Index

Absolute value, 19
Absolute value penalty function, 561
Activation function, 254
Active constraint, 483
Adaline, 257, 271
Adaptive linear element, 257
Additivity, 19
Affine function, 62, 63, 247, 524
Affine matrix inequality, 524
Affine scaling, 406, 407
Affine scaling method, 406–411
  artificial problem, 410
  stopping criterion, 409
  strictly interior feasible point, 406
Algebraic Riccati inequality, 544
Algorithm
  affine scaling, 406–411
  backpropagation, 253, 258–269
  BFGS, 207–211
  Broyden-Fletcher-Goldfarb-Shanno, see BFGS algorithm
  complexity of, 306, 402
  conjugate gradient, see Conjugate gradient algorithm
  convergence of, see Convergence
  Davison-Fletcher-Powell, see DFP algorithm
  DFP, 202–207
  ellipsoid, see Khachiyan's method
  exponential complexity, 402
  fixed step size, 146, 151, 261
  for constrained optimization, 545–567
  genetic, 285–298
  globally monotone, 157
  gradient, 131–153
  Gram-Schmidt, 177, 189
  interior-point, 307, 403, 406, 411, 423
  iterative, 124, 159. See also Search methods
  Kaczmarz's, 232–236, 257
  Karmarkar's, see Karmarkar's method
  Khachiyan's, 306, 402–405, 418
  Lagrangian, 553–560
  naive random search, 279
  Nelder-Mead, 274–278
  particle swarm optimization, 282–285
  polynomial complexity, 402
probabilistic search, 278
projected, 546, 556
projected gradient, 407, 547, 549–553
projected steepest descent, 551
quasi-Newton, see Quasi-Newton methods
randomized search, 278
rank one, 187–202
rank two, 202
RLS, 227–232, 250
secant method, 120–122, 172
simplex, see Simplex method
simulated annealing, 278–282
single-rank symmetric, 197
SRS, 197
steepest descent, 133–141
symmetric Huang family, 215
variable metric, 202
Widrow-Hoff, 257, 271
zero finding, 118, 155
Allocation, 541
Alphabet in genetic algorithm, 286
Argmin, 82
Armijo backtracking algorithm, 126
Armijo condition, 125
Armijo-Goldstein condition, 125
Artificial neural networks, see Feedforward neural networks
Artificial problem
in affine scaling method, 410
in Karmarkar’s method, 418
in simplex method, 361
Associative, 8, 9
Asymmetric duality, 379
Augmented matrix, 325
Backpropagation algorithm, 253, 258–259
as a gradient algorithm, 261
forward pass, 263
reverse pass, 263
Ball, 50
Banana (Rosenbrock’s) function, 68, 159, 173, 191, 215
Basic columns, 325
Basic feasible solution, 305, 325, 327, 331, 349
Basic solutions, 324–327
Basic variables, 325, 347
Basis
definition of, 11
entering, 351
in linear equations, 325, 347, 348
leaving, 351
natural, 11
orthogonal, 29
Beltrami, 562
Best-so-far, 280, 283, 290
BFGS algorithm, 207–211
Big-oh notation, 74, 149
Bisection method, 116
Bland’s rule, 360, 375
Boltzmann, 281
Bolzano-Weierstrass theorem, 58, 234
Boundary, 51
Boundary point, 51
Bounded above, 57
Bounded below, 57
Bounded sequence, 56, 57
Bounded set, 51
Box constraint, 546
Bracketing, 123, 129
Brent’s method, 122
Broyden, 207
Broyden-Fletcher-Goldfarb-Shanno algorithm, see BFGS algorithm
Canonical augmented matrix, 346–348
Canonical form, 346
Canonical representation, 347
Canonical representation of LMI, 529
Canonical tableau, 358
Carrier of polyhedron, 53
Cauchy-Schwarz inequality, 20, 132, 207, 551
Center of gravity, 102, 275
Centroid, 102, 274
Chain rule, 67
Characteristic equation, 26
Characteristic polynomial, 26
Chromosome in genetic algorithm, 286
Circuit, 121, 253, 254, 311, 487
Citation style, 6
Clairaut’s theorem, 65
Closed set, 51
Column vector, 7
Combinatorial optimization, 273, 282, 297
Commutative, 8
Compact set, 51
Compatible matrix norm, 36
Complementarity, 207
Complementary slackness, 390, 539
Complex inner product, 21
Complex vector space, 12
Complexity of algorithm, 306, 402
exponential, 402
polynomial, 402
Component of vector, 7
Composite function, 67, 85, 90, 459
Concave function, 512, see Convex function
Condition number, 148
Conjugate direction methods, 175–188
Conjugate gradient algorithm
Fletcher-Reeves formula, 187
Hestenes-Stiefel formula, 187
nonquadratic problems, 186–188
Polak-Ribière formula, 187
Powell formula, 188
quadratic problems, 182–186
stopping criterion, 188
Consistent linear inequalities, 404
Constrained optimization, 82, 305, 449
Constraint
active, 483
box, 546
convex, 517
equality, 450, 504
functional, 82
inactive, 483
inequality, 483, 561
set, 82
Constraint set, 81. See also Feasible set
Continuity, 21, 60, 453, 565
Continuous function, 21, 60, 453, 565
Continuously differentiable function, 65, 451, 513
Contradiction, proof, 5
Contraposition, proof, 4
Contrapositive, 4
Control system, 102, 472, 474, 475, 481, 553
Convergence
fixed-step-size gradient algorithm, 146
globally convergent, 141
gradient algorithms, 143
Kaczmarz's algorithm, 233
linear, 148
locally convergent, 141, 554, 556
Newton's method, 165
of sequence of matrices, 59
order of, 148, 149, 152, 153, 157, 165
penalty method, 555
quadratic (second-order), 148
rate of, 141, 148
ratio, 148
steepest descent algorithm, 145
sublinear, 148
superlinear, 148
Convergent sequence, 56
Convex combination, 48, 297, 582
Convex constraint, 517
Convex function, 508–517
definition of, 509
differentiable, 513
equivalent definition of, 509
minimizers of, 517
optimization of, 517–536
quadratic, 512
strict, 512
twice differentiable, 515
Convex optimization, 517–536
Convex programming, see Convex optimization
Convex set, 48–50
definition of, 48, 330, 509
extreme point, 50, 331, 332, 335
in definition of convex function, 509
polyhedron, 52, 317
polytope, 52, 316
properties of, 49
supporting hyperplane, 52, 316
Cooling schedule, 281
Coordinates, 11
Cost function, 81
Courant-Beltrami penalty function, 562, 570
Cramer's rule, 33
Crossing site, 287
Crossover in genetic algorithm, 287
crossing site, 287
multiple-point crossover, 287, 300
one-point crossover, 287
Cubic fit, 122
Curve, 453
Cutting-plane method, 435
Cycling in simplex method, 360, 375
Dantzig, 306
Davidon, 202
Davidon-Fletcher-Powell algorithm, see DFP algorithm
Decision variable, 81, 305, 541
Decomposition
direct sum, 30, 458
orthogonal, 30, 226
Decreasing sequence, 55
Degenerate basic feasible solution, 325, 328, 360
DeMorgan's law, 3
Derivative, 82, 83
partial, 64
Derivative descent search, 154
Derivative matrix, 64
Descent property, 135, 141, 144, 167, 168, 193, 552
Determinant, 14
DFF algorithm, 202–207
Diagonal matrix, 27, 407, 421, 541, 571
Diet problem, 308, 381, 388
Differentiable curve, 453
Differentiable function, 62, 63, 453
Dimension, 11, 452
Direct sum decomposition, 30, 458
Directional derivative, 84
Discrete Fourier series, 225
Discrete-time linear system, 102, 248, 474, 481
Distributive, 9
Domination, 575
Dual linear program, 378, 403
Dual nonlinear program, 543
Dual quadratic program, 399
Duality
asymmetric, 379
dual nonlinear program, 543
dual problem, 378, 403
dual quadratic program, 399
dual vector, 378
duality theorem, 387, 543
in quasi-Newton methods, 207
Karush-Kuhn-Tucker conditions, 395, 539
linear programming, 377, 539
nonlinear programming, 542
primal nonlinear program, 542
primal problem, 378, 403
primal quadratic program, 399
quadratic programming, 399
symmetric, 378, 403
weak duality lemma, 385, 399, 543
Duality theorem, 387, 543
Dyadic product, 197
Eberhart, Russell, 282
Edge of polyhedron, 53
Eigenvalue
definition of, 26
maximal, 144
minimal, 144
of symmetric matrix, 28, 35
Eigenvalue
definition of, 26
of symmetric matrix, 28
orthogonal, 28
relation to Q-conjugacy, 189
Electric circuit, 311, 487
Elementary matrix
elementary row operation, 341
first kind, 340
second kind, 340
third kind, 341
Elementary row operation, 341
Elitism in genetic algorithm, 290
Ellipsoid, 405
Ellipsoid algorithm, see Khachyan’s method
Encoding in genetic algorithm, 285, 290, 297
Entry of matrix, 12
Epigraph, 508
Equality constraint, 450, 504
Estimation, 121, 223, 244
Euclidean inner product, 19
Euclidean norm, 20
Evolution in genetic algorithm, 287
Exact penalty function, 556
Exclusive OR, see XOR
Expanding subspace theorem, 181
Exponential complexity, 402
Extreme point, 50, 331, 332, 334
Extremizer, 82
Face of polyhedron, 53, 316
Farrel’s transposition theorem, 396
Feasibility problem, 398, 526, 529
Feasible direction, 84, 519, 566
Feasible point, 305, 450
Feasible set, 81, 450
Feedforward neural networks, 253–269
activation function, 254
Adaline, 257
backpropagation algorithm, 253, 258–269
function approximation, 255
hidden layer, 254
input layer, 254
learning, 253
neuron, 253
output layer, 254
single-neuron training, 256–258
supervised learning, 256
training, 253
training set, 255
unsupervised learning, 256
weights, 253, 254
Fibonacci method, 108–115
Fibonacci sequence, 109
First-order Lagrangian algorithm, 553
First-order necessary condition
equality constraint (Lagrange), 460, 461
        in convex optimization, 518
inequality constraint (KKT), 484
interior case, 86
set constraint, 85
Fitness in genetic algorithm, 286
Fitting straight line, 101, 221, 227, 246, 247
Fixed point, 553
Fixed step size, 146, 151, 261
Fletcher, 187, 202, 207
Fletcher-Reeves formula, 187
Floor, 435
FONC, see First-order necessary condition
Fourier series, 225
Probenius norm, 36
Pull-rank factorization, 236
Function
        affine, 62, 63, 247, 524
        banana, 68, 159, 173, 191, 215
        composite, 67, 85, 90, 459
        concave, 512, see Convex function
        continuous, 21, 60, 453, 565
        continuously differentiable, 65, 451, 513
        convex, 508–517
        cost, 81
        derivative matrix of, 64
        derivative of, 62, 83
        differentiable, 62, 63, 453
        directional derivative of, 84
        gradient of, 65, 71, 83, 131
        graph of, 71, 508
        Jacobian matrix of, 64
        Lagrangian, 464, 468, 543, 553
        linear, see Linear transformation
        matrix-valued, 60
        maximum rate of decrease, 132
        maximum rate of increase, 71, 131
        notation, 5
        objective, 81
        partial derivative of, 64
        penalty, 560
        Powell, 162
        Rosenbrock’s, 68, 159, 173, 191, 215
        sigmoid, 266
        twice continuously differentiable, 65, 515
        twice differentiable, 65, 454
        uniformly continuous, 23
        unimodal, 104
utility, 542
        Function approximation, 255
        Functional constraint, 82
        Fundamental theorem of algebra, 26
        Fundamental theorem of linear algebra, 41
        Fundamental theorem of LP, 328
        Fuzzy linear programming, 585
        Gale’s transposition theorem, 397
        Gauss-Newton method, 171
        Generalized eigenvalue, 468, 534
        Generalized inverse, 236, 238
        Genetic algorithm, 285–298
        alphabet, 286
        analysis of, 291–297
        best-so-far chromosome, 290
        chromosome, 286
        crossover, 287
        elitism, 290
        encoding, 285, 290, 297
        evolution, 287
        fitness, 266
        initial population, 285
        length of schema, 294
        mating pool, 286
        mutation, 288
        offspring, 287
        order of schema, 294
        parents, 287
        population size, 286
        real-number, 297–298
        representation scheme, 286
        roulette-wheel scheme, 286
        schema, 292
        selection, 286
        stopping criterion, 290
        tournament scheme, 286
        Gibbs, 281
        Global minimizer, 82, 94, 473, 517–519, 521, 537, 552
        Globally convergent, 141
        Globally monotone algorithm, 157
        Golden section, 106
        Golden section search, 104–108
        Goldfarb, 207
        Goldstein condition, 125
        Gomery cut, 435
        Gomory cutting-plane method, 435
        Jordan’s transposition theorem, 397
        Gradient, 65, 71, 83, 131
        Gradient descent algorithm, see Algorithm, gradient
        Gradient methods, 131–153
backpropagation algorithm, 253, 258–260
constrained optimization, see Projected gradient method
convergence of, 141–147
convergence rate of, 147–153
descent property, 135, 141, 144
equality constraints, see Lagrangian algorithms
fixed step size, 145
inequality constraints, see Lagrangian algorithms
Lagrangian, 553–560
order of convergence, 152
projected, 547, 549–553
stopping criterion, 135
Gram matrix, 220
Gram-Schmidt, 177, 189
Grammian, 299
Graph, 71, 508
Greatest lower bound, 57
Hačijan, see Khachiyan
Hadamard product, 283
Hajek, 282
Half-space, 46, 316
negative, 46
positive, 46
Hessian, 65, 468, 515
Hessian matrix, 83
Hestenes, Magnus, 187, 188
Hestenes-Stiefel formula, 187
Hidden layer in neural network, 254
Hoff, 257
Holland, John, 285
Homogeneity, 19, 21
Huang family, 215
Hyperplane
definition of, 46
supporting, 52, 316
tangent to graph, 71
Identity matrix, 16
ILP, see Integer linear programming
Image of matrix, see Range of matrix
Implicit function theorem, 457
Impulse response, 248
Inactive constraint, 483
Inconsistent system of equations, 217
Increasing sequence, 55
Indefinite matrix, 35
Induced matrix norm, 36, 480
Induction, principle of, 5
Inequality constraint, 483, 561
Infimum, see Greatest lower bound
Inner product
complex, 21
Euclidean, 19
properties of, 19
Innovation, 229
Input layer in neural network, 254
Integer linear programming, 427–444
Integer programming, see Integer linear programming
Interior, 51
Interior point, 51
Interior-point method, 307, 403, 406, 411, 423
Inverse
continuity of, 60
matrix, 16
Inverse Hessian, 191
Inverse parabolic interpolation, 122
Invertible matrix, see Nonsingular matrix
Iterative algorithm, 124, see Search methods, 159
Jacobian matrix, 64
Jordan form, 50
Kaczmarz’s algorithm, 232–236, 257
Kantorovich, 306
Karmarkar, 306, 403
Karmarkar’s method, 306, 403, 411–423
artificial problem, 418
complexity, 403
Karmarkar’s canonical form, 413–415
Karmarkar’s restricted problem, 414–415
projective transformation, 416, 424
simplex, 412
stopping criterion, 415, 420
strictly interior feasible point, 407, 416
Karush-Kuhn-Tucker condition, see KKT condition
Karush-Kuhn-Tucker multiplier, see KKT multiplier
Karush-Kuhn-Tucker theorem, 484
Kennedy, James, 282
Kernel of matrix, see Nullspace of matrix
Khachiyan, 306, 402
Khachiyan’s method, 306, 402–405, 418
KKT condition, 484, 485, 489, 521, 556, 594
KKT multiplier, 484, 490
KKT theorem, 484
Klee-Minty problem, 401
Koopmans, 306
Krylov subspace, 188
Kuhn-Tucker condition, see KKT condition

Lagrange condition, 460, 464, 520, 550, 553, 593
Lagrange multiplier, 460, 463
Lagrange's theorem, 460, 461
Lagrangian algorithms, 553–560
Lagrangian function, 464, 468, 543, 553
Lanczos, Cornelius, 188
Leading principal minor, 31
Learning in neural network, 253
Least squares, 217–227, 238 nonlinear, 169
Least upper bound, 57
Left pseudoinverse, 238
Level set, 68, 131, 134
Levenberg-Marquardt algorithm, 171
Levenberg-Marquardt modification, 168
Limit of sequence, 55
Line fitting, 101, 221, 227, 246, 247
Line search, 103, 124, 133, 167, 186, 188, 194, 209
Line segment, 45, 48
Linear combination, 10
Linear convergence, 143
Linear dynamical system, see Discrete-time linear system

Linear equations
augmented matrix, 325
basic solution, 325
basis, 325, 347, 348
canonical augmented matrix, 347
canonical form, 368
degenerate basic solutions, 325
existence of solution, 17
inconsistent, 217
Kaczmarz's algorithm, 232–236
least-squares solution, 217, 218, 221
minimum-norm solution, 231, 241, 257, 473
 overdetermined, 217
particular solution, 346
pivot, 349, 352, 364
solving in general, 217–243
solving using row operations, 339–346

Linear function, see Linear transformation

Linear inequalities consistent, 404
in linear programming, 305, 307, 316
Linear least squares, 217–227, 238
Linear matrix inequality, 524, 541
Linear programming
artificial scaling method, 406–411
artificial problem in affine scaling method, 410
artificial problem in Karmarkar's method, 418
artificial problem in simplex method, 361
artificial variables in simplex method, 361
as constrained problem, 450
asymmetric duality, 379
basic columns, 325
basic feasible solution, 305, 325, 327, 331, 349
basic solutions, 324–327
basic variables, 325, 347
Bland's rule, 360, 375
brief history of LP, 305
canonical augmented matrix, 347
canonical tableau, 358
complementary slackness, 390, 539
cycling, 360, 375
degenerate basic feasible solution, 325, 328, 360
dual problem, 378, 403
duality, see Duality
duality theorem, 387
equations of, 100, 307–314
extreme point, 331, 332, 334
feasible solution, 325
fundamental theorem of LP, 328
fuzzy, 585
geometric view of, 330
integer linear programming, 427–444
interior-point method, 307, 403, 406, 411, 423
Karmarkar's method, see Karmarkar's method
Karush-Kuhn-Tucker condition, 395, 501, 539
Khachiyan's method, 306, 403–405, 418
Klee-Minty problem, 401
optimal basic feasible solution, 328
optimal feasible solution, 328
primal problem, 378, 403
reduced cost coefficient, 353, 357, 358, 391
revised simplex method, 364–368
sensitivity, 398
simplex method, 306, 339–368
slack variable, 319
standard form, 318, 324
surplus variable, 319
symmetric duality, 378, 403
tableau, 358
two-dimensional, 314
two-phase affine scaling method, 409
two-phase simplex method, 360–364
uncertain, 584–591
weak duality lemma, 385, 399
Linear quadratic regulator, 474
Linear regression, see Line fitting
Linear space, see Vector space
Linear transformation, 25, 63
Linear variety, 47
Linear-fractional LMIs, 534
Linearly dependent, 10
Linearly independent, 9, 176, 324, 451, 484
Little-o notation, 74, 85
LMI, see Linear matrix inequality
LMI solvers, 629
LMI toolbox for MATLAB, 529, 536
LMITOOL, 536
Local minimizer, 82, 83, 85, 90, 92, 461, 469, 471, 484, 494, 517
Locally convergent, 141, 554, 556
Location parameter, 267
Lower bound, 57
LP, see Linear programming
LQR, 474
Lyapunov inequality, 527, 543
MacDuffee, 241
Markov chain, 397
Mating pool in genetic algorithm, 286
LMI toolbox, 529, 536
Matrix
affine matrix inequality, 524
compatible norm, 36
condition number, 148
continuous, 60
convergence of sequence, 59
definition of, 12
derivative, 64
determinant, 14
diagonal, 27, 407, 421, 541, 571
eigenvalue of, see Eigenvalue
eigenvector of, see Eigenvector
elementary, see Elementary matrix
entry of, 12
full-rank factorization, 235
function, matrix-valued, 60
game theory, 314
generalized inverse, 236, 238
Gram, 220
Hadamard product, 283
Hessian, 65, 83, 408, 515
identity, 16
image of, see Range of matrix
indefinite, 35
induced norm, 36, 480
inverse, 16
invertible, see Nonsingular matrix
Jacobian, 64
Jordan form, 59
kernel of, see Nullspace of matrix
leading principal minor of, 31
left pseudoinverse, 238
linear matrix inequality, 524, 541
minor of, 15, 428
Moore-Penrose inverse, 236, 237
negative definite, 35
negative semidefinite, 35
nonsingular, 16, 208, 218, 229, 325, 342
notation, 12
nullspace of, 30, 41, 227, 372, 406, 454
orthogonal, 29, 571
orthogonal projector, 30, 226, 406, 408, 423, 549
Penrose generalized inverse, 243
positive definite, 35
positive semidefinite, 35
principal minor of, 31
pseudoinverse, 236, 238
range of, 30, 41
rank of, 13–16
representation of linear transformation, 25
right pseudoinverse, 238
Schur complement, 526
Schur product, 283
sequence of, 59
series of, 60
similar, 26
square, 14
stochastic, 397
submatrix of, 432
Sylvester's criterion, 31
symmetric, 28, 35, 139
totally unimodular, 432
trace, 534
transformation, 26
transpose of, 12
unimodular, 428
Matrix norm, 35–39
Matrix-valued function, 60
Max, 21
Maximizer, 82
Mean value theorem, 76, 554, 558
MILP, see Mixed integer linear programming
Min, 15, 82
Minimax, 582, 587, 592
Minimizer
description of, 81
global, 82, 94, 473, 517–519, 521, 537, 552
local, 82, 83, 85, 90, 461, 469, 484, 494, 517
Pareto, 575
strict global, 82
strict local, 82, 92, 102, 471, 494
Minimum norm, 231, 242, 257, 584, 592
Minor
definition of, 15, 428
leading principal, 31
principal, 31
Minty, 401
Mixed integer linear programming, 444
Monotone sequence, 55, 57
Moore–Penrose inverse, 236, 237
Morrison, 208, 229
Multicriteria optimization, 573
Multiobjective optimization, 573, 586
Mutation in genetic algorithm, 288
Naive random search, 279
Natural basis, 11
Negative definite
matrix, 35
quadratic form, 31
Negative half-space, 46
Negative semidefinite
matrix, 35
quadratic form, 31
Neighborhood, 50
Nelder-Mead algorithm, 274–278
centroid, 274
contraction, 276
extension, 275
Neural networks, see Feedforward neural networks
Neuron, 253
Newton's method
collection of, 165
descent direction, 167
descent property, 167
for nonlinear least squares, 168–171
Gauss-Newton method, 171
general, 161–171
Levenberg-Marquardt modification of, 168
modification of, 167
of tangents, 119
one-dimensional, 116–119
order of convergence, 165
Newton-Raphson method, see Newton's method
Non-strict inequality, 528
Nondecreasing sequence, 55
Nondifferentiable optimization, 523
Nondifferentiable penalty function, 586
Nonincreasing sequence, 55
Nonlinear least squares, 169
Nonsingular matrix, 16, 208, 218, 229, 325, 342
Norm
collection of, 36
Euclidean, 20
Frobenius, 36
general vector norm, 21
induced, 36, 480
matrix, 35–39
p-norm, 21, 584
properties of, 21
Normal, 47, 70
Normal plane, 458
Normal space, 457, 458
Notation, 5
Nullspace of matrix, 30, 41, 227, 372, 406, 454
Objective function, 81
Offspring in genetic algorithm, 287
One-dimensional search methods, 103–126
Open set, 51
Optimal basic feasible solution, 328
Optimal control, 472, 475, 481, 482, 583
Optimal feasible solution in LP, 328
Optimization
combinatorial, 273, 282, 297
constrained, 82, 305, 449
convex, 517–536
linear, see Linear programming
multicriteria, 573
multiobjective, 573, 586
nondifferentiable, 523
semidefinite, 523
unconstrained, see Unconstrained optimization
vector, 573
with equality constraints, 449, 553
with inequality constraints, 483, 556
with set constraint, 82, 502
Optimization algorithm, see Search methods
Order of convergence, 148, 149, 152, 153, 157, 165
Order symbol, 73, 149
Orthant, 416
Orthogonal, 70
Orthogonal basis, 29
Orthogonal complement, 29, 226, 458
Orthogonal decomposition, 30, 226
Orthogonal matrix, 29, 571
Orthogonal projection, 30, 219, 406, 408, 423
Orthogonal projector, 30, 226, 406, 408, 423, 549
Orthogonal vectors, 20
Outer product, 197
Output layer in neural network, 254
Overdetermined system of equations, 217
Parents in genetic algorithm, 287
Pareto front, 575
Pareto minimizer, 575
Partial derivative, 64
Particle swarm optimization, 282–285
Particular solution, 346
Penalty function, 560
Penalty method, 560–567
  absolute value penalty function, 561
  convergence, 565
  Courant-Beltrami penalty function, 562, 570
  exact penalty function, 566
  nondifferentiable penalty function, 566
  penalty function, 560
  penalty parameter, 560
Penalty parameter, 560
Penrose, see Moore-Penrose inverse
Penrose generalized inverse, 243
Perp, see Orthogonal complement
Pivot, 349, 352, 364
Polak-Ribière formula, 187
Polyhedron
carrier of, 53
definition of, 52
definition of, 53
definition of, 53, 316
in linear programming, 316–318
vertex of, 53
Polynomial, characteristic, 26
Polynomial complexity, 402
Polytope
definition of, 52
in linear programming, 316
Population in genetic algorithm, 285, 286
Positive definite
  matrix, 35
  quadratic form, 31
  relation to eigenvalues, 35
  Sylvester’s criterion, 31
Positive half-space, 46
Positive orthant, 416
Positive semidefinite
  matrix, 35
  quadratic form, 31
  relation to eigenvalues, 35
  relation to principal minors, 35
Positivity, 19, 21
Powell, 162, 188, 202
Powell formula, 188
Powell function, 162
Primal linear program, 378, 403
Primal nonlinear program, 542
Primal quadratic program, 399
Primal-dual method, 377
Principal minor, 31
Principle of induction, 5
Probabilistic search, 278
Probability vector, 397, 540
Product
dyadic, 197
inner, 19, 21
outer, 197
Product rule, 68
Projected algorithm, 546, 556
Projected gradient method, 407, 547, 549–553
  stopping criterion, 552
Projected steepest descent algorithm, 551
Projection, 297, see Orthogonal projection, 546
Projective transformation, 416, 424
INDEX 619

Proof
contradiction (reductio ad absurdum), 5
contraposition, 4
direct method, 4
methods of, 3–5
principle of induction, 5
Proportional fairness, 541
Pseudoinverse, 236, 238
Pythagorean theorem, 21

Q-conjugate
definition of, 176
linear independence, 176
relation to eigenvectors, 189
relation to orthogonality, 189
Quadratic convergence, 148
Quadratic fit, 122, 128
Quadratic form
convex, 512
definition of, 31
maximising, 467, 471
negative definite, 31
negative semidefinite, 31
positive definite, 31
positive semidefinite, 31, 35
Sylvester’s criterion, 31
Quadratic programming, 399, 472, 481, 500
Quasi-Newton methods, 193–211
approximating inverse Hessian, 194
BFGS algorithm, 207–211
complementarity, 207
conjugate direction property, 196
descent property, 193
DFP algorithm, 202–207
duality, 207
rank one formula, 197–202
rank two update, 202
single-rank symmetric, 197
symmetric Huang family, 215
variable metric algorithm, 202
 Recursive least-squares, see RLS algorithm
Reduced cost coefficient, 353, 357, 358, 391
Reductio ad absurdum, 5
Reeves, 187
Regular point, 451, 456, 461, 464, 484,
554, 556
Relative cost coefficient, see Reduced cost coefficient
Representation scheme in genetic algorithm, 286
Revised simplex method, 364–368
Revised tableau, 365
Ribière, 187
Riccati inequality, 544
Right pseudoinverse, 238
RLS algorithm, 227–232, 250
Rosenbrock’s function, 68, 159, 173, 191, 215
Roulette-wheel scheme, 286
Row operations, 339–346
Row vector, 7

Scalar, 9
Scale parameter, 267
Schema in genetic algorithm, 292
length of, 294
order of, 294
Schmidt, see Gram-Schmidt
Schur complement, 526
Schur product, 283
Schwarz, see Cauchy-Schwarz inequality
Schwarz’s theorem, 68
Scilab Consortium, 536
Search direction, 124, 128, 167, 168
Search methods
bisection method, 116
conjugate direction methods, 175–188
conjugate gradient algorithm, 182–188
constrained optimization, 545–567
derivative descent search, 154
Fibonacci, 108–115
genetic algorithm, 211
general algorithm, 211
gradient algorithms, 285–298
Goldberg’s search, 104–108
gradient methods, 131–153
Kaczmarz’s algorithm, 232–238, 257
Lagrangian, 553–560
line search, 103, 124, 133, 167, 186,
188, 194, 209
naïve random search, 279
Nelder-Mead algorithm, 274-278
neural network training, 355
Newton's method, 116-119, 161-171
Newton-Raphson method, see Newton's method
one-dimensional, 103-126
particle swarm optimization, 282-285
penalty method, 560-567
probabilistic, 278
projected, 546, 556
projected gradient methods, 547, 549-553
quasi-Newton methods, 193-211
randomized, 278
secant method, 120-122, 172
simulated annealing algorithm, 278-282
steepest descent method, 133-141
Secant method, 120-122, 172
Second-order necessary condition
equality constraints, 469
inequality constraints, 494
interior case, 90
set constraint, 90
Second-order sufficient condition
equality constraints, 470
inequality constraints, 494
interior case, 92
set constraint, 102
Selection in genetic algorithm, 286
Semidefinite programming, 523
Sensitivity, 398
Sequence
bounded, 56, 57
bounded above, 57
bounded below, 57
convergent, 56
decreasing, 55
Fibonacci, 109
greatest lower bound, 57
increasing, 55
least upper bound, 57
limit, 55
lower bound, 57
monotone, 55, 57
nondecreasing, 55, 564
nonincreasing, 55
of matrices, 59
of real numbers, 55
order of convergence, 148, 149, 152, 153, 157, 165
subsequence of, 58
upper bound, 57
Set
boundary of, 51
bounded, 51
closed, 51
compact, 51
constraint, 81, see Feasible set
convex, see Convex set
feasible, 81, 450
interior of, 51
minus, 5
notation, 5
open, 51
simplex, 274, 412
subset of, 5
Set constraint, 82, 502
Shannon, 207
Sherman-Morrison formula, 208, 229
Sherman-Morrison-Woodbury formula, 229
Shift parameter, 267
Sigmoid, 266
Signal-to-interference ratio, 88, 98
Similar matrices, 26
Simplex, 274, 412
Simplex algorithm, see Simplex method
Simplex method, 306, 339-368
algorithm, 349-356
artificial problem, 361
artificial variables, 361
Bland's rule, 360, 375
canonical augmented matrix, 346-348
canonical tableau, 358
complexity, 402
cycling, 360, 375
exponential complexity, 402
integer linear programming, 427-444
matrix form, 356-360
reduced cost coefficient, 353, 357, 368, 391
revised simplex method, 364-368
revised tableau, 365
row operations, 339-346
stopping criterion, 351, 372
tableau, 358
two-phase, 360-364
updating augmented matrix, 348-349
updating canonical tableau, 358
Simulating annealing algorithm, 278-282
Simultaneous equations, see Linear equations
Single-rank symmetric algorithm, 197
Singular value decomposition, 571
Slack variable, 319
SONC, see Second-order necessary condition
SOSC, see Second-order sufficient condition
Span, 10
Sphere, 425
Square matrix, 14
SRS algorithm, 197
Standard form linear program, 318, 324
Statement
  biconditional, 4
  conditional, 3
Steepest ascent, 71
Steepest ascent method, see Steepest descent method
Steepest descent
  order of convergence, 152
  Steepest descent method, 133–141
    for constrained optimization, 551
    for quadratic, 139
    projected, 551
Step response, 248
Step size, 132, 146, 156, 168, 233, 261, 547, 551
Stiefel, Eduard, 187, 188
Stochastic matrix, 397
Stopping criterion
  affine scaling method, 409
  conjugate gradient method, 188
  genetic algorithm, 290
  gradient method, 135
  Karmarkar’s method, 415, 420
  line search, 128
  projected gradient method, 552
  simplex method, 361, 372
Strict Inequality, 528
Strictly interior feasible point, 406, 407, 416
Strong Wolfe condition, 126
Structured representation of LMI, 529
Subgradient, 515, 537
Sublinear convergence, 148
Submatrix, 432
Subsequence, 58
Subset, 5
Subspace, 10
Superlinear convergence, 148
Supervised learning, 256
Supporting hyperplane, 52, 316

Supremum, see Least upper bound
Surface, 451, 454
Surplus variable, 319
SVD, see Singular value decomposition
Sylvester’s criterion, 31
Symmetric duality, 378, 403
Symmetric Huang family, 215
Symmetric matrix, 28, 35, 139
Symmetry, 19

Tableau in linear programming, 358
Tangent line, 70
Tangent plane, 70, 454
Tangent space, 454, 455, 458
Tangent vector, 65, 70, 454, 456
Taylor series, 72–76, 162, 165, 551. See also Taylor’s theorem
Taylor’s formula, 72, 75. See also Taylor’s theorem
Taylor’s theorem, 72, 85, 90, 92, 132, 513, 516
Temperature schedule, 281
Termination criterion, see Stopping criterion
Third-order necessary condition, 94
Third-order sufficient condition, 94
Threshold, 267
Totally unimodular, 432
Tournament scheme, 286
Trace, 534
Training of neural network, 253
Training set, 255
Transformation
  affine scaling, 407
  linear, 25, 63
  matrix, 26
  matrix representation of, 25
  projective, 418, 424
Transportation problem, 306, 310
Transpose
  matrix, 12
  vector, 8
Transposition theorems, 396
Traveling salesperson problem, 282
Triangle inequality, 21
Truth table, 3
Tucker, see KKT condition
Twice continuously differentiable function, 65, 515
Twice differentiable function, 65, 454
Two-dimensional linear program, 314
Two-phase affine scaling method, 409
Two-phase simplex method, 360–364
Uncertainty range, 106
Unconstrained optimization
  basics of, 82
  conditions for, 83-92
Uniform continuity, 23
Uniformly continuous function, 23
Unimodal, 104
Unimodular, 428
Unimodular, totally 432
Unsupervised learning, 256
Upper bound, 57
Utility function, 542

Variable metric algorithm, 202
Variety, linear, 47
Vector
  column, 7
  complex, 12
  component of, 7
  convex combination, 46, 297, 582
  definition of, 7
  difference, 8
  field, 65
  linear combination, 10
  linearly dependent, 10
  linearly independent, 9, 176, 324,
    451, 484
  normal, 47
  orthogonal, 20
  probability, 397, 540
  row, 7
  tangent, 65, 70, 454, 456
  transpose of, 8
  zero vector, 8
Vector field, 65
Vector optimization, 573
Vector space
  basis for, 11
  complex, 12
  definition of, 7
  dimension of, 11
  real, 7
  subspace of, 10
Vertex, 53, 274, 406

Weak duality lemma, 385, 399, 543
Weierstrass theorem, 36, 51
Weighted sum, 474, 582, 592
Weights in neural network, 253, 254
Widrow, 257
Widrow-Hoff algorithm, 257, 271
Wiener filter, 188, 245
Wolfe condition, 125
Woodbury, 229

XOR, 268, 271

YALMIP, 536
Yet Another LMI Package (YALMIP), 536

Zero finding, 118, 155
Zero matrix, 22, 30, 36
Zero vector, 8