Biological membranes display a wealth of physical phenomena including phase transitions, propagating voltage pulses, variable permeability, structural transitions (as seen in endo- and exocytosis), and domain formation that is thought to have an important influence on signal cascades. The title of this book “Thermal Physics of Membranes” indicates that it deals in particular with the thermodynamics of such systems. Thermodynamics is always true because it is based on only two basic and intuitive laws: the conservation of energy and the maximum entropy principle. Beyond that it is free of any approximations and assumptions. One therefore finds thermodynamics as a basis for physics on all length scales from atomic dimensions up to cosmological scales. Naturally, thermodynamics is also true on the level of biological membranes. We wish to introduce the reader to some of these principles and their consequences concerning the behavior of membranes. Important topics in this book are “phase diagrams” including domain formation and rafts, elasticity and the related changes in vesicular shape, pulse propagation, permeability as well as protein binding and electrostatics.

Biology deals with complex ensembles of organic molecules including proteins, nucleic acids, and lipids, but also salts and water. Proteins often display unique molecular surfaces that give rise to specific interactions. Much of biophysical research therefore has been dedicated to the study of structures and interactions between individual molecules. Cells and their compartments are defined by a large variety of membranes that not only surround the cell as a whole but also each organelle as the nucleus, mitochondria, or the endoplasmic reticulum. On average 50% of the biomembrane mass stems from proteins. The human genome contains about 30,000 genes encoding at least as many proteins, many or most of those being membrane proteins.

The major building blocks of membranes, however, are hundreds or thousands of different lipid species. The human body contains several kg of membrane lipids with a total surface on the order of 0.4 km² per kg. The plasma membranes of one eucariot cell contains about $10^{10}$ lipid molecules. Although
the diversity of lipids is seemingly smaller than that of proteins, lipid membranes contain many molecules and are thus large ensembles.

Biological molecules usually do not only interact with one specific binding partner but also with the abundant lipid surfaces, with protons (because macromolecules contain protonable groups), ions and, very importantly, with water. Therefore one typically deals not with one interaction but rather with many. Even if only a few of these interactions have a strength that is of interest and even if one takes into account that one cell usually does not express all the proteins that are encoded in the genome, it is immediately obvious that it is impossible to investigate all possible interactions. One further has to take into account that the molecules may have different orientations and different conformations further increasing the complexity. We leave it to the reader to figure out how many different arrangements of, say, 200 lipid species in variable concentrations and conformations in an ensemble of $10^{10}$ molecules are possible—but the number is beyond any range that can ever be accessed by computers. One must come to the conclusion that life will never be understood on the basis of binary molecular interactions alone. In particular, many cooperative phenomena such as the melting of lipid membranes are beyond the scope of single molecule physics.

Thermodynamics is a fundamental discipline of physics that describes the behavior of assemblies of molecules. It solely relies on two basic principles: the law of the conservation of energy (first law) and the seemingly tautological principle that a most likely state exists that is assumed with the highest probability (second law). The latter principle is also known as the principle of maximum entropy. These two principles are so general and universal that the thermodynamic relations that are derived from them are also fundamentally true. In the case of biological systems, the variety of proteins, lipids, and ions is taken into account by their chemical potentials that are a function of the concentrations of other molecules as well as of temperature, pressure, voltage, or other intensive variables. In thermal equilibrium a multimolecular ensemble like a membrane fluctuates around the state of maximum entropy. If the system is not in equilibrium, the first derivative of the entropy constitutes the thermodynamic forces, which are the forces that drive a system back to equilibrium. The second derivatives of the entropy are related to susceptibilities, for example, to the heat capacity or the elastic constants of membranes. These properties of membranes are often easier to measure, for example with calorimeters (heat capacity), ultrasonic velocity measurements (volume compressibility) or by vesicular shape fluctuations (bending elasticity). Even though in thermal equilibrium the thermodynamics forces are zero, the susceptibilities generally assume nonzero values. Since the different susceptibilities are all second derivatives of the same thermodynamic function (the entropy), they are not independent of each other, but one can find surprising
relationships between various thermodynamic susceptibilities that can provide insights into the behavior of membranes that one would never be able to predict on the basis of single molecule interactions. Many such relations stem from the so-called Maxwell relation. We show two examples:

\[
\left( \frac{dS}{dp} \right)_{T,n_i} = -\left( \frac{dV}{dT} \right)_{p,n_i}
\]

(0.1)

where \( S \) and \( V \) are the entropy and the volume of an ensemble, respectively, including all their proteins and lipids—and all their conformations. This equation implies that the term on the left-hand side that is experimentally difficult to access is identical to the volume expansion coefficient that is very easy to measure. A second example is

\[
\left( \frac{d\mu_i}{dn_j} \right)_{S,V,n_i \neq j} = \left( \frac{d\mu_j}{dn_i} \right)_{S,V,n_j \neq i}
\]

(0.2)

This relation couples the chemical potential of one component to the variation of another and demonstrates the symmetry of the coupling. In biochemical textbooks such couplings usually do not play a role. This implies that the findings shown in such books are not necessarily incorrect but definitely incomplete. However, there are also examples where the molecular textbook models are clearly in conflict with the laws of thermodynamics. The application of thermodynamics therefore should not be considered as a method averaging out the molecular details (and thereby losing information) but rather as a means to gain considerable insight into all the couplings between seemingly different processes.

In this textbook we will introduce the reader to the thermodynamic concepts. Overall, our intention is to show the beautiful manner by which thermodynamics can link seemingly unrelated membrane processes resulting in a unified picture of the behavior of membranes as a whole. Our aim therefore is to present a coherent concept rather than achieving a complete presentation of the field. This approach takes the risk that important results of respected colleagues are not presented to the extent that they deserve.

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