sweeps out equal areas in equal intervals of time.

- **Kepler's third law** states that the squares of the periods of the planets are proportional to the cubes of their semi-major axes:

$$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3} \tag{1.2}$$

where T_1 and T_2 are the sidereal periods of orbit, and R_1 and R_2 are the lengths of the semi-major axes of planets 1 and 2 respectively. Kepler also proposed a theory for the tides when he wrote, in 1609, that 'The sphere of influence of the attraction which is in the Moon extends as far as the Earth, and incites the waters up from the torrid zone...', thus invoking the force that Isaac Newton would later call gravity.

1.3.4 Galileo's differential motion tidal theory

Although Galileo Galilei (Figure 1.7) probably developed his explanations for tidal phenomenon within the new heliocentric Copernican system in the closing years of

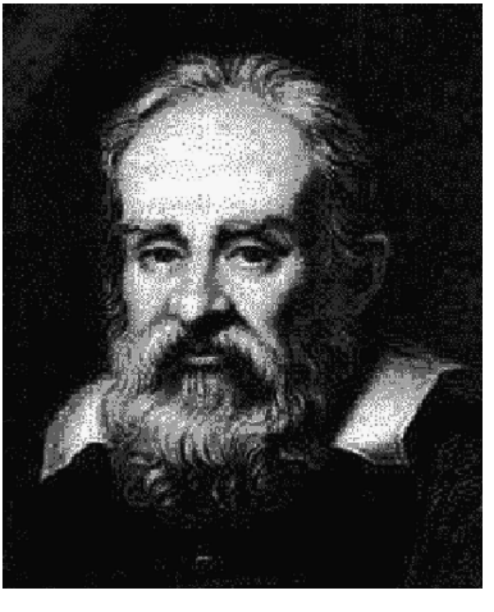

Figure 1.7 Galileo Galilei (1564–1642).

the sixteenth century, they were not formally published until 1616 in his *Treatise on the Tides* (Tyson, 2007). The idea is said to have occurred to him while travelling on a barge ferrying fresh water to Venice, close to his home in Padua. He noticed that, whenever the barge's speed or direction altered, the fresh water inside sloshed around accordingly. If the vessel suddenly ground to a halt on a sandbar, for instance, the water pushed up towards the bow then bounced back towards the stern. Galileo proposed that the Earth's dual motion – its daily rotation around its axis and its annual rotation around the Sun – might have the same effect on oceans and other bodies of water as the barge had on its freshwater cargo. The key, as Galileo saw it, was that different parts of the planet moved at different speeds depending on the time of day, as if the Earth were a barge that accelerated and decelerated and periodically changed direction.

1.3.5 Descartes' universal matter tidal theory

By the mid-seventeenth century. three different theories for the origins of the tides were being seriously considered:

1. Kepler was one of the originators of the idea that the Moon exerted a gravitational attraction on the water of the ocean, drawing it towards the place where it was overhead. This attraction was balanced by the Earth's attraction on the waters, for 'If the Earth should cease to attract its waters, all marine waters would be elevated and would flow into the body of the Moon'.

2. Galileo proposed that the rotations of the Earth produced motions of the sea, which were modified by the shape of the seabed to give the tides.

3. The French mathematician, philosopher, and scientist Rene Descartes (Figure 1.8) introduced the third theory in the early part of the seventeenth century. Descartes argued that space was full of invisible matter. As the Moon

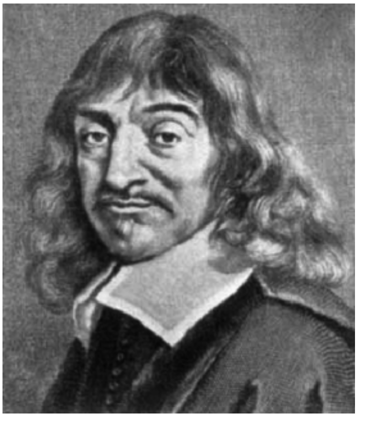

Figure 1.8 Renee Descartes (1596–1650).

travelled around the Earth, it compressed the matter in a way that transmitted pressure to the sea, hence forming the tides.

Although there was merit in each of these approaches, it was Isaac Newton's introduction of the concept of force in a heliocentric solar system that was finally to resolve the differences between them.

1.4 Isaac Newton and the equilibrium theory

The newly established Royal Society of London was a rich and fertile environment for the development of tidal science in the middle years of the seventeenth century. Association d'Oceanographique Physique (1955) lists a number of papers on tidal phenomena in the first volumes of the *Transactions* between the years 1665 and 1668, including Moray (1665, 1666a and b), Wallis (1666), Colepresse (1668), Norwood (1668), Philips (1668), Stafford (1668), and Wallis (1668).

A major step forward in the understanding of the processes by which tides are generated was made by the British mathematician, Isaac Newton (Figure 1.9). Newton was able to apply his formulation of the law of gravitational attraction to show why there were two tides for each lunar transit, why the Spring to Neap cycle occurred, and why daily tides were a maximum when the Moon was furthest from the plane of the equator (Pugh, 1996).

Newton's seminal work, which transformed the worlds of physics and mathematics, begins with the concept that there is an inverse and linear relationship between the acceleration and the mass of a body. This is captured by the simple proportionality

$$a\alpha\frac{1}{m} \tag{1.3}$$

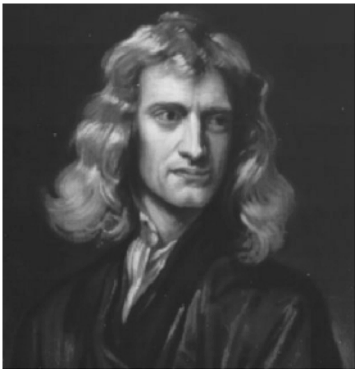

Figure 1.9 Isaac Newton (1643–1727).

The coefficient of proportionality was called the 'force', so that the relationship is captured in Newton's first two laws of motion:

- **Newton's first law** states that every object in a state of uniform motion tends to remain in that state of motion unless an external force is applied to it.

- **Newton's second law** states that the relationship between an object's mass, its acceleration, and the applied force is

$$F = ma \tag{1.4}$$

The unit of force is, of course, the newton. Newton's *Principia* was published in 1687 and proposed and then used the problem of tides to prove that the gravitational force between two masses depends upon the product of the masses and the inverse square of their separation:

$$F = \frac{GMm}{R^2} \tag{1.5}$$

where G is the universal gravitational constant which is now known to have a value of $6.67300 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$. Equations (1.4) and (1.5) are combined to derive Newton's equilibrium theory of the tides (e.g. Andrews & Jelley, 2007). We now know that Kepler was right and that the main cause of the tides is the gravitational effect of the Moon. The effect of the Sun is about half that of the Moon but increases or decreases the size of the lunar tide according to the positions of the Moon and the Sun relative to the Earth. Galileo was wrong; the daily rotation of the Earth about its own axis only affects the location of the high tides. Galileo was right, however, in his advocacy of the heliocentric solar system. We initially ignore the effect of the Sun in the following derivation.

For simplicity, we consider that the Earth is covered by water, as shown schematically in Figure 1.10. Consider a unit mass of water situated at some point P. The gravitational potential due to the Moon is then given by

$$\phi = -\frac{GM_{\mathrm{M}}}{s} \tag{1.6}$$

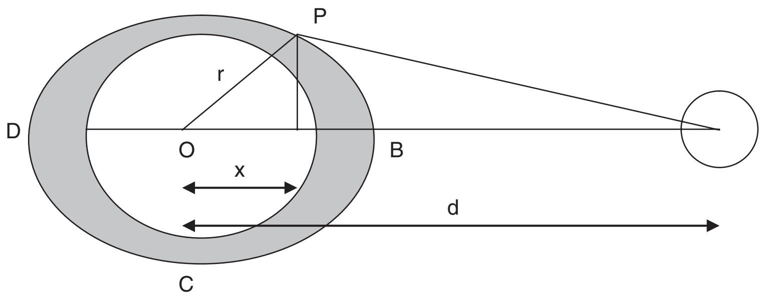

Figure 1.10 Tidal effect of the Moon (not to scale).

where s is the distance between the centre of mass of the Moon and point P. For $d >> r$ we can expand $1/s$ as follows by applying Pythagoras' theorem to the triangle OPN in the figure:

$$\begin{aligned}\frac{1}{s} &= \frac{1}{\left[d^2 + r^2 - 2dr\cos\theta\right]^{1/2}} = \frac{1}{d}\left[1 + \left(-\frac{2r}{d}\cos\theta + \frac{r^2}{d^2}\right)\right]^{1/2} \\ &= \frac{1}{d}\left[1 + \frac{r}{d}\cos\theta + \frac{r^2}{d^2}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right) + \mathrm{L}\right]\end{aligned} \tag{1.7}$$

The first term in the expansion does not yield a force and can be ignored. The second term corresponds to a constant force Gm/d^2 directed towards N which acts on the Earth as a whole and is balanced by the centrifugal force due to the rotation of the Earth–Moon system. The third term describes the variation of the Moon's potential around the Earth. The surface profile of the water in this simplified system is an equipotential surface due to the combined effects of the Moon and the Earth. The potential of a unit mass of water owing to the Earth's gravitation is gh and $g = GM_E/r^2$. Hence, the height of the tide at θ is given by

$$gh(\theta) - \frac{Gmr^2}{d^3}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right) = 0 \tag{1.8}$$

or

$$h(\theta) = h_{\max}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right) \tag{1.9}$$

where

$$h_{\max} = \frac{mr^4}{Md^3} \tag{1.10}$$

The maximum height of the tide, $h_{\max}$, occurs at points P and D, where $\theta = 0$ and $\theta = \pi$. Putting $m/M = 0.0123$, $d = 384\ 000$ km, and $r = 6378$ km, we obtain $h_{\max} = 0.36$ m, which is roughly in accord with the observed mean tidal height. Equation (1.9) was incorporated into a software routine (which is available from this book's online website) and produces the global variation in the tidal heights shown in Figure 1.11, with the two maxima corresponding to high water on the near side and far side of the Earth from the Moon.

Newton's equilibrium theory finally reconciled science and religion and, through the introduction of the new physics of force and response, explained, in a quasi-static sense, the behaviour of the waters of the world's oceans and the ebbing and flowing of the tides. Although the explanation was correct, it could not be applied to particular sites without the advantage of long-term datasets, as described in the following section.

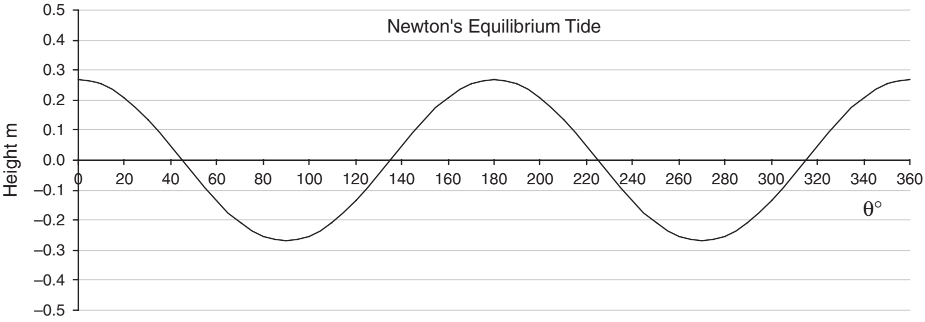

Figure 1.11 The global variation in the height of the lunar ride (from equation (1.9)), utilizing the model from this book's online website.

1.5 Measurement after Moray

The development of observational techniques has to be viewed as a necessary progression that happened alongside the establishment of the theoretical framework. The problem was, of course, that, although Newton's gravitational theory provided the fundamental explanation for tidal phenomena, it was not possible to apply the theory on a local basis to predict the height and timing of high water and low water at a specific site because of the complications of bathymetry and coastal form. Instead, long continuous records of water depth (as opposed to approximate measurements at points in time) were to lead, eventually, to the identifications of the tidal components and to the harmonic analysis that powerfully provided accurate local predictions.

The early Royal Society of London was strongly inclined to experimentation, and was the first institution to encourage careful series of tidal measurements as distinct from the collection of casual reports from mariners. A formal set of recommendations was assigned to Sir Robert Moray who, in 1666, described a stilling well with a small connection to the sea and within which was a float connected to a pen recorder (Moray, 1666a). It was more than 150 years later that the first continuous sea level record to cover a Spring–Neap cycle was taken (Figure 1.12). The measurements were obtained by a civil engineer named Mitchell and appear to have been based upon a design suggested to the Royal Society by Henry Palmer (1831) which had its antecedents in Moray's work. These gauges are therefore known as Palmer–Moray devices.

At the same time as these temporal variations in tidal heights were being determined, workers such as the Reverend William Whewell and J.W. Lubbock were involved in defining spatial variations in the range and progress of the tides. Admiral Sir Francis Beaufort (1774–1857) (Figure 1.13), hydrographer to the British Royal Navy, assisted in their work. As well as providing staff to assist Lubbock's efforts, Beaufort arranged for tides to be observed at over 100 British coastguard stations for 2 weeks in June 1884. The following year, another observational exercise was achieved, covering 101 ports in seven European countries, 28 in America from the Mississippi to Nova Scotia, and 537 ports in the British Isles including Ireland.

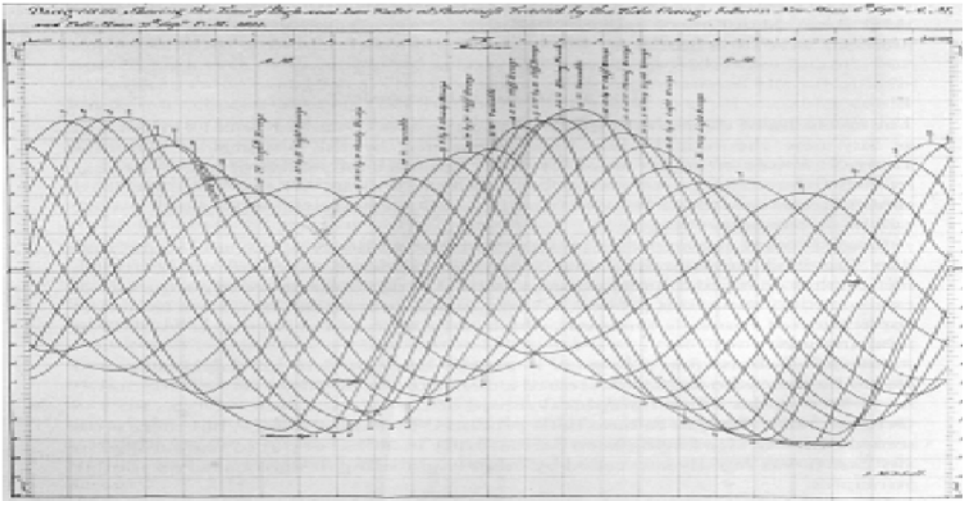

Figure 1.12 The first known record of a continuous Spring–Neap cycle taken at Sheerness Dockyard on the River Thames in England from 6 to 21 September 1831. From Cartwright (1999).

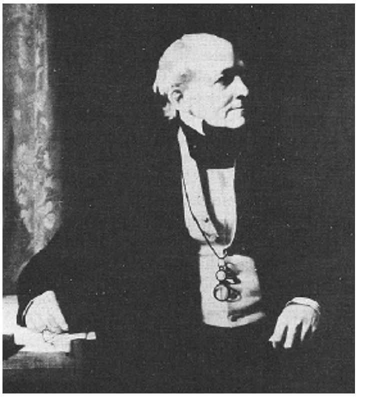

Figure 1.13 Admiral Sir Francis Beaufort (1774–1857).

After more than 100 years of such observations around the world, Figure 1.14 has been constructed to show the global distribution of tidal range and the cotidal lines. Much of the tidal motion has the character of rotary waves. In the south and equatorial Atlantic Ocean the tide mainly takes the form of north–south oscillation on east–west lines. This complex reality may usefully be compared with the simple equilibrium concept which pictures the tide as a sinusoidal wave progressing around the Earth in the easterly direction.

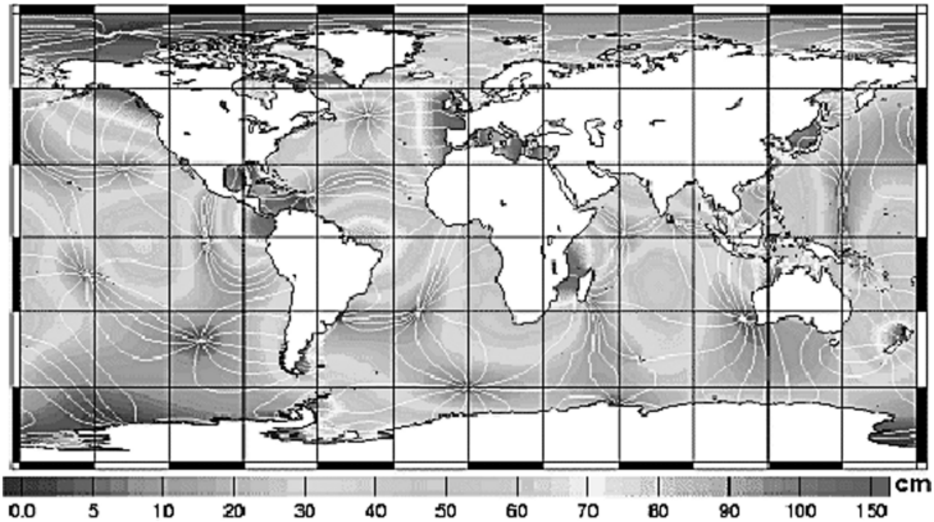

Figure 1.14 Global distribution of tidal range and cotidal lines (LEGOS, 2008). A URL link to the full colour version of this image is available on the book's website.

1.6 Eighteenth and nineteenth centuries: Laplace to Kelvin

1.6.1 Laplace tidal equations

Pierre Simon Laplace (Figure 1.15) was born at Beaumont-en-Auge in Normandy (Rouse, 1908). He produced much of his original work in astronomy during 1771–1787, commencing with a 1773 memoir showing that the planetary motions were stable. This was followed by several papers on points in the integral calculus, finite differences, differential equations, and astronomy. During the years 1784–1787 he

Figure 1.15 Pierre Simon de Laplace (1749–1827).

produced some memoirs that were reprinted in the third volume of the *Méchanique Céleste*, in which he completely determined the attraction of a spheroid on a particle outside it. This is memorable for the introduction of spherical harmonics which are now known as Laplace's coefficients.

Cartwright (1999) explains that Laplace's major theoretical advance was the formulation of a system of three linear, partial differential equations for the horizontal components of ocean velocity relative to the Earth, and the vertical displacement of the ocean surface. Known to tidal scientists as the Laplace tidal equations (LTEs), the equations remain the basis of tidal computation to this day. Laplace also showed that the equations were dynamically stable for realistic values of the physical constants, and therefore the frequencies of any solutions had to coincide with frequencies present in the forcing term. There were many such frequencies, due to various astronomical periodicities. From the fact that the trigonometric expansion contained terms involving the cosines of t and $2t$, as well as time-independent arguments, Laplace concluded that there were at least three 'species' of tidal waves. The first species, corresponding to terms independent of time, were long-period tides – typically of low amplitude – which would not be apparent to the casual dockside observer but which careful measurements had already begun to detect. The second, corresponding to terms involving t, were the main cause of the long-noted differences between successive high- or low-water heights. The third, corresponding to terms involving $2t$, caused the familiar semi-diurnal tides.

1.6.2 Kelvin and the harmonic species

From about 1833, the British Admiralty first began to publish tide tables with the times and heights of the tidal extrema, high water and low water, based upon the 'synthetic method' of J.W. Lubbock which involved very long and laborious cross-correlation of the extrema with lunar constants and timings. The first official British Admiralty predictions covered the four English ports of Plymouth, Portsmouth, Sheerness, and London Bridge.

The subject was taken forward by William Thompson, the 1st Baron Kelvin (Figure 1.16). Kelvin appreciated that the new continuous tidal records would permit the analysis of the whole tidal profile and more accurate and easier tidal predictions. Kelvin was a mathematical physicist, engineer, and outstanding leader in the physical sciences of the ninteenth century. He did important work in the mathematical analysis of electricity and thermodynamics and in unifying the emerging discipline of physics in its modern form. He is widely known for developing the Kelvin scale of absolute temperature measurement. The title Baron Kelvin was given in honour of his achievements, and named after the River Kelvin, which flowed past his university in Glasgow, Scotland.

Kelvin devised the method of reduction of tides by harmonic analysis about the year 1867. Harmonic analysis is based on the principle that any periodic motion or oscillation can always be resolved into the sum of a series of simple harmonic motions (Phillips, 2007). The principle is said to have been discovered by Eudoxas in 356 BC, when he explained the apparently irregular motions of the planets by combinations of uniform circular motions. In the early part of the nineteenth century, Laplace recognized the existence of partial tides that might be expressed by

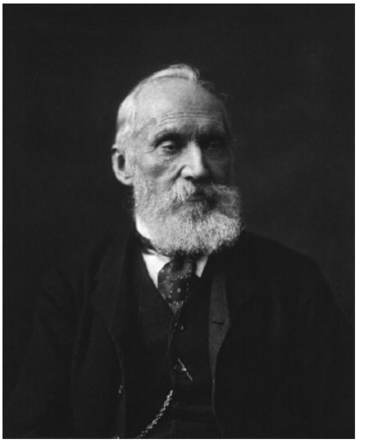

Figure 1.16 Lord Kelvin (1824–1907).

Table 1.1 The astronomical time constants which give the fundamental frequencies for gravitational tides (MSD = mean solar day)

Name			Period
Mean lunar day	T_1	The time of the rotation of the Earth with respect to the Moon, or the interval between two upper transits of the Moon over the meridian of a place	1.035050 MSD
Tropical month	s	The average period of the revolution of the Moon around the Earth with respect to the vernal equinox	27.321582 MSD
Tropical year	h	The average period of the revolution of the Earth around the Sun with respect to the vernal equinox	365.2422 MSD
Rotation of Moon's perigee	p	Change in mean longitude of the lunar perigee	8.85 years
Rotation of Moon's node	N	Change in mean longitude of the Moon's node	18.61 years
Rotation of Earth's perigee	p_1	Change in mean longitude of the solar perigee	20 900 years

the cosine of an angle increasing uniformly with time, and also applied the essential principles of harmonic analysis to the reduction of high and low waters. The earliest analysis by Kelvin made use of combinations of the six main periods shown in Table 1.1 to integrate the effect of the 12 main harmonics shown in Table 1.2.

1.6.3 Fennel and Harris

The American mathematicians who have had an important part in the development of this subject include William Ferrel and Rollin Harris, both of whom were associated with the US Coast and Geodetic Survey. *The Tidal Researches*, by Ferrel, was

Table 1.2 Selected main constituents of the tide and their periods from Table 1.1 and Groves-Kirkby *et al*. (2006)

	Name		Period
S_{sa}	Solar semi-annual constituent	Accounts for the non-uniform changes in the Sun's declination and distance. Mostly reflects yearly meteorological variations influencing the sea level	$2h$
M_m	Lunar monthly constituent	The effect of irregularities in the Moon's rate of change of distance and speed in orbit	$s - p$
M_f	Lunar fortnightly constituent	The effect of departure from a sinusoidal declinational motion	$2s$
O_1	Lunar diurnal constituent	See K_1	$T_1 - 2s + p$
Q_1	Larger lunar elliptic diurnal constituent	Modulates the amplitude and frequency of the declinational O_1	$T_1 - s$
K_1	Lunisolar diurnal constituent	Expresses, with O_1, the effect of the Moon's declination. They account for diurnal inequality and, at extremes, diurnal tides. With P_1, it expresses the effect of the Sun's declination	$T_1 + s$
P_1	Solar diurnal constituent	See K_1	$T_1 + s - 2h$
L_2	Smaller lunar elliptic semi-diurnal constituent	Modulates, with N_2, the amplitude and frequency of M_2 for the effect of variation in the Moon's orbital speed owing to its elliptical orbit	$2T_1 + s - p$
N_2	Larger lunar elliptic semi-diurnal constituent	See L_2	$2T_1 - s + p$
M_2	Principal lunar semi-diurnal constituent	The rotation of the Earth with respect to the Moon	$2T_1$
S_2	Principal solar semi-diurnal constituent	The rotation of the Earth with respect to the Sun	$2T_1 + 2s + 2h$
K_2	Lunisolar semi-diurnal constituent	Modulates the amplitude and frequency of M_2 and S_2 for the declinational effect of the Moon and Sun respectively.	$2T_1 + 2s$

published in 1874, and additional articles on harmonic analysis by the same author appeared from time to time in the annual reports of the Superintendent of the Coast and Geodetic Survey. The best known work of Harris is his *Manual of Tides*, which was published in several parts as appendices to the annual reports of the Superintendent of the Coast and Geodetic Survey. The subject of harmonic analysis was treated principally in Part II of the *Manual* which appeared in 1897.

1.7 Tide-predicting machines

Lord Kelvin also designed the first tide-predicting machine, which was built in 1873. It was based on a suggestion by Beauchamp Towers for summing several trigonometric functions with independent periods and under the auspices of the British

Association for the Advancement of Science (Kelvin, 1911; FWERI, 2007; NOAA, 2007a, 2007b). The origins of the idea are described by AMS (2007) as follows:

> In 1872 Kelvin was preoccupied with the tidal prediction, and in particular with the problem of summing a large number of harmonic motions with irrationally related frequencies. Here is how the solution came to him, in his own words (*Mathematical and Physical Papers, Vol. VI*. Cambridge, 1911, p. 286):
>
>> On his way to attend the British Association in 1872, with Mr. Tower for a fellow-passenger, the Author was deeply engaged in trying to find a practical solution for the problem. Having shown his plans and attempts to Mr. Tower, whose great inventiveness is well known, Mr. Tower suggested: '*Why not use Wheatstone's plan of the chain passing around a number of pulleys, as in his alphabetic telegraph instrument?*' This proved the very thing wanted. The plan was completed on the spot; with a fine steel hair-spring, or wire, instead of the chain which was obviously too frictional for the tide predictor. Everything but the precise mode of combining the several simple harmonic motions had, in fact, been settled long before. At the Brighton meeting . . . the Author described minutely the tide-predicting machine thus completed in idea, and obtained the sanction of the Tidal Committee to spend part of the funds then granted to it on the construction of a mechanism to realise the design for tidal investigation by the British Association. Before the end of the meeting he wrote from Brighton to Mr. White at Glasgow, ordering the construction of a model to help in the designing of the finished mechanism for the projected machine.

The device was an integrating machine designed to compute the height of the tide in accordance with

$$h = H_0 + \sum \left\{ fH \cos\left[a_A t + (V_0 + u) - K \right] \right\} \tag{1.11}$$

Tide-predicting machines are simple devices that essentially use mechanical means to represent the trigonometric functions (Figure 1.17). Kelvin's first machine provided for the summation of 10 of the principal constituents M_2, S_2, N_2, K_1, O_1, K_2, L_2, P_1, M_4, and MS_4, and the resulting predicted heights were registered by a pen trace. The machine is described in Part I of Thomson and Tait's *Natural Philosophy*, 1879 edition. Kelvin's second machine was a smaller model completed in about 1880 and was in turn succeeded by a third in about 1883 and a 'Fourth British Tide Predictor' in about 1910.

The first tide-predicting machine used in the United States was designed by William Ferrel, of the US Coast and Geodetic Survey, and completed in 1882. Ferrel's machine summed 19 harmonic tidal constituents and gave direct readings of the predicted times and heights of the high and low waters. Predictions were made on this machine from 1885 until 1991. Ferrel's machine did not trace an output curve, and the times and heights of the high and low waters were indicated directly by scales on the machine. The Coast and Geodetic Survey in the United States constructed a second machine that provided continuous, graphical outputs of the predictions in 1910. The machine was about 11 ft long, 2 ft wide, and 6 ft high and weighed approximately 2500 lb. NOAA (2007a, 2007b) also reports tide-predicting machines that were built in Brazil and Germany.

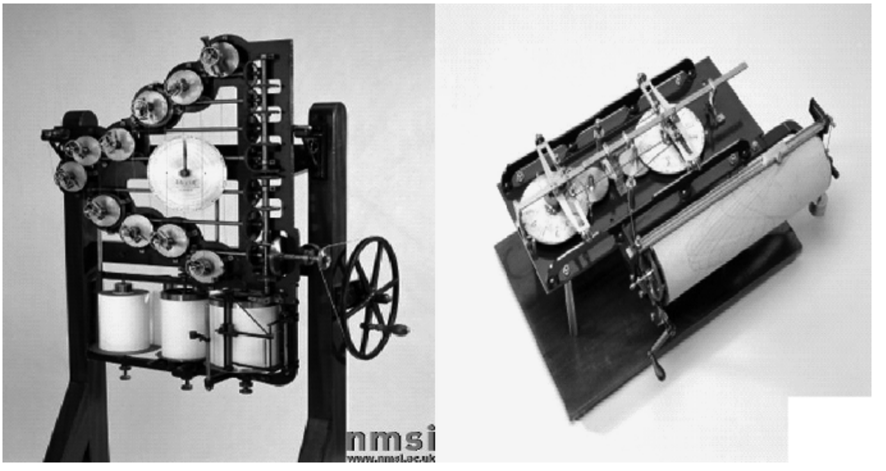

Figure 1.17 The first and second Kelvin tidal machines.

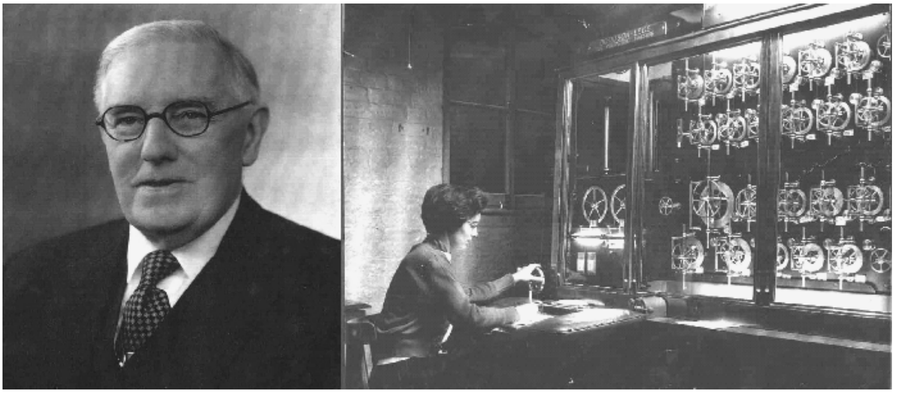

Figure 1.18 (Left) Arthur Doodson (1890–1968) and (right) Arthur Doodson's daughter-in-law, Valerie, operates the Doodson–Légé tidal machine at the Bidston Observatory.

In the United Kingdom, Kelvin's machine continued to be used and was more permanently installed at the Bidston Tidal Observatory on Merseyside until the Doodson–Légé tide-predicting machine was designed by one of the Observatory's directors, A.T. Doodson, and built by the Liverpool firm Légé during 1948/1949. The machine resolved up to 42 constituents and was in daily use from then until the early 1960s, when it was superseded by the electronic computer (Figure 1.18).

1.8 Tidal currents

The measurement of a ship's speeds through the water, and conversely the measurement of tidal currents past a moored ship, probably date back into antiquity.

Tidal currents are routinely reported in units of knots. The definition of the knot (symbolized by kn or kt) is based on the internationally agreed length of the nautical mile, since a speed of 1 knot is equivalent to one nautical mile per hour and exactly equal to 1.852 km. The international definition was adopted by the USA in 1954 (which previously used the US nautical mile of 1853.248 m). The international definition was adopted by the UK in 1970. The knot is approximately equivalent to 101.268591 feet per minute, 1.687810 feet per second, 1.150779 miles (statute) per hour (mph), or 0.5144444 recurring metres per second. For convenience, equivalence to 0.51 m s^{-1} is usually used.

Until the mid-nineteenth century, vessel speed at sea (or current speed past a moored vessel) was measured using a chip log (Art of Navigation, 2007). This device consisted of a wooden panel, weighted on one edge to float upright, attached by line to a reel. The chip log was 'cast' over the stern of the moving vessel and the line allowed to pay out. Knots placed at a distance of 47 ft 3 in (14.4018 m) passed through a sailor's fingers, while another sailor used a 30 s sandglass to time the operation. This method gives a value for the knot of 20.25 in s^{-1}, or 1.85166 km h^{-1}. The difference from the modern definition is less than 0.02 %.

The Proudman Oceanographic Laboratory (2007a, 2007b) reports on the more recent and instrumented history of tidal-current metering. Figure 1.19 (left) shows Arthur Doodson with an early current meter. The water movements turn the large fan at the front, and the vane at the back orients the meter with the direction of the current flow.

Figure 1.19 (right) shows a current meter record which was probably written by a mechanical stylus onto a smoked glass slide. It is about 2 min long. The average current is about 0.7 kt (0.36 m s^{-1}): this will be due to a steady tidal current. The average current is modulated by a signal of about 15 cycles per minute: this will be caused by small surface waves with a 4 s period.

NOAA (2007a, 2007b) notes that, since the tidal-current velocities in any locality may be expressed by the sum of a series of harmonic terms involving the same periodic constituents as those found in the tidal elevations, the tide-predicting machine may also be used for tidal-current prediction. The harmonic constants for the prediction of current velocities are derived from current observations by an analysis

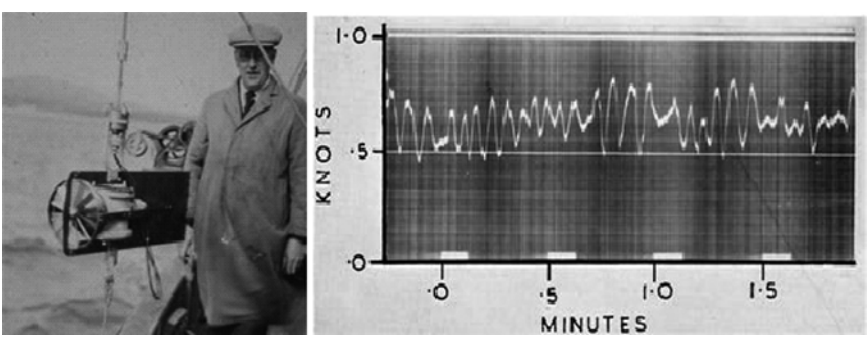

Figure 1.19 (Left) Arthur Doodson with an early current meter and (right) an early current meter record.

similar to that used in obtaining the harmonic constants from tidal height observations. For the currents, however, consideration must be given to the direction of flow.

The machine can be used for the prediction of reversing currents in which the direction of the flood current is taken as positive and the ebb current as negative. Rotary currents may be predicted by taking the north and east components separately, but the labour of obtaining the resultant velocities and directions from these components would be very great without a machine especially designed for the purpose.

Kelvin's equation is then rewritten as the harmonic current equation:

$$U(t) = U_0 + \sum\left[U_A \cos\left(\frac{2\pi t}{T_A} + \rho_A\right)\right] \tag{1.12}$$

Equation (1.12) is, in effect, the result of all of the work stretching for more than two millennia and described in Part I of this chapter. We have seen how a rudimentary knowledge of astronomy developed by the Ancient Greeks has developed into a very detailed understanding of planetary motion. Newton has then used the gravitational attraction of these orbiting masses to explain the equilibrium theory of the tides, and Laplace and Kelvin have then developed the harmonic analysis of tidal datasets to explain and to predict the vertical and now the horizontal motions of tidal waters. In the following sections, we examine the development of turbine science to utilize the water currents and develop the analysis of tidal stream power.

Part 2 Turbine science

1.9 Antiquity: the Romans and Chinese

From classical times (Hansen, 2007) there have existed three general types of water wheel (Figure 1.20). The Norse wheel has vanes protruding from a wooden rotor which is turned by a jet of water. The undershot wheel is rotated by the impulse

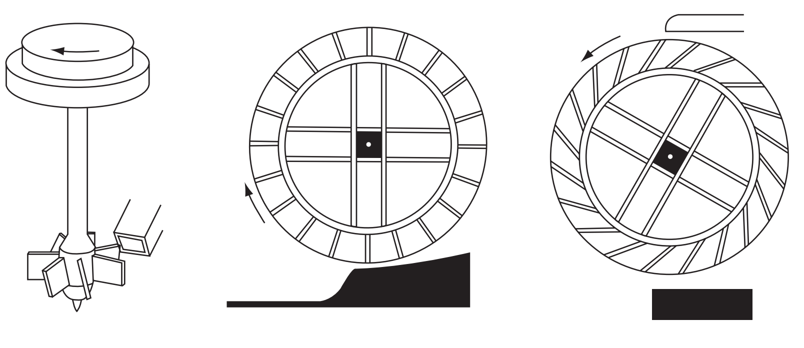

Figure 1.20 Water wheels. Norse wheels (left) turn millstones directly, undershot wheels (centre) require gears, and overshot wheels (right) also require an elevated stream (after Hansen, 2007).

Figure 1.21 Marcus Vitruvius Pollio (ca 80 to ca 25 BC).

of a water current. The overshot wheel receives water from above and utilizes both the potential energy of the mass of the water and the kinetic energy of the current. It seems probable (Ramage, 2004) that the earliest water device was an undershot wheel known as the noria and used to raise water for irrigation.

The first description of a water wheel is from Vitruvius (Figure 1.21), an engineer of the Augustan Age (31 BC to AD 14) who composed a 10-volume treatise on all aspects of Roman engineering. Vitruvius described the workings of an undershot wheel. One innovation occurred when Rome was under siege in AD 537 and the besieging Goths shut off the aqueducts that drove the city's mills. Belisarius, the Byzantine general defending the city, ordered floating mills to be installed close to the Tiber bridges, the piers of which constricted and accelerated the current. Two rows of boats were anchored, with water wheels suspended between them. The arrangement worked so well that cities all over Europe were soon copying it.

Water power was also an important source of energy in ancient Chinese civilizations. A contemporary text reports that, in AD 31, the engineer Tu Shih invented a water-powered reciprocator for the casting of agricultural implements. Smelters and casters were 'instructed to use the rushing of water to operate their billows'. Water power was also applied at an early date to the grinding of grain. Large mills appeared in China in about the second century BC and at about the same time in Europe. However, in China the water wheel was a critical power supply, while, for hundreds of years, Europe relied heavily on slave- and donkey-powered mills.

1.10 Middle Ages: the Syrians and Agricola

The fall of the western part of the Roman Empire in about AD 400 was followed by a transitional period of about 1000 years. Advances in hydraulics that had occurred under such notables as Archimedes effectively ceased as Europe became fragmented

into small states. In the Arab world, however, knowledge of water supply using combinations of water wheels and Archimedean screws flourished. The water wheels that had been originally introduced into Europe by the Romans were improved during this period, and the Middle East developed many ingenious combinations of screws and undershot and overshot water wheels (Sarton, 1931). In contrast, in the Western World, the structures that had been built by the Romans were allowed to deteriorate.

There was, however, a gradual spread in the use of traditional water wheels. Areas in northern and western Europe came under cultivation. Grain was an important crop, and most of it was ground by water mills (e.g. ETMTL, 2007). Historic records provide useful insights. The Doomsday Book, a survey prepared in England in AD 1086, lists 5624 water mills, whereas, only a century earlier, fewer than 100 mills were counted. French records tell a similar story. In the Aube district, 14 mills operated in the eleventh century, 60 in the twelth, and nearly 200 in the thirteenth century. In Picardy, 40 mills in 1080 grew to 245 by 1175. Boat mills, moored under the bridges of medieval Paris and other cities, began in the twelth century to be replaced by structures permanently joined to bridges. A good picture of metallurgy and water wheels can be obtained from *De Re Metallica*, by Georgius Agricola, published in 1556. An excellent translation of this work was prepared by Herbert Hoover (a mining engineer and future President) and his wife (the first woman geologist to graduate from Stanford University). Agricola was one of the first to record mining and metallurgical practices, and in so doing has left us with impressive images of water wheel technology.

1.11 Eighteenth and nineteenth centuries: Smeaton to Kaplan

1.11.1 John Smeaton

Major improvements were made in water wheel design from the 1700s onwards. John Smeaton (Figure 1.22) made a series of experiments on model water wheels in 1752 and was elected Fellow of the Royal Society in 1753. In 1759 he published a paper on water wheels and windmills (Smeaton, 1759), for which he received the Copley Medal of the Royal Society. Among his results, he noted that overshot wheels were twice as efficient as undershot wheels and that the impact of streams of water on a flat plate dissipated a large amount of energy. He was a member of the Royal Society Club, an occasional guest at meetings of the Lunar Society, and a charter member of the first professional engineering society, the Society of Civil Engineers. The latter became known as the Smeatonian Society after his death.

1.11.2 Benoît Fourneyron

In 1832 the young French engineer Benoît Fourneyron (Figure 1.23) patented the first water turbine (from the Latin *turbo*, meaning something that spins). Fourneyron's turbine incorporated guide vanes to direct the water towards the centre across the curved faces of the fixed vanes so that it travelled almost parallel to the curve of the runner blades as it reached them. The water was deflected as it crossed the

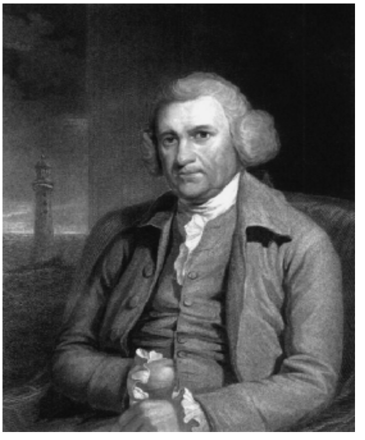

Figure 1.22 John Smeaton (1724–1792).

Figure 1.23 Benoît Fourneyron (1802–1867).

runner blades, exerting a rotational force, and then, having given up its energy, it fell away into the outflow. Tests showed that Fourneyron's turbine converted as much as 80 % of the energy in the water into useful mechanical output.

1.11.3 James Bicheno Francis

Soon after Fourneyron's turbine developments, the British–American engineer James Bicheno Francis (Figure 1.24) invented the Francis inward flow radial turbine (combining radial and axial flow) for low-pressure installations. He had joined the Locks and Canal Company of Lowell, Massachusetts, shortly after emigrating to the United States at the age of 18. He became Chief Engineer in 1837 and remained

Figure 1.24 James Bicheno Francis (1815–1892).

with the Company for his entire career. The Company owned and operated Lowell's canal system, providing water power for the local textile mills.

1.11.4 Lester Allen Pelton

During the nineteenth century, a number of impulse turbines were also invented. Lester Pelton (Figure 1.25) developed the first water wheel to take advantage of the kinetic energy of water rather than the weight or pressure of a stream. The speed and efficiency of Pelton's wheel made it ideal for generating electricity. Pelton designed a wheel with split buckets that harnessed the kinetic energy of a small volume of water flowing at high speed. Properly adjusted, Pelton's wheel could achieve efficiencies in

Figure 1.25 Lester Allen Pelton (1829–1908).

Figure 1.26 Victor Kaplan (1876–1934).

excess of 90 %. Pelton's wheel permitted low-cost hydroelectric power to replace expensive steam engines in mining operations in the western United States where streams rarely flowed at high enough volumes to turn traditional water wheels.

1.11.5 Victor Kaplan

Victor Kaplan (Figure 1.26) created the Kaplan turbine, a propeller-type machine, in 1913. It was an evolution of the Francis turbine but revolutionized the ability to develop low-head hydro sites.

1.12 Modern turbines

A modern water turbine is a device that converts the energy in a stream of fluid into mechanical energy by passing the stream through a system of fixed and moving blades and causing the latter to rotate (Electrical Engineering Tutorials, 2008).

Turbines may be classified according to the direction of the water flow into four groups (cf. Chapter 3):

1. **Tangential** or **peripheral turbines** have flow along the tangential direction, such as the Pelton wheel.

2. **Inward radial-flow turbines** have flow along the radius, such as the crossflow turbine.

3. **Mixed flow turbines** have a radial inlet and axial outlet, such as the Francis turbine.

4. **Axial-flow turbines** have flow along the shaft axis, such as the Kaplan turbine.

Turbines are also classified into impulse turbines and reaction turbines:

- **Impulse turbines** change the direction of the flow of a high-velocity fluid jet. The resulting impulse spins the turbine and reduces the kinetic energy of the flow. There is no pressure change of the fluid in the turbine rotor blades. Before reaching the turbine, the fluid's pressure head is changed to a velocity head by accelerating the fluid with a nozzle. Pelton wheels use this process exclusively. Impulse turbines do not require a pressure casement around the runner, since the fluid jet is prepared by a nozzle prior to reaching the turbine.

- **Reaction turbines** develop torque by reacting to the fluid's pressure or weight. The pressure of the fluid changes as it passes through the turbine rotor blades. A pressure casement is needed to contain the working fluid as it acts on the turbine stage(s), or the turbine must be fully immersed in the fluid flow. The casing contains and directs the working fluid and, for water turbines, maintains the suction imparted by the draft tube. Francis and Kaplan turbines are different types of reaction turbine. For compressible working fluids such as gases, multiple turbine stages may be used to harness the expanding gas efficiently.

1.13 Summary

Although the connection between the tides and the Moon had been recognized by Aristotle, three centuries before Christ, Earth-centred astronomy persisted. Newton wrote the equilibrium theory in the late seventeenth century, and this was developed by Laplace who recognized that the response functions would have the same frequencies as the astronomical drivers. Kelvin took these forward, applied harmonic analyses, and introduced the naming convention for the constituents.

The harmonic current equation might be rewritten as

$$U(t) = U_0 + \sum \left[U_A \cos\left(\frac{2\pi t}{T_A} + \rho_A \right) \right] \tag{1.12}$$

The first two constituents are the lunar semi-diurnal constituent, with a symbol M_{U2} and a period of 12.4206012 h, and the solar semi-diurnal S_{U2}, with a period of 12.00 h.

1.14 Bibliography

AMS (2007) http://www.ams.org/featurecolumn/archive/tidesIII2.html [accessed 8 May 2007].

Andrews, J. & Jelley, N. (2007) *Energy Science: Principles, Technologies and Impacts*. OUP, Oxford, UK, 328 pp.

Art of Navigation (2007) http://www.abc.net.au/navigators/navigation/history.htm [accessed 14 November 2007].

Association d'Oceanographique Physique (1955) *Bibliography on Tides 1665–1939. Publication Scientifique 15*. Bergen, Norway. http://www.airmynyorks.co.uk/BibMod.html [accessed 1 May 2007].

Cartwright, D.E. (1999) *Tides: A Scientific History*. Cambridge University Press. Cambridge, UK, 292 pp.

Charlier, R.H. (2003) Sustainable co-generation from the tides: bibliography. *Renewable and Sustainable Energy Reviews* **7**, 561–563.

Colepresse, S. (1668) Tides observed at Plymouth. *Phil. Trans. R. Soc. Lond.* **2**, 632.

Defant, A. (1961) *Physical Oceanography*. Pergamon Press, Oxford, UK.

Drennon, W. (2007) http://home.cvc.org/science/kepler.htm [accessed 5 May 2007].

Electrical Engineering Tutorials (2008) http://powerelectrical.blogspot.com/2007/04/water-turbines-and-its-classification.html [accessed 7 March 2008].

ETMTL (2007) http://www.elingtidemill.wanadoo.co.uk/sitem.html [accessed 8 May 2007].

FWERI (2007) http://www.baw.de/vip/en/departments/department_k/methods/kenn/frqw/frqw-en.html#glo [accessed 7 May 2007].

Groves-Kirkby, C.J., Denman, A.R., Crockett, R.G.M., Phillips, P.S., & Gillmore, G.K. (2006) *Science of the Total Environment* **367**, 191–202.

Hansen, R.D. (2007) *Water Wheels*. http://www.waterhistory.org/histories/waterwheels/[accessed 5 March 2008].

Kelvin (1911) *Mathematical and Physical Papers (Vol. VI)*, pp. 272–305 [from the *Minutes of the Proceedings of the Institution of Civil Engineers*, 11 March 1882.]

LEGOS (2008) http://www.legos.obs-mip.fr/en/[accessed 19 June 2008].

Moray, R. (1665) Extraordinary tides in the west Isles of Scotland. *Phil. Trans. R. Soc. Lond.* **1**, 53.

Moray, R. (1666a) Considerations and inquiries concerning tides. *Phil. Trans. R. Soc. Lond.* **1**, 298.

Moray, R. (1666b) Patterns of the tables proposed to be made for observing tides. *Phil. Trans. R. Soc. Lond.* **1**, 311.

NOOA (2007a) http://www.co-ops.nos.noaa.gov/predhist.html [accessed 5 May 2007].

NOAA (2007b) http://tidesandcurrents.noaa.gov/predmach.html [accessed 5 May 2007].

Norwood, R. (1668) Of the tides at Bermuda. *Phil. Trans. R. Soc. Lond.* **2**, 565.

Palmer, H. (1831) A description of a graphical register for tides and wind. *Proc. R. Soc. Lond.* **121**, 209–213.

Pannikar, N.K. & Srinivasan, T.M. (1971) The concept of tides in ancient India. *Indian J. Hist. Sci.* **6**, 36–50.

Philips, H. (1668) Time of the tides observed at London. *Phil. Trans. R. Soc. Lond.* **2**, 656.

Phillips, A. (2007) *Harmonic Analysis and Prediction of Tides*. http://www.math.sunysb.edu/~tony/tides/harmonic.html [accessed 7 May 2007].

Proudman Oceanographic Laboratory (2007a) http://www.pol.ac.uk/home/insight/tideinfo.html [accessed 7 May 2007].

Proudman Oceanographic Laboratory (2007b) http://www.abc.net.au/navigators/navigation/history.htm [accessed 14 November 2007].

Pugh, D. (1996) *Tides, Surges and Mean Sea Level*. John Wiley & Sons, Ltd, Chichester, UK.

Ramage, J. (2004) Hydroelectricity, in *Renewable Energy*, 2nd edition, ed. by Boyle, G., Oxford University Press, Oxford, pp. 148–194.

Round, D.G. (2004) *Incompressible Flow Turbomachines*. Butterworth-Heinemann, Oxford, 352 pp.

Rouse, W.W. (1908) A *Short Account of the History of Mathematics*, 4th edition. Macmillan, London, UK.

Sarton, G. (1931) *Introduction to the History of Science, Vol. II, Part II*. Carnegie Institution of Washington, Washington, DC.

Smeaton, J. (1759) An experimental enquiry concerning the natural powers of water and wind to turn mills, and other machines, depending on a circular motion. *Phil. Trans. R. Soc. Lond.* **51**, 100–174.

Stafford, R. (1668) Of the tides at Bermuda. *Phil. Trans. R. Soc. Lond.* **2**, 792.

The Galileo Project (2007) http://galileo.rice.edu/sci/brahe.html [accessed 5 May 2007].

Tyson (2007) http://www.pbs.org/wgbh/nova/galileo/mistake.html [accessed 1 May 2007].

Wallis, J. (1666) Hypothesis on the flux and reflux of the sea. *Phil. Trans. R. Soc. Lond.* **1**, 263–281.

Wallis, J. (1668) On the variety of the annual tides in several places of England. *Phil. Trans. R. Soc. Lond.* **2**, 652.