CONTENTS

Preface xxvii
Acknowledgments xxxi
Acronyms and Abbreviations xxxiii

1 Introduction, 1

1.1 Navigation, 1
  1.1.1 Navigation-Related Technologies, 1
  1.1.2 Navigation Modes, 2
1.2 GNSS Overview, 4
  1.2.1 GPS, 4
    1.2.1.1 GPS Orbits, 4
    1.2.1.2 GPS Signals, 4
    1.2.1.3 Selective Availability (SA), 5
    1.2.1.4 Modernization of GPS, 6
  1.2.2 Global Orbiting Navigation Satellite System (GLONASS), 6
    1.2.2.1 GLONASS Orbits, 6
    1.2.2.2 GLONASS Signals, 6
    1.2.2.3 Next Generation GLONASS, 7
  1.2.3 Galileo, 7
    1.2.3.1 Galileo Navigation Services, 7
    1.2.3.2 Galileo Signal Characteristics, 8
    1.2.3.3 Updates, 9
1.2.4 Compass (BeiDou-2), 10
  1.2.4.1 Compass Satellites, 10
  1.2.4.2 Frequency, 10

1.3 Inertial Navigation Overview, 10
  1.3.1 Theoretical Foundations, 10
  1.3.2 Inertial Sensor Technology, 11
    1.3.2.1 Sensor Requirements, 12
    1.3.2.2 Motivation, 13
    1.3.2.3 Inertial Sensors Prior to Newton, 13
    1.3.2.4 Early Momentum Wheel Gyroscopes (MWGs), 14
    1.3.2.5 German Inertial Technology: 1930s–1945, 15
    1.3.2.6 Charles Stark Draper (1901–1987), “The Father of Inertial Navigation”, 19
    1.3.2.7 Aerospace Inertial Technology, 20
    1.3.2.8 Developments Since the Cold War, 30

1.4 GNSS/INS Integration Overview, 30
  1.4.1 The Role of Kalman Filtering, 30
  1.4.2 Implementation, 31
  1.4.3 Applications, 31
    1.4.3.1 Military Applications, 31
    1.4.3.2 Civilian and Commercial Applications, 31

Problems, 32
References, 32

2 Fundamentals of Satellite Navigation Systems, 35

2.1 Navigation Systems Considered, 35
  2.1.1 Systems Other than GNSS, 35
  2.1.2 Comparison Criteria, 36

2.2 Satellite Navigation, 36
  2.2.1 Satellite Orbits, 36
  2.2.2 Navigation Solution (Two-Dimensional Example), 36
    2.2.2.1 Symmetric Solution Using Two Transmitters on Land, 36
    2.2.2.2 Navigation Solution Procedure, 40
  2.2.3 Satellite Selection and Dilution of Precision (DOP), 41
  2.2.4 Example Calculation of DOPS, 45
    2.2.4.1 Four Satellites, 45

2.3 Time and GPS, 46
  2.3.1 Coordinated Universal Time (UTC) Generation, 46
  2.3.2 GPS System Time, 46
  2.3.3 Receiver Computation of UTC, 47
3 Fundamentals of Inertial Navigation, 54

3.1 Chapter Focus, 54
3.2 Basic Terminology, 55
3.3 Inertial Sensor Error Models, 59
    3.3.1 Zero-Mean Random Errors, 60
        3.3.1.1 White Sensor Noise, 60
        3.3.1.2 Exponentially Correlated Noise, 60
        3.3.1.3 Random Walk Sensor Errors, 60
        3.3.1.4 Harmonic Noise, 61
        3.3.1.5 “1/f” Noise, 61
    3.3.2 Fixed-Pattern Errors, 61
    3.3.3 Sensor Pattern Errors, 62
3.4 Sensor Calibration and Compensation, 63
    3.4.1 Sensor Biases, Scale Factors, and Misalignments, 63
        3.4.1.1 Compensation Model Parameters, 63
        3.4.1.2 Calibrating Sensor Biases, Scale Factors, and Misalignments, 64
    3.4.2 Other Calibration Parameters, 65
        3.4.2.1 Nonlinearities, 65
        3.4.2.2 Sensitivities to Other Measurable Conditions, 65
        3.4.2.3 Other Accelerometer Models, 66
    3.4.3 Calibration Parameter Instabilities, 66
        3.4.3.1 Calibration Parameter Changes between Turn-Ons, 67
        3.4.3.2 Calibration Parameter Drift, 67
    3.4.4 Auxiliary Sensors before GNSS, 67
        3.4.4.1 Attitude Sensors, 67
        3.4.4.2 Altitude Sensors, 68
    3.4.5 Sensor Performance Ranges, 68
3.5 Earth Models, 68
    3.5.1 Terrestrial Navigation Coordinates, 69
    3.5.2 Earth Rotation, 70
    3.5.3 Gravity Models, 70
        3.5.3.1 GNSS Gravity Models, 71
        3.5.3.2 INS Gravity Models, 71
        3.5.3.3 Longitude and Latitude Rates, 73
3.6 Hardware Implementations, 77
   3.6.1 Gimbaled Implementations, 78
   3.6.2 Floated Implementation, 80
   3.6.3 Carouseling and Indexing, 81
      3.6.3.1 Alpha Wander and Carouseling, 81
      3.6.3.2 Indexing, 81
   3.6.4 Strapdown Systems, 82
   3.6.5 Strapdown Carouseling and Indexing, 82

3.7 Software Implementations, 83
   3.7.1 Example in One Dimension, 83
   3.7.2 Initialization in Nine Dimensions, 84
      3.7.2.1 Navigation Initialization, 84
      3.7.2.2 INS Alignment Methods, 84
      3.7.2.3 Gyrocompass Alignment, 85
   3.7.3 Gimbal Attitude Implementations, 87
      3.7.3.1 Accelerometer Recalibration, 87
      3.7.3.2 Vehicle Attitude Determination, 87
      3.7.3.3 ISA Attitude Control, 88
   3.7.4 Gimbaled Navigation Implementation, 89
   3.7.5 Strapdown Attitude Implementations, 90
      3.7.5.1 Strapdown Attitude Problems, 90
      3.7.5.2 Coning Motion, 90
      3.7.5.3 Rotation Vector Implementation, 93
      3.7.5.4 Quaternion Implementation, 95
      3.7.5.5 Direction Cosines Implementation, 96
      3.7.5.6 MATLAB® Implementations, 97
   3.7.6 Strapdown Navigation Implementation, 97
   3.7.7 Navigation Computer and Software Requirements, 99
      3.7.7.1 Physical and Operational Requirements, 100
      3.7.7.2 Operating Systems, 100
      3.7.7.3 Interface Requirements, 100
      3.7.7.4 Software Development, 100

3.8 INS Performance Standards, 101
   3.8.1 Free Inertial Operation, 101
   3.8.2 INS Performance Metrics, 101
   3.8.3 Performance Standards, 102

3.9 Testing and Evaluation, 102
   3.9.1 Laboratory Testing, 102
   3.9.2 Field Testing, 103

3.10 Summary, 103

Problems, 104

References, 106
4  GNSS Signal Structure, Characteristics, and Information Utilization, 108

4.1 Legacy GPS Signal Components, Purposes, and Properties, 109
  4.1.1 Mathematical Signal Models for the Legacy GPS Signals, 109
  4.1.2 Navigation Data Format, 112
    4.1.2.1 Z-Count, 114
    4.1.2.2 GPS Week Number (WN), 115
    4.1.2.3 Information by Subframe, 116
  4.1.3 GPS Satellite Position Calculations, 117
    4.1.3.1 Ephemeris Data Reference Time Step and Transit Time Correction, 119
    4.1.3.2 True, Eccentric, and Mean Anomaly, 119
    4.1.3.3 Kepler’s Equation for the Eccentric Anomaly, 120
    4.1.3.4 Satellite Time Corrections, 121
  4.1.4 C/A-Code and Its Properties, 122
    4.1.4.1 Temporal Structure, 124
    4.1.4.2 Autocorrelation Function, 124
    4.1.4.3 Power Spectrum, 125
    4.1.4.4 Despreading of the Signal Spectrum, 126
    4.1.4.5 Role of Despreading in Interference Suppression, 127
    4.1.4.6 Cross-Correlation Function, 128
  4.1.5 P(Y)-Code and Its Properties, 129
    4.1.5.1 P-Code Characteristics, 129
    4.1.5.2 Y-Code, 130
  4.1.6 L1 and L2 Carriers, 130
    4.1.6.1 Dual-Frequency Operation, 130
  4.1.7 Transmitted Power Levels, 131
  4.1.8 Free Space and Other Loss Factors, 131
  4.1.9 Received Signal Power, 132

4.2 Modernization of GPS, 132
  4.2.1 Areas to Benefit from Modernization, 133
  4.2.2 Elements of the Modernized GPS, 134
  4.2.3 L2 Civil Signal (L2C), 135
  4.2.4 L5 Signal, 136
  4.2.5 M-Code, 138
  4.2.6 L1C Signal, 139
  4.2.7 GPS Satellite Blocks, 140
  4.2.8 GPS III, 141

4.3 GLONASS Signal Structure and Characteristics, 141
  4.3.1 Frequency Division Multiple Access (FDMA) Signals, 142
4.3.1.1 Carrier Components, 142
4.3.1.2 Spreading Codes and Modulation, 142
4.3.1.3 Navigation Data Format, 142
4.3.1.4 Satellite Families, 143
4.3.2 CDMA Modernization, 143
4.4 Galileo, 144
4.4.1 Constellation and Levels of Services, 144
4.4.2 Navigation Data and Signals, 144
4.5 Compass/BD, 146
4.6 QZSS, 146
Problems, 148
References, 150

5 GNSS Antenna Design and Analysis, 152
5.1 Applications, 152
5.2 GNSS Antenna Performance Characteristics, 152
5.2.1 Size and Cost, 153
5.2.2 Frequency and Bandwidth Coverage, 153
5.2.3 Radiation Pattern Characteristics, 155
5.2.4 Antenna Polarization and Axial Ratio, 156
5.2.5 Directivity, Efficiency, and Gain of a GNSS Antenna, 159
5.2.6 Antenna Impedance, Standing Wave Ratio, and Return Loss, 160
5.2.7 Antenna Bandwidth, 161
5.2.8 Antenna Noise Figure, 163
5.3 Computational Electromagnetic Models (CEMs) for GNSS Antenna Design, 164
5.4 GNSS Antenna Technologies, 166
5.4.1 Dipole-Based GNSS Antennas, 166
5.4.2 GNSS Patch Antennas, 166
5.4.2.1 Edge-Fed, LP, Single-Frequency GNSS Patch Antenna, 168
5.4.2.2 Probe-Fed, LP, Single-Frequency GNSS Patch Antenna, 170
5.4.2.3 Dual Probe-Fed, RHCP, Single-Frequency GNSS Patch Antenna, 171
5.4.2.4 Single Probe-Fed, RCHP, Single-Frequency GNSS Patch Antenna, 172
5.4.2.5 Dual Probe-Fed, RHCP, Multifrequency GNSS Patch Antenna, 175
5.4.3 Survey-Grade/Reference GNSS Antennas, 176
5.4.3.1 Choke Ring-Based GNSS Antennas, 176
5.4.3.2 Advanced Planner-Based GNSS Antennas, 177
5.5 Principles of Adaptable Phased-Array Antennas, 180
   5.5.1 Digital Beamforming Adaptive Antenna Array Formulations, 182
   5.5.2 STAP, 185
   5.5.3 SFAP, 185
   5.5.4 Configurations of Adaptable Phased-Array Antennas, 185
   5.5.5 Relative Merits of Adaptable Phased-Array Antennas, 186

5.6 Application Calibration/Compensation Considerations, 187

Problems, 189

References, 190

6 GNSS Receiver Design and Analysis, 193

6.1 Receiver Design Choices, 193
   6.1.1 Global Navigation Satellite System (GNSS) Application to be Supported, 193
   6.1.2 Single or Multifrequency Support, 194
      6.1.2.1 Dual-Frequency Ionosphere Correction, 194
      6.1.2.2 Improved Carrier Phase Ambiguity Resolution in High-Accuracy Differential Positioning, 194
   6.1.3 Number of Channels, 195
   6.1.4 Code Selections, 195
   6.1.5 Differential Capability, 196
      6.1.5.1 Corrections Formats, 197
   6.1.6 Aiding Inputs, 198

6.2 Receiver Architecture, 199
   6.2.1 Radio Frequency (RF) Front End, 199
   6.2.2 Frequency Down-Conversion and IF Amplification, 201
      6.2.2.1 SNR, 202
   6.2.3 Analog-to-Digital Conversion and Automatic Gain Control, 203
   6.2.4 Baseband Signal Processing, 204

6.3 Signal Acquisition and Tracking, 204
   6.3.1 Hypothesize about the User Location, 205
   6.3.2 Hypothesize about Which GNSS Satellites Are Visible, 205
   6.3.3 Signal Doppler Estimation, 206
   6.3.4 Search for Signal in Frequency and Code Phase, 206
      6.3.4.1 Sequential Searching in Code Delay, 208
      6.3.4.2 Sequential Searching in Frequency, 209
      6.3.4.3 Frequency Search Strategy, 209
      6.3.4.4 Parallel and Hybrid Search Methods, 210
6.3.5 Signal Detection and Confirmation, 210
   6.3.5.1 Detection Confirmation, 211
   6.3.5.2 Coordination of Frequency Tuning and Code Chipping Rate, 213
6.3.6 Code Tracking Loop, 213
   6.3.6.1 Code Loop Bandwidth Considerations, 217
   6.3.6.2 Coherent versus Noncoherent Code Tracking, 217
6.3.7 Carrier Phase Tracking Loops, 218
   6.3.7.1 PLL Capture Range, 221
   6.3.7.2 PLL Order, 221
   6.3.7.3 Use of Frequency-Lock Loops (FLLs) for Carrier Capture, 221
6.3.8 Bit Synchronization, 222
6.3.9 Data Bit Demodulation, 222
6.4 Extraction of Information for User Solution, 223
   6.4.1 Signal Transmission Time Information, 223
   6.4.2 Ephemeris Data for Satellite Position and Velocity, 224
   6.4.3 Pseudorange Measurements Formulation Using Code Phase, 224
   6.4.3.1 Pseudorange Positioning Equations, 226
   6.4.4 Measurements Using Carrier Phase, 226
   6.4.5 Carrier Doppler Measurement, 228
   6.4.6 Integrated Doppler Measurements, 229
6.5 Theoretical Considerations in Pseudorange, Carrier Phase, and Frequency Estimations, 231
   6.5.1 Theoretical Error Bounds for Code Phase Measurement, 232
   6.5.2 Theoretical Error Bounds for Carrier Phase Measurements, 233
   6.5.3 Theoretical Error Bounds for Frequency Measurement, 234
6.6 High-Sensitivity A-GPS Systems, 235
   6.6.1 How Assisting Data Improves Receiver Performance, 236
   6.6.1.1 Reduction of Frequency Uncertainty, 236
   6.6.1.2 Determination of Accurate Time, 237
   6.6.1.3 Transmission of Satellite Ephemeris Data, 238
   6.6.1.4 Provision of Approximate Client Location, 238
   6.6.1.5 Transmission of the Demodulated Navigation Bit Stream, 239
   6.6.1.6 Server-Provided Location, 240
6.6.2 Factors Affecting High-Sensitivity Receivers, 240
6.6.2.1 Antenna and Low-Noise RF Design, 240
6.6.2.2 Degradation due to Signal Phase Variations, 240
6.6.2.3 Signal Processing Losses, 241
6.6.2.4 Multipath Fading, 241
6.6.2.5 Susceptibility to Interference and Strong Signals, 241
6.6.2.6 The Problem of Time Synchronization, 242
6.6.2.7 Difficulties in Reliable Sensitivity Assessment, 242

6.7 Software-Defined Radio (SDR) Approach, 242
6.8 Pseudolite Considerations, 243

References, 246

7 GNSS Data Errors, 250

7.1 Data Errors, 250
7.2 Ionospheric Propagation Errors, 251
7.2.1 Ionospheric Delay Model, 252
7.2.2 GNSS SBAS Ionospheric Algorithms, 254
7.2.2.1 L1L2 Receiver and Satellite Bias and Ionospheric Delay Estimations for GPS, 256
7.2.2.2 Kalman Filter, 259
7.2.2.3 Selection of Q and R, 261
7.2.2.4 Calculation of Ionospheric Delay Using Pseudoranges, 262

7.3 Tropospheric Propagation Errors, 263
7.4 The Multipath Problem, 264
7.4.1 How Multipath Causes Ranging Errors, 264

7.5 Methods of Multipath Mitigation, 266
7.5.1 Spatial Processing Techniques, 267
7.5.1.1 Antenna Location Strategy, 267
7.5.1.2 Ground Plane Antennas, 267
7.5.1.3 Directive Antenna Arrays, 267
7.5.1.4 Long-Term Signal Observation, 267

7.5.2 Time-Domain Processing, 269
7.5.2.1 Narrow-Correlator Technology (1990–1993), 269
7.5.2.2 Leading-Edge Techniques, 270
7.5.2.3 Correlation Function Shape-Based Methods, 271
7.5.2.4 Modified Correlator Reference Waveforms, 271
7.5.3 Multipath Mitigation Technology (MMT) Technology, 272
7.5.3.1 Description, 272
7.5.3.2 Maximum-Likelihood (ML) Multipath Estimation, 272
7.5.3.3 The Two-Path ML Estimator (MLE), 273
7.5.3.4 Asymptotic Properties of ML Estimators, 274
7.5.3.5 The MMT Multipath Mitigation Algorithm, 274
7.5.3.6 The MMT Baseband Signal Model, 274
7.5.3.7 Baseband Signal Vectors, 275
7.5.3.8 The Log-Likelihood Function, 275
7.5.3.9 Secondary-Path Amplitude Constraint, 277
7.5.3.10 Signal Compression, 277
7.5.3.11 Properties of the Compressed Signal, 279
7.5.3.12 The Compression Theorem, 280
7.5.4 Performance of Time-Domain Methods, 281
7.5.4.1 Ranging with the C/A-Code, 281
7.5.4.2 Carrier Phase Ranging, 282
7.5.4.3 Testing Receiver Multipath Performance, 283
7.6 Theoretical Limits for Multipath Mitigation, 283
7.6.1 Estimation-Theoretic Methods, 283
7.6.1.1 Optimality Criteria, 284
7.6.2 Minimum Mean-Squared Error (MMSE) Estimator, 284
7.6.3 Multipath Modeling Errors, 284
7.7 Ephemeris Data Errors, 285
7.8 Onboard Clock Errors, 285
7.9 Receiver Clock Errors, 286
7.10 SA Errors, 288
7.11 Error Budgets, 288

8 Differential GNSS, 293
8.1 Introduction, 293
8.2 Descriptions of Local-Area Differential GNSS (LADGNSS), Wide-Area Differential GNSS (WADGNSS), and Space-Based Augmentation System (SBAS), 294
8.2.1 LADGNSS, 294
8.2.2 WADGNSS, 294
8.2.3 SBAS, 294
8.2.3.1 Wide-Area Augmentation System (WAAS), 294
8.2.3.2 European Global Navigation Overlay System (EGNOS), 298
8.2.3.3 Other SBAS, 299
8.3 GEO with L1L5 Signals, 299
  8.3.1 GEO Uplink Subsystem Type 1 (GUST) Control Loop Overview, 302
    8.3.1.1 Ionospheric Kalman Filters, 303
    8.3.1.2 Range Kalman Filter, 303
    8.3.1.3 Code Control Function, 304
    8.3.1.4 Frequency Control Function, 304
    8.3.1.5 L1L5 Bias Estimation Function, 305
    8.3.1.6 L1L5 Bias Estimation Function, 305
    8.3.1.7 Carrier Frequency Stability, 306
  8.4 GUS Clock Steering Algorithm, 307
    8.4.1 Receiver Clock Error Determination, 308
    8.4.2 Clock Steering Control Law, 310
  8.5 GEO Orbit Determination (OD), 310
    8.5.1 OD Covariance Analysis, 312
  8.6 Ground-Based Augmentation System (GBAS), 316
    8.6.1 Local-Area Augmentation System (LAAS), 316
    8.6.2 Joint Precision Approach and Landing System (JPALS), 317
    8.6.3 Enhanced Long-Range Navigation (eLoran), 318
  8.7 Measurement/Relative-Based DGNSS, 319
    8.7.1 Code Differential Measurements, 319
    8.7.1.1 Single-Difference Observations, 320
    8.7.1.2 Double-Difference Observations, 320
    8.7.2 Carrier Phase Differential Measurements, 321
    8.7.2.1 Single-Difference Observations, 321
    8.7.2.2 Double-Difference Observations, 321
    8.7.2.3 Triple-Difference Observations, 322
    8.7.2.4 Combinations of L1 and L2 Carrier Phase Observations, 322
    8.7.3 Positioning Using Double-Difference Measurements, 322
    8.7.3.1 Code-Based Positioning, 322
    8.7.3.2 Carrier Phase-Based Positioning, 322
    8.7.3.3 Real-Time Processing versus Postprocessing, 323
  8.8 GNSS Precise Point Positioning Services and Products, 323
    8.8.1 The International GNSS Service (IGS), 323
    8.8.2 Continuously Operating Reference Stations (CORSs), 324
    8.8.3 GPS Inferred Positioning System (GIPSY) and Orbit Analysis Simulation Software (OASIS), 324
    8.8.4 Australia’s Online GPS Processing System (AUPOS), 325
    8.8.5 Scripps Coordinate Update Tool (SCOUT), 325
    8.8.6 The Online Positioning User Service (OPUS), 325
9 **GNSS and GEO Signal Integrity, 328**

9.1 **Introduction, 328**
- 9.1.1 Range Comparison Method, 329
- 9.1.2 Least-Squares Method, 330
- 9.1.3 Parity Method, 331

9.2 **SBAS and GBAS Integrity Design, 332**
- 9.2.1 SBAS Error Sources and Integrity Threats, 333
- 9.2.2 GNSS-Associated Errors, 334
  - 9.2.2.1 GNSS Clock Error, 334
  - 9.2.2.2 GNSS Ephemeris Error, 335
  - 9.2.2.3 GNSS Code and Carrier Incoherence, 335
  - 9.2.2.4 GNSS Signal Distortion, 335
  - 9.2.2.5 GNSS L1L2 Bias, 336
  - 9.2.2.6 Environment Errors: Ionosphere, 336
  - 9.2.2.7 Environment Errors: Troposphere, 336
- 9.2.3 GEO-Associated Errors, 336
  - 9.2.3.1 GEO Code and Carrier Incoherence, 336
  - 9.2.3.2 GEO-Associated Environment Errors: Ionosphere, 337
  - 9.2.3.3 GEO-Associated Environment Errors: Troposphere, 337
- 9.2.4 Receiver and Measurement Processing Errors, 337
  - 9.2.4.1 Receiver Measurement Error, 337
  - 9.2.4.2 Intercard Bias, 337
  - 9.2.4.3 Multipath, 338
  - 9.2.4.4 L1L2 Bias, 338
  - 9.2.4.5 Receiver Clock Error, 338
  - 9.2.4.6 Measurement Processing Unpack/Pack Corruption, 338
- 9.2.5 Estimation Errors, 338
  - 9.2.5.1 Reference Time Offset Estimation Error, 338
  - 9.2.5.2 Clock Estimation Error, 339
  - 9.2.5.3 Ephemeris Correction Error, 339
  - 9.2.5.4 L1L2 Wide-Area Reference Equipment (WRE) and GPS Satellite Bias Estimation Error, 339
- 9.2.6 Integrity-Bound Associated Errors, 339
  - 9.2.6.1 Ionospheric Modeling Errors, 339
  - 9.2.6.2 Fringe Area Ephemeris Error, 340
  - 9.2.6.3 Small-Sigma Errors, 340
9.2.6.4 Missed Message: Old But Active Data (OBAD), 340
9.2.6.5 Time to Alarm (TTA) Exceeded, 340
9.2.7 GEO Uplink Errors, 340
9.2.7.1 GEO Uplink System Fails to Receive SBAS Message, 340
9.2.8 Mitigation of Integrity Threats, 340
9.2.8.1 Mitigation of GNSS Associated Errors, 341
9.2.8.2 Mitigation of GEO-Associated Errors, 343
9.2.8.3 Mitigation of Receiver and Measurement Processing Errors, 343
9.2.8.4 Mitigation of Estimation Errors, 344
9.2.8.5 Mitigation of Integrity-Bound-Associated Errors, 345

9.3 SBAS Example, 346
9.4 Summary, 347
9.5 Future: GIC, 348
Problem, 348
References, 348

10 Kalman Filtering, 350

10.1 Introduction, 350
10.1.1 What Is a Kalman Filter?, 351
10.1.2 How Does It Work?, 352
10.1.2.1 Prediction and Correction, 353
10.1.3 How Is It Used?, 353
10.2 Kalman Filter Correction Update, 354
10.2.1 Deriving the Kalman Gain, 354
10.2.1.1 Approaches to Deriving the Kalman Gain, 355
10.2.1.2 Gaussian Probability Density Functions, 355
10.2.1.3 Properties of Likelihood Functions, 356
10.2.1.4 Solving for Combined Information Matrix, 358
10.2.1.5 Solving for Combined Argmax, 359
10.2.1.6 Noisy Measurement Likelihoods, 360
10.2.1.7 Gaussian Maximum-Likelihood Estimate (MLE), 362
10.2.1.8 Estimate Correction, 364
10.2.1.9 Kalman Gain Matrix for MLE, 364
10.2.2 Estimate Correction Using the Kalman Gain, 364
10.2.3 Covariance Correction for Using Measurements, 365
10.3 Kalman Filter Prediction Update, 365
10.3.1 Stochastic Systems in Continuous Time, 365
10.3.1.1 White-Noise Processes, 365
10.3.1.2 Stochastic Differential Equations, 365
10.3.1.3 Systems of First-Order Linear Differential Equations, 367
10.3.1.4 Representation in Terms of Vectors and Matrices, 368
10.3.1.5 Eigenvalues of Dynamic Coefficient Matrices, 369
10.3.1.6 Matrix Exponential Function, 371
10.3.1.7 Forward Solution, 371
10.3.1.8 Time-Invariant Systems, 371
10.3.2 Stochastic Systems in Discrete Time, 372
  10.3.2.1 Zero-Mean White Gaussian Noise Sequences, 372
  10.3.2.2 Gaussian Linear Stochastic Processes in Discrete Time, 372
10.3.3 State Space Models for Discrete Time, 373
10.3.4 Dynamic Disturbance Noise Distribution Matrices, 374
10.3.5 Predictor Equations, 374
10.4 Summary of Kalman Filter Equations, 375
  10.4.1 Essential Equations, 375
  10.4.2 Common Terminology, 375
  10.4.3 Data Flow Diagrams, 376
10.5 Accommodating Time-Correlated Noise, 377
  10.5.1 Correlated Noise Models, 378
    10.5.1.1 Autocovariance Functions, 378
    10.5.1.2 Random Walks, 378
    10.5.1.3 Exponentially Correlated Noise, 379
    10.5.1.4 Harmonic Noise, 379
    10.5.1.5 Selective Availability (SA), 379
    10.5.1.6 Slow Variables, 380
  10.5.2 Empirical Modeling of Sensor Noise, 380
    10.5.2.1 Spectral Characterization, 381
    10.5.2.2 Shaping Filters, 381
  10.5.3 State Vector Augmentation, 382
    10.5.3.1 Correlated Dynamic Disturbance Noise, 382
    10.5.3.2 Correlated Sensor Noise, 383
    10.5.3.3 Correlated Noise in Continuous Time, 383
10.6 Nonlinear and Adaptive Implementations, 384
  10.6.1 Assessing Linear Approximation Errors, 384
    10.6.1.1 Statistical Measures of Acceptability, 384
    10.6.1.2 Sampling for Acceptability Testing, 385
  10.6.2 Nonlinear Dynamics, 390
    10.6.2.1 Nonlinear Dynamics with Control, 390
    10.6.2.2 Propagating Estimates, 390
    10.6.2.3 Propagating Covariances, 390
10.6.3 Nonlinear Sensors, 391
  10.6.3.1 Predicted Sensor Outputs, 391
  10.6.3.2 Calculating Kalman Gains, 391
10.6.4 Linearized Kalman Filter, 391
10.6.5 Extended Kalman Filtering (EFK), 392
10.6.6 Adaptive Kalman Filtering, 393

10.7 Kalman–Bucy Filter, 395
  10.7.1 Implementation Equations, 395
  10.7.2 Kalman–Bucy Filter Parameters, 396

10.8 Host Vehicle Tracking Filters for GNSS, 397
  10.8.1 Vehicle Tracking Filters, 397
  10.8.2 Dynamic Dilution of Information, 397
  10.8.2.1 Effect on Position Uncertainty, 398
  10.8.3 Specialized Host Vehicle Tracking Filters, 399
    10.8.3.1 Unknown Constant Tracking Model, 401
    10.8.3.2 Damped Harmonic Resonator, 401
    10.8.3.3 Type 2 Tracking Model, 402
    10.8.3.4 DAMP1 Tracking Model: Velocity Damping, 403
    10.8.3.5 DAMP2 Tracking Model: Velocity and Acceleration Damping, 403
    10.8.3.6 DAMP3 Tracking Model: Position, Velocity, and Acceleration Damping, 405
    10.8.3.7 Tracking Models for Highly Constrained Trajectories, 408
    10.8.3.8 Filters for Spacecraft, 409
    10.8.3.9 Other Specialized Vehicle Filter Models, 409
    10.8.3.10 Filters for Different Host Vehicle Types, 409
    10.8.3.11 Parameters for Vehicle Dynamics, 409
    10.8.3.12 Empirical Modeling of Vehicle Dynamics, 409
  10.8.4 Vehicle Tracking Filter Comparison, 411
    10.8.4.1 Simulated Trajectory, 411
    10.8.4.2 Results, 412
    10.8.4.3 Model Dimension versus Model Constraints, 412
    10.8.4.4 Role of Model Fidelity, 413

10.9 Alternative Implementations, 413
  10.9.1 Schmidt–Kalman Suboptimal Filtering, 413
    10.9.1.1 State Vector Partitioning, 414
    10.9.1.2 Implementation Equations, 414
    10.9.1.3 Simulated Performance in GNSS Position Estimation, 415
  10.9.2 Serial Measurement Processing, 416
    10.9.2.1 Measurement Decorrelation, 416
    10.9.2.2 Serial Processing of Decorrelated Measurements, 417
10.9.3 Improving Numerical Stability, 417
  10.9.3.1 Effects of Finite Precision, 417
  10.9.3.2 Alternative Implementations, 418
  10.9.3.3 Conditioning and Scaling Considerations, 419
10.9.4 Kalman Filter Monitoring, 421
  10.9.4.1 Rejecting Anomalous Sensor Data, 421
  10.9.4.2 Monitoring Filter Health, 423
10.10 Summary, 425
Problems, 426
References, 428

11 Inertial Navigation Error Analysis, 430
11.1 Chapter Focus, 430
11.2 Errors in the Navigation Solution, 432
  11.2.1 The Nine Core INS Error Variables, 432
  11.2.2 Coordinates Used for INS Error Analysis, 432
  11.2.3 Model Variables and Parameters, 432
    11.2.3.1 INS Orientation Variables and Errors, 433
  11.2.4 Dynamic Coupling Mechanisms, 439
    11.2.4.1 Dynamic Coupling, 439
11.3 Navigation Error Dynamics, 442
  11.3.1 Error Dynamics due to Velocity Integration, 442
  11.3.2 Error Dynamics due to Gravity Calculations, 443
    11.3.2.1 INS Gravity Modeling, 443
    11.3.2.2 Navigation Error Model for Gravity Calculations, 444
  11.3.3 Error Dynamics due to Coriolis Acceleration, 445
  11.3.4 Error Dynamics due to Centrifugal Acceleration, 446
  11.3.5 Error Dynamics due to Earthrate Leveling, 447
  11.3.6 Error Dynamics due to Velocity Leveling, 448
  11.3.7 Error Dynamics due to Acceleration and Misalignments, 449
  11.3.8 Composite Model from All Effects, 450
  11.3.9 Vertical Navigation Instability, 452
    11.3.9.1 Altimeter Aiding, 454
  11.3.10 Schuler Oscillations, 457
  11.3.11 Core Model Validation and Tuning, 459
    11.3.11.1 Horizontal Inertial Navigation Model, 459
11.4 Inertial Sensor Noise, 459
  11.4.1 CEP Rate versus Sensor Noise, 461
11.5 Sensor Compensation Errors, 461
  11.5.1 Sensor Compensation Error Models, 462
    11.5.1.1 Exponentially Correlated Parameter Drift Model, 463
12 GNSS/INS Integration, 472

12.1 Chapter Focus, 472
  12.1.1 Objective, 472
  12.1.2 Order of Presentation, 473

12.2 GNSS/INS Integration Overview, 473
  12.2.1 Historical Background, 473
  12.2.2 The Loose/Tight Ranking, 475
    12.2.2.1 Loosely Coupled Implementations, 476
    12.2.2.2 More Tightly Coupled Implementations, 476
    12.2.2.3 Ultratightly Coupled Integration, 477
    12.2.2.4 Limitations, 477
  12.2.3 Unified Navigation Model, 477

12.3 Unified Model for GNSS/INS Integration, 479
  12.3.1 GNSS Error Models, 479
    12.3.1.1 Receiver Clock Error Model, 479
    12.3.1.2 Atmospheric Propagation Delay Model, 480
    12.3.1.3 Pseudorange Measurement Noise, 481
  12.3.2 INS Error Models, 481
    12.3.2.1 Navigation Error Model, 481
    12.3.2.2 Sensor Compensation Errors, 481
  12.3.3 GNSS/INS Error Model, 482
    12.3.3.1 State Variables, 482
    12.3.3.2 Numbers of State Variables, 482
    12.3.3.3 Dynamic Coefficient Matrix, 483
    12.3.3.4 Process Noise Covariance, 484
    12.3.3.5 Measurement Sensitivities, 484

12.4 Performance Analysis, 485
  12.4.1 Dynamic Simulation Model, 485
    12.4.1.1 State Transition Matrices (STMs), 485
    12.4.1.2 Dynamic Simulation, 486
  12.4.2 Results, 486
    12.4.2.1 Stand-Alone GNSS Performance, 486
    12.4.2.2 Stand-Alone INS Performance, 488
    12.4.2.3 Integrated GNSS/INS Performance, 488

12.5 Other Integration Issues, 490
  12.5.1 Antenna/ISA Offset Correction, 490
  12.5.2 Influence of Trajectories on Performance, 491
12.6 Summary, 492
Problem, 493
References, 494

Appendix A  Software, 495
A.1 Software Sources, 495
A.2 Software for Chapter 3, 496
A.3 Software for Chapter 4, 496
A.4 Software for Chapter 7, 496
A.5 Software for Chapter 10, 497
A.6 Software for Chapter 11, 498
A.7 Software for Chapter 12, 498
A.8 Almanac/Ephemeris Data Sources, 499

Appendix B  Coordinate Systems and Transformations, 500
B.1 Coordinate Transformation Matrices, 500
B.1.1 Notation, 500
B.1.2 Definitions, 501
B.1.3 Unit Coordinate Vectors, 501
B.1.4 Direction Cosines, 502
B.1.5 Composition of Coordinate Transformations, 503
B.2 Inertial Reference Directions, 503
B.3 Application-Dependent Coordinate Systems, 504
B.3.1 Cartesian and Polar Coordinates, 504
B.3.2 Celestial Coordinates, 505
B.3.3 Satellite Orbit Coordinates, 505
B.3.4 ECI Coordinates, 507
B.3.5 Earth-Centered, Earth-Fixed (ECEF) Coordinates, 508
B.3.5.1 Longitudes in ECEF Coordinates, 508
B.3.5.2 Latitudes in ECEF Coordinates, 508
B.3.5.3 Latitude on an Ellipsoidal Earth, 509
B.3.5.4 Parametric Latitude, 509
B.3.5.5 Geodetic Latitude, 510
B.3.5.6 WGS84 Reference Geoid Parameters, 513
B.3.5.7 Geocentric Latitude, 513
B.3.5.8 Geocentric Radius, 514
B.3.6 Ellipsoidal Radius of Curvature, 515
B.3.7 Local Tangent Plane (LTP) Coordinates, 515
B.3.7.1 Alpha Wander Coordinates, 516
B.3.7.2 ENU/NED Coordinates, 516
B.3.7.3 ENU/ECEF Coordinates, 516
B.3.7.4 NED/ECEF Coordinates, 517
B.3.8 RPY Coordinates, 518
B.3.9 Vehicle Attitude Euler Angles, 518
  B.3.9.1 RPY/ENU Coordinates, 519
B.3.10 GNSS Navigation Coordinates, 521
B.4 Coordinate Transformation Models, 523
  B.4.1 Euler Angles, 523
  B.4.2 Rotation Vectors, 524
    B.4.2.1 Rotation Vector to Matrix, 525
    B.4.2.2 Matrix to Rotation Vector, 527
    B.4.2.3 Special Cases for $\sin(\theta) \approx 0$, 528
    B.4.2.4 Time Derivatives of Rotation Vectors, 529
    B.4.2.5 Time Derivatives of Matrix Expressions, 534
    B.4.2.6 Partial Derivatives with Respect to Rotation Vectors, 537
  B.4.3 Direction Cosine Matrix, 539
    B.4.3.1 Rotating Coordinates, 540
  B.4.4 Quaternions, 543
    B.4.4.1 Quaternion Matrices, 543
    B.4.4.2 Addition and Multiplication, 544
    B.4.4.3 Conjugation, 545
    B.4.4.4 Representing Rotations, 545
B.5 Newtonian Mechanics in Rotating Coordinates, 548
  B.5.1 Rotating Coordinates, 548
  B.5.2 Time Derivatives of Matrix Products, 549
  B.5.3 Solving for Centrifugal and Coriolis Accelerations, 549

Index 551