Atoms and Energy

“Nothing in life is to be feared. It is only to be understood.” M. Curie

Physics underpins radiation protection. It is necessary for describing the origins of radiation, the types and properties of emitted radiation(s), and the mechanisms by which radiant energy is deposited in various media. It is standard practice, which is perhaps unique for radiation protection, to optimize protection as far below established safe levels as reasonably achievable, and an understanding of the physics of the various elements is fundamental to its accomplishment.

Four basic forces of nature control the dynamics (i.e., position, energy, work, etc.) of all matter, including the constituents of atoms – protons, neutrons, and electrons. These forces, along with their magnitude relative to gravity are:

- gravity, which is an attractive force between masses $= G$;
- the weak force, which influences radioactive transformation $\equiv 10^{24} G$;
- the electromagnetic force, which exists between electric charges $\equiv 10^{37} G$;
- the nuclear force, which is strongly attractive between nucleons only $\equiv 10^{39} G$.

These forces range over some 40 orders of magnitude; however, two of them largely determine the energy states of particles in the atom (gravitational forces are insignificant for the masses of atom constituents, and the weak force is a special force associated with the process of radioactive transformation of unstable atoms):

- the nuclear force between neutrons and protons which is so strong that it overcomes the electrical repulsion of the protons (which is quite strong at the small dimensions of the nucleus) and holds the nucleus and its constituent protons and neutrons together;
- the force of electrical attraction between the positively charged nucleus and the orbital electrons which not only holds the electrons within the atom, but influences where they orbit.
The nuclear force, or strong force, is amazing and a bit strange. It exists only between protons and neutrons or any combination of them; consequently it exists only in the nucleus of atoms. The nuclear force is not affected by the charge on neutrons and protons, nor the distance between them. It is strongly attractive, so much so that it overcomes the natural repulsion between protons at the very short distances in the nucleus since it is about 100 times stronger than the electromagnetic force.

The electromagnetic force, on the other hand, exists between charged particles no matter where they are (a nucleus can also be thought of as a large charged particle although it contains several protons, each of which has a unit positive charge). The electromagnetic force is inversely proportional to the square of the distance, $r$, between two particles with a charge of $q_1$ and $q_2$:

$$F_{em} = \frac{1}{4 \pi \varepsilon_0} \frac{q_1 q_2}{r^2} = k_0 \frac{q_1 q_2}{r^2}$$

where the charges on each particle are expressed in coulombs and the separation distance $r$ is in meters. The constant $k_0$ is for two charges in a vacuum and has the value $k_0 = 8.9876 \times 10^9$ N m$^2$/C$^2$. This fundamental relationship is called Coulomb’s law after its developer, and is referred to as the coulomb force. If $q_1$ and $q_2$ are of the same sign (i.e., positive or negative), $F$ will be a repulsive force; if they are of opposite signs, $F$ will be attractive.

1.1 Structure of Atoms

Atoms contain enormous amounts of energy distributed among the energy states of the constituent parts. Some of this energy is emitted from the atom if an overall decrease occurs in the potential energy states of one or more of the constituents, and similarly absorption of radiant energy by an atom yields an increase in the potential energy of one or more states. Atom constituents are primarily neutrons, protons, and electrons, and their number and array establish:

- what the element is and whether its atoms are stable or unstable;
- if unstable, how the atoms will emit energy (we will deal with energy later).
Modern theory has shown that protons and neutrons are made up of more fundamental particles, or quarks, but it is not necessary to go into such depth to understand the fundamental makeup of atoms and how they behave to produce radiant energy. Atoms are bound systems – they only exist when protons and neutrons are bound together to form a nucleus and when electrons are bound in orbits around the nucleus. The particles in atoms are bound into such an array because nature forces atoms toward the lowest potential energy possible; when they attain it they are stable, and until they do they have excess energy and are thus unstable, or radioactive.

The proton has a reference mass of about 1.0. It also has a positive electrical charge of plus 1 (+1).

\[
\text{Proton} \\
\begin{array}{c}
+ \\
\text{Mass} = 1 \quad \text{Charge} = +1
\end{array}
\]

The electron is much lighter than the proton. Its mass is about 1/1840 of that of the proton and it has an electrical charge of minus one (−1).

\[
\text{Electron} \\
\begin{array}{c}
- \\
\text{Mass} = 1/1800 \quad \text{Charge} = -1
\end{array}
\]

The neutron is almost the same size as the proton, but slightly heavier. It has no electrical charge.

\[
\text{Neutron} \\
\begin{array}{c}
0 \\
\text{Mass} = 1 \quad \text{Charge} = 0
\end{array}
\]

When these basic building blocks are put together, which is what happened at the beginning of time, very important things become evident. First, a proton will attract a free electron to form an atom:
The resulting atom is electrically neutral. That is, each $-1$ charge on an orbital electron is matched by a $+1$ positively charged proton in the nucleus. The total atom (proton plus an electron) has a diameter of about $10^{-10}$ m (or $10^{-8}$ cm) and is much bigger than the central nucleus which has a radius of about $10^{-15}$ m or ($10^{-13}$ cm); thus the atom is mostly empty space. The radius of the nucleus alone is proportional to $A^{1/3}$, where $A$ is the atomic mass number of the atom in question or

$$r = r_o A^{1/3}$$

The constant $r_o$ varies according to the element but has an average value of about $1.3 \times 10^{-15}$ m, or femtometers. The femtometer ($10^{-15}$ m) is commonly referred to as a fermi in honor of the great Italian physicist and nuclear navigator, Enrico Fermi.

A free neutron is electrically neutral and, in contrast to a proton, an atom does not form; i.e., it is just a free neutron subject to thermal forces of motion.

Likewise, two or more neutrons are also unaffected by any electrons present. However, if left alone for a while, a free neutron will undergo transformation (commonly referred to as decay) into a proton and an electron; therefore, in a free state, the neutron, though not an atom (no orbiting electrons), behaves like a radioactive atom by emitting a negatively charged electron.

A neutron can, however, be bound with one or more protons to form a nucleus, and in this state it does not undergo transformation but will maintain its identity as a fundamental particle. When this occurs, an electron will join with the proton–neutron nucleus to form an electrically neutral atom:
However, it now weighs about twice as much as the other one because of the added neutron mass. And, if another (a second) neutron is added, a heavier one-proton atom is formed:

\[ _1^1 \text{H} \]

\[ _1^2 \text{H} \]

\[ _1^3 \text{H} \]

This atom is the same electrically neutral atom (one proton balanced by one electron) we started with, but it weighs about three times as much due to the two extra neutrons, and because of the array of the particles in the nucleus it is an atom with excess energy, i.e., it is radioactive. Each of these one-proton atoms is an atom of hydrogen because hydrogen is defined as any atom containing one proton balanced by one electron. Each atom has a different weight because of the number of neutrons it contains, and these are called isotopes (Greek: “iso” = same; “tope” = place) of hydrogen to recognize their particular features. These three isotopes of hydrogen are denoted by the following symbols:

These symbols establish the nomenclature used to identify atoms: the subscript on the lower left denotes the number of protons in the atom; the superscript on the upper left refers to the mass number, an integer that is the sum of the number of protons and neutrons in the nucleus. It is common practice to leave off the subscript for the number of protons because the elemental symbol, H, defines the substance as hydrogen with only one proton. The isotopes of hydrogen are identified as protium (or hydrogen), deuterium, and tritium; the first two are stable and exist in nature, but tritium is radioactive and will be converted to an isotope of helium (He) through radioactive transformation. Almost all elements exist, or can be produced, with several different mass numbers yielding several isotopes. A par-
1.1.1
Two-proton Atoms

If we try to put two protons together, the repulsive coulomb force between them at the very short distance required to form a nucleus is so great that it even overcomes the strongly attractive nuclear force between the protons; thus, an atom (actually a nucleus) cannot be assembled from just these two particles.

If, however, a neutron is added, it tends to redistribute the forces, and a stable nucleus can be formed. Two electrons will then join up to balance the two positive (+) charges of the protons to create a stable, electrically neutral atom of helium.

This atom is defined as helium because it has two protons. It has a mass of 3 (2 protons plus 1 neutron) and is written as helium-3 or \(^{3}\text{He}\). Because neutrons provide a cozy effect, yet another neutron can be added to obtain \(^{4}\text{He}\).

Although extra mass was added in forming \(^{4}\text{He}\), only two electrons are needed to balance the two positive charges. This atom is the predominant form of helium (isotope if you will) on earth, and it is very stable (we will see later that this same atom, minus the two orbital electrons, is ejected from some radioactive atoms as an alpha particle, i.e., a charged helium nucleus).

If yet another neutron is stuffed into helium to form helium-5 \(^{5}\text{He}\), the atom now contains more mass than it can handle and it breaks apart very fast (in \(10^{-21}\) s or so); it literally spits the neutron back out. There is just not enough room for the
third neutron, and by putting it in we create a highly unstable atom. But, as we observed for hydrogen and as we will see for other atoms, adding an extra neutron (or proton) to a nucleus only destabilizes it; i.e., it will often stay for quite a while as an unstable, or radioactive, atom due to the “extra” particle mass, identified by the “isotope” of a given element.

1.1.2

Three-proton Atoms

Atoms with three protons can be assembled with three neutrons to form lithium-6 \(^{6}\text{Li}\) or with four neutrons, lithium-7 \(^{7}\text{Li}\), or

Since lithium contains three protons, it must also have three orbital electrons, but another orbit further away is required for the third electron because the first orbit can only hold two electrons (there is an important reason for this which is explained by quantum theory).

If we keep combining protons and neutrons we get heavier and heavier atoms, but they obey the same general rules. The ratio of neutrons to protons is fairly high in heavy atoms because the extra neutrons are necessary to distribute the nuclear force and moderate the repulsive electrostatic force between protons in such a way that the atoms stay together. The heaviest element in nature is \(^{238}\text{U}\) with 92 protons and 146 neutrons; it is radioactive, but very long-lived. The heaviest stable element in nature is \(^{209}\text{Bi}\) with 83 protons and 126 neutrons. Lead with 82 protons is much more common in nature than bismuth and for a long time was thought to be the heaviest of the stable elements; it is also the stable endpoint of the radioactive transformation of uranium and thorium, two primordial naturally occurring radioactive elements (see Chapter 6).

1.2

Nuclide Chart

This logical pattern of atom building can be plotted in terms of the number of protons and neutrons in each to create a chart of the nuclides, a portion of which is shown in Figure 1-1.
The chart of the nuclides contains basic information on each element, how many isotopes it has (atoms on the horizontal lines), and which ones are stable (shaded) or unstable (unshaded). A good example of such information is shown in Figure 1-2 for four isotopes of carbon, which has 6 protons, boron (5 protons), and nitrogen (7 protons). Actually there are 8 measured isotopes of carbon but these 4 are the most important. They are all carbon because each contains 6 protons, but each of the 4 has a different number of neutrons: hence they are distinct isotopes with different weights. Note that the two blocks in the middle for $^{12}$C and $^{13}$C are shaded which indicates that these isotopes of carbon are stable (as are two shaded blocks for each of boron and nitrogen). The nuclides in the unshaded blocks (e.g., $^{11}$C and $^{14}$C) on each side of the shaded (i.e., stable) blocks are unstable simply because they do not have the right array of protons and neutrons to be stable (we will use these properties later to discuss radioactive transformation). The dark band at the top of the block for $^{14}$C denotes that it is a naturally occurring radioactive isotope, a convention used for several other such radionuclides. The block to the far left contains information on naturally abundant
1.3 Atom Measures

Many radiation protection problems require knowledge of how many atoms there are in common types of matter, the total energy represented by each atom, and the energy of its individual components, or particles, which can be derived from the mass of each atom and its component particles. Avogadro's number and the atomic mass unit are basic to these concepts, especially the energy associated with mass changes that occur in and between atoms. Other key parameters related to atoms and radiation physics are listed in Appendix A.
1.3.1 Avogadro’s Number

In 1811 an Italian physicist, Amedeo Avogadro, reasoned that the number of atoms or molecules in a mole of any substance is a constant, independent of the nature of the substance; however, he had no knowledge of its magnitude only that the number was very large. Because of this insight, the number of atoms or molecules in a mole of a substance is called Avogadro’s number, $N_A$. It has the following value:

$$N_A = 6.0221415 \times 10^{23} \text{ atoms/mol}$$

Example 1-1. Calculate the number of atoms of $^{13}$C in 0.1 g of natural carbon.

Solution. From Figure 1-1, the atomic weight of carbon is 12.0107 g and the atom percent abundance of $^{13}$C is 1.10%. Thus

$$\text{Number of atoms of } ^{13}\text{C} = \frac{0.1 \text{ g} \times N_A \text{ atoms/mol}}{12.0107 \text{ g/mol}} \times 0.011$$

$$= 5.5154 \times 10^{19} \text{ atoms}$$

1.3.2 Atomic Mass Unit (u)

Actual weights of atoms and constituent particles are extremely small and difficult to relate to, except that they are small. Thus, a natural unit, the atomic mass unit, or amu, that approximates the weight of a proton or neutron is used to express the masses of particles in individual atoms (since the sum of these is the mass number, one of these must be a “mass unit”). To be precise, the unit is the unified mass unit, denoted by the symbol u, but by precedent most refer to it as the atomic mass unit, or amu. The masses of stable and unstable isotopes of the elements contained in Appendix B and the chart of the nuclides are listed in unified mass units.

One amu, or u, is defined as one-twelfth the mass of the neutral $^{12}$C atom. One mole of $^{12}$C by convention is defined as weighing exactly 12.000000... g. All other elements and their isotopes are assigned weights relative to $^{12}$C. The amu was originally defined relative to oxygen-16 at 16.000000 g/mol but carbon-12 has proved to be a better reference nuclide, and since 1962 atomic masses have been based on the unified mass scale referenced to the carbon-12 mass at exactly 12.000000... g. The mass of a single atom of $^{12}$C can be obtained from the mass of one mole of $^{12}$C which contains Avogadro’s number of atoms as follows:

$$m^{(12}\text{C}) = \frac{12.000000 \text{ g/mol}}{6.0221415 \times 10^{23} \text{ atoms/mol}} = 1.9926466 \times 10^{-23} \text{ g/atom}$$

This mass is shared by 6 protons and 6 neutrons; thus, the average mass of each of the 12 building blocks of the carbon-12 atom, including the paired electrons,
can be calculated by distributing the mass of one atom of carbon-12 over the 12 nucleons. This quantity is defined as one amu, or u, and has the value

\[ 1 \text{ u} = 1.992646636 \times 10^{-23} \text{ g}/12 = 1.66053886 \times 10^{-24} \text{ g} \]

which is close to the actual mass of the proton (actually \( 1.6726231 \times 10^{-24} \text{ g} \)) or the neutron (actually \( 1.6749286 \times 10^{-24} \text{ g} \)). In unified mass units, the mass of the proton is \( 1.00727647 \text{ u} \) and that of the neutron is \( 1.008664923 \text{ u} \). Each of these values is so close to unity that the mass number of an isotope is thus a close approximation of its atomic weight. Measured masses of elements are given in unified mass units, u, and are some of the most precise measurements in physics with accuracies to six or more decimal places. Mass changes in nuclear processes represent energy changes; thus, accurate masses, as listed in Appendix B, are very useful in calculations of energies of nuclear events such as radioactive transformation.

### 1.4 Energy Concepts for Atoms

When we consider the energy of the atom, we usually focus on the energy states of the orbital electrons (referred to by many as atomic physics) or the arrangement of neutrons and protons in the nucleus (or nuclear physics). In any given atom, the electromagnetic force between the positively charged nucleus and the negatively charged electrons largely establishes where the electrons will orbit. An electron orbiting a nucleus experiences two forces: a centripetal (or pulling) force induced by the electromagnetic force between the electron and the positively charged nucleus and a centrifugal force (one that falls away) due to its angular rotation. The array of neutrons and protons in the nucleus is similarly determined by a balancing of the nuclear force and the electromagnetic force acting on them.

Since atoms are arrays of particles bound together under the influence of the nuclear force and the electromagnetic force, the particles in atoms (or anywhere else for that matter) have energy states that are directly related to how the force fields act upon them. These concepts lead naturally to the question: what is energy? Energy is the ability to do work. This then leads to the question, what is work? Work is a force acting through some distance. This appears to be a circular argument which can perhaps be better illustrated with a macro-world example of lifting a rock that weighs 1 kg (2.2 lb) from ground level onto a perch 1 m (about 3.28 ft) high.

The work done by the lifter is obviously related to the amount of effort to lift the weight of the rock against the force due to gravity and how high it is to be lifted. The physicist would characterize this effort as:

\[ \text{Work} = \text{force} \times \text{height of perch} \]

or \( W = mgh \)
where \( m \) is the mass of the rock in kilograms, \( g \) is the acceleration due to gravity (9.81 meters per second per second), and \( h \) is the height in meters. In this example, the amount of work done would be

\[
\text{Work} = 1 \text{ kg} \times 9.81 \text{ m/s}^2 \times 1 \text{ m} = 9.81 \text{ N m} = 9.81 \text{ J}
\]

In this example, work is expressed in the energy unit of joules which represents a force of one newton (kg m/s²) acting through a distance of one meter; i.e., energy and work are interrelated. The key concept is that 9.81 J represents an effort expended against gravity to raise a mass of 2.2 lb (1 kg) over a distance of 3.28 ft (1 m); i.e., work was done or energy was expended to get it there – if it were raised to another perch, one meter higher, the same amount of work would need to be done again.

The basic concepts of energy, work, and position associated with a rock can be extrapolated to atoms, which are what they are because their constituent particles exist at distinct energy levels. A rock on the perch can be thought of as “bound” with the ground; it has positive energy relative to the ground by virtue of the work done on it to get it up there. The stored work in the rock can be recovered by pushing it off the perch and allowing it to fall under the force of gravity. The stored energy (work = \( mgh \)) in the rock is called potential energy – it is perched there ready to do work.

As the rock falls back to the ground its potential energy is converted to kinetic energy, or energy due to its motion as it falls under the influence of the gravitational force. At any point above the ground the falling rock has both potential energy (yet to be recovered) due to its height (or \( PE = mgh \)) and kinetic energy due to its velocity or \( KE = \frac{1}{2} mv^2 \). Its total energy at any point is the sum of the two, or

\[
E = mgh + \frac{1}{2} mv^2
\]
where \( h \) and \( v \) are both variables. When the rock is at rest on the perch, all the energy is potential energy; when it strikes the ground, \( h \) is zero and all the potential energy has been converted to kinetic energy such that

\[
\frac{1}{2} mv^2 = mgh
\]

which can be used to determine the velocity the rock would have after falling through the distance \( h \) and striking the ground, a quantity called the terminal velocity:

\[
v = \sqrt{2gh}
\]

This expression can also be used to determine the velocity at any intermediate height, \( h \) minus the distance traveled, above the ground, and because of the conservation of energy the relative proportions of potential energy and kinetic energy (always equal to the total energy) along the path of the falling rock.

Another good example of potential (stored) energy and kinetic energy is a pebble in a slingshot. Work is done to stretch the elastic in the slingshot, so that the pebble has potential energy (stored work). When let go, the pebble is accelerated by the elastic returning to its relaxed position and it gains kinetic energy due to the imparted velocity, \( v \). The main point of both of these examples is that a body (particle) with potential energy has a stored ability to do work; when the potential energy is released, it shows up in the motion of the body (particle) as kinetic, or “active,” energy.

Potential energy can be positive or negative. A rock on the perch above the ground (or in a slingshot) has positive potential energy that can be recovered as kinetic energy by letting it go. But if the rock were in a hole, it would have negative potential energy of \(-mgh\) relative to the ground surface. Energy would need to be supplied (i.e., do work on it) to get it out of the hole and back to the ground surface.

Of course, the deeper the hole the more negative would be the potential energy and the more tightly bound it would be relative to the ground surface. Kinetic energy is, however, recovered as the rock falls into the hole and the amount is proportional to the distance it fell.

These same concepts can also be applied to the particles in an atom, and are very important because the way an atom behaves is determined by the potential energy states of its various particles. Perhaps the most important concept for particles in atoms is that they will always have a total energy which is the sum of the potential energy (PE), which is determined by their position, and the kinetic energy (KE) associated with their motion, or
Particles that change potential energy states gain (or lose) kinetic energy which can be released from the atom either as a particle or as a wave. For example, an electron “free” of the nucleus will experience a decrease in potential energy in the process of becoming bound into a position near the nucleus. This phenomenon can be represented as perches in a hole into which the electron might fall with the release of energy.

The perches in the hole represent negative energy states because it would take work to get them out and return them to the region where they would be considered “free” electrons. The amount of energy required to get an electron out of an atom is determined by the energy level it occupies; this amount is also the amount of energy released as the electron goes from the surface (the “free” state) to level 2, or on down to level 1. These energy changes have been observed by measuring the photons (more about these later) emitted when electrons are disturbed in atoms of an element such as hydrogen, helium, neon, nitrogen, etc. The atom with bound electrons has been determined to be slightly lighter than without; the loss of mass exactly matches the energy of the photon emitted.

**Checkpoints**

Consideration of energy states yields two very important concepts directly applicable to atoms:

- an electron bound to an atom has less potential energy than if it were floating around “free”;
- the process by which an atom constituent (electron, proton, or neutron) becomes bound causes the emission of energy as it goes to a lower potential energy state; the same amount of energy must be supplied to free the particle from its bound state and move it to another.

These concepts can be extended in a similar way to protons and neutrons which are bound in the nucleus at different energy levels represented schematically as:
1.5 Relativistic Energy

Rearranging the neutrons and protons to lower potential energy states yields energy in the form of an ejected particle (mass = energy), or pure electromagnetic radiation, or both. Processes that change the array of particles in the nucleus are fundamental to radioactive transformation (see Chapter 3).

1.5 Relativistic Energy

The energy associated with atoms is governed by Einstein’s special theory of relativity which states that mass and energy are one and the same. The rock on the perch gained potential energy from muscles doing the work necessary to lift it there; in an atom the energy of a particle is determined by the mass that is available to be transformed into energy as an electron or a nucleon (proton or neutron) undergoes a change in its bound state; i.e., as it becomes more tightly bound it gives up energy in relation to the change in its potential energy state or consumes energy in becoming less so.

The special theory of relativity developed by Einstein in 1905 describes the role of mass and energy and how mass changes at high speeds. In developing the special theory of relativity, Einstein showed that the mass \( m \) of a body varies with its speed \( v \) according to

\[
m = \frac{m_0}{\sqrt{1 - (v^2/c^2)}}
\]

where \( m_0 \), the rest mass, is the mass of the body measured when it is at rest and \( c \) is the velocity of light in a vacuum, which is a constant. This means that the mass of a body will increase with velocity, and thus momentum (or \( mv \)) must be treated...
in a way that recognizes that mass changes as the velocity changes. In doing so, it is necessary to state Newton’s second law precisely as he stated it, i.e., that the net force on a body is equal to the time rate of change of the momentum of the body, or

\[ F = \frac{d(mv)}{dt} \]

If the mass can be assumed to remain constant, this reduces to

\[ F = m \frac{dv}{dt} = ma \]

which is the classical relationship used to calculate the dynamics of most objects in the macro-world. \( F = ma \) provides reasonably accurate calculations of the force on objects at low speeds (less than 10% of the speed of light) and is thus very practical to use. However, many particles in and associated with atoms move at high speeds, and Newton’s second law must be stated in terms of the relativistic mass which changes with velocity, or

\[ F = \frac{d(mv)}{dt} = \frac{d}{dt} \left( \frac{m_0v}{\sqrt{1 - (v^2/c^2)}} \right) \]

In relativistic mechanics, as is classical mechanics, the kinetic energy, KE, of a body is equal to the work done by an external force in increasing the speed of the body by the value \( v \) in a distance \( ds \):

\[ KE = \int Fds \]

Using \( ds = vdt \) and the relativistic generalization of Newton’s second law, \( F = d(mv)/dt \), this expression becomes

\[ KE = \int_0^v \frac{d(mv)}{dt} vdt = \int_0^v vd(mv) \]

or if the term in the parentheses is differentiated, and the integration performed,

\[ KE = \int_0^v \frac{m_0v}{\sqrt{1 - v^2/c^2}} \]

since \( m \) in relativistic terms is a function of the rest mass, \( m_0 \), which is a constant, and the velocity of the particle; therefore,

\[ KE = \frac{m_0c^2}{\sqrt{1 - v^2/c^2}} - m_0c^2 \]
which reduces to

\[ KE = mc^2 - m_0c^2 = (m - m_0)c^2 \]

The kinetic energy (KE) gained by a moving particle is due to the increase in its mass; it is also the difference between the total energy, \( mc^2 \), of the moving particle and the rest energy, \( m_0c^2 \), of the particle when at rest. This is the same logical relationship between potential energy and kinetic energy we developed for a macro-world rock, but in this case the key variable is the change in mass of the moving body. Rearrangement of this equation yields Einstein’s famous equivalence of mass and energy:

\[ E = mc^2 = KE + m_0c^2 \]

More importantly it states that the total energy of a body is, similar to classical principles, the sum of the kinetic and potential energy at any given point in time and space with the important distinction that its potential energy is a property of its rest mass. Thus, even when a body is at rest it still has an energy content given by \( E_0 = m_0c^2 \), so that in principle the potential energy inherent in the mass of an object can be completely converted into kinetic energy. Atomic and nuclear processes routinely convert mass to energy and vice versa, thus nuclear processes that yield or consume energy can be conveniently described by measuring the mass changes that occur. Such measurements are among the most accurate in science.

Although Einstein’s concepts are fundamental to atomic phenomena, they are even more remarkable because when he stated them in 1905 he had no idea of atom systems, and no model of the atom existed. He had deduced the theory in search of the basic laws of nature that govern the dynamics and motion of objects. Einstein’s discoveries encompass Newton’s laws for the dynamics of macro-world objects but more importantly also apply to micro-world objects where velocities approach the speed of light; Newton’s laws break down at these speeds, but Einstein’s relationships do not. Einstein believed there was a more fundamental connection between the four forces of nature and he sought, without success, a unified field theory to elicit an even more fundamental law of nature. Even though his genius was unable to find the key to interconnect the four forces of nature, or to perhaps describe a unified force that encompassed them all, his brilliant and straightforward mass/energy concepts provided the foundation for later descriptions and understanding of the origins of atomic and nuclear phenomena, including the emission of radiation and its energy.
1.5.1
Momentum and Energy

Since momentum is conserved, but not velocity, it is often useful to express the energy of a body in terms of its momentum rather than its velocity. To this end, if the expression

\[ m = \frac{m_0}{\sqrt{1 - (v^2/c^2)}} \]

is squared, both sides are multiplied by \( c^4 \), and terms are collected,

\[ m^2 c^4 - m^2 v^2 c^2 = m_0^2 c^4 \]

Since \( E = mc^2 \), \( E_0 = m_0 c^2 \), and \( p = mv \), \( E \) is related to \( p \) as follows:

\[ E^2 = (pc)^2 + E_0^2 = (pc)^2 + (m_0 c^2)^2 \]

1.5.2
Effects of Velocity

First, it can be shown that the following expression for kinetic energy of a particle is universally applicable for any velocity, including bodies that move at ordinary speeds as well as those at relativistic speeds:

\[ KE = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}} - m_0 c^2 \]

Expanding this expression by the binomial theorem yields:

\[ KE = m_0 c^2 \left[ 1 + \frac{v^2}{2c^2} + \frac{3v^4}{8c^4} + \cdots \right] - 1 \]

If the velocity of a particle \( v \) is 0.1\( c \) or less, this expression reduces to \( KE = \frac{1}{2} m v^2 \) which means that the rest mass can be used to calculate the kinetic energy of a particle with an error of less than 1%. Also, when \( v \) is 0.1\( c \) or less,

\[ m \equiv m_0 \text{ and } p \equiv m_0 v \]

When \( v \) approaches \( c \), \( E \) (or \( mc^2 \)) and \( KE \) are very large compared with \( m_0 c^2 \), and the relativistic equations reduce to the following simplified form:

\[ p \equiv E/c \text{ and } KE \equiv E \]
If \( v/c > 0.99 \), \( E = pc \) with an error of less than 1%; i.e., at such velocities the total energy \( E \) is almost all kinetic energy since the rest mass energy, though still present, is negligible by comparison.

1.5.3
A Natural Limit

Einstein’s discoveries also yield a natural limit for particle dynamics. For a particle that contains a positive rest mass \( m_0 \) to achieve a velocity \( v \) equal to \( c \), its rest mass would be

\[
m = \frac{m_0 v}{\sqrt{1 - v^2/c^2}} = \frac{m_0}{0} = \infty
\]

which is not possible; therefore, the velocity of light is a natural limit that no particle with mass can reach. Since it is impossible to provide infinite mass for a material particle, then it is impossible for it to move with a velocity equal to that of the speed of light (in a vacuum). However, if it is assumed that the rest mass \( m_0 = 0 \), then \( m = 0/0 \) when \( v = c \). This is an indeterminate quantity, mathematically. Thus only those particles that have zero rest mass can travel at the speed of light. When such a “particle” exists:

\[
m_0 = 0, \quad E = pc, \quad KE = E
\]

That is, the “particle” has momentum and energy, but no rest mass. According to classical mechanics, there can be no such particle. But, according to Einstein’s special theory of relativity, a “particle” with such characteristics is indeed a reality; it is called a photon. A few examples illustrate these important relationships for nuclear particles.

**Example 1-2.** What is the maximum speed that a particle can have such that its kinetic energy can be written as \( \frac{1}{2} m v^2 \) with an error no greater than 0.5%?

**Solution.** The kinetic energy, \( KE \), when \( m_0/m = 0.005 \),

\[
\frac{KE - \frac{1}{2} m_0 v^2}{KE} = 0.005
\]

but

\[
KE = m_0 c^2 \left( \frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right) = m_0 c^2 \left[ \left( 1 + \frac{v^2}{2c^2} + \frac{3v^4}{8c^4} + \cdots \right) - 1 \right] = \frac{1}{2} \left( m_0 v^2 \right)
\]

if higher-order terms are neglected, thus

\[
v \approx 0.082c
\]
Example 1-3. Calculate the velocity of an electron which has a kinetic energy of 2 MeV (rest mass energy = 0.511 MeV).

Solution.

\[ KE = \frac{m_0c^2}{\sqrt{1 - \frac{v^2}{c^2}}} - m_0c^2 \]

\[ 2\text{MeV} = \frac{0.511\text{MeV}}{\sqrt{1 - \frac{v^2}{c^2}}} - 0.511\text{MeV} \]

\[ v = 0.979c \]

Example 1-4. Compute the effective mass of a 2 MeV photon.

Solution.

\[ E_{\text{photon}} = m_{\text{eff}}c^2 \]

For \( E \) in J, mass in kg, and the speed of light in m/s:

\[ 2\text{MeV} \times 1.6022 \times 10^{-13} \frac{\text{J}}{\text{MeV}} = m_{\text{eff}} (3 \times 10^8 \text{m/s})^2 \]

\[ m_{\text{eff}} = \frac{3.2044 \times 10^{-13}\text{J}}{(3 \times 10^8 \text{m/s})^2} = 3.56 \times 10^{-30}\text{kg} \]

1.5.4 Mass-energy

Since the masses of isotopes of atoms and all the constituent particles are known to better than six decimal places, the energy changes in nuclear processes are readily determined by the mass changes (expressed in u) that occur. For this reason, a most useful quantity is the energy equivalent of the atomic mass unit, u:

\[ E = m_0c^2 = \frac{1.660539 \times 10^{-27}\text{kg/u} \times (2.99792458 \times 10^8 \text{m/s})^2}{1.6021892 \times 10^{-13}\text{J/MeV}} = 931.494\text{ MeV/u} \]

A similar calculation can be performed to determine the energy equivalent of the electron mass:

\[ E = m_0c^2 = 9.1093826 \times 10^{-31}\text{kg} \times (2.99792458 \times 10^8 \text{m/s})^2 \]

\[ = 8.187 \times 10^{-14}\text{J}/1.6022 \times 10^{-13}\text{J/MeV} \]

\[ = 0.511\text{ MeV} \]
Various nuclear processes occur in which electron masses are converted to 0.511 MeV photons or in which photons of sufficient energy are transformed to electron masses. For example, a photon with $E = h\nu > 1.022$ MeV can, in the vicinity of a charged body (a nucleus or an electron), vanish yielding two electron masses (a process called pair production) and kinetic energy, which is the energy of the photon minus that consumed in forming the two electron masses; i.e. $KE = h\nu - 1.022$ MeV. This net kinetic energy is shared by the particles thus produced. And in similar fashion, the amount of energy potentially available from the complete annihilation of 1 g of matter would be

$$E = mc^2 = 1 \text{ g} \times (2.99792458 \times 10^{10} \text{ cm/s})^2 = 8.988 \times 10^{20} \text{ g cm}^2/\text{s}^2$$

which is equal to $8.988 \times 10^{20}$ erg, or $8.988 \times 10^{13}$ J, or about $25 \times 10^6$ kWh. This is an enormous amount of energy, and probably explains why scientists could not resist releasing the energy locked up in atoms after fission and fusion were discovered. Both fission and fusion cause atoms to become more tightly bound (see below) with significant changes in mass that is converted to energy and released.

**Checkpoints**

Application of the concepts of special relativity leads to several important properties of mass and energy and their relationship to each other:

- An increase in the kinetic energy of a body moving at relativistic speeds is due to an increase in its mass, which in turn is due to an increase in velocity, or $KE = mc^2 - m_0c^2 = (m - m_0)c^2$

  where

  $$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

- The expression $E = mc^2$ represents the total energy of a body due to its motion (i.e., kinetic energy) plus that due to its rest mass (i.e., potential energy), or $E = mc^2 = KE + m_0c^2$

  therefore, energy changes in an atom can be described by the change in mass that occurs during certain processes like radioactive transformation, interactions of bombarding particles, and fission and/or fusion of nuclei.

- No body with rest mass can reach or exceed the speed of light because to do so would require it to have infinite mass; i.e.,
1 Atoms and Energy

\[ m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \]

when \( v = c \)

\[ m_0 = \infty \]

a feat that is impossible to accomplish.

- Photons, which always travel at the speed of light, must have zero rest mass; however, they have a mass equivalence associated with their energy. This energy can in turn be added to an atom in such a way that it changes its mass, or it can be converted to a particle mass (e.g., an electron) in nuclear processes.

1.6 Electron Volt (eV)

The electron volt (eV) is a very useful and practical unit for characterizing the energy in atoms, groups of atoms, or their constituent particles. The electron volt is defined as the increase in kinetic energy of a particle with one unit of electric charge (e.g., an electron) when it is accelerated through a potential difference of 1 V. This can be represented schematically as:

The stationary electron at the negative electrode in the figure has a potential energy of 1 eV; when released it will be repelled by the negatively charged electrode and attracted to the positively charged one gaining in the process an energy of motion such that when it slams into the positive electrode all of the potential energy will be converted to kinetic energy. This energy, which is defined as 1 eV, is calculated by multiplying the unit charge of the electron by the voltage across the electrodes, i.e.

\[ 1 \text{ eV} = (\varnothing)(\Delta V) = (1.60217653 \times 10^{-19} \text{ C}) \times (1 \text{ V}) \]

\[ = 1.60217653 \times 10^{-19} \text{ V C} \]

\[ = 1.60217653 \times 10^{-19} \text{ J} \]

\[ = 1.60217653 \times 10^{-12} \text{ erg} \]

The relationship 1 eV = 1.60217653 \times 10^{-19} J = 1.60217653 \times 10^{-12} erg is used frequently in calculations of the amount of energy deposited in a medium. In absolute terms, 1 eV is not very much energy. In fact, the energy of atomic changes is commonly expressed in keV (10^3 eV) and MeV (10^6 eV). The concept of representing the energy of small particles by the energy they possess in motion is, however, very useful in describing how they interact. For example, one can think of the energy of a 1 MeV beta particle or proton as each being a unit-charged particle that gained an acceleration equal to being subjected to a jolt of a million volts of electrical energy.
Example 1-5. An x-ray tube accelerates electrons from a cathode into a tungsten target anode to produce x-rays. If the electric potential across the tube is 90 kV, what will be the energy of the electrons when they hit the target in eV, joules, and ergs?

Solution. $E = eV = 90,000$ eV

The energy in joules is

$$E = 90,000 \text{ eV} \times 1.6022 \times 10^{-19} \text{ J/eV} = 1.442 \times 10^{-14} \text{ J}$$

and in ergs

$$E = 1.442 \times 10^{-14} \text{ J} \times 10^7 \text{ erg/J} = 1.442 \times 10^{-7} \text{ erg}$$

1.7 Binding Energy of Nuclei

The equivalence of mass and energy takes on special significance for atoms, which are bound systems in which the potential energy of the bound particles is negative due to the forces that hold the masses in the system together. The potential energy states of the particles that make up the atom are less than they would be if they were separate; i.e., if two masses $m_1$ and $m_2$ are brought together to form a larger mass $M$, it will hold together only if

$$M < m_1 + m_2$$

This circumstance can be illustrated by introducing a quantity of energy, $E_b$, that is released as follows:

$$m_1 + m_2 \rightarrow M + E_b \text{ (released)}$$

where $E_b$ is the amount of energy released when the two masses are bound in such a way to decrease the rest-mass of the system. In other words, the amount of energy released in binding the masses together is simply the net decrease in the mass or

$$E_b = (m_1 + m_2 - M)c^2 = \Delta mc^2$$

$E_b$ is called the binding energy (a negative quantity since $m_1 + m_2$ is greater than the mass $M$ of the combined system) because it is responsible for holding the parts of the system together. It is also the energy that must be supplied to break $M$ into separate masses, $m_1$ and $m_2$:

$$M + E_b \rightarrow m_1 + m_2$$
These relationships are perhaps the most important consequences of Einstein’s mass–energy relation. They are most useful in understanding the energetics of particles and electromagnetic radiations, and are in fact fundamental properties of matter that are used constantly in atomic and nuclear physics.

1.7.1 Calculation of Binding Energy

The total binding energy of bound atoms can be calculated by first finding the mass difference between that of the assembled atom and that of the individual particles that make up the atom. For example, an atom of deuterium, $^2\text{H}$, consists of a proton and a neutron with an orbital electron. It is one of the simplest bound stable atoms with a rest mass of 2.01410178 u. The mass of the proton is 1.00727647 u, the electron is 0.00054858 u, and the neutron is 1.008664904 u (note the 8-figure accuracy of the masses). The masses of the individual particles are

\[
\begin{align*}
1.00727647 \text{ u} & \quad \text{(proton)} \\
1.00866491 \text{ u} & \quad \text{(neutron)} \\
0.00054858 \text{ u} & \quad \text{(electron)} \\
2.01648996 \text{ u} & \quad \text{(total)}
\end{align*}
\]

which is less than the measured mass of deuterium at 2.01410178 u by 0.00238818 u; therefore the binding energy holding the deuterium atom together is

\[
E_b = 0.00238818 \text{ u} \times 931.5 \text{ MeV/u} = 2.2246 \text{ MeV}
\]

which is released as photon energy as the constituents become arrayed in their bound (i.e., lower potential energy) states in the newly formed atom. This calculation could also be written as a nuclear reaction:

\[
\begin{align*}
\text{\textbf{\text{H}}}_1^{1} + \text{\textbf{n}}_0 \rightarrow \text{\textbf{H}}_2^{2} + \gamma + Q
\end{align*}
\]

where the mass of $^1\text{H}$ is used instead of summing the masses of the proton nucleus and the orbiting electron. $Q$ represents the net energy change, and in this case has a positive value of 2.225 MeV that is released as a gamma ray as the deuterium atom is formed. The binding energy of the orbital electron is implicitly ignored in this example; however, it is only about 13.6 eV which is negligible compared to 2.225 MeV.

1.7.2 Q-value Calculations

Calculations of binding energy are a subset of $Q$-value calculations, so called because they involve a net energy change due to changes in the nuclear masses of the constituents. The $Q$-value represents the amount of energy that is gained
when atoms change their bound states or that must be supplied to break them apart in a particular way. For example, in order to break $^2\text{H}$ into a proton and a neutron, an energy equal to 2.225 MeV (the $Q$-value) would need to be added to the $^2\text{H}$ atom, which in this case would be represented by the reaction

$$^2\text{H} + Q \rightarrow ^1\text{H} + ^1\text{n}$$

In this case, an energy equal to $Q = 2.225$ MeV, usually as a photon, must be supplied to break apart the bound system (because it has negative potential energy). If the photon energy is greater than 2.225 MeV the excess energy will exist as kinetic energy shared by the proton and the neutron.

**Example 1-6.** Find the binding energy for tritium, which contains one proton, one electron, and two neutrons.

**Solution.** The constituent masses of the tritium atom are

- mass of proton = 1.00727647 u
- mass of neutrons = $2 \times 1.00866491$ u
- mass of electron = 0.0005485799 u
- Total = 3.025155 u

which is larger than the measured mass of 3.016049 u (see Appendix B) for $^3\text{H}$ by 0.0091061 u, and the total binding energy is

$$E_b = 0.009106 \text{ u} \times 931.5 \text{ MeV/u} = 8.48 \text{ MeV}$$

and since tritium ($^3\text{H}$) contains three nucleons, the average binding energy for each nucleon ($E_b/A$) is

$$\frac{E_b}{A} = \frac{8.48\text{MeV}}{3} = 2.83\text{MeV/nucleon}$$

The binding energy per nucleon ($E_b/A$) can be calculated from the isotopic masses of each of the elements (as listed in Appendix B), and when plotted for each of the stable elements plus uranium and thorium as shown in Figure 1-3, several significant features become apparent.

- The elements in the middle part of the curve are the most tightly bound, the highest being $^{62}\text{Ni}$ at 8.7945 MeV/nucleon and $^{58}\text{Fe}$ at 8.7921 MeV/nucleon.
- Certain nuclei, $^4\text{He}$, $^{12}\text{C}$, $^{16}\text{O}$, $^{28}\text{Si}$, and $^{32}\text{S}$, are extra stable because they are multiples of helium atoms (mass number of 4), with one important exception, $^8\text{Be}$, which breaks up very rapidly ($10^{-16}$ s or so) into two atoms of $^4\text{He}$. 
- Fission of a heavy nucleus like $^{235}$U and $^{239}$Pu produces two lighter atoms (or products) with nucleons that are, according to Figure 1-3, more tightly bound resulting in a net energy release.
- Fusion of light elements such as $^2$H and $^3$H produces helium with nucleons that are more tightly bound causing a release of energy that is even larger per unit mass than that released in fission.

![Figure 1-3](image)

**Fig. 1-3** Curve of binding energy per nucleon versus atomic mass number (plotted from data in Appendix B).

### 1.8 Summary and Checkpoints

The dynamics of all objects in the universe from subatomic particles to stars and planets are governed by four fundamental forces of nature that have field strengths that vary by some 40 orders of magnitude: these are the gravitational, electromagnetic, weak, and nuclear (or strong) forces. The electromagnetic force fields of charged objects extend over all space, as does the gravitational force field associated with the mass of an object; as far as is known, the weak force and nuclear force exist only between nuclear particles. Although physicists have begun to delve deeper into the structure of matter and to develop and support a grand unification theory based on smaller and more energetic constituents (quarks, leptons, bosons, hadrons, etc.), the purposes of radiation physics can be represented...
with simplified models based on electrons, protons, neutrons, and their respective energy states.
The dynamics of the forces of nature yield several important concepts related to atoms:

- Each atom has a nucleus which contains one or more protons (to establish the identity of the atom) and, except for hydrogen-1, one or more neutrons.
- Hydrogen, by definition, has one proton. Helium has two protons, lithium has three, and so on.
- Isotopes are atoms of the same element, but with different weights. Hydrogen ($^1\text{H}$) with a weight of one, deuterium ($^2\text{H}$) with a weight of two, and tritium ($^3\text{H}$) with a weight of three are isotopes of hydrogen. Similarly, $^3\text{He}$ and $^4\text{He}$ are isotopes of helium.
- Atoms left to themselves are electrically neutral. Electrons will form in orbits around a nucleus to balance the positive charge of each proton in the nucleus.
- The nucleus is very small when compared to the electron orbits, which are about $10^{-8}$ cm in size; the nucleus, which is at the center of the atom, is of the order of $10^{-13}$ cm in diameter, so the atom is mostly empty space.
- The nucleus is also very dense because of its size; it contains essentially all the mass of the atom because the protons and neutrons in the nucleus each weigh about 1800 times more than the electrons orbiting about it.
- According to Avogadro, the number of atoms (or molecules) in a mole of a substance is the same, given by Avogadro’s number, $N_A = 6.0221415 \times 10^{23}$ atoms/mol.
- The unified mass unit, u, has a mass of $1.66053886 \times 10^{-24}$ g and an energy equivalence of 931.494 MeV. The unified mass unit, often referred to as the atomic mass unit, has a mass of the order of a neutron or proton (called nucleons because they are constituent parts of the nucleus of atoms); thus, the mass number, $A$, of the isotope of an element is close to, but not exactly, the atomic mass of the individual atoms.
- The electron volt is a reference amount of energy that is used to describe nuclear and atomic events. It is defined as the energy that would be gained by a particle with a unit charge when it is accelerated through a potential difference of 1 V. It is equivalent to $1.60217653 \times 10^{-19}$ J or $1.60217653 \times 10^{-12}$ erg.

Einstein’s special theory of relativity is applicable to atom systems where particles undergo interchangeable mass/energy processes. These processes yield “radiation” which can be characterized as the emission of energy in the form of particles or electromagnetic energy; the amount of energy emitted (or consumed) is due to the change in mass that occurs as atom constituents change potential energy states. The mass values are very exact and the calculated energy change (or Q-value) is also. Binding energy is one important result of such calculations; it
denotes the amount of energy that is given off as constituents come together to form atoms, or is a measure of energy that must be supplied to disengage constituents from an atom.

**Other Suggested Sources**

National Nuclear Data Center, Brookhaven National Laboratory, Upton, Long Island, NY 11973. Data resources are available at: www.nndc.bnl.gov

**Problems – Chapter 1**

1–1. How many neutrons and how many protons are there in (a) 14C, (b) 27Al, (c) 133Xe, and (d) 209Bi?

1–2. If one were to base the atomic mass scale on 16O at 16.000000 atomic mass units (amu), calculate the mass of the 16O atom and the mass of one amu. Why is its mass on the 12C scale different from 16.000000?

1–3. Calculate the number of atoms of 1/2H in 1 g of natural hydrogen.

1–4. Hydrogen is a diatomic molecule, H2. Calculate the mass of one molecule of H2.

1–5. Calculate the radius of the nucleus of 27Al in meters and fermis.

1–6. A 1 g target of natural lithium is to be put into an accelerator for bombardment of 6Li to produce 3H. Use the information in Figure 1-1 to calculate the number of atoms of 6Li in the target.

1–7. A linear accelerator is operated at 700 kV to accelerate protons. What energy will the protons have when they exit the accelerator in (a) eV and (b) joules. If deuterium ions are accelerated what will be the corresponding energy?

1–8. A rowdy student in a food fight threw a fig (weight of 200 g) so hard that it had an acceleration of 10 m/s² when it was released. What “fig newton” force was applied?

1–9. Calculate the velocity required to double the mass of a particle.

1–10. Calculate from the masses in the mass table of Appendix B the total binding energy and the binding energy per nucleon of (a) beryllium-7, (b) iron-56, (c) nickel-62, and (d) uranium-238.