1

Physics of Radiation and Matter

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1.1 Introduction

Medical imaging at its core relies on foundational physics of electromagnetic radiation, atomic structure of stable and radioactive matter, and the interaction of radiation with matter. Associated with these topics are the process of X-ray formation and detection. Most medical imaging technologies exploit the principles associated with these foundational physics to form the medical image. These principles also impose fundamental limitations of what is possible; nature never meets our expectations for the ideal. This necessitates innovation to find technological solutions to mitigate fundamental challenges. Most advances in imaging technology have in fact been founded on innovations that best address these fundamental limitations. Therefore, insights into medical imaging and its progress necessitate an understanding of the physics of radiation and matter. Thus, it is most appropriate that we first address the foundational physics of radiation and matter in this chapter followed in Chapter 2 by a foundation of human anatomy and physiology.

1.2 Electromagnetic Radiation

Electromagnetic radiation consists of oscillating electric and magnetic fields. An electromagnetic wave requires no medium for propagation; that is, it can travel in a vacuum as well as through matter. In the simplified diagram in Figure 1.1, the wavelength of an electromagnetic wave is depicted as the distance between the adjacent crests of the oscillating fields. The wave is moving from left to right in the diagram.

The constant speed \( c \) of electromagnetic radiation in a vacuum is the product of the frequency \( \nu \) and the wavelength \( \lambda \) of the electromagnetic wave,

\[
c = \nu \lambda. \tag{1.1}
\]

Often it is convenient to assign wavelike properties to electromagnetic energy. At other times, it is useful to regard these radiations as discrete bundles of energy-termed photons or quanta. The two interpretations of electromagnetic radiation are united by the equation

\[
E = h \nu, \tag{1.2}
\]

where \( E \) represents the energy of a photon and \( \nu \) represents the frequency of the electromagnetic wave. The symbol \( h \) represents Planck’s constant, \( 6.62 \times 10^{-34} \) Js.

From Eq. (1.1),

\[
\nu = \frac{c}{\lambda}, \tag{1.3}
\]

and the photon energy may therefore be written as

\[
E = \frac{hc}{\lambda}. \tag{1.4}
\]

Using this equation, the energy in units of kiloelectron volts (keV) of a photon of wavelength \( \lambda \) in nanometers (nm) may be computed by

\[
E = \frac{1.24}{\lambda}. \tag{1.5}
\]

Electromagnetic waves ranging in energy from a few nanoelectron volts up to the order of a gigaelectron volt make up the electromagnetic spectrum in Table 1.1 (definition of electron volt can be found in Section 1.2.2.3).

1.2.1 The Atom

All matter is composed of atoms. A sample of a pure element is composed of a single type of atom, and chemical compounds are composed of more than one type of atom. Atoms themselves are complicated entities with detailed internal structure, but they are the smallest unit of matter that retains the chemical properties of a material. In that sense, an atom is a fundamental building block of matter. In the case of a compound, the fundamental building block is a molecule consisting of one or more
Atoms bound together by electrostatic attraction and/or the sharing of electrons by more than one nucleus.

### 1.2.2 Structure of the Atom

In its simplest depiction, the Bohr model of the atom is a positively charged nucleus, containing electrically neutral neutrons and positively charged protons, surrounded by one or more negatively charged electrons. The number and distribution of electrons in the atom determine the chemical properties of the atom, and the number and configuration of neutrons and protons (collectively, nucleons) in the nucleus determine the stability of the atom in its nuclear configuration.

The positive charge and roughly half the mass of the nucleus are contributed by protons. Each proton possesses a positive charge of \( +e \) and a mass of \( 1.6734 \times 10^{-27} \) kg. Neutrons have a mass of \( 1.6747 \times 10^{-27} \) kg. Clearly, expressing the mass of atomic particles in kilograms is unwieldy, so the atomic mass unit (amu) is a more convenient unit for atomic particles. An amu is defined as 1/12 the mass of the carbon atom, which has six protons, six neutrons, and six electrons. Therefore,

- 1 amu = \( 1.6605 \times 10^{-27} \) kg,
- Electron = 0.00055 amu,
- Proton = 1.00727 amu, and
- Neutron = 1.00866 amu.

#### 1.2.2.1 Atomic Nomenclature

The standard form used to denote the composition of a specific atom or nucleus is \( {}^{A}_{Z}X \), where \( X \) is the chemical symbol (e.g. H, He, and Li) and \( A \) and \( Z \) are as defined above. There is some redundancy in this symbolism. The atomic number, \( Z \), is uniquely associated with the chemical symbol, \( X \). For example, when \( Z = 6 \), the chemical symbol is always C, i.e. the element carbon. As such, the \( Z \) subscript is often omitted.

Isotopes of a particular element are atoms that possess the same number of protons but a varying number of neutrons. For example, \(^1\text{H}(\text{protium}), \(^2\text{H}(\text{deuterium}), \) and \(^3\text{H}(\text{tritium})\) are isotopes of the element hydrogen and \(^6\text{C}, \(^{10}\text{C}, \(^{11}\text{C}, \(^{12}\text{C}, \(^{13}\text{C}, \(^{14}\text{C}, \(^{15}\text{C}, \) and \(^{16}\text{C}\) are isotopes of carbon. Isotones are atoms that possess the same number of neutrons but a different number of protons. For example, \(^{2}\text{He}, \(^{3}\text{Li}, \(^{4}\text{Be}, \(^{8}\text{B}, \) and \(^{9}\text{C}\) are isotones, each contains three neutrons. Isobars are atoms with the same number of nucleons but a different number of protons and a different number of neutrons. For example, \(^{2}\text{He}, \(^{4}\text{Li}, \) and \(^{6}\text{Be}\) are isobars, with six nucleons each. Finally, isomers represent different energy states for nuclei with the same number of neutrons and protons. Differences between isotopes, isotones, isobars, and isomers are illustrated in Table 1.2.

#### 1.2.2.2 Electrons

Atoms in their normal state are neutral because the number of electrons outside the nucleus (i.e. the negative charge in the atom) equals the number of protons (i.e. the positive charge in the atom) of the nucleus. Electrons are positioned in energy levels, termed shells that surround the nucleus. The first or K-shell contains no more than two electrons, the second or L shell contains no more than eight electrons, and the third or M shell contains...
1.2 Electromagnetic Radiation

Table 1.2 Nucleus variability based on the numbers of protons and neutrons

<table>
<thead>
<tr>
<th>Atomic number Z</th>
<th>Neutron number N</th>
<th>Mass number A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isotopes</td>
<td>Same</td>
<td>Different</td>
</tr>
<tr>
<td>Isotones</td>
<td>Different</td>
<td>Same</td>
</tr>
<tr>
<td>Isobars</td>
<td>Different</td>
<td>Same</td>
</tr>
<tr>
<td>Isomers</td>
<td>Same</td>
<td>Same (different nuclear energy states)</td>
</tr>
</tbody>
</table>

no more than 18 electrons (see Figure 1.2). The outermost electron shell of an atom, no matter which shell it is, never contains more than eight electrons. Electrons in this shell are called valence electrons and determine, to a large degree, the chemical properties of the atom. For example, atoms with an outer shell entirely filled with electrons seldom react chemically; these atoms constitute inert elements known as the noble gases (i.e. helium, neon, argon, krypton, xenon, and radon).

The energy levels for electrons are divided into sub-levels separated from each other. To describe the position of an electron in the extranuclear structure of an atom, the electron is assigned four quantum numbers designated below as \( n \), \( l \), \( m_l \), and \( m_s \).

The principal quantum number, \( n \), defines the main energy level or shell within which the electron resides (\( n = 1 \) for the K-shell, \( n = 2 \) for the L shell, etc.). The orbital angular momentum (also called azimuthal) quantum number, \( l \), describes the electron’s angular momentum (\( l = 0, 1, 2, \ldots n - 1 \)). The orientation of the electron’s magnetic moment in a magnetic field is determined by the magnetic quantum number, \( m_l \) (\( m_l = -l, -l+1, \ldots -1, 1 \)). The direction of the electron’s spin upon its own axis is specified by the spin quantum number, \( m_s \) (\( m_s = \pm \frac{1}{2} \)). The Pauli exclusion principle states that no two electrons in the same atomic system may be assigned identical values for all four quantum numbers. Listed in Table 1.3 are quantum numbers for electrons in a few atoms with low atomic numbers.

The values of the orbital angular momentum quantum number, \( l = 0, 1, 2, 3, 4, 5, \) and \( 6 \), are also identified with the symbols, \( s, p, d, f, g, h, \) and \( i \), respectively. This notation is known as “spectroscopic” because it is used to describe the separate emission lines observed when light emitted from a heated metallic vapor lamp is passed through a prism. From 1890s onward, observation of these spectra provided major clues about the binding energies of electrons in atoms of the metals under study. By the 1920s, it was known that the spectral lines above “s” (i.e. \( l = 0 \)) could be split into multiple lines in the presence of a magnetic field. The lines were thought to correspond to “orbitals” or groupings of similar electrons within orbits. The modern view of this phenomenon is that while the “s” orbital is spherically symmetric (see Figure 1.3), the other orbitals are not. In the presence of a magnetic field, the “p” orbital can be in alignment along any one of the three axes of space: \( x, y, \) or \( z \). Each of these three orientations has a slightly different energy corresponding to the three possible values of \( m_l \) (\( -1, 0, \) and \( 1 \)). According to the Pauli exclusion principle, each orbital may contain two electrons, one with \( m_s = +\frac{1}{2} \) and the other with \( m_s = -\frac{1}{2} \).

The K-shell of an atom consists of one orbital, 1s, containing two electrons. The L shell consists of the 2s sub-shell that contains one orbital (two electrons) and the 2p subshell containing a maximum of three orbitals (i.e. six electrons). If the L shell of an atom is filled, its electrons will be noted in spectroscopic notation as \( 2s^22p^6 \). This notation is summarized for three atoms: helium, carbon, and sodium, in Table 1.3.

Since the late 1920s, it has been understood that electrons in an atom do not behave exactly like tiny moons orbiting a planet-like nucleus. It is more accurate to define them as entities whose behavior is described by wave functions, rather than as point particles in orbits. Although a wave function itself is not directly observable, calculations performed with this function predict the location of the electron. In contrast to the calculations of classical mechanics, in which properties such as force, mass, and acceleration are used in equations to yield a definite answer for a quantity such as position in space,
Table 1.3  Quantum numbers for electrons in helium, carbon, and sodium

<table>
<thead>
<tr>
<th>Element</th>
<th>n</th>
<th>l</th>
<th>m_l</th>
<th>m_s</th>
<th>Orbital</th>
<th>Shell</th>
</tr>
</thead>
<tbody>
<tr>
<td>Helium ((Z = 2))</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>−½</td>
<td>1s²</td>
<td>K</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>+½</td>
<td>1s²</td>
<td>K</td>
</tr>
<tr>
<td>Carbon ((Z = 6))</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>−½</td>
<td>1s²</td>
<td>K</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>+½</td>
<td>1s²</td>
<td>K</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>−½</td>
<td>2s²</td>
<td>L</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>+½</td>
<td>2s²</td>
<td>L</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1</td>
<td>−1</td>
<td>−½</td>
<td>2p²</td>
<td>L</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1</td>
<td>−1</td>
<td>+½</td>
<td>2p²</td>
<td>L</td>
</tr>
<tr>
<td>Sodium ((Z = 11))</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>−½</td>
<td>1s²</td>
<td>K</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>+½</td>
<td>1s²</td>
<td>K</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>−½</td>
<td>2s²</td>
<td>L</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>+½</td>
<td>2s²</td>
<td>L</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1</td>
<td>−1</td>
<td>−½</td>
<td>2p⁶</td>
<td>L</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1</td>
<td>−1</td>
<td>+½</td>
<td>2p⁶</td>
<td>L</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>−½</td>
<td>2p⁶</td>
<td>L</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>−½</td>
<td>2p⁶</td>
<td>L</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>−½</td>
<td>3s</td>
<td>M</td>
</tr>
</tbody>
</table>

quantum mechanical calculations yield probabilities. At a particular location in space, for example, the square of the amplitude of a particle’s wave function yields the probability that the particle will appear at that location.

In Figure 1.3, this probability is depicted for several possible energy levels of a single electron surrounding a hydrogen nucleus (i.e. a single proton). In this illustration, a brighter shading implies a higher probability of finding the electron at that location. Locations at which the probability is maximized correspond roughly to the electron shell model discussed previously. However, it is important to emphasize that the probability of finding the electron at other locations, even in the middle of the nucleus, is not zero. This particular result explains a certain form of radioactive decay in which a nucleus captures an electron, a phenomenon that is better explainable by quantum mechanics.

1.2.2.3 Electron Energy Levels and Binding Energies

The extent to which electrons are bound to the nucleus determines several energy absorption and emission phenomena. The binding energy of an electron \((E_b)\) is defined as the energy required to completely remove the electron from the atom. When energy is measured in the macroscopic world of everyday experience, units such as joules (J) and kilowatt hours (kWh) are used. In the microscopic world, the electron volt (eV) is a more convenient unit of energy. One electron volt is the kinetic energy imparted to an electron accelerated across a potential difference (i.e. voltage) of 1 V. One eV is equal to \(1.6 \times 10^{-19} \text{ J}\) or \(4.4 \times 10^{-26} \text{ kWh}\). As a unit of energy, the eV describes potential energy as well as kinetic energy; the binding energy of an electron in an atom, for example, is a form of potential energy.

An electron in an inner shell of an atom is attracted to the nucleus by a force greater than that exerted by the nucleus on an electron farther away. An electron may be moved from one shell to a more distant shell only if energy is supplied by an external source. Binding energy is a negative value because it represents an amount of energy that must be supplied to remove an electron from an atom. The energy that must be imparted
1.2 Electromagnetic Radiation

to an atom to move an electron from an inner shell to an outer shell is equal to the arithmetic difference in binding energy between the two shells. For example, the binding energy for an electron in the K-shell of hydrogen is $-13.5$ and $-3.4$ eV for an electron in the L shell. The energy required to move an electron from the K to the L shell in hydrogen is $(-3.4 \text{ eV}) - (-13.5 \text{ eV}) = 10.1 \text{ eV}$. Electrons in inner shells of high-$Z$ atoms are bound to the nucleus with a force much greater than that exerted upon the solitary electron in hydrogen (Table 1.4).

The electrons within a particular electron shell do not have exactly the same binding energy. Differences in binding energy among the electrons in a particular shell are determined by the orbital magnetic and spin quantum numbers, $l$, $m_l$, and $m_s$. The combinations of these quantum numbers provide three subshells for the L shell ($L_1$ to $L_{III}$) and five subshells for the M shell ($M_{I}$–$M_{V}$), where the M subshells occur only if a magnetic field is present. Energy differences between the subshells are small when compared to differences between shells, but these differences are important in medical imaging as they explain certain properties of the emission spectra of X-ray tubes. Table 1.5 gives values for the binding energies of K, L, and M shell electrons for selected elements.

<table>
<thead>
<tr>
<th>Atomic number</th>
<th>Element</th>
<th>K</th>
<th>L$_I$</th>
<th>L$_{II}$</th>
<th>L$_{III}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Hydrogen</td>
<td>0.0136</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Carbon</td>
<td>0.283</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Oxygen</td>
<td>0.531</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Sodium</td>
<td>1.08</td>
<td>0.055</td>
<td>0.034</td>
<td>0.034</td>
</tr>
<tr>
<td>13</td>
<td>Aluminum</td>
<td>1.559</td>
<td>0.087</td>
<td>0.073</td>
<td>0.072</td>
</tr>
<tr>
<td>14</td>
<td>Silicon</td>
<td>1.838</td>
<td>0.118</td>
<td>0.099</td>
<td>0.098</td>
</tr>
<tr>
<td>19</td>
<td>Potassium</td>
<td>3.607</td>
<td>0.341</td>
<td>0.297</td>
<td>0.294</td>
</tr>
<tr>
<td>20</td>
<td>Calcium</td>
<td>4.038</td>
<td>0.399</td>
<td>0.352</td>
<td>0.349</td>
</tr>
<tr>
<td>26</td>
<td>Iron</td>
<td>7.111</td>
<td>0.849</td>
<td>0.721</td>
<td>0.708</td>
</tr>
<tr>
<td>29</td>
<td>Copper</td>
<td>8.980</td>
<td>1.100</td>
<td>0.953</td>
<td>0.933</td>
</tr>
<tr>
<td>31</td>
<td>Gallium</td>
<td>10.368</td>
<td>1.30</td>
<td>1.134</td>
<td>1.117</td>
</tr>
<tr>
<td>32</td>
<td>Germanium</td>
<td>11.103</td>
<td>1.42</td>
<td>1.248</td>
<td>1.217</td>
</tr>
<tr>
<td>39</td>
<td>Yttrium</td>
<td>17.037</td>
<td>2.369</td>
<td>2.154</td>
<td>2.079</td>
</tr>
<tr>
<td>42</td>
<td>Molybdenium</td>
<td>20.002</td>
<td>2.884</td>
<td>2.627</td>
<td>2.523</td>
</tr>
<tr>
<td>47</td>
<td>Silver</td>
<td>25.517</td>
<td>3.810</td>
<td>3.528</td>
<td>3.352</td>
</tr>
<tr>
<td>53</td>
<td>Iodine</td>
<td>33.164</td>
<td>5.190</td>
<td>4.856</td>
<td>4.559</td>
</tr>
<tr>
<td>54</td>
<td>Xenon</td>
<td>34.570</td>
<td>5.452</td>
<td>5.104</td>
<td>4.782</td>
</tr>
<tr>
<td>56</td>
<td>Barium</td>
<td>37.410</td>
<td>5.995</td>
<td>5.623</td>
<td>5.247</td>
</tr>
<tr>
<td>57</td>
<td>Lanthanum</td>
<td>38.931</td>
<td>6.283</td>
<td>5.894</td>
<td>5.489</td>
</tr>
<tr>
<td>58</td>
<td>Cerium</td>
<td>40.449</td>
<td>6.561</td>
<td>6.165</td>
<td>5.729</td>
</tr>
<tr>
<td>74</td>
<td>Tungsten</td>
<td>69.508</td>
<td>12.090</td>
<td>11.535</td>
<td>10.198</td>
</tr>
<tr>
<td>79</td>
<td>Gold</td>
<td>80.713</td>
<td>14.353</td>
<td>13.733</td>
<td>11.919</td>
</tr>
<tr>
<td>82</td>
<td>Lead</td>
<td>88.001</td>
<td>15.870</td>
<td>15.207</td>
<td>13.044</td>
</tr>
<tr>
<td>92</td>
<td>Uranium</td>
<td>115.591</td>
<td>21.753</td>
<td>20.943</td>
<td>17.163</td>
</tr>
</tbody>
</table>

Source: Data from Fine [1].
1.2.2.4 Electron Transitions, Characteristic, and Auger Emission

Various processes can cause an electron to be ejected from an electron shell. When an electron is removed from a shell a vacancy or hole is left in the shell (i.e. a unique quantum “address” is left vacant). An electron may move from another shell to fill the vacancy. This movement, called an electron transition, involves a change in the binding energy of the moving electron. To move an inner shell electron to an outer shell, some external source of energy is required.

Alternatively, an outer shell electron may drop spontaneously to fill a vacancy in an inner shell. This spontaneous transition results in the release of energy. Spontaneous transitions of outer shell electrons falling to inner shells are depicted in Figure 1.4.

The energy released when an outer electron falls to an inner shell equals the difference in binding energy between the two shells involved in the transition, for example, an electron moving from the M to the K-shell of tungsten releases \((-69,500 \text{ eV}) - (-28,100 \text{ eV}) = -66,690 \text{ eV} \) or \(-66.69 \text{ keV}\). The energy is released in one of the two forms. In its first form, the transition energy is released as a photon (see Figure 1.4b). As the binding energy of electron shells is a unique characteristic of each element, the emitted photon is called a characteristic photon or characteristic X-ray. The emitted photon may be described as a K, L, or M characteristic photon denoting the destination of the transition electron.

An electron transition creates a vacancy in the outer shell where the electron originated, and this vacancy may be filled by an electron transition from another shell, leaving yet another vacancy and so on. Thus, a vacancy in an inner electron shell produces a cascade of electron transitions that yield a range of characteristic photon energies. Electron shells farther from the nucleus are more closely spaced in terms of binding energy. Therefore, characteristic photons produced by transitions among outer shells have less energy than do those produced by transitions involving inner shells. For transitions to shells beyond the M shell, characteristic photons are no longer energetic enough to be considered X-rays.

An alternative to the release of the energy through characteristic X-ray production is for the atom to “eject” another electron to compensate for an electron vacancy (Figure 1.4c). In this process, the energy released during an electron transition is transferred to another electron. This energy is sufficient to eject the electron from its shell. The ejected electron is referred to as an Auger electron. The kinetic energy of the ejected electron will not equal the total energy released during the transition because some of the transition energy is used to free the electron from its shell. The Auger electron is usually ejected from the same shell that held the electron that made the transition to an inner shell, as shown in Figure 1.4c. In this case, the kinetic energy of the Auger electron is calculated by twice subtracting the binding energy of the outer shell electron from the binding energy of the inner shell electron. The first subtraction yields the transition energy and the second subtraction accounts for the liberation of the ejected electron.

Either characteristic photon emission or Auger electron emission may occur during an electron transition, and although it is impossible to predict which process will occur for a specific atom, the probability of characteristic emission is termed the fluorescence yield, \(\omega\), defined as

\[
\omega = \frac{\text{Number of characteristic photons emitted}}{\text{Number of electron shell vacancies}}.
\]

The fluorescence yield increases with atomic number as depicted in Figure 1.5. For a transition to the K-shell of calcium, for example, the probability is 0.19 that a K characteristic photon will be emitted and 0.81 that an Auger electron will be emitted. The fluorescence yield is a factor to consider in the selection of materials used to produce X-ray imaging system and also for radioactive sources for nuclear imaging where Auger electrons result...
in increased radiation dose to the patient because they do not travel far in tissue.

### 1.2.2.5 Electronic Conduction

Electrons in individual atoms have specific binding energies described by quantum mechanics, but when atoms bind together into solids, the energy levels change as the electrons influence each other. Just as each atom has a unique set of quantum energy levels, a solid also has a particular set of energy levels, called energy bands, which are determined by the combination of atoms composing the solid and by bulk properties of the material such as temperature and pressure. Furthermore, similar to quantum vacancies or holes that may exist when an allowable energy state in a single atom is not filled, energy bands in a solid may or may not be fully populated with electrons. This enables the movement of charge between bands.

Two electron energy bands of a solid are depicted in Figure 1.6. The lower energy band, called the valence band, consists of electrons that are tightly bound in the chemical structure of the material. The upper energy band, called the conduction band, contains electrons that are relatively loosely bound. Conduction band electrons can move in the material and may constitute an electrical current under the proper conditions. If no electrons populate the conduction band, then the material cannot support an electrical current under normal circumstances. However, if enough energy is imparted to an electron in the valence band to raise it to the conduction band, then the material can support an electrical current.

Solids can be separated into three categories based on the difference in energy between electrons in the valence and conduction bands. In conductors, there is little energy difference between the bands, so the electrons are continuously promoted from the valence to the conduction band by routine collisions between electrons. In insulators, the conduction and valence bands are separated by a wide band gap, also known as the “forbidden zone,” of 3 eV or more. Under this condition, application of voltage to the material usually will not provide enough energy to promote electrons from the valence band to the conduction band. Therefore, insulators do not support an electrical current under normal circumstances. Of course, there is always a "breakdown voltage" above which an insulator will support a current, although probably not without structural damage to the material.

If the band gap of the material is between 0 and 3 eV, the material exhibits electrical properties between those of an insulator and a conductor. Such a material, termed a semiconductor, will conduct electricity under certain conditions and act as an insulator under others. The conditions may be altered by the addition to the material of trace amounts of impurities that have allowable energy levels within the band gap of the solid, creating electron traps as depicted in Figure 1.6. Semiconductors have significant applications in radiation detection.

As previously stated, very little energy is required to promote electrons to the conduction band in a conductor. However, the electrons in the conduction band do not fully move freely. As electrons attempt to move through the material, they interact with other electrons and with imperfections such as impurities in the solid. At each encounter, some energy is transferred to atoms and molecules and ultimately transferred to heat. This transfer of energy is the basis of electrical resistance and is usually viewed as energy loss, unless explicitly intended for heat production as in an electric heating element. Otherwise, the loss or energy and mitigation for that loss can create a significant technological challenge as in an X-ray tube.

There is a class of material, called superconductors, in which there is very little resistance to the flow of electrons under certain conditions. First discovered in mercury[2],

![Figure 1.5 Fluorescence yield for the K-shell vs. atomic number.](image)

![Figure 1.6 Energy level diagram for solids. An electron promoted from the valence band to the conduction band may move freely in the material to constitute an electric current.](image)
it has been described using the formalism of quantum mechanics \[3,4\] and is applicable to other materials at very low temperature. In a superconductor, the passage of an electron disturbs the structure of the material in such a way as to encourage the passage of another electron arriving after exactly the right interval of time. The passage of the first electron establishes an oscillation in the positive charge of the material that pulls the second electron through. This behavior has been analogized as "electronic waterskiing" where one electron is swept along in another electron’s “wake.” Thus, electrons tend to travel in what are known as Cooper pairs. Cooper pair electrons form a lower energy bound state where they can move as a unit with essentially no resistance to flow. Currents in superconducting loops of wire can continue for several years with no additional input of energy.

Many types of materials exhibit superconductive behavior when cooled to temperatures in the range of a few degrees Kelvin (room temperature is approximately 295 K); in fact, 26 elements and thousands of alloys and compounds exhibit this behavior. Lowering the temperature promotes superconductivity by decreasing molecular motion, thereby decreasing the kinetic energy of the material. However, maintenance of materials at very low temperatures often requires liquid helium as a cooling agent. Helium liquefies at 23 K, is relatively expensive, and is usually insulated from ambient conditions with another refrigerant such as liquid nitrogen, which adds to the cost. Superconducting technology is a foundation of modern magnetic resonance imaging (MRI) systems, where a string magnetic field is produced and maintained by superconducting coils. The science of superconducting has been leading to higher temperature possibilities in new material (e.g. barium–lanthanum–copper oxide and magnesium diboride) \[5,6\], with potential profound impact across electronic technology, which is sure to have a significant influence on medical imaging technology as well.

### 1.2.3 The Nucleus

In a model analogous to that for electrons, the shell model of the nucleus considers nucleons to be distributed between shells and was introduced to explain the existence of discrete nuclear energy states. Furthermore, nuclei are extraordinarily stable if they contain 2, 8, 20, 82, or 126 protons or similar numbers of neutrons. These numbers are termed magic numbers and may reflect full occupancy of nuclear shells. Nuclei with odd numbers of neutrons or protons tend to be less stable than nuclei with even numbers of neutrons and protons. The pairing of similar nucleons increases the stability of the nucleus, as illustrated in Table 1.6.

<table>
<thead>
<tr>
<th>Number of protons</th>
<th>Number of neutrons</th>
<th>Number of stable nuclei</th>
</tr>
</thead>
<tbody>
<tr>
<td>Even</td>
<td>Even</td>
<td>165</td>
</tr>
<tr>
<td>Even</td>
<td>Odd</td>
<td>57</td>
</tr>
<tr>
<td>Odd</td>
<td>Even</td>
<td>53</td>
</tr>
<tr>
<td>Odd</td>
<td>Odd</td>
<td>6</td>
</tr>
</tbody>
</table>

### 1.2.3.1 Nuclear Forces and Stability

Protons have “like” charges, i.e. each has the same positive charge and as such repel each other by the electrostatic force of repulsion. One may then wonder how a nucleus stays together. The answer is that when protons are very close together, there is also an attractive force. This force, called the strong nuclear force, is one hundred times stronger than the electrostatic force of repulsion. However, it acts only over distances on the order of magnitude of the diameter of the nucleus. Therefore, protons can stay together in the nucleus once they are there but assembling a nucleus by forcing protons together requires the expenditure of significant energy to overcome the electrostatic repulsion.

Neutrons have no electrostatic charge and do not experience the electrostatic force. Therefore, adding neutrons to a nucleus requires much less energy. Neutrons are, however, governed by the weak nuclear force, a force through which a neutron is believed to change spontaneously into a proton, an electron, and an antineutrino (an almost massless, virtually noninteracting particle) and back (i.e. a proton to a neutron, a positron, and a neutrino). These processes, called beta decay, are described in greater detail in the Section 1.3.1. Gravity, as the fourth of the traditional four fundamental forces of nature (i.e. electromagnetism, strong nuclear force, weak nuclear force, and gravity), is so overshadowed in terms of magnitude by the other forces within an atom that it is believed to play no essential role in nuclear stability or instability.

### 1.2.3.2 Nuclear Fusion and Fission

The mass of an atom is less than the sum of the masses of its neutrons, protons, and electrons. This seeming paradox reflects Einstein’s famous equation \( E = mc^2 \) depicting the exchangeability of the binding energy of the nucleus and the masses of its constituent particles. When the nucleons are separate, they have their own individual masses, but when they are combined in a nucleus, some of their mass is converted into binding energy. The mass difference between the sum of the masses of
1.2 Electromagnetic Radiation

Figure 1.7 Average binding energy per nucleon vs. mass number and its exploitation in the processes of fission and fusion.

the atomic constituents and the mass of the assembled atom is termed the mass defect. In Einstein’s equation, an energy \( E \) is equivalent to mass, \( m \), multiplied by the speed of light in a vacuum, \( c \), (i.e. \( 2.998 \times 10^8 \text{ m/s} \)) squared. Because of the significant magnitude of the proportionality constant, \( c^2 \), in this equation, 1 kg of mass is equal to \( 9 \times 10^{16} \text{ J} \), roughly equivalent to the energy released during detonation of 30 megatons of C\(_7\)H\(_5\)N\(_3\)O\(_6\) (i.e. TNT).

The energy equivalent of even 1 amu is significant:

\[
\frac{(1 \text{ amu})(1.660 \times 10^{-27} \text{ kg/amu})(2.998 \times 10^8 \text{ m/s})^2}{(1.602 \times 10^{-13} \text{ J/MeV})} = 931 \text{ MeV}
\]

Figure 1.7 depicts the average binding energy per nucleon as a function of the mass number \( A \).

The potential energy bound in the nucleus can be released if a nucleus with a high mass number separates or fissions into two parts, each with an average binding energy per nucleon greater than that of the original nucleus. The energy release occurs because this split produces low-Z products with a higher average binding energy per nucleon than the original high-Z nucleus (see Figure 1.7). A transition from a state of lower binding energy per nucleon to a state of higher binding energy per nucleon results in the release of energy. This is reminiscent of the previous discussion of energy release that accompanies an L to K electron transition. However, the energy available from transition between nuclear energy levels is orders of magnitude greater than the energy released during electron transitions.

Certain high-\( A \) nuclei (e.g. \(^{233}\text{U}\), \(^{235}\text{U}\), and \(^{239}\text{Pu}\)) fission spontaneously after absorbing a slowly moving neutron. For \(^{235}\text{U}\), a typical fission reaction is as follows:

\[
^{235}\text{U} + \text{neutron} \rightarrow ^{236}\text{U} \rightarrow ^{92}\text{Kr} + ^{141}\text{Ba} + 3 \text{ neutrons} + Q.
\]

The energy released is designated as \( Q \) and averages more than 200 MeV per fission. The energy is liberated primarily as \(^{235}\text{U}\) radiation, kinetic energy of radioactive fission products (e.g. \(^{36}\text{Kr}\) and \(^{141}\text{Ba}\)), neutrons, and of course heat and light. Neutrons released during fission may interact with other nuclei and create the possibility of a chain reaction, provided that sufficient mass of fissionable material is contained within a small volume. The rate at which fission can occur in a material may be regulated by controlling the number of neutrons available to interact with fissionable nuclei; this is the method used to control nuclear reactors. Uncontrolled nuclear fission results in an “atomic explosion.”

The counterpart to fission is fusion, which is the combination of certain low-mass nuclei to produce a nucleus with an average binding energy per nucleon greater than that for either of the original nuclei. This process is accompanied by the release of large amounts of energy. A typical reaction is as follows:

\[
^1\text{H} + ^1\text{H} \rightarrow ^4\text{He} + \text{neutron} + Q.
\]

In this particular reaction, \( Q = 18 \text{ MeV} \) per fusion. To form products with higher average binding energy per nucleon, nuclei must be brought sufficiently near one another, so the strong nuclear force can initiate fusion. In the process, the electrostatic force of repulsion must be overcome as the two nuclei approach each other. Nuclei moving at very high velocities possess enough momentum to overcome this repulsive force. Adequate velocities may be attained by heating a sample containing low-Z nuclei to a temperature greater than \( 12 \times 10^6 \) K, roughly equivalent to the temperature in the inner region of the Sun. Fusion is in fact the primary reaction fueling the energy production in the stars including our sun. Fusion has been used for military purposes where the high temperatures necessary in initiating a fusion (e.g. in a hydrogen bomb) are triggered with a fission igniter explosion. Although still out of reach, however, there is much hope and effort for harnessing fusion for safe energy production.

1.2.3.3 Nuclear Spin and Nuclear Magnetic Moments

Protons and neutrons behave like small magnets and have associated magnetic moments. The term moment has a strict meaning in physics. When a force is applied on a wrench to turn a bolt (see Figure 1.8a), for example,
The mechanical moment is the product of force and length. The mechanical moment can be increased by increasing the length of the wrench, applying more force to the wrench, or a combination of the two. A magnetic moment (see Figure 1.8b) is likewise the product of the current in a circuit (i.e. a path followed by electrical charges) and the area encompassed by the circuit, so the magnetic moment is increased by increasing the current, the area, or a combination of the two. The magnetic moment is a vector, a quantity having both magnitude and direction. Like electrons, protons have a characteristic called spin, which can be explained by treating the proton as a small object spinning on its axis. In this model, the spinning charge of the proton produces a magnetic moment (see Figure 1.8c).

The spinning–charge model of the proton has two marked limitations. First, the mathematical prediction for the value of the magnetic moment is not quite what has been measured experimentally. From the model, a proton would have a fundamental magnetic moment known as the nuclear magneton, $\mu_n$, as

$$\mu_n = \frac{e\hbar}{2M_p} = 3.1525 \times 10^{-12} \frac{\text{eV}}{\text{gauess}}.$$ (1.7)

Here, $e$ is the charge of the proton in coulombs, $\hbar$ is Planck’s constant divided by $2\pi$, and $M_p$ is the mass of the proton. The magnetic moment magneton, $\mu_n$, of the proton, however, has been measured as $\mu_p = 2.79\mu_n$. This classical observation has been explained using quantum mechanics and is used to describe the fundamental concepts of MRI (see Chapter 10). The unit of the nuclear magneton, energy (i.e. electron volt) per magnetic field strength (i.e. gauss), expresses the fact that a magnetic moment has a certain potential energy in a magnetic field. The second difficulty of the spinning proton model is that the neutron, an uncharged particle, exhibits a magnetic moment equal to $1.91\mu_n$. An explanation for these limitations is that neutrons and protons are composed of other more fundamental particles called quarks [7,8]. Quarks have fractional charges adding up to a unit charge. The nonuniform distribution of spinning charges of the quarks is the basis for the observed magnetic moments.

When magnetic moments exist near each other, as in the nucleus, they tend to form pairs with the vectors of the moments pointed in opposite directions. In nuclei with even numbers of neutrons and protons, this pairing cancels out the magnetic properties of the nucleus as a whole. Thus, an atom such as $^{12}$C with six protons and six neutrons has no net magnetic moment because the neutrons and protons are paired up. An atom with an odd number of either neutrons or protons will have a net magnetic moment. For example, $^{13}$C with six protons and seven neutrons has a net magnetic moment because it contains an unpaired neutron. Also, $^{14}$N with seven protons and seven neutrons has a net magnetic moment because both proton and neutron numbers are odd. Table 1.7 lists several nuclides with net magnetic moments. The presence of a net magnetic moment for the nucleus is essential to MRI, as only nuclides with net magnetic moments can interact with the magnetic field of an MRI system to provide a signal to form an image.

### Table 1.7 Nuclides with a net magnetic moment

<table>
<thead>
<tr>
<th>Nuclide</th>
<th>Number of protons</th>
<th>Number of neutrons</th>
<th>Magnetic moment (multiple of $\mu_n$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^1$H</td>
<td>1</td>
<td>0</td>
<td>2.79</td>
</tr>
<tr>
<td>$^2$H</td>
<td>1</td>
<td>1</td>
<td>0.86</td>
</tr>
<tr>
<td>$^{13}$C</td>
<td>6</td>
<td>7</td>
<td>0.70</td>
</tr>
<tr>
<td>$^{14}$N</td>
<td>7</td>
<td>7</td>
<td>0.40</td>
</tr>
<tr>
<td>$^{17}$O</td>
<td>8</td>
<td>9</td>
<td>−1.89</td>
</tr>
<tr>
<td>$^{19}$F</td>
<td>9</td>
<td>10</td>
<td>2.63</td>
</tr>
<tr>
<td>$^{23}$Na</td>
<td>11</td>
<td>12</td>
<td>2.22</td>
</tr>
<tr>
<td>$^{31}$P</td>
<td>15</td>
<td>16</td>
<td>1.13</td>
</tr>
<tr>
<td>$^{39}$K</td>
<td>19</td>
<td>20</td>
<td>0.39</td>
</tr>
</tbody>
</table>

Source: Data from Heath [9].
1.3 Radioactivity

level may fall to fill a vacancy at a lower energy level. This transition releases energy and yields a more stable nucleus. The amount of energy released is again related to the difference in binding energy between the higher and lower levels, but the binding energy is much greater for neutrons and protons inside the nucleus than for electrons outside the nucleus. Hence, energy released during nuclear transitions is much greater than that released during electron transitions. This process is known as radioactivity, first discovered in 1896 by Henri Becquerel who observed the emission of radiation, later shown to be beta particles, from uranium salts. Becquerel experienced a skin burn from carrying a radioactive sample in his vest pocket, which is also the first known bioeffect of radiation exposure.

In all radioactive transitions, the nucleus loses energy and gains stability, which is the essence of radioactive decay. In any radioactive process, the mass number of the decaying nucleus (i.e. parent) equals the sum of the mass numbers of the product nucleus (i.e. progeny) and the ejected particle. That is, mass number \( A \) is conserved in radioactive decay. The energy is released in the form of the kinetic energy of the progeny, radiation, and heat.

1.3.1 Decay Schemes

Radioactive transitions are often depicted by decay schemes. A decay scheme depicts the decay processes specific for a nuclide, a generic term for any nuclear schemes. A decay scheme shows the decay processes of a nuclide, i.e. Pathway 1 in Figure 1.9. In isomeric transition, the nucleus gains stability by transition of a neutron between neutron energy levels or a proton between proton energy levels. In this process, the energy is released by either an electromagnetic particle, i.e. gamma ray (\( \gamma \)-ray), or by an ejected electron, by internal conversion. Note that no radioactive nuclide decays solely by an isomeric transition. Isomeric transitions are always preceded by either electron capture or emission of an alpha (\( \alpha \)) or beta (\( \beta \)) particle.

The specific decay scheme for a given nuclide is strongly influenced by the ratio of \( N \) to \( Z \). In low-\( Z \) stable nuclei, the number of neutrons is about equal to the number of protons. As \( Z \) increases, the number of neutrons increases more rapidly than the number of protons in stable nuclei, as depicted in Figure 1.10. The shell model of the nucleus accounts for this finding by suggesting that at higher \( Z \), the energy difference is slightly less between neutron levels than between proton levels.

Nuclei with too many or too few neutrons to reside on or close to the line of stability depicted in Figure 1.10 are unstable and therefore undergo radioactive decay. Nuclei above and to the left of the line of stability (i.e. those with \( N/Z \) ratio too high for stability) tend to emit neutrons by the process of beta (\( \beta^- \)) decay, i.e. Pathway 3. Nuclei below and to the right of the line of stability (i.e. those with \( N/A \) ratio too low for stability) tend to undergo the competing processes of positron (\( \beta^+ \)) decay and electron capture, i.e. Pathway 2. In both cases, the total number of nucleons \( A \) remains the same in the parent and progeny nuclei. Table 1.8 offers a summary of the key attributes of various radioactive transitions, which we discuss in more detail below.

1.3.1.1 Alpha Decay

Some heavy nuclei gain stability by the emission of an alpha (\( \alpha \)) particle, which is two protons and two neutrons.

---

**Figure 1.9** A generic radioactive decay scheme representing alpha decay (1), positron decay and electron capture (2), beta decay (3), and isomeric transition (4).
tightly bound as a nucleus of helium, $^{4}_{2}$He [10]. The alpha particle is a relatively massive, poorly penetrating type of radiation that can be stopped by a sheet of paper or paint. An example of alpha decay is the decay of naturally occurring radium into the inert gas radon by the emission of an alpha particle:

$$^{226}_{88}\text{Ra} \rightarrow ^{222}_{86}\text{Rn} + ^{4}_{2}\text{He}.$$  

A decay scheme for alpha decay is depicted in Figure 1.11.

**Figure 1.10** Number of neutrons ($N$) for stable (or least unstable) nuclei plotted as a function of the number of protons ($Z$).

**Figure 1.11** Radioactive decay scheme for $\alpha$ decay of $^{226}$Ra.

### 1.3.1.2 Beta Decay

Beta decay can be envisioned as a neutron falling to a lower energy level reserved for protons and “becoming” a proton. In this process, the $Z$ and $N$ of the nucleus change, and the nucleus transmutes from one element to another. The product is most stable if it contains even numbers of protons and neutrons and least stable if it contains an odd number for either [11].

Nuclei with an $N/Z$ ratio too high for stability (see Figure 1.10) decay as

$$^0_n \rightarrow ^1_p + ^0_\beta + \bar{\nu},$$

where $^0_\beta$ is an electron ejected from the nucleus and $\bar{\nu}$ is an antineutrino, a massless neutral particle that is required for the conservation of energy and is the antimatter counterpart of the neutrino. An example of beta negative decay is that of $^{137}$Cs:

$$^{137}_{55}\text{Cs} \rightarrow ^{137}_{56}\text{Ba} + ^0_\beta + \bar{\nu} + \text{isomeric transition}.$$  

### Table 1.8 Summary of radioactive decay in terms of changes in $A$, $Z$, and $N$ from parent to progeny

<table>
<thead>
<tr>
<th>Type of transition</th>
<th>Final $A$</th>
<th>Final $Z$</th>
<th>Final $N$</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alpha ($\alpha$) decay</td>
<td>$A - 4$</td>
<td>$Z - 2$</td>
<td>$N - 2$</td>
<td>Fixed amount of released energy per mass–energy conservation</td>
</tr>
<tr>
<td>Positron ($\beta^+$) decay</td>
<td>$A$</td>
<td>$Z - 1$</td>
<td>$N + 1$</td>
<td>Beta particles have an energy distribution with mean energy equal to a third of the maximum energy</td>
</tr>
<tr>
<td>Electron capture</td>
<td>$A$</td>
<td>$Z - 1$</td>
<td>$N + 1$</td>
<td>Produces characteristic X-rays and Auger electrons</td>
</tr>
<tr>
<td>Beta ($\beta^-$) decay</td>
<td>$A$</td>
<td>$Z + 1$</td>
<td>$N - 1$</td>
<td>Beta particles have an energy distribution with mean energy equal to a third of the maximum energy</td>
</tr>
<tr>
<td>Isomeric transition – gamma ($\gamma$) emission</td>
<td>$A$</td>
<td>$Z$</td>
<td>$N$</td>
<td>Metastable if $T_{1/2} &gt; 10^{-6}$ s</td>
</tr>
<tr>
<td>Isomeric transition (IT) or internal conversion (IC)</td>
<td>$A$</td>
<td>$Z$</td>
<td>$N$</td>
<td>IT and IC are competing processes; IC electrons produce characteristic X-rays and Auger electrons</td>
</tr>
</tbody>
</table>
1.3 Radioactivity

A decay scheme for this transition is shown in Figure 1.12. A beta (β⁻) particle with a maximum energy ($E_{\text{max}}$) of 1.17 MeV is released in 5% of all decays; in the remaining 95%, a beta (β⁻) particle with an $E_{\text{max}}$ of 0.51 MeV is accompanied by an isomeric transition of 0.66 MeV, where either a $\gamma$-ray is emitted or an electron is ejected by internal conversion. The transition energy is 1.17 MeV for the decay of $^{137}$Cs.

Beta (β⁻) decay pathways are characterized by specific maximum energies; however, most beta (β⁻) particles are ejected with energies lower than these maxima. The average energy of beta (β⁻) particles is about $1/3$ $E_{\text{max}}$ along a specific pathway. The energy distribution of beta (β⁻) particles emitted during the decay of $^{32}$P is shown in Figure 1.13. Spectra of similar shape but with different values of $E_{\text{max}}$ and $E_{\text{mean}}$ exist for the decay pathways of every beta particle emitting radioactive nuclides. In each particular decay, the difference in energy between $E_{\text{max}}$ and the specific energy of the beta (β⁻) particle is carried away by the antineutrino. That is,

$$E_{\nu} = E_{\text{max}} - E_{k}, \quad (1.8)$$

where $F_{\text{max}}$ is the energy released during the beta (β⁻) decay process, $E_{k}$ is the kinetic energy of the beta (β⁻) particle, and $E_{\nu}$ is the energy of the antineutrino.

1.3.1.3 Positron Decay and Electron Capture

Analogous to beta decay, positron decay or electron capture (EC) can be thought as a proton falling to a lower energy level reserved for neutrons, in which case the proton “becomes” a neutron, although conservation of mass dictates that a lower mass proton cannot truly change to a neutron. As in the case of beta rays, the progeny is most stable if it contains even numbers of protons and neutrons and least stable if it contains an odd number for either [10].

Positron decay results from the nuclear transition

$$^1_1p \rightarrow ^0_0n + ^0_1\beta + \nu,$$

where $^0_1\beta$ represents a positron ejected from the nucleus during decay and $\nu$ is a neutrino that accompanies the positron. The neutrino and antineutrino are similar, except that they have opposite spin and share the same energy balance with their corresponding positron or electron emission.

In positron decay, the $N/Z$ ratio increases; hence, positron-emitting nuclides tend to be positioned below the $N/Z$ stability curve shown in Figure 1.10. The decay of $^{30}$P is representative of a positron decay:

$$^{30}_{15}\text{P} \rightarrow ^{30}_{14}\text{Si} + ^0_1\beta + \nu.$$

The $N/Z$ ratio of a nuclide may also be increased by electron capture, in which an electron is captured by the nucleus to yield the following transition:

$$^{1}_1p + ^{-1}_{-1}e \rightarrow ^1_0n + \nu.$$

Most electrons are captured from the K electron shell, although occasionally an electron may be captured from the L shell or a shell even farther from the nucleus. During electron capture, a hole is created in an electron shell within the atom. This vacancy is filled by an electron cascading from a higher shell, resulting in the release of characteristic radiation or one or more Auger electrons.

Many nuclei decay by both electron capture and positron emission, as illustrated below for $^{22}$Na:

$$^{22}_{11}\text{Na} \left\{ \begin{array}{c} ^{-1}_{-1}e \rightarrow ^{22}_{10}\text{Ne} + \nu \hfill \\
- ^0_1\beta + ^{22}_{10}\text{Ne} + \nu \end{array} \right\}$$

The electron capture branching ratio is the probability of electron capture per decay for a particular nuclide. For $^{22}$Na, the branching ratio is 10% for electron capture and 90% of the nuclie decay by positron emission. Generally, positron decay prevails over electron capture when both decay modes occur.

A decay scheme for $^{22}$Na is shown in Figure 1.14. The $2m_{\nu}c^2$ listed alongside the vertical portion of the
This amount of energy must be supplied by the transition energy during positron decay. Nuclei that cannot furnish at least 1.02 MeV for the transition do not decay by positron emission. These nuclei increase their $N/Z$ ratios solely by electron capture.

There are a few nuclides that can decay by beta ($\beta^-$) emission, positron emission, or electron capture. An example is $^{74}$As (see Figure 1.15), which undergoes beta ($\beta^-$) emission 32% of the time, positron emission 30%, and electron capture 38%. All modes of decay result in the transformation of the highly unstable "odd-odd" $^{74}$As nucleus ($Z = 33, N = 41$) into a progeny nucleus with even $Z$ and even $N$.

### 1.3.1.4 Isomeric Transition

Radioactive decay often leaves the progeny in an exited state necessitating an isomeric transition. A common form of isomeric transition is gamma decay, in which the energy is released as a $\gamma$-ray. $\gamma$-rays are electromagnetic radiation of similar energy to X-rays and are differentiated only by their origin: X-rays result from electron interactions outside the nucleus, whereas $\gamma$-rays result from nuclear transitions. The gamma emission process competes with another isomeric transition, internal...
1.3 Radioactivity

Conversion, in which an electron from an extra nuclear shell carries the energy out of the atom.

Sometimes, one or more of the excited states of a progeny nuclide may exist for a finite lifetime. An excited state is termed a metastable state if its half-life exceeds $10^{-6}$ seconds (see Section 1.3.2.1). For example, the decay scheme for $^{99}$Mo shown in Figure 1.16 exhibits a metastable energy state, $^{99m}$Tc, that has a half-life of six hours.

Just as electron transitions emit characteristic X-rays, metastatic nuclides emit $\gamma$-rays with characteristic energies. For example, $^{99m}$Tc emits photons of 142 and 140 keV and beta ($\beta^−$) decay of $^{60}$Co releases photons of 1.17 and 1.33 MeV (see Figure 1.17). In the latter case, the photons are released during a cascade of isomeric transitions of progeny $^{60}$Ni nuclei from excited states to the ground energy state.

An isomeric transition can also occur by the interaction of the nucleus with an electron in one of the electron shells, i.e. internal conversion. When this happens, the electron is ejected with kinetic energy $E_k$ equal to the energy $E_\gamma$ released by the nucleus, reduced by the binding energy $E_b$ of the electron:

$$E_k = E_\gamma - E_b. \quad (1.9)$$

The ejected electron is accompanied by X-rays and Auger electrons as the extra nuclear structure of the atom resumes a stable configuration.

The internal conversion coefficient for an electron shell is the ratio of the number of conversion electrons from the shell compared with the number of $\gamma$-rays emitted by the nucleus. The probability of internal conversion increases rapidly with increasing $Z$ and with the lifetime of the excited state of the nucleus.

1.3.2 Mathematics of Radioactive Decay

Radioactivity can be described mathematically without regard to specific mechanisms of decay. The rate of decay, i.e. the number of decays per unit time, of a radioactive sample depends on the number $N$ of radioactive atoms in the sample. This concept can be stated as

$$\frac{\Delta N}{\Delta t} = -\lambda N, \quad (1.10)$$

where $\Delta N/\Delta t$ is the rate of decay and the constant $\lambda$ is called the decay constant. The minus sign indicates that the number of parent atoms in the sample and therefore the number of atoms decaying per unit time are decreasing. The decay constant of a nuclide is truly a constant; it is not affected by external influences such as temperature and pressure or by magnetic, electrical, or gravitational fields. However, Eq. (1.10) only describes the expected decay rate of a radioactive sample. At any moment, the actual decay rate may differ somewhat from the expected rate because of statistical fluctuations in the decay rate; therefore, Eq. (1.10) describes a reaction known as a first-order reaction.

By rearranging Eq. (1.10) to

$$-\frac{\Delta N}{\Delta t} = \frac{\lambda}{N}, \quad (1.11)$$

the decay constant can be seen to be the fractional rate of decay of the atoms. The decay constant has units of $(\text{time})^{-1}$, such as $s^{-1}$ or $h^{-1}$, and it has a characteristic value for each nuclide. It also reflects the nuclide’s degree of instability; a larger decay constant implies a more unstable nuclide and more rapid decay.

![Figure 1.16](image1.png)

Simplified radioactive decay scheme for $^{99}$Mo.

![Figure 1.17](image2.png)

Radioactive decay scheme for $^{60}$Co.
The rate of decay is a measure of a sample’s activity $A$, defined as
\[ A = -\frac{\Delta N}{\Delta t} = \lambda N. \] (1.12)

The activity of a sample depends on the number of radioactive atoms in the sample and the decay constant of the atoms. A sample may have a high activity because it contains a few highly unstable (i.e. large decay constant) atoms or because it contains many atoms that are only moderately unstable (i.e. small decay constant).

The System International unit of activity is the becquerel (Bq) defined as 1 disintegration per second (dps). An older unit of activity is the curie (Ci) defined as $3.7 \times 10^{10}$ dps. The curie was defined in 1910 as the activity of 1g of radium. Although subsequent measures revealed that 1g of radium actually has a decay rate of $3.61 \times 10^{10}$ dps, the definition of the curie was left as $3.7 \times 10^{10}$ dps.

1.3.2.1 Decay Equations and Half-life

Equation (1.10) is a differential equation, and it can be solved to yield an expression for the number $N$ of parent atoms present in the sample at any time $t$ as
\[ N = N_0 e^{-\lambda t}, \] (1.13)

where $N_0$ is the number of atoms present at time $t = 0$ and $e$ is the exponential quantity 2.7183. Equation (1.13) reveals that the number $N$ of parent atoms decreases exponentially with time.

By multiplying both sides of Eq. (1.13) by $\lambda$, the expression can be rewritten as
\[ A = A_0 e^{-\lambda t}, \] (1.14)

where $A$ is the activity remaining after time $t$ and $A_0$ is the original activity. The specific activity of a radioactive sample is the activity per unit mass of sample. The number of atoms $N^*$ decaying in time $t$ is $N_0 - N$, or
\[ N^* = N_0(1 - e^{-\lambda t}). \] (1.15)

Radioactive decay must always be described in terms of the probability of decay; whether any particular radioactive nucleus will decay within a specific time period is never certain. The probability that any particular atom will not decay during time $t$ is $N/N_0$, or
\[ P(\text{no decay}) = e^{-\lambda t}, \] (1.16)

and thus, the probability that a particular atom will decay during time $t$ is
\[ P(\text{decay}) = 1 - e^{-\lambda t}. \] (1.17)

The physical half-life, $T_{1/2}$, of a radioactive nuclide is the time required for the decay of half of the atoms in a sample of the nuclide. In Eq. (1.13), $N = 2N_0$ when $t = T_{1/2}$, with the assumption that $N = N_0$ when $t = 0$. By substitution in Eq. (1.13),
\[ \frac{1}{2} = \frac{N}{N_0} = e^{-\lambda T_{1/2}} \]
\[ \ln \left(\frac{1}{2}\right) = -\lambda T_{1/2} \]
\[ T_{1/2} = \frac{0.693}{\lambda}, \] (1.18)

where 0.693 is the natural logarithm of 2.

The mean (average) life $\tau$ is the average expected lifetime for atoms of a radioactive sample and is related to the decay constant $\lambda$ by
\[ \tau = \frac{1}{\lambda}. \] (1.19)

Since $T_{1/2} = \frac{0.693}{\lambda}$, the mean life may be calculated from the expression
\[ \tau = 1.44T_{1/2}. \] (1.20)

The percent of original activity of a radioactive sample is pictured in Figure 1.18 as a function of time expressed in units of physical half-life.

1.3.2.2 Transient Equilibrium

Progeny atoms produced during radioactive decay are sometimes also radioactive. For example, $^{226}$Ra decays to $^{222}$Rn, which decays to $^{218}$Po, which decays to $^{214}$Po, which decays to $^{210}$Po, which decays to $^{210}$Pb, which decays to $^{206}$Pb. The percent of original activity of a radioactive sample is expressed as a function of time in units of half-life.

![Figure 1.18](image-url)
1.3 Radioactivity

222Rn (with half-life of 1600 years), 222Rn then decays by to 218Po (half-life of 3.82 days), an unstable nuclide that in turn decays to 214Pb (with half-life of about three minutes), and so on. If the half-life of the parent is significantly longer than the half-life of the progeny (i.e. by a factor of 10 or more), then a condition of transient equilibrium may be established, where the rate of the delay of the progeny is governed by that of the parent. After the time required to attain transient equilibrium has elapsed, the activity of the progeny decreases with an apparent half-life equal to the physical half-life of the parent. The apparent half-life reflects the simultaneous production and decay of the progeny atoms. If no progeny atoms are present initially when \( t = 0 \), the number \( N_2 \) of progeny atoms present at any later time is

\[
N_2 = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_0 (e^{-\lambda_2 t} - e^{-\lambda_1 t}). \tag{1.21}
\]

Equation (1.21) is a Bateman equation. More complex Bateman equations describe progeny activities for sequential phases of multiple radioactive nuclides in transient equilibrium. In Eq. (1.21), \( N_0 \) is the number of parent atoms present when \( t = 0 \), \( \lambda_1 \) is the decay constant of the parent and \( \lambda_2 \) is the decay constant of the progeny. If \( (N_2)_0 \) progeny atoms are present when \( t = 0 \), then Eq. (1.21) may be rewritten as

\[
N_2 = (N_2)_0 e^{-\lambda_2 t} + \frac{\lambda_1}{\lambda_2 - \lambda_1} N_0 (e^{-\lambda_2 t} - e^{-\lambda_1 t}). \tag{1.22}
\]

Transient equilibrium for the transition

\[
\text{132Te} (T_{1/2} = 78 \text{ h}) \rightarrow \text{132I} (T_{1/2} = 2.3 \text{ h})
\]

is illustrated in Figure 1.19. Note that the plot is a semilogarithmic format where the log of the dependent (y-axis) variable is plotted as a function of the linear independent (x-axis) variable. The exponential decay relationship yields a linear form in this format. The activity of \( \text{132I} \) is greatest at the moment of transient equilibrium when parent \( \text{132Te} \) and progeny \( \text{132I} \) activities are equal. At later times, the activity of the progeny \( (A_2) \) exceeds the activity of the parent \( (A_1) \):

\[
A_2 = A_1 \frac{\lambda_2}{\lambda_2 - \lambda_1}. \tag{1.23}
\]

Transient equilibrium is the principle underlying the production of short-lived isotopes (e.g. \( ^{99m}\text{Tc} \), \( ^{113m}\text{In} \), and \( ^{68}\text{Ga} \)) in generators for radioactive nuclides used in nuclear medicine. As an example, the activities of \( ^{99m}\text{Tc} \) \( (T_{1/2} = 6.0 \text{ hours}) \) and \( ^{99}\text{Mo} \) \( (T_{1/2} = 66 \text{ hours}) \) are plotted in Figure 1.20 as a function of time. The \( ^{99m}\text{Tc} \) activity remains less than that of \( ^{99}\text{Mo} \) because about 14% of the \( ^{99}\text{Mo} \) nuclei decay promptly to \( ^{99m}\text{Tc} \) without passing through the isomeric \( ^{99m}\text{Tc} \) state. The abrupt decrease in activity at 48 hours reflects the removal of \( ^{99m}\text{Tc} \) from the generator that would be used for nuclear imaging (see Chapter 8).

1.3.2.3 Secular Equilibrium

If the half-life of the parent is much greater than the half-life of the progeny (e.g. by a factor of \( 10^4 \) or more),
a condition known as the secular equilibrium occur. As $\lambda_1 \ll \lambda_2$ if $T_{1/2 \text{parent}} \gg T_{1/2 \text{progeny}}$, Eq. (1.21) may be simplified by assuming that $\lambda_2 - \lambda_1 = \lambda_2$ and that $e^{-\lambda_1 t} = 1$:

$$N_2 = \frac{\lambda_1}{\lambda_2} N_0 (e^{-\lambda_1 t} - e^{-\lambda_2 t}) \cong \frac{\lambda_1}{\lambda_2} N_0 (1 - e^{-\lambda_2 t}).$$

(1.24)

After several half-lives of the progeny have elapsed, $e^{-\lambda_2 t} \cong 0$ and

$$N_2 = \frac{\lambda_1}{\lambda_2} N_0$$

(1.25)

The terms $\lambda_1 N_0$ and $\lambda_2 N_2$ in Eq. (1.25) describe the activities of parent and progeny, respectively. These activities are equal after secular equilibrium has been achieved. Illustrated in Figure 1.21 are the growth of activity and the activity at equilibrium for $^{222}\text{Rn}$ produced in a closed environment by decay of $^{226}\text{Ra}$. Units for the $x$-axis are multiples of the physical half-life of $^{222}\text{Rn}$. The growth curve for $^{222}\text{Rn}$ approaches the decay curve for $^{226}\text{Ra}$ asymptotically. Several half-lives of $^{222}\text{Rn}$ must elapse before the activity of the progeny equals 99% of the parent activity.

If the parent half-life is less than that for the progeny ($T_{1/2 \text{parent}} < T_{1/2 \text{progeny}}$, or $\lambda_1 > \lambda_2$), then a constant relationship is not achieved between the activities of the parent and the progeny. Instead, the activity of the progeny increases initially, reaches a maximum, and then decreases with a half-life intermediate between the parent and the progeny half-lives.

Radioactive nuclides in secular equilibrium are often used in radiation oncology. One example is energetic $\beta$ particles from $^{90}\text{Y}$ in secular equilibrium with $^{90}\text{Sr}$. The activity of the $^{90}\text{Sr}$-$^{90}\text{Y}$ irradiator decays with the physical half-life of $^{90}\text{Sr}$ (i.e. 28 years), whereas a source of $^{90}\text{Y}$ alone decays with the 64-hour half-life of $^{90}\text{Y}$. Radium needles and capsules containing long-lived $^{226}\text{Ra}$ with many decay products in secular equilibrium have been used widely for superficial and intracavitary radiation treatments.

### 1.3.2.4 Natural Radioactivity and Decay Series

Almost every radioactive nuclide found in nature is a member of one of three radioactive decay series. Each series consists of sequential transformations that begin with a long-lived parent and end with a stable nuclide. In a closed environment, products of intermediate transformations within each series are in secular equilibrium with the long-lived parent. These products decay with an apparent half-life equal to the half-life of the parent. All radioactive nuclides found in nature decay by emitting either $\alpha$ particles or beta ($\beta^-$) particles. Consequently, each transformation in a radioactive decay series changes the mass number of the nucleus either by 4 ($\alpha$ decay) or by zero ($\beta^-$ decay).

The uranium series begins with $^{238}\text{U}$ ($T_{1/2} = 4.5 \times 10^9$ years) and ends with stable $^{206}\text{Pb}$. The mass number 238 of the parent nuclide is divisible by 4 with remainder of 2. In fact, all members of the uranium series, including stable $^{206}\text{Pb}$, also possess mass numbers divisible by 4 with remainder of 2. Consequently, the uranium decay series is sometimes referred to as the “4$n$ + 2 series,” where $n$ is an integer between 51 and 59 (see Figure 1.22). The nuclide $^{226}\text{Ra}$ and its decay products are members of the uranium decay series. A sample of $^{226}\text{Ra}$ decays with a half-life of 1600 years. However, $4.5 \times 10^9$ years are required for the earth’s supply of $^{226}\text{Ra}$ to decrease to half-life because $^{226}\text{Ra}$ in nature is in secular equilibrium with its parent $^{238}\text{U}$.

Other radioactive series are the actinium or “4$n$ + 3 series” ($^{235}\text{U}$-$^{207}\text{Pb}$) and the thorium or “4$n$ series” ($^{232}\text{Th}$-$^{208}\text{Pb}$). Members of the hypothetical neptunium or “4$n$ + 1 series” ($^{241}\text{Am}$-$^{206}\text{Pb}$) are not found in nature because there is no long-lived parent for this series.

![Figure 1.21](image.png)

**Figure 1.21** Growth of activity and secular equilibrium of $^{222}\text{Rn}$ produced by the decay of $^{226}\text{Ra}$. 
Fourteen naturally occurring radioactive nuclides are not members of a decay series. These nuclides, all with relatively long half-lives, are $^3$H, $^{14}$C, $^{40}$K, $^{50}$V, $^{87}$Rb, $^{115}$In, $^{130}$Te, $^{138}$La, $^{142}$Ce, $^{144}$Nd, $^{147}$Sm, $^{176}$Lu, $^{187}$Re, and $^{192}$Pt.

### 1.3.3 Artificial Production of Radionuclides

Nuclides may be produced artificially in order to have properties desirable for clinical medicine, research, or other purposes. Nuclear transmutation by particle bombardment was first observed by Ernest Rutherford in 1919 in his studies of $\alpha$ particles traversing an air-filled chamber [12]. The observed transmutation was

$$ ^2\text{He} + ^{14}\text{N} \rightarrow ^{17}\text{O} + ^1\text{H}, $$

where He and H represent $\alpha$ particles and protons, respectively, detected during the experiment. This reaction can be written more concisely as $^{14}\text{N}(\alpha, p)^{17}\text{O}$, where $^{14}\text{N}$ is the bombarded nucleus, $^{17}\text{O}$ the product nucleus, and $(\alpha, p)$ the incident and ejected particles, respectively.

Nuclides with excess neutrons, which will therefore emit beta ($\beta^-$) particles, are created by bombarding nuclei with neutrons from a nuclear reactor. Examples include the following:

$$ ^{12}\text{N}(n, p)^{14}\text{C} $$

$$ ^{31}\text{P}(n, y)^{32}\text{P} $$

Isotopes produced by this method include the following:

$^3$H, $^{35}$S, $^{51}$Cr, $^{60}$Co, $^{92}$Mo, $^{133}$Xe, $^{198}$Au.

Nuclides with excess protons may be produced by bombarding nuclei with protons, $\alpha$ particles, deuterons, or other charged particles from high-energy particle accelerators. These nuclides then decay by positron decay or electron capture with the emission of positrons, photons, etc. Representative transitions include the following:

$$ ^{68}\text{Zn}(p, 2n)^{67}\text{Ga} $$

$$ ^{14}\text{N}(p, \alpha)^{11}\text{C} $$

Useful radioisotopes produced by particle accelerators include the following:

$$ ^{13}\text{N}, ^{15}\text{O}, ^{18}\text{F}. $$

Other nuclides are obtained as by-products of fission in nuclear reactors. The process

$$ ^{235}\text{U} + ^0\text{n} \rightarrow ^{A_1}\text{X} + ^{A_2}\text{Y} + \text{neutrons} $$

produces a range of isotopes X and Y, including the following:

$$ ^{90}\text{Sr}, ^{99}\text{Mo}, ^{131}\text{I}, ^{137}\text{Cs}. $$

Finally, some radionuclides are produced as decay products of isotopes created by the above methods. For example, $^{201}$Pb is formed by bombardment of $^{205}$Tl by protons:

$$ ^{203}\text{Tl} + p \rightarrow ^{201}\text{Pb} + 3\text{n} $$

$^{201}$Pb ($T_{1/2} = 9.4$ hours) decays by electron capture to the useful radionuclide $^{201}$Tl. Additional description of radionuclide production for nuclear medicine procedures is offered in Chapter 8.
1.4 Radiation Interactions with Matter

Medical imaging is made possible through the interaction of radiation with matter, either biological tissue or a detector material. Radiation–matter interactions follow the fundamental principles of physics such as the conservation of energy and momentum in the same manner as interactions in the macroscopic world.

In a radiation interaction, the radiation and the material with which it interacts may be considered as a single system. When the system is compared before and after the interaction, certain quantities will be found to be invariant or conserved in the interaction. One quantity that is always conserved in an interaction is the total energy of the system, with the understanding that mass is a form of energy. Other quantities that are conserved include momentum and electric charge. However, some quantities are not necessarily conserved during an interaction. For example, the number of particles may not be conserved because particles may be fragmented, fused, “created” (energy converted to mass), or “destroyed” (mass converted to energy) during an interaction.

Interactions may be classified as either elastic or inelastic. An interaction is elastic if the kinetic energies of the interacting entities are conserved during the interaction. If some energy is used to free an electron or nucleon from a bound state, kinetic energy is not conserved and the interaction is inelastic. Total energy is conserved in all interactions, but kinetic energy is conserved only in interactions designated as elastic.

### 1.4.1 Ionizing Radiation Interactions

When an electron is ejected from an atom the process is termed ionization and the atom is left in an ionized state. Hydrogen is the element with the smallest atomic number and requires the least energy (i.e. binding energy of 13.6 eV) to eject its K-shell electron. Radiation of energy less than 13.6 eV is termed nonionizing radiation because it cannot eject this most easily removed electron, but radiation with energy above 13.6 eV is not always ionizing. If electrons are not ejected from atoms but are merely raised to higher energy levels (i.e. outer shells), the process is termed excitation and the atom is said to be excited. Charged particles such as electrons, protons, and atomic nuclei are directly ionizing radiations because they can eject electrons from atoms through charged particle interactions. Photons (either X- or γ-rays) and neutrons can set charged particles into motion, but they do not produce significant ionization directly because they are uncharged. These radiations are said to be indirectly ionizing.

Energy transferred to an electron in excess of its binding energy appears as kinetic energy of the ejected electron. An ejected electron and the residual positive ion constitute an ion pair. An average energy of 33.85 eV, termed the W-quantity or \( W \), is expended by charged particles per ion pair produced in air \([13]\); however, the average energy required to remove an electron from nitrogen or oxygen (the most common atoms in air) is much less than 33.85 eV. The W-quantity includes not only the electron’s binding energy but also the average kinetic energy of the ejected electron and the average energy lost as incident particles excite atoms, interact with nuclei, and increase the rate of vibration of nearby molecules. On average, 2.2 atoms are excited per ion pair produced in air.

The specific ionization (SI) is the number of primary and secondary ion pairs produced per unit length of the incident radiation’s path. The specific ionization of α particles in air varies from about 30000 to 70000 ion pairs/cm, and the specific ionization of protons and deuterons is slightly less. This value is four orders of magnitude smaller for electrons. The linear energy transfer (LET) is the average loss in energy per unit length of path of the incident radiation. The LET is the product of the specific ionization and the W-quantity.

#### 1.4.1.1 Interactions of Electrons

As electrons (and positrons) traverse a substance, they interact with atoms of the material via excitations and ionizations and transfer their energy to the material in three different ways: scattering with other electrons, elastic scattering with nuclei, and inelastic scattering with nuclei. In the process, the path of the electron changes direction a number of times. The interactions that take place along the path of the particle is summarized as specific ionization (SI, ion pairs/cm) or as linear energy transfer (LET, keV/cm) along the straight-line continuation of the particle’s trajectory beyond its point of entry.

#### 1.4.1.1.1 Scattering by Electrons

Electrons traversing an absorbing medium transfer energy to electrons of the medium, thereby losing energy and deflecting at some angle with respect to their original direction. An electron receiving energy may be raised to a shell farther from the nucleus or may be ejected from the atom. The kinetic energy \( E_k \) of an ejected electron equals the energy of the impinging electron \( E \) minus the binding energy \( E_b \) of the electron in the atom:

\[
E_k = E - E_b. \tag{1.26}
\]

If the binding energy is negligible compared with the energy received by the impinging electron, then the interaction may be considered an elastic collision between free particles. If the binding energy is not negligible, then the interaction is considered inelastic.
Incident electrons and positrons are scattered by interactions with electrons with a probability that increases with the atomic number of the absorber and decreases rapidly with increasing kinetic energy of the incident particles. Low-energy electrons and positrons interact frequently with electrons of an absorber; the frequency of interaction diminishes rapidly as the kinetic energy of the incident particles increases [14].

Ion pairs are produced by electrons and positrons during both elastic and inelastic interactions. The specific ionization (ion pairs per meter [IP/m]) in air at standard temperature and pressure (STP, i.e., standard temperature &= 0\, ^\circ\text{C}, \text{standard pressure} &= 760 \text{ mm Hg}) may be estimated by

$$SI = \frac{4500}{(v/c)^2},$$ \hspace{1cm} (1.27)

for electrons and positrons with kinetic energies between 0 and 10 MeV, where \(v\) represents the velocity of an incident electron or positron and \(c\) represents the speed of light in vacuum \((3 \times 10^8 \text{ m/s})\).

Incident positrons combine with electrons in the absorbing medium after expending their kinetic energy. The particles annihilate each other, and their mass appears as electromagnetic radiation, usually two 0.51 MeV photons moving in opposite directions. These photons are termed annihilation radiation, and the interaction is referred to as pair annihilation. Pair production is the foundation of positron emission tomography (PET) discussed in Chapter 8.

### 1.4.1.1.2 Elastic Scattering by Nuclei

Electrons are deflected with reduced energy during elastic interactions with nuclei of an absorbing medium. The probability of elastic interactions with nuclei varies with \(Z^2\) of the absorber and approximately with \(1/E_k^2\), where \(E_k\) represents the kinetic energy of the incident electrons. The probability for elastic scattering by nuclei is slightly less for positrons than for electrons with the same kinetic energy.

Probabilities for elastic scattering of electrons by electrons and nuclei of an absorbing medium are about equal if the medium is hydrogen \((Z = 1)\). In absorbers with higher atomic number, elastic scattering by nuclei occurs more frequently than electron scattering by electrons because the nuclear scattering cross section, an expression of interaction probability, varies with \(Z^2\), whereas the cross section for scattering by electrons varies with \(Z\). Generally, the bigger the cross section, the higher the probability an interaction will occur. This is the same principle as the observation "the bigger the target, the easier it is to hit." The unit name was first used by American scientists during the Manhattan Project, the project in which the atomic bomb was developed during World War II. In 1950, the Joint Commission on Standards, Units, and Constants of Radioactivity recommended international acceptance because of its common usage in the United States [15].

### 1.4.1.1.3 Inelastic Scattering by Nuclei

An electron or positron passing near a nucleus may be deflected with reduced velocity; the interaction is inelastic if energy is released as electromagnetic radiation during the encounter. This radiated energy is known as bremsstrahlung, for the German word meaning braking radiation. A bremsstrahlung photon may possess any energy up to the entire kinetic energy of the incident particle. For low-energy electrons, bremsstrahlung photons are radiated predominantly at right angles to the motion of the particles. The angle narrows as the kinetic energy of the electrons increases.

The probability of bremsstrahlung production varies with \(Z^2\) of the absorbing medium, but the relative shape of the spectrum is independent of the atomic number of the absorber. A typical bremsstrahlung spectrum is illustrated in Figure 1.23.

The ratio of radiation energy loss (the result of inelastic interactions with nuclei) to the energy lost by excitation and ionization (the result of interactions with electrons) is approximately

$$\frac{\text{Radiation energy loss}}{\text{Ionization energy loss}} = \frac{E_k Z}{820},$$ \hspace{1cm} (1.28)

where \(E_k\) represents the kinetic energy of the incident electrons in MeV and \(Z\) is the atomic number of the

![Figure 1.23 Bremsstrahlung spectrum for a molybdenum target bombarded by electrons accelerated through 20 kV plotted as a function of wavelength in angstroms ( Å) = 10⁻¹⁰ m. Source: From Wehr and Richards 1960 [16]. Used with permission.](image-url)
absorbing medium. This ratio is important to the design of X-ray tubes.

1.4.1.2 Interactions of Heavy-Charged Particles

Protons, deuterons, \( \alpha \) particles, and other heavy-charged particles lose kinetic energy rapidly as they penetrate matter. Most of the energy is lost as the particles interact inelastically with electrons of the absorbing medium. The transfer of energy is accomplished by interacting electrical fields, and physical contact is not required between the incident particles and absorber electrons. It is apparent that \( \alpha \) particles produce dense patterns of interaction but have limited range. Deuterons, protons, and other heavy-charged particles also exhibit high SI and a relatively short range. The density of soft tissue (1 g/cm\(^3\)) is much greater than the density of air (1.29 \( \times \) \( 10^{-3} \) g/cm\(^3\)). Hence, \( \alpha \) particles of a few mega-electronvolts or less from radioactive nuclei penetrate soft tissue to depths of only a few microns. For example, \( \alpha \) particles from a radioactive source near or on the body penetrate only the most superficial layers of the skin.

The SI and LET are not constant along the entire path of charged particles traversing a homogeneous medium. The SI of 7.7 MeV for \( \alpha \) particles from \(^{214}\)Po is plotted in Figure 1.24 as a function of the distance traversed in air. The increase of SI near the end of the path of the particles reflects the decreased velocity of the \( \alpha \) particles. As the particles slow down, the SI increases dramatically because nearby atoms are influenced by the electromagnetic force for a longer period of time. The region of increased SI is termed the Bragg peak. The rapid decrease in SI beyond the peak is primarily due to the capture of electrons by slowly moving \( \alpha \) particles. Captured electrons reduce the charge of the \( \alpha \) particles and decrease their ability to produce ionization. The increased energy deposition of heavy particle at the end of their range is a favorable property for designing proton and charged particle-based radiation therapy techniques where the deposition of radiation energy (i.e. dose) can be more precisely targeted in a tumor while sparing the intermediary healthy tissues.

1.4.2 Indirectly Ionizing Radiation Interactions

Uncharged particles such as neutrons and photons (i.e. electromagnetic radiation) are also a form of ionizing process, though the ionization takes place indirectly. Photons of ionizing energy are of prime relevance to imaging as the primary agents of most imaging processes. Neutrons are a radiation safety concern in radiation treatment facilities, but they have also been explored for therapeutic applications (e.g. boron neutron capture therapy) as well as imaging (e.g. neutron-stimulated emission computed tomography).

1.4.2.1 Interactions of Neutrons

Neutrons may be produced by a number of sources, and the distribution of energies depends on the method of production. Slow, intermediate, and fast neutrons (see Table 1.9) are present within the core of a nuclear reactor. Neutrons with various kinetic energies are emitted by \(^{252}\)Cf, a nuclide that fissions spontaneously and has been encapsulated into needles and used experimentally for implant therapy. Neutrons are also produced in cyclotrons and other accelerators in which low-\( Z \) nuclei (e.g. \(^3\)H or \(^9\)Be) are bombarded by positively charged particles (e.g. nucleus of \(^1\)H, \(^2\)H, and \(^3\)H) moving at high velocities. The energy distribution of neutrons from these devices depends on the target material and on the type and energy of the bombarding particle.

As uncharged particles, neutrons interact primarily by “billiard ball” or “knock-on” collisions with absorber nuclei. A knock-on collision is elastic if the kinetic energy of the particles is conserved and the collision is inelastic if part of the kinetic energy is used to excite the nucleus. During an elastic knock-on collision, the energy transferred from a neutron to the nucleus is maximized if the mass of the nucleus equals the neutron mass. If the absorbing medium is tissue, then the energy transferred per collision is greatest for collisions of neutrons with nuclei of hydrogen because the mass of the hydrogen

<table>
<thead>
<tr>
<th>Type</th>
<th>Energy range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slow</td>
<td>0–0.1 keV</td>
</tr>
<tr>
<td>Intermediate</td>
<td>0.1–20 keV</td>
</tr>
<tr>
<td>Fast</td>
<td>20 keV–10 MeV</td>
</tr>
<tr>
<td>High energy</td>
<td>&gt;10 MeV</td>
</tr>
</tbody>
</table>

Table 1.9 Classification of neutrons according to kinetic energy
nucleus (i.e. a proton) is close to the mass of a neutron. Many molecules in tissue have hydrogen, and the cross section for elastic collision is greater for hydrogen than for other constituents of tissue. For these reasons, elastic collisions with hydrogen nuclei account for most of the energy deposited in tissue by neutrons with kinetic energies less than 10 MeV.

For neutrons with kinetic energy greater than 10 MeV, inelastic scattering also contributes to the energy lost in tissue. Most inelastic interactions occur with nuclei other than hydrogen. Energetic charged particles (e.g. protons or α particles) are often ejected from nuclei excited by inelastic interactions with neutrons.

### 1.4.2.2 Interaction of X- and γ-Radiation

When an X- or γ-ray impinges upon a material, there are three possible outcomes. The photon may be absorbed (i.e. transfer its energy to atoms of the target material) during one or more interactions, scattered during one or more interactions, or traverse the material without interaction. If the photon is absorbed or scattered, it is said to have been attenuated. If 1000 photons impinge on a slab of material, for example, and 200 are scattered and 100 are absorbed, then 300 photons have been attenuated from the beam and 700 photons have been transmitted without interaction.

The number of photons attenuated in a medium depends on the number traversing the medium. If all photons have the same energy (i.e. the beam is monenergetic) and if the photons are attenuated under conditions of good geometry (i.e. the beam is narrow and the transmitted beam contains no scattered photons), then the number of photons penetrating a thin slab of material of thickness \( x \) is

\[
I = I_0 e^{-\mu x},
\]

where \( \mu \) is the linear attenuation coefficient of the medium for the photons and \( I_0 \) represents the number of photons in the beam before the thin slab of matter is placed into position. The number \( I_{at} \) of photons attenuated (absorbed or scattered) from the beam is

\[
I_{at} = I_0 - I = I_0 - I_0 e^{-\mu x} = I_0(1 - e^{-\mu x}).
\]

The exponent \( e^{-\mu x} \) must be unitless. Therefore, the unit for \( \mu \) is cm\(^{-1}\) if the thickness \( x \) is expressed in cm.

The mean path length, sometimes termed the mean free path or relaxation length, is the average distance traveled by X- or γ-rays before interaction in a particular medium and it equals \( 1/\mu \). Occasionally, the thickness of an attenuating medium may be expressed in multiples of the mean free path of photons of a particular energy in the medium.

The probability that a photon traverses a slab of thickness \( x \) without interacting is \( e^{-\mu x} \). This probability is the product of probabilities that the photon does not interact by any of five interaction processes of coherent scattering, photoelectric absorption, Compton scattering, pair production, or photodisintegration as

\[
e^{-\mu x} = (e^{-\omega_0}) (e^{-\tau_0}) (e^{-\sigma_0}) (e^{-\kappa_0}) (e^{-\pi_0}),
\]

where coefficients \( \omega_0, \tau_0, \sigma_0, \kappa_0, \) and \( \pi_0 \) represent the attenuation coefficient associated with these five processes, respectively, as described below. Thus,

\[
\mu = \omega + \tau + \sigma + \kappa + \pi.
\]

At photon energies most commonly used in medical imaging coherent scattering, photodisintegration, and pair production are negligible and \( \mu \) is thus written as

\[
\mu = \tau + \sigma.
\]

In general, attenuation coefficients vary with the energy of the X- or γ-rays and with the atomic number of the absorber. Linear attenuation coefficients also depend on the density of the absorber. Mass attenuation coefficients, obtained by dividing linear attenuation coefficients by the density \( \rho \) of the attenuating medium, do not vary with the density of the medium:

\[
\mu_m = \frac{\mu}{\rho}, \quad \tau_m = \frac{\tau}{\rho}, \quad \sigma_m = \frac{\sigma}{\rho}, \quad \kappa_m = \frac{\kappa}{\rho}, \quad \pi_m = \frac{\pi}{\rho}.
\]

Mass attenuation coefficients usually have units of m\(^2\)/kg, although sometimes cm\(^2\)/mg are used. Total mass attenuation coefficients for air, water, sodium iodide, and lead are plotted in Figure 1.25 as a function of the energy of incident photons.

When mass attenuation coefficients are used, thicknesses \( \tau_m \) are expressed in units such as kg/m\(^2\), g/cm\(^2\), or mg/cm\(^2\). These units may seem unusual to express thickness but can be understood by recognizing that they describe the mass of a 1 m\(^2\) cross section of an absorber. That is, the amount of material traversed by a beam is indirectly measured by the mass per unit area of the slab. The thickness of the attenuating medium may also be expressed as the number of atoms or electrons per unit area. Symbols \( x_a \) and \( x_e \) denote thicknesses in units of atoms/m\(^2\) and electrons/m\(^2\). These thicknesses may be computed from the linear thickness \( x \) by

\[
x_a (\text{atoms/m}^2) = \frac{x m \rho \text{(kg/m}^3) N_a \text{(atoms/gram-atOMIC mass)}}{M \text{(kg/gram-atOMIC mass)}}
\]

\[
x_e = \frac{x p N_a}{M}.
\]
and

\[
x_e (\text{electrons/m}^2) = [x_a (\text{atoms/m}^2)] \times [Z(\text{electrons/atom})]
= x_a Z.
\] (1.36)

In these equations, \( M \) is the gram atomic mass of the attenuating medium, \( Z \) is the atomic number of the medium, and \( N_a \) is the Avogadro’s number (i.e. \( 6.02 \times 10^{23} \)), which is the number of atoms/g atomic mass. Total atomic and electronic attenuation coefficients \( \mu_a \) and \( \mu_e \), to be used with thicknesses \( x_a \) and \( x_e \), may be computed from the linear attenuation coefficient \( \mu \) by

\[
\mu_a (\text{m}^2/\text{atom}) = \frac{\mu (\text{m}^{-1}) M (\text{g/gram-atomic mass})}{\rho (\text{kg/m}^2) N_a (\text{atoms/gram-atomic mass})}
= \frac{\mu M}{\rho N_a}
\] (1.37)

and

\[
\mu_e (\text{m}^2/\text{electron}) = \frac{\mu_a (\text{m}^2/\text{atom})}{Z(\text{electrons/atom})}
= \frac{\mu_a}{Z}.
\] (1.38)

Therefore, the number \( I \) of photons penetrating a thin slab of matter may be computed with any of the following expressions:

\[
I = I_0 e^{-\mu x_a}
I = I_0 e^{-\mu_e x_a}
I = I_0 e^{-\mu_w x_m}
= I_0 e^{-\mu_e x_e}.
\]

### 1.4.2.2.1 Half-value Layer

The thickness of a slab of matter required to reduce the intensity (or exposure rate) of an X- or \( \gamma \)-ray beam to half its incident value is called the half-value layer (HVL) for the beam. The HVL describes the quality, or penetrating ability, of the beam. The HVL of a monoenergetic beam of X- or \( \gamma \)-rays in any medium is

\[
HVL = \frac{\ln 2}{\mu}
\] (1.39)

where \( \mu \) is the total linear attenuation coefficient of the medium for photons striking it, collectively called a beam of photons.

A narrow beam of monoenergetic photons is attenuated exponentially as described by Eq. (1.29). From this, an equivalent relationship can be found:

\[
\ln \frac{I}{I_0} = -\mu x
\] (1.40)

The logarithm of the number \( I \) of photons decreases linearly with increasing thickness of the attenuating slab. Hence, a straight line is obtained when the logarithm of the number of X- or \( \gamma \)-rays is plotted as a function of thickness.

If the measured transmitted beam contains scatter, termed broad-beam geometry, the falloff of the number of photons as a function of slab thickness would be less rapid because some photons that interacted in the material and are scattered would strike the detector and would be considered still part of the primary photon beam, i.e. unattenuated. A broad-beam measurement would change with orientation and size of the detector and has limited practicality, whereas narrow-beam geometry is more relevant in understanding how a detector or radiation shielding attenuates radiation.

When an X-ray beam is polyenergetic, as is the beam emitted from an X-ray tube, a plot of the number of photons remaining in the beam as a function of the thickness of the attenuating material it has traversed does not yield a straight line on a semilogarithmic plot (see Figure 1.26). This is because the attenuation of a polyenergetic beam cannot be represented by a single simple exponential equation with a single attenuation coefficient. Instead, it is represented by a weighted average of exponential equations for each of the different photon energies in the beam.

### 1.4.2.2.2 Filtration

In a polyenergetic beam, lower energy photons are more likely to be attenuated, and higher energy photons are more likely to pass through without interaction. Therefore, after passing through some attenuating material, the distribution of photon energies contained within the emergent beam is different than that of the beam.
1.4 Radiation Interactions with Matter

1.4.2.3 Energy Absorption and Energy Transfer

The attenuation coefficient $\mu$ (or $\mu_a$, $\mu_e$, and $\mu_m$) refers to total attenuation (i.e. absorption plus scatter). Sometimes, however, it is necessary to determine the energy absorbed in a material and not simply scattered from it. Here, the energy absorption coefficient $\mu_{en}$ is used, where $\mu_{en}$ is given by the expression

$$\mu_{en} = \frac{E_a}{\Delta\nu} \quad (1.42)$$

In this expression, $\mu$ is the attenuation coefficient, $E_a$ is the average energy absorbed in the material per photon interaction, and $\Delta\nu$ is the photon energy. Thus, the energy absorption coefficient is equal to the attenuation coefficient times the fraction of energy truly absorbed. Energy absorption coefficients may also be expressed as mass-energy absorption coefficients ($\mu_{en}^{\text{m}}$), atomic energy absorption coefficients ($\mu_{en}^{\text{a}}$), or electronic energy absorption coefficients ($\mu_{en}^{\text{e}}$) by dividing the energy absorption coefficient $\mu_{en}$ by the physical density, the number of atoms/m$^3$, or the number of electrons/m$^3$, respectively.

Energy absorption coefficients and attenuation coefficients for air, as a function of photon energy, are plotted in Figure 1.27.

1.4.2.4 Coherent Scattering: Ionizing Radiation

Photons are deflected or scattered with negligible loss of energy by the process of coherent or Rayleigh scattering.
Coherent scattering is sometimes referred to as classical scattering because the interaction may be completely described by methods of classical physics. The classical description assumes that a photon interacts with electrons of an atom as a group rather than with a single electron within the atom. Thus, the process can be considered as more of molecular scattering, where the photons interact with the molecules and not the individual electrons. In coherent scattering, photon are usually scattered in an approximately forward direction indicative of the molecular structure. Thus, they have been used extensively to study the crystalline structure and is being explored for imaging applications as well.

1.4.2.2.5 Photoelectric Effect

During a photoelectric interaction, the total energy of an X- or γ-ray is transferred to an inner electron of an atom. The electron is ejected from the atom with kinetic energy $E_k$, where $E_k$ equals the photon energy $h\nu$ minus the binding energy $E_b$ required to remove the electron from the atom:

$$E_k = h\nu - E_b.$$  \hspace{1cm} (1.43)

The ejected electron is called a photoelectron (see Figure 1.28).

The average binding energy is approximately 0.5 keV for K-shell electrons in soft tissue. Consequently, a photoelectron ejected from the K-shell of an atom in tissue possesses a kinetic energy about 0.5 keV less than the energy of the incident photon. Photoelectrons resulting from the interaction of low-energy photons are released approximately at a right angle to the motion of the incident photons. As the energy of the photons increases, the average angle between incident photons and released photoelectrons decreases (see Figure 1.29).

An electron ejected from an inner shell leaves a vacancy or hole that is filled immediately by an electron from an energy level farther from the nucleus. Only rarely is a hole filled by an electron from outside the atom. Instead, electrons usually cascade from higher to lower energy levels and produce a number of characteristic photons and Auger electrons with energies that, when added together, equal the binding energy of the ejected photoelectron. Characteristic photons and Auger electrons released during photoelectric interactions in tissue possess energy less than 0.5 keV. These lower energy photons and electrons are absorbed rapidly in surrounding tissue.

The probability of photoelectric interaction decreases rapidly as the photon energy increases. In general, the mass attenuation coefficient for photoelectric absorption varies roughly as $1/E^3$, where $E$ is the photon energy. Figure 1.30 shows the photoelectric mass attenuation coefficients of muscle and lead as a function of the energy

![Figure 1.28](image-url) Photoelectric absorption of a photon with energy $\nu$. The photon disappears and is replaced by an electron ejected from the atom; characteristic radiation and Auger electrons are emitted as electrons cascade to replace the ejected photoelectron.

![Figure 1.29](image-url) Electrons are ejected approximately at a right angle as low-energy photons undergo photoelectric interactions. As the energy of the photons increases, the angle between incident photons and ejected electrons decreases.
of incident photons. Discontinuities in the curve for lead are termed absorption edges and occur at photon energies equal to the binding energies of electrons in inner electron shells. Photons with energy less than the binding energy of K-shell electrons undergo photoelectric interactions only with electrons in the L shell or shells farther from the nucleus. Photons with energy equal to or greater than the binding energy of K-shell electrons interact predominantly with K-shell electrons. Similarly, photons with energy less than the binding energy of L-shell electrons interact only with electrons in M and more distant shells. That is, most photons undergo photoelectric interactions with electrons that have a binding energy nearest to, but less than, the energy of the photons. Hence, the photoelectric attenuation coefficient increases abruptly at photon energies equal to the binding energies of electrons in different shells.

Absorption edges for photoelectric attenuation in soft tissue occur at photon energies that are too low to be shown in Figure 1.30. Iodine and barium exhibit K-absorption edges at energies of 33 and 37 keV. Compounds containing these elements are routinely used as contrast agents in diagnostic radiology.

At all photon energies depicted in Figure 1.30, the photoelectric attenuation coefficient for lead ($Z = 82$) is greater than that for soft tissue ($Z_{\text{eff}} = 7.4$), where $Z_{\text{eff}}$ represents the effective atomic number of a mixture of elements (in a composite material, the effective atomic number is a weighted average of the atomic numbers of the different elements of which it is composed). As the photoelectric mass attenuation coefficient varies with $Z^3$, the knowledge of the effective atomic number is important. For example, the number of 15-keV photons absorbed primarily by photoelectric interaction in bone ($Z_{\text{eff}} = 11.6$) is approximately four times greater than the number of 15-keV photons absorbed in an equal mass of soft tissue with effective atomic number of 7.4: $(11.6/7.4)^3 = 3.8$. Selective attenuation of photons in media with different atomic numbers and different physical densities is the principal reason for the usefulness of low-energy X-rays for producing images in diagnostic radiology.

### 1.4.2.2.6 Compton (Incoherent) Scattering

X- and γ-rays with energy between 30 keV and 30 MeV interact in soft tissue predominantly by Compton scattering. During a Compton interaction, part of the energy of an incident photon is transferred to a loosely bound or free electron within the attenuating medium. The kinetic energy of the recoil (Compton) electron equals the energy lost by the photon, with the assumption that the binding energy of the electron is negligible. Although the photon may be scattered at any angle $\phi$, with respect to its original direction, the Compton electron is confined to an angle $\theta$, which is 90° or less with respect to the motion of the incident photon. Both $\theta$ and $\phi$ decrease with increasing energy of the incident photon (see Figures 1.31 and 1.32).

During a Compton interaction, the change in wavelength ($\Delta \lambda$ in nm) of the X- or γ-ray is

$$
\Delta \lambda = 0.00243(1 - \cos \phi).
$$

The Compton mass attenuation coefficient decreases with increase in photon energy (see Figure 1.33). The Compton attenuation varies directly with the electron density (electrons/kg) of the absorbing medium because Compton interactions occur primarily with loosely bound electrons. As such, Compton mass attenuation coefficient is nearly independent of the atomic
number of the attenuating medium. However, a medium with more unbound electrons will attenuate more photons by Compton scattering than will a medium with fewer electrons.

As Compton scattering is nearly independent of the atomic number of the attenuating medium, X-ray images exhibit very poor contrast when exposed to high-energy photons. When most of the photons in a beam of X- or γ-rays interact by Compton scattering, little selective attenuation occurs in materials with different atomic numbers. The image in a radiograph obtained by exposing a patient to high-energy photons is therefore not the result of differences in atomic number between different regions of the patient. Instead, the image reflects differences in physical density between the different regions (e.g. bone and soft tissue), as shown in Figure 1.34.

### 1.4.2.2.7 Pair Production

An X- or γ-ray may interact by pair production while near a nucleus in an attenuating medium. An electron and a positron appear in place of the photon. As the energy equivalent to the mass of an electron/positron is 0.51 MeV, the creation of the two requires a minimum of 1.02 MeV of energy. Consequently, photons with energy less than 1.02 MeV do not interact by pair production. This energy requirement makes pair production irrelevant to conventional radiographic imaging, but they are relevant to shielding considerations for high-energy radiation therapy systems. During pair production, energy in excess of 1.02 MeV is released as kinetic energy of the two electrons.

Although the nucleus recoils slightly during pair production, the small amount of energy transferred to the recoiling nucleus may usually be neglected. Pair production is depicted in Figure 1.35.

Occasionally, pair production occurs near an electron rather than near a nucleus. For 10-MeV photons in soft tissue, for example, about 10% of all pair production interactions occur in the vicinity of an electron. An interaction near an electron is termed triplet production because the interacting electron receives energy from the photon and is ejected from the atom. Three ionizing particles, two electrons and one positron, are released during triplet production. To conserve momentum, the threshold energy for triplet production must be 2.04 MeV. The ratio of triplet to pair production increases with the energy of incident photons and decreases as the atomic number of the medium is increased.

The mass attenuation coefficient $\kappa_m$ for pair production varies almost linearly with the atomic number of the attenuating medium but increases slowly with energy of the incident photons. In soft tissue, pair production accounts for only a small fraction of the interactions of X- and γ-rays with energy between 1.02 and 10 MeV. Positrons released during pair production produce annihilation radiation identical to that produced by positrons released from radioactive nuclei.

The relative importance of photoelectric, Compton, and pair production, as three principle interactions of photons with matter, is depicted in Figure 1.36. In muscle or water ($Z_{eff} = 7.4$), the probabilities of photoelectric
interaction and Compton scattering are equal at a photon energy of 35 keV. However, equal energies are not deposited in tissue at 35 keV by each of these modes of interaction, as all of the photon energy is deposited during a photoelectric interaction, whereas only part of the photon energy is deposited during a Compton interaction. Equal deposition of energy in tissue by photoelectric and Compton interactions occurs for 60-keV photons rather than 35-keV photons.

1.4.3 Nonionizing Electromagnetic Radiation

As mentioned previously, radiation with energy less than 13.6 eV is classified as nonionizing radiation. Two types of nonionizing radiations (electromagnetic waves and mechanical vibrations) are of interest. Electromagnetic waves are discussed here, and mechanical vibrations, or sound waves, are discussed in Chapter 9. As no ionization is produced by the radiation, quantities such as specific ionization, LET, and \( W \) do not apply. However, the concepts of attenuation and absorption are still applicable.

For those parts of the electromagnetic spectrum with energy less than X- and \( \gamma \)-rays, mechanisms of interaction involve a direct interplay between the electromagnetic field of the wave and molecules of the target material. The energy of the electromagnetic field and hence the

![Figure 1.34](image1.png) Radiographs taken at 70 keV, 250 keV, and 1.25 MeV (\(^{60}\)Co). The radiographic contrast diminishes as the energy of the incident photons increases.

![Figure 1.35](image2.png) Pair production interaction of a high-energy photon near a nucleus. Annihilation photons are produced when the positron and electron annihilate each other.

![Figure 1.36](image3.png) Relative importance of the three principal interactions of X- and \( \gamma \)-rays. The lines represent energy/atomic number combinations for which the two interactions on either side of the line are equally probable.
energy of the wave is diminished when electrons or other charge carriers are set into motion in the target. Energy that electrons absorb directly from the wave is referred to as conduction loss, whereas energy that produces molecular rotation is referred to as dielectric loss. These properties of a material are described by its conductivity $\sigma$ and dielectric constant $k_e$. The degree to which a material loses energy to electromagnetic waves (that is, the material absorbs energy and is therefore heated) is a complex function of the frequency of the waves [17].

Some generalizations about low-energy electromagnetic radiation are possible. For a given material, for example, the increase in conductivity with frequency tends to limit penetration of the radiation. Compared with lower frequency electromagnetic waves, waves of higher frequency tend to be attenuated more severely in biological materials. Conductivity and dielectric constants are shown in Figure 1.37 for typical biological tissues, with high water content, as a function of frequency.

The rate at which energy is deposited in a material by nonionizing radiation is described by the specific absorption rate (SAR) of the material, usually stated in units of W/kg. Calculation of the SAR involves consideration of the geometry of the target material as well as energy absorption parameters including conductivity, dielectric constant, and energy loss mechanisms. SARs in MRI pulse sequences are discussed in Chapter 10.

The interactions of nonionizing electromagnetic radiation vary greatly from one end of the electromagnetic spectrum to the other. In medical imaging, MRI exploits the interaction of electromagnetic waves with resonance absorption properties of nuclei in the body to form medical images (see Chapter 10). Other uses of nonionizing electromagnetic radiation, including microwave radiation, for medical imaging are currently under investigation.

1.5 Production of X-rays

To produce medical images with X-rays, a source is required that can

1. produce enough X-rays in a short time,
2. allow the user to vary the beam energy,
3. provide X-rays in a reproducible manner and
4. meet standards of safety and economy of operation.

X-ray tubes meet all the requirements mentioned above. X-ray tubes can be viewed as a special purpose particle accelerator in which bremsstrahlung and characteristic X-rays are produced as accelerated high-speed electrons are slowed by interactions in a target. Although the physical design of X-ray tubes has been altered significantly over a century, the basic principles of operation have not changed.

Early X-ray studies were performed with a cathode ray tube in which electrons liberated from residual gas atoms in the tube were accelerated toward a positive electrode (anode). These electrons produced X-rays as they interacted with components of the tube. Initial cathode ray tubes were unreliable and inefficient in producing X-rays. In 1913, William Coolidge improved the X-ray tube by heating a wire filament with an electric current to release electrons (called thermionic emission) [18]. The liberated electrons were repelled by the negative charge of the filament (cathode) and accelerated toward a positive target (the anode). X-rays were produced as the electrons struck the target. The Coolidge tube was the prototype for hot-cathode X-ray tubes in wide use today.

1.5.1 Conventional X-ray Tubes

Figure 1.38 shows the cross section of a modern X-ray tube and Figure 1.39 shows its main components. A heated filament releases electrons that are accelerated
1.5 Production of X-rays

The electrons are directed to a specific location on the anode, and this location is called the focal spot. The focal spot defines the physical size of the region on the target from which the majority of X-rays originate. Characterization of the geometric properties of the imaging study and subsequently the sharpness associated with anatomical boundaries in an image are partially defined by the size of the focal spot (see Section 1.5.1.2). Control of the focusing of the electrons onto the focal spot of the anode is accomplished by applying a negative charge to a focusing cup or cathode cup surrounding the cathode. The negative charge on the focusing cup deflects the electrons as they move from the cathode to the anode and causes them into a tight stream.

1.5.1.2 Anode

The anode functions as the positive electrode in the X-ray circuit. Placing a positive voltage on the anode causes the negatively charged electrons from the cathode to be accelerated to the focal spot. The anode also contains the target material that will define the characteristics of the X-ray spectrum (see Section 1.5.1.5). To ensure that an adequate amount of X-rays can be produced without damaging the X-ray tube, the anode must be a good conductor of heat to remove the large amount of thermal energy produced from the electron interactions. It must also be a good conductor of electricity so that electrons do not build up on the surface and repel the incoming electron stream. Since most of the energy in an X-ray tube is converted to heat, the anode material must also have a high melting point. Anode materials have a relatively high atomic number as the efficiency of X-ray production is proportional to atomic number. Therefore, the most important properties of the anode are

1. Good conductor of heat
2. Good conductor of electricity
3. High melting point
4. High atomic number
Because of the conduction properties, metals should be good for anode materials. Unfortunately, a lot of metals do not have a very high melting point. The fluorescent yield increases as the $Z$ of the material increases, so higher $Z$ materials are better for anode materials (see Figure 1.5). Some of the traditional target materials are gold ($Z = 79$), tungsten ($Z = 74$), and molybdenum ($Z = 42$). As tungsten has both a high $Z$ and a high melting point, it is the most used target material for X-ray tube targets.

For X-ray images of highest clarity (i.e., spatial resolution), electrons should be absorbed within a small focal spot (see Chapter 6). To achieve this, the electrons should be emitted from a small or fine filament. In addition, in clinical practice, images are expected to be acquired with the shortest exposure time so as to reduce the influence of voluntary or involuntary motion of the patient on image quality (i.e., temporal resolution). This effect can be decreased by using X-ray exposures of high intensity and short duration. However, these high-intensity exposures may require an electron emission rate that exceeds the capacity of a small filament. Consequently, many X-ray tubes have two filaments. A smaller, fine filament is used when the body region of interest is small and/or spatial resolution is a strong priority. The size of a focal spot utilized for diagnostic imaging can range from 0.1 mm for mammography X-ray tubes to over 2.0 mm for CT X-ray tubes. Typical diagnostic X-ray tubes for general imaging have a large 1.2 mm focal spot size and a small 0.6 mm focal spot size.

It is the physical size of the filament, the charge on the focusing cup, and the angle of the anode that determines the size of the focal spot on the anode surface. The target of an X-ray tube is mounted at a steep angle with respect to the direction of impinging electrons (see Figure 1.40). With the target at this angle, X-rays appear to originate within a focal spot much smaller than the volume of the target absorbing energy from the impinging electrons. This apparent reduction in size of the focal spot is termed the line focus principle. Most diagnostic X-ray tubes use a target angle between $6^\circ$ and $17^\circ$. In the illustration, side $a$ of the projected or apparent focal spot may be calculated by

$$a = A \sin \theta,$$

where $A$ is the corresponding dimension of the true focal spot and $\theta$ is the target angle.

Side $b$ of the apparent focal spot equals side $B$ of the true focal spot because side $B$ is perpendicular to the electron beam. However, side $B$ is shorter than side $A$ of the true focal spot because the width of a filament is always less than its length. However, in the center of the field of view, the apparent focal spot is usually approximately square.

As mentioned earlier, dual-focus (i.e., focal spot) diagnostic X-ray tubes furnish two apparent focal spots, one for fine focus imaging (e.g., 0.6-mm$^2$ or less in radiography) produced with a smaller filament and another for coarse focus imaging (e.g., 1.2-mm$^2$ in radiography) produced with a larger filament. The apparent focal spot to be used is determined by the desired electron flow (tube current) between the filament and target. The small filament is used when a low tube current is satisfactory. The coarse filament is used when roughly twice the low tube current is required to reduce exposure time. Apparent focal spots of very small dimensions (e.g., 0.1 mm) are available with certain X-ray tubes used for specialized radiography.

For most X-ray tubes, the size of the focal spot is not constant. Instead, it varies with both the tube current and the voltage applied between the filament and the target of the X-ray tube [19, 20]. This influence is shown in Figure 1.41 for the dimension of the focal spot parallel to the motion of impinging electrons. On the left, the growth or “blooming” of the focal spot with tube current is illustrated. The gradual reduction of the same focal spot dimension with increasing levels of peak voltage is shown on the right. In most advanced X-ray tube designs, such as those used in CT imaging, magnetic fields are deployed to focus the electron beam and maintain a consistent focal spot size.

Electrons accelerated from the filament to the target are absorbed in the first 0.5 mm of target. In this distance, a typical electron will experience 1000 or more interactions with target atoms. Low-energy X-rays...
generated in a tungsten target are highly attenuated during their escape from the target. For targets mounted at a small angle, the attenuation is greater for X-rays emerging along the anode (target) side of the X-ray beam than for those emerging along the side of the beam nearest the cathode (filament). Consequently, the X-ray intensity decreases from the cathode to the anode side of the beam. This variation in intensity across an X-ray beam is termed the heel effect and is demonstrated in Figure 1.42. This effect is noticeable for X-ray beams used in diagnostic radiology, particularly for those generated at low voltage, because the X-ray energy is relatively low and the target angles are steep. To compensate for the heel effect, a filter may be installed in the tube housing near the exit portal of the X-ray beam, the thickness of which increases from the anode to the cathode side of the beam, to decrease the flux from the cathode side. Furthermore, positioning thicker portions of a patient near the cathode side of the X-ray beam also helps to compensate for the heel effect.

The heel effect increases with the steepness of the target angle. This increase limits the maximum useful field size obtainable with a particular target angle. For example, a target angle no steeper than 12° is recommended for X-ray examinations using 14 × 17-in. detector at a 40-in. distance from the X-ray tube, whereas target angles of 7° may be used if field sizes no larger than 10 × 10 in. are required at the same distance.

1.5.1.3 Tube Voltage and Voltage Waveforms

The intensity and energy distribution of X-rays emerging from an X-ray tube are influenced by the potential difference (voltage) between the filament and the target of the tube. The tube voltage (in kV) dictates the maximum energy of X-rays generated by an X-ray tube. Higher voltages (and thus energies) are used and required for larger body parts that need more penetration. Lower voltages and energies increase the material differentiation ability of the resultant images. For a given X-ray tube, the maximum allowable voltage is governed by the heat capacity of the tube. The lowest voltage is dictated by the required potential to efficiently pull the generated electrons from the cathode. If the voltage is too low, the electrons form an electronic cloud between the cathode and the anode, the so-called space charge, severely limiting the
movement of electrons from cathode to anode and the production of X-rays.

The source of electrical power for radiographic equipment is usually alternating current (AC). This type of electricity is by far the most common form available for general use because it can be transmitted with little energy loss through power lines that span large distances. X-ray tubes are designed to operate at a single polarity, with a positive target (anode) and a negative filament (cathode). X-ray production is most efficient (more X-rays are produced per unit time) if the potential of the target is always positive and if the voltage between the filament and the target is kept at its maximum value.

In most of today’s X-ray equipment, AC is converted to direct current (DC), and the voltage between the filament and the target is kept at or near its maximum value. This conversion of AC to DC, called rectification, is done at the generator that is supplied with the X-ray tube. The X-ray tubes and generators of higher quality generally offer a more stable DC voltage and thus more stable and manageable X-ray beam.

1.5.1.4 Heat Management

The efficiency of X-ray production is the ratio of energy emerging as X-rays from the target material divided by the energy deposited by electrons impinging on the target. The rate at which electrons deposit energy in a target is termed the power deposition $P_d$ (in W) and is given by

$$P_d = VI,$$

(1.46)

where $V$ is the tube voltage in V (or in kV), and $I$ is the tube current in A (or in mA). X-ray production is a very inefficient process. For X-ray tubes operated at conventional voltages, less than 1% of the energy deposited in the target appears as X-rays because almost all the energy delivered by impinging electrons is converted to heat within the target. As a result, a significant component of an X-ray tube design is devoted to managing the generated heat and prevent it from melting the anode. The angulation of the anode is in fact a strategy to enable spreading the heat of the bombarding electrons into a larger area of the anode while still managing a small focal spot. Another strategy is rotating the anode and using a large mass for the anode to spread the heat. Dissipation of the heat out of the tube through special ball-bearings and an oil envelope around the tube are also key features of modern X-ray tubes.

The heat dissipation and management of an X-ray tube determines how frequently a tube be used and how much time gap might be needed between subsequent image acquisitions. Those limitations are specified for a given X-ray tube through its specified heating chart. Figure 1.43 shows an example indicating that for a single
exposure of a given kV, mAs, and focal spot size, how much wait might be necessary to dissipate the generated heat before a second exposure can be initiated. The timing limitations of X-ray tubes are usually programmed in modern generator to prevent unintentional damage to the tube.

1.5.1.5 X-ray Spectra
As the electrons strike the anode, they produce electromagnetic X-rays that encompass a wide range of energies. The energy distribution of the photons, the so-called X-ray spectrum, is governed by four factors:

1. The energy of the bombarding electrons as a function of tube voltage.
2. Bremsstrahlung X-rays due to the gradual stopping of electrons in the target, representing a range of energies even though electrons of a single energy bombard the target.
3. X-rays are released as characteristic X-rays associated with the target material, so long as the threshold energy for characteristic X-ray emission in the target material is exceeded.
4. Self-absorption of X-rays in the target. X-rays are produced at a range of depths in the target material of the X-ray tube. These X-rays travel through different thicknesses of the target and lose energy through one or more interactions.

The distribution of photon energies produced by a typical X-ray tube is shown in Figure 1.44. The maximum energy of the spectrum is determined by the energy of the electrons. The spread is formed and influenced by the above factors. Although these factors dictate the inherent spectrum emerging from an X-ray tube, the spectrum can be manipulated by changes in tube filtration, target material, peak tube voltage, current, and exposure time.

1.5.1.5.1 Effects of Filtration
An X-ray beam traverses several attenuating materials before it reaches the patient, including the glass envelope of the X-ray tube, the oil surrounding the tube, and the exit window in the tube housing. These attenuators are referred to collectively as the inherent filtration of the X-ray tube (Table 1.10). The aluminum equivalent for each component of inherent filtration is the thickness of aluminum that would reduce the exposure rate by an amount equal to that provided by the component. The inherent filtration is approximately 0.9-mm Al equivalent for the tube described in Table 1.10, with the glass envelope contributing most of the inherent filtration. The inherent filtration of most X-ray tubes is about 1-mm Al.

As mentioned earlier, the probability that incident X-rays undergo a photoelectric interaction in a given medium varies roughly as $1/E^3$, where $E$ is the energy of the incident photons (see Section 1.4.2.2.5). That is, low-energy X-rays are attenuated to a greater extent than those of high energy. After passing through a material, an X-ray beam has a higher average energy per photon (termed a harder X-ray beam), even though the total number of photons in the beam has been reduced, because more low-energy photons than high-energy photons have been removed from the beam. Therefore, inherent filtration of an X-ray tube hardens the X-ray beam equivalent.

Table 1.10 Contributions to inherent filtration in typical diagnostic X-ray tube

<table>
<thead>
<tr>
<th>Component</th>
<th>Thickness (mm)</th>
<th>Aluminum-equivalent thickness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass envelope</td>
<td>1.40</td>
<td>0.78</td>
</tr>
<tr>
<td>Insulating oil</td>
<td>2.36</td>
<td>0.07</td>
</tr>
<tr>
<td>Bakelite window</td>
<td>1.02</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Source: Data from Trout [21].
beam. Additional hardening may be achieved by purposefully adding filters of various composition to the beam. The total filtration in the X-ray beam is the sum of the inherent and added filtration. Usually, additional hardening is desirable because it removes low-energy X-rays that, if left in the beam, would increase the radiation dose to the patient without contributing substantially to image formation, as shown in Figure 1.44.

The filtered spectra for a tungsten target X-ray tube are shown in Figure 1.45 for various thicknesses of filtration through human tissue. The spectra are normalized to the same number of photons. The added filtration decreases the total number of photons but also increases the average energy of photons in the beam, reflected in the overall shift of the peak of the spectrum toward higher energies.

**1.5.1.5.2 Tube Voltage**

As the energy of the electrons bombarding the target increases, the high-energy limit of the X-ray spectrum increases correspondingly. The height of the spectrum also increases with increasing tube voltage because the efficiency of bremsstrahlung production increases with electron energy (see Figure 1.46).

**1.5.1.5.3 Tube Current and Time**

The product of tube current in milliamperes (mA) and exposure time in seconds (s) dictate the total number of electrons bombarding the target. Other factors being equal, more X-rays are produced if more electrons bombard the target of an X-ray tube. Hence, the number of X-rays produced is directly proportional to the product of tube current and exposure time (i.e. mAs). Spectra from the same X-ray tube operated at different values of mAs are shown in Figure 1.47. The overall shape of the spectrum (specifically, the upper and lower limits of energy and the position of characteristic peaks) remains unchanged. However, the height of the spectrum and the area under it increase with increasing mAs. These increases reflect the greater number of X-rays produced at higher values of mAs.

**1.5.1.5.4 Target Material**

The choice of target material in an X-ray tube affects the efficiency of X-ray production and the energy at which characteristic X-rays appear. If technique factors (i.e. tube voltage, milliamperage, and time) are fixed, a target material with a higher atomic number (Z) will produce more X-rays by the process of bremsstrahlung. With the $P_d = VI$ of an X-ray tube, the rate at which energy is released as X-rays [20], termed the radiated power $P_r$, is

$$P_r = 0.9 \times 10^{-9} Z V^2 I,$$

(1.47)

where $P_r$ is in watts and $Z$ is the atomic number of the target. Hence, the efficiency of X-ray production is

$$\text{Efficiency} = \frac{P_r}{P_d} = \frac{0.9 \times 10^{-9} Z V^2 I}{VI} = 0.9 \times 10^{-9} ZV.$$
The characteristic radiation produced by an X-ray target is usually dominated by one or two peaks with specific energies slightly less than the binding energy of the K-shell electrons. The most likely transition involves an L-shell electron dropping to the K-shell to fill a vacancy there. This transition yields a photon of energy equal to the difference in electron binding energies of the K and L shells. A characteristic X-ray released during the transition of an electron between adjacent shells is known as a Kα ray. For example, a Kα X-ray is one produced during transition of an electron from the L to the K-shell. A Kβ X-ray is an X-ray produced by an electron transition among nonadjacent shells. For example, a Kβ X-ray reflects a transition of an electron from the M- to the K-shell. A characteristic photon with energy equal to the binding energy of the K-shell alone is produced only when a free electron from outside the atom fills the vacancy. The probability of such an occurrence is vanishingly small.

Most X-ray tubes use tungsten as their target material, thanks to its high X-ray production yield, high melting point, and rigidity. In addition, filters are used that primarily reduce low-energy radiation. However, the combination of target and filter material used in an X-ray tube can allow more flexibility in modifying the spectrum produced. This is demonstrated in Figure 1.48 where the X-ray spectrum from a molybdenum (Mo) target is shown before and after it has passed through a filter material made also of Mo. Note how the low energies are attenuated in a similar pattern as seen with the tungsten target and aluminum filtration (see Figure 1.44). However, above the K-shell binding energy for Mo, there is a large increase in attenuation that removes a large percentage of the X-rays with energies at this value. Note that the characteristic X-rays that are produced in the Mo target material are not attenuated much. This is because the attenuation curve for Mo is at a minimum value just below its K-shell binding energy and the characteristic X-rays that were formed in the Mo target are at energies that are the difference between the K-shell and the L-shell binding energies, i.e. they are below the energy where increased attenuation occurs. Because the attenuation curve increases dramatically at the K-shell binding energies, this type of filtration is called "K-edge filtration." By using K-edge filtration in this manner, the spread of the energy of spectrum can be limited. Further refinement of the energy spectrum can be done by using other filter materials with different K-shell binding energies.

1.6 Radiation Detectors

An imaging system, apart from its source of energy, requires a mechanism for the capture of that energy for
the purpose of image formation. All image capture receptors used for imaging are, in a sense, radiation detectors. As imaging involves the capture of varying spatial patterns of radiation intensity, a multiplicity of radiation detectors are combined into a form factor that constitutes the imaging field of view. The specific properties of image receptors used for specific imaging modalities are discussed in chapters where the individual imaging techniques are covered. In this section, we review the general principles of imaging detectors used for ionizing radiation-based modalities as well as those of radiation detectors used to make quantitative measurement of radiation for dosimetry and monitoring purposes. Energy receptors used on modalities that do not use ionizing radiation, namely ultrasound and MRI, are covered in their respective Chapters 9 and 10.

### 1.6.1 General Detector Properties

Radiation detectors are used for either point measurements, of specific value in radiation protection and monitoring applications, or for area measurements. In the latter, the detector encompasses an array of point detectors, thus forming an image sensor. Generally speaking, a radiation detector, particularly the one used for imaging purposes, need to have three desirable attributes: (i) high signal from a high-efficiency photon absorption with minimal energy loss, (ii) low noise in either form of quantum noise from the limited number of quanta detected and negligible instrument noise, and (iii) in the case of imaging detectors, high resolution through having small detection elements and minimal signal blur across the elements. These attributes form the essential properties and requirements expected from an effective radiation detector device. How these attributes pertain to the properties in image production are covered in Chapter 3.

The first two requirements directly relate to the radiation absorption in the detector. X-ray and γ-ray absorption in the detector varies significantly with the energy of the incident radiation, the material of the detectors, and the active volume of the absorption. Lower photon energy, higher atomic number, and higher volume and density of the absorption element of the detector lead to higher detector signal and potentially lower detector noise. X-ray and γ-ray interaction in the detector is through either photoelectric or Compton interactions with subsequent secondary radiation production effecting the total energy deposition. For each incident X-ray, a sequence of radiation transport events (cascade) results in the production of numerous electrons. Figure 1.49 offers an example of such a cascade. For a set of X-ray photons of a given energy incident on a detector, there will be a spectrum of deposited energies reflecting the photoelectric interaction where all the photon energy is deposited in the detector, interactions where characteristic X-ray produced escape from the detector, Compton interactions where the spectrum of absorbed energies varies based on the scatter angle, and characteristic X-ray peaks that do not escape the detector. This is reflected in the example of Figure 1.50.

The radiation energy deposited in the detector, $S_e$, is converted to electrical charge, $q_e$, often collected in an associated capacitor and voltage as

$$q_e = \frac{S_e}{\varepsilon_e} \quad \text{and} \quad S_v = q/C,$$  \hspace{1cm} (1.49)

where $S_v$ is the detector signal in terms of energy in units of eV, $\varepsilon_e$ is the energy required to release an electron in...
The detector, $S$, is the detector signal in volts, $q$ is the charge in coulombs equaling $1.602 \times 10^{-19} \, q_e$, $C$ is the capacitance in farads, and $q_e$ is the released electrons in the detector.

The example of Figure 1.50 reflects how even a monoenergetic photon can leave a broad spectrum in the detector. However, this spectrum is still further altered; the charge deposited in a detector may vary because of the statistical fluctuations in the number of electrons produced, $q_e$, for a specific energy deposition $E$. The dispersion of $q_e$ values resulting from an energy deposition, $e$, can be described by Poisson statistics, as illustrated in Figure 1.51. As a result, the idealistic spectrum of energy absorption of Figure 1.50 takes a more blurred form of Figure 1.52.

The overall probability for producing a charge $q_e$ by radiation of energy $E$ is essentially the convolution of the energy deposition probability, $p(e,E) \, de$, and the charge dispersion probability, $p(q_e,e) \, dq_e$ as

$$p(q_e,E) \, dE = \int_0^E p(q_e,e) \, p(e,E) \, de \, dE. \quad (1.50)$$

For monoenergetic radiation of energy $E_i$, the charge signal is deduced from the integration of the charge production probability as

$$Q_e = N_i \int_0^{q_{max}} p(q_e,E_i) \, dq_e = N_i \bar{Q}_{E_i}. \quad (1.51)$$

This is equivalent to considering the average deposited charge from the discrete sum of all events as

$$Q_e = N_i \sum_{n=1}^{N_i} \frac{q_n}{N_i}. \quad (1.52)$$

The deposited charges in the detector, reflected in the detector voltage signal, is processed through a preamplifier circuit that amplifies and often digitizes the

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**Figure 1.49** A cascade of energy deposition and transfer ensuing a photoelectric absorption in a detector.

**Figure 1.50** Probability of energy deposition from the absorption of the photons of a single energy in a detector.

**Figure 1.51** The dispersion of energy deposition in a detector because of statistical fluctuations in the produced electrons.
signal using an analog to digital converter (ADC). This produces the so-called “raw” image values. The preamplifiers typically use a nonlinear, often logarithmic, process so that the raw image values can represent a wide range of exposures. Alternatively, a nonlinear input Look Up Table (LUT) can transform the ADC values. In the logarithmic preamplification, small relative changes in exposure due to small tissue structures produce a fixed change in raw values regardless of the total tissue transmission. For example, the detector raw value, $Q$, can be related to the radiation exposure, $X$, as $Q = 1000\log_{10}(X) + \text{constant}$. One major manufacturer uses preamplifiers that make the raw values proportional to the square root of exposure as $Q = 1250X^{1/2}$. This makes the relative noise in $Q$ constant for all incident exposure values.

### 1.6.2 Gas-Filled Detectors

Gas-filled detectors form the simplest type of radiation sensing. Ionizing radiation generates ion pairs as it deposits its energy in a medium. Using a gas as the attenuating medium, gas-filled detectors collect the ion pairs by charged electrodes. The ion pairs migrate toward the charged electrodes with a drift velocity that depends on the type and pressure of the gas between the electrodes and on the potential difference and distance between the electrodes. Depending on the voltage between the electrodes, a gas-filled detector may operate in three different regions: ionization chamber, proportional counter, and Geiger–Müller (G-M) counter.

Figure 1.53 illustrates the number of ion pairs produced in a gas-filled chamber as a function of the voltage applied across the electrodes. At very low voltages, the electrons and positive ions are not strongly attracted to the electrodes and some of the ion pairs are lost because of recombination. The attraction for ion pairs increases with the voltage between the electrodes and the fewer ion pairs recombine. When the voltage between the electrodes exceeds the saturation voltage, the electrodes collect all ion pairs produced by the radiation. The saturation voltage for a particular ionization chamber depends on the design of the chamber, the shape and spacing of the electrodes, and the type and pressure of the gas in the chamber. No increase in ionization current is observed as the electrode voltage is raised a few hundred volts above saturation because all ion pairs produced by the radiation are collected. This region of voltage is referred to as the ionization chamber plateau. When the voltage between the electrodes is increased just to the saturation level at which all ion pairs produced by the impinging radiation are collected, the chamber is called an ionization chamber.

The ionization current increases abruptly at the end of the saturation plateau. This amplification of the signal reflects the production of additional ion pairs as electrons liberated by the incident radiation gain energy on their way to the anode. This amplification is generally proportional to the magnitude of the initial ionization, and the chamber is considered a proportional counter. In the proportional region, the amount of charge collected by the electrodes increases with the number of ion pairs initially produced by the impinging radiation. Consequently, the size of the signal from a proportional chamber increases with the amount of ionization produced by radiation that traverses the chamber.
In both ionization chamber and proportional counter regions, at any particular voltage, the ionization current produced by an α-emitting sample is much greater than that produced by a sample that emits β particles, so the reduced signal for the β-emitting sample reflects the reduced ionization produced by β particles.

At voltages beyond proportion region, the chamber goes into a limited proportionality region and eventually chamber reaches a region where the additional produced ion pairs are no longer proportional to the pairs generated by impinging radiation. When the magnification is increased to this saturation level, the detector is known as a G-M counter, which only reflect the presence of the radiation quanta and not their energy. At voltages beyond the G-M region, the chamber discharges.

1.6.2.1 Ionization Chamber

In ionization chambers, the electrodes receive only ion pairs that result directly from interactions of ionizing radiation with gas in the chamber. An ionization chamber generally uses a parallel plate or coaxial design in a volume occupied by a filling gas. A typical coaxial chamber is composed of a central electrode in the form of a straight wire or wire loop that is positively charged with respect to the surrounding cylindrical case (see Figure 1.54). The entrance of radiation into the chamber results in an electric current or voltage pulses produced as ion pairs are collected by the electrodes. Ionization chambers are usually operated in the pulse mode for the detection of α particles; other radiation with lower specific ionization are usually measured by operation of the chamber in the current mode.

The operation of the ionization chamber in pulse mode is perhaps best illustrated with an example. Consider a 1.75-MeV α particle traversing the collecting volume of an ionization chamber. If the gas in the collecting volume is air, then the α particle loses an average energy of 33.85 eV for each ion pair (IP) produced. If the kinetic energy of the α particle is dissipated completely within the collecting volume, then \(1.75 \times 10^6 \text{ eV}/(33.85 \text{ eV/IP}) = 5 \times 10^8\) ion pairs are produced. Electrons liberated by the radiation migrate rapidly to the central electrode (anode) and reduce the positive charge of this electrode. Usually, electrons are collected within a microsecond after their liberation, but the heavier, positively charged ions migrate more slowly toward the negative case (cathode) of the chamber. As the positive ions approach the case, they induce a negative charge on the case that masks the total reduction in voltage between the electrodes. Hence, the total reduction in voltage between the electrodes is not attained until all positive ions within the chamber have been neutralized. Usually, a few 100 ms are required to neutralize the positive ions. In most pulse-type ionization chambers, only that portion of the reduction in voltage that is created by the collection of electrons is utilized in forming a voltage pulse.

In a pulse-type chamber designed to produce voltage pulses by the rapid collection of electrons, no interference with the migration of these electrons can be tolerated. Gases such as oxygen, water vapor, and the halogens have an affinity for electrons and so should not be present in the collecting volume of an ionization chamber. Other gases such as helium, neon, argon, hydrogen, nitrogen, carbon dioxide, and methane rarely form negative ions by combining with electrons, so these gases have been used to fill ionization chambers.

The ionization chamber is used in the pulse mode primarily for the detection of α particles, which expend their entire kinetic energy by interacting with the gas in the collecting volume of an ionization chamber. β particles (electrons and positrons), with an average kinetic energy lower by orders of magnitude, usually strike the wall of the chamber before dissipating all of their kinetic energy. For these reasons, voltage pulses produced as β particles traverse an ionization chamber are much smaller than those produced by α particles. Pulses produced by X- and γ-rays are even smaller, necessitating the use of the chamber in the current mode.

In the more common, current mode operation of an ionization chamber, electrons collected by the anode of an ionization chamber constitute a DC that may be amplified and measured with a conventional DC meter. In general, this approach is unsatisfactory because instability and measurement offset are introduced by the DC amplifier, affecting the accuracy of the measurements. However, small currents from an ionization chamber may be measured more accurately with an electrometer, which converts the signal from the ionization chamber into an AC that may be amplified with an AC amplifier. An AC amplifier is not subject to the problems of instability and zero drift encountered with a DC amplifier. The amplified AC may then be measured to within ±0.05% precision by one of two methods, referred to as the voltage-drop method and the rate-of-charge method.

With the voltage-drop method, the amplified AC is rectified and directed through a precision resistance;
the voltage developed across the resistance is proportional to the current. With the rate-of-charge method, the current is rectified and collected by a precision capacitor; the rate of collection of electrical charge on the capacitor plates is proportional to the current. The rate-of-charge method may be used with currents smaller than those measured by the voltage-drop method. The voltage-drop method without current amplification is used in portable survey meters such as the “cutie pie” (see Figure 1.55).

Ionization chambers are typically used to detect radiation from low-activity radioactive sources or serve as so-called free-air ionization chambers to calibrate X- and γ-ray beams [22].

Radiation from solid, liquid, and gaseous samples may be measured with an ionization chamber and electrometer. The activity of liquid samples prepared for administration to patients is often determined by placing the vial or syringe containing the sample into a well-type ionization chamber referred to as an isotope calibrator (see Figure 1.56). Gaseous samples may be counted by filling an ionization chamber with the radioactive gas. For example, ionization chambers may be used to measure the amount of $^{14}$CO$_2$ in air expired by patients metabolizing compounds labeled with $^{14}$C.

Portable survey instruments are frequently used in nuclear medicine to monitor exposure rates in the vicinity of radioactive sources and patients receiving therapeutic quantities of radioactive material. A neutron detector may be constructed by filling an ionization chamber with BF$_3$ gas or by coating the wall of an ionization chamber with lithium or boron. Ionization is produced within the chamber by α particles and recoil nuclei that are liberated during interactions of neutrons with the lithium or boron.

### 1.6.2.2 Proportional Counters

The small signals from an ionization chamber must be greatly amplified before they are measured. Because of the introduction of electronic noise and instability, amplification by electrical circuits is difficult to achieve without distortion of the signal. This problem is reduced if the signal is amplified within the counting volume of the chamber. If the potential difference between the electrodes of a chamber is raised beyond a certain voltage, electrons liberated by radiation traversing the chamber are accelerated to a velocity great enough to produce additional ionization (by an order of 2–4), mostly near the anode of the chamber. This process is referred to as gas amplification or the Townsend effect.

The amplification factor for a chamber is the ratio of the total number of ion pairs produced within the chamber to the number liberated directly by radiation entering the chamber; it depends on the construction of the chamber and the type of gas enclosed within the chamber. The amplification factor varies from $10^2$ to $10^4$ for most proportional counters. As an example, for a smaller number ($10^3$ to $10^5$) of ion pairs produced directly by radiation entering the chamber, many ($10^6$ to $10^7$) electrons and positive ions are collected by the electrodes. This provides a signal in the millivolt range that requires only a small amount of external amplification. However, in proportional counter, the voltage between electrodes must be regulated closely because the amplification factor is greatly affected by small changes in voltage.
1.6 Radiation Detectors

Proportional counters have a rapid response to ionizing events, as such a type of these counters known as multiwire proportional chambers has been considered as imaging sensors for nuclear medicine and for radiography [23, 24]. By raising the pressure of the counting gas to 10 atm or more, the sensitivity of the chamber to X- and γ-rays can be improved. The sensitivity can also be improved by placing high-Z foils in front of the chamber to convert incoming photons to photoelectrons and Compton electrons.

At voltages higher than the proportional region, α particles and other densely ionizing radiations initiate ionization of most of the atoms of gas in the vicinity of the anode. If the chamber is operated at a voltage in this region, then the number of ion pairs collected is not strictly proportional to the ionization produced directly the radiation. Therefore, this voltage region is referred to as the region of limited proportionality. Unsurprisingly, proportional chambers are not operated routinely in the region of limited proportionality.

1.6.2.3 Geiger–Müller Tubes

If the potential difference between the electrodes of a gas-filled detector exceeds the region of limited proportionality, then the interaction of a charged particle or X- or γ-ray within the chamber initiates an avalanche of ionization, which represents almost complete ionization of the counting gas in the vicinity of the anode. Because of this cascade process, the number of ion pairs collected by the electrodes is independent of the amount of ionization produced directly the impinging radiation. This means the voltage pulses (usually 1–10 V) emerging from the detector are similar in size and independent of the type of radiation that initiates the signal. The range of voltage where this is true is referred to as the G-M region. For detectors operating in this voltage region, the amplification factor is $10^6$–$10^8$.

In Figure 1.57, the number of pulses (or counts) recorded per minute is plotted as a function of the voltage across the electrodes of a G-M detector exposed to a radioactive source. No counts are recorded if the voltage applied to the detector is raised beyond the G-M plateau. This region of voltage if referred to as the region of spontaneous discharge or region of continuous discharge because the counting gas may be ionized even in the absence of radiation. Note that a G-M tube may be permanently damaged by the application of voltages higher than the G-M plateau.

A few G-M detectors are illustrated in Figure 1.58. The anode is a thin wire of tungsten or stainless steel in the center of the detector and is surrounded by a metal or glass cathode that is coated internally with a conducting layer of graphite or evaporated metal. The efficiency of a G-M detector for high-energy X- and γ-rays may be increased by coating the cathode with a heavy metal such as bismuth or lead. The end-window G-M detector (see Figure 1.58a) is often used for the assay of radioactive samples. The window is usually constructed from split mica with a thickness of a few milligrams per square centimeter. The thin window admits α particles and low-energy β particles into the counting volume. Ultrathin Mylar windows between 100 and 200 μg/cm² thick are available, but they must be used with a flow counter and a continuous supply of counting gas because windows this thin are permeable to the counting gas.

Detectors with thin walls are used primarily with portable survey meters (Figure 1.54b). The walls are metal or glass and are about 30 mg/cm² thick. With the
Various G-M detectors: (a) end window; (b) side window; (c) windowless flow; (d) thin-wall dipping tube; (e) needle probe; and (f) heavy-metal wall.

When an ionizing event is initiated in a G-M detector, an avalanche of electrons is created along the entire length of the anode. The residual positive ions require 200 μs or longer to migrate to the cathode. During this time, the detector will not respond fully to additional radiation that enters the counting volume. The curve in Figure 1.59 depicts the response of the detector as a function of time after an ionizing event. During the dead time, the detector is completely unresponsive to additional radiation, whereas an ionizing event occurring within the recovery time produces a voltage pulse that is smaller than normal. The resolving time is the time between an ionizing event and second event that furnishes a pulse large enough to pass the discriminator.

Positive ions that approach the cathode of a G-M detector dislodge electrons from the wall of the chamber. As these electrons combine with the positive ions, ultraviolet and X-ray photons are released. Some of these photons strike the chamber wall and release electrons that cause the chamber to remain discharged. If this secondary release of electrons is permitted to occur, the detector will be unresponsive to radiation after the first ionizing event.

Various methods have been devised to quench the self-perpetuating discharge of G-M detectors. The most common method is to add a small concentration (about 0.1%) of a selected gas to the counting gas. The gases used most often are polyatomic organic gases (e.g. amyl acetate or ethyl alcohol vapor) or halogens (e.g. Br₂ or Cl₂). As the positive ions move toward the cathode, they collide with and transfer charge to molecules of quench gas. The charged molecules of quench gas migrate to the cathode and dislodge electrons from the chamber wall. Energy released as the dislodged electrons combine with the charged molecules causes the dissociation of molecules of the quench gas. The dissociation is irreversible with a polyatomic organic gas, and the useful life is 10⁸ to 10¹⁰ pulses for a G-M detector quenched with one of these agents. Halogen-quenched tubes have an infinite useful life, theoretically, because the molecules recombine after dissociation.

The counting gas typically used in G-M detectors is an inert gas such as argon, helium, or neon. “Geiger gas” used in flow counters is composed of 99% helium and about 1% butane or isobutane. For G-M tubes quenched with an organic gas, the plateau should be 200–300 V.
1.6 Radiation Detectors

1.6.2 Gas-Filled Detectors

Long and should have a slope not greater than 1–2% per 100 V. Halogen-quenched tubes have a shorter plateau (100–200 V) and a plateau slope of 3–4% per 100 V.

The detection efficiency of a G-M counter is about 1% for X- and γ-rays and nearly 100% for α and β particles that enter the counting volume. Of course, many α and low-energy β particles are absorbed by the window of the detector, so windowless flow counters are often used to detect these particles. Figure 1.60 shows a survey meter equipped with an end-window G-M tube for detecting the presence of radioactive contamination.

1.6.3 Solid Scintillation Detectors

Gas-filled chambers are not efficient detectors for X- and γ-ray photons because these radiations pass through the low-density gas without interacting. The probability of X- and γ-ray interaction is increased if a solid detector with a high density and atomic number is used. Atoms of a solid are immobile; however, an interaction cannot be readily and rapidly registered by the collection of electrons and positive ions. Instead, the interaction must be detected by some alternate method.

In a scintillation detector, light is released as radiation is absorbed. When an X- or γ-ray interacts within a scintillation material, electrons are raised from one energy state to a higher one. The number of electrons raised to a higher energy level depends on the energy deposited in the crystal by the incident radiation. Light is released as these electrons return almost instantaneously to the lower energy state; in most scintillation detectors, about 20–30 photons of light photons are released for every kiloelectron volts of the energy absorbed. The light is then subsequently detected using a photosensitive detector, such as a photomultiplier tube, which can convert lights to electronic signal.

Scintillation detectors are used to detect particulate radiation as well as X- and γ-ray photons. Their applications range from sensors for nuclear medicine imaging; indirect detectors for flat panel radiographic, mammographic, and fluoroscopic imaging; gemstone detectors for CT and radiation monitoring; and dosimetry sensors. We cover the principles of these detectors in this chapter with additional modality-specific discussion in the subsequent chapters.

1.6.3.1 Scintillation Materials

The γ-rays from radioactive samples are often detected with a scintillation crystal. Alkali halide crystals are commonly used because the probability of photoelectric interactions is increased by the presence of the high-Z halide component.

Sodium iodide is the most ubiquitous alkali halide, although more expensive crystals of cesium iodide and potassium iodide are available. High thickness of the material is preferred to increase the efficiency of the energy absorption, specifically in the applications with higher energy photons. Crystals of sodium iodide up to 9 in. in diameter by 9 in. thick, or 20 in. in diameter by 0.5 in. thick, are available commercially. Smaller crystals (e.g. 2 in. in diameter by 2 in. thick) are routinely used for the assay of γ-emitting samples. The efficiency of a crystal of detecting X- and γ-rays increases with the size of the crystal.

The detection efficiency of a NaI(Tl) scintillation detector decreases with an increasing energy of impinging γ-rays (see Figure 1.61). However, the efficiency of

![Figure 1.60](image1.png) Portable survey meter equipped with an end-window G-M tube.

![Figure 1.61](image2.png) The detection efficiency of 3 × 3-in. and 1.5 × 1-in. NaI(Tl) crystals is plotted as a function of the energy of incident γ-rays. The radioactive sources were positioned 7 cm from the 1.5 × 1-in. crystal and 9.3 cm from the 3 × 3-in. crystal. Source: From Lazar et al. 1956 [25]. Used with permission.
a scintillation detector for detection of γ-rays may be improved by using a larger crystal and by improving the counting geometry. For example, a crystal into which the radioactive sample may be inserted (i.e. a well detector) may enable a detection efficiency greater than that provided by a crystal that receives at best no more than half the γ-rays from a radioactive source.

To be used as a scintillation detector, an alkali halide crystal must be activated with an impurity, which is usually thallium iodide at a concentration of about 0.1%; the resulting crystals are denoted as NaI(Tl), CsI(Tl), or KI(Tl).

Highly purified organic crystals (e.g. anthracene and trans-stilbene) are used to detect β-particles. The atomic number of these crystals is relatively low, and the probability that β-particles will be scattered out of the detector after dissipating only part of their energy is reduced. The sensitivity of an anthracene or trans-stilbene detector to γ-rays is low, particularly if the crystal is thin. Consequently, β-particles may be detected with limited interference from γ-rays.

In X-ray imaging applications, the material of choice is cesium iodide or gadolinium oxi-sulfide (GSO), often with thallium activation. As the energy of the photons used is lower than those of γ-rays used in nuclear medicine, the thickness of the material can be less, reducing the cost of the detector, thus CsI tend to be a more manageable choice for X-ray applications. Such applications, however, require a significantly higher spatial and temporal resolution than that of nuclear medicine imaging. As such, scintillators are not used as a single crystal, as in most nuclear medicine detectors, rather formed into needle-like structures to minimize the dissipation of the generated light photons and direct them to photodiode light sensors used in radiographic applications.

CT applications use crystalline gemstone scintillators that, although tend to be expensive, offer significantly shorter response time compared to other scintillators. This is of notable advantage in meeting the demanding high-speed need of modern CT systems.

1.6.3.2 Scintillator Sealing and Coupling

Many scintillating materials are hydrophobic. Sodium iodide crystals attract and hold water molecules and must be protected from moisture. If exposed to moisture, an NaI(Tl) crystal turns yellow from the release of free iodine and absorbs much of the radiation-induced fluorescence. Therefore, crystals are mounted in a dry atmosphere and sealed to prevent the entrance of moisture. Sometimes, a NaI crystal is enclosed within an aluminum canister, perhaps 1/32-in. thick. The canister prevents moisture from reaching the crystal and ambient light from reaching the photomultiplier tube. The crystal or light pipe is coupled to the glass face of the photomultiplier tube with a transparent viscous medium such as silicone fluid.

The light from the scintillator needs to efficiently reach the light sensor, although maintaining its seal against elements. A light coupling or pipe of Lucite, clear glass, or quartz is sometimes attached to the side of the scintillator nearest the light sensor. Other surfaces of the scintillator are coated with a light-reflecting material such as Al₂O₃, MgO, or aluminum foil. The light coupling may also use fiber optic tapers or optical lenses.

1.6.3.3 Light Sensors

1.6.3.3.1 Photomultiplier Tubes

In nuclear medicine detectors, the so-called gamma cameras (see Chapter 8), the light photons, transmitted through the scintillation material, are captured by photomultiplier tubes. A photomultiplier has a light-sensitive photosensitive cathode (photocathode). If the wavelength of light striking the photocathode matches the spectral sensitivity of this surface, then electrons are ejected. The number of electrons is multiplied by various stages (dynodes) of the photomultiplier tube and a signal can be measured at the photomultiplier anode (and amplified electronically, if required). The size of the signal at the anode is proportional to the energy dissipated in the detector by the incident radiation.

A photomultiplier tube is depicted in Figure 1.62. The photocathode is usually an alloy of cesium and antimony, often mixed with sodium and potassium (i.e. a bi-alkali photocathode), from which an acceptable number of electrons are released per light photon absorbed. The spectral sensitivity of the alloy must match the wavelength of the light emerging from the crystal; the spectral sensitivity of a photocathode with an “S-11 response” is compared with the emission spectrum of light from irradiated NaI(Tl) in Figure 1.63. Only 10–30% of the light photons that strike the photocathode cause the ejection of electrons. These electrons are then accelerated to the first dynode, a positively charged electrode positioned a short distance from the photocathode. For each electron absorbed by the first dynode, three or four electrons are ejected and accelerated to the second dynode, where more electrons are released. As photomultiplier tubes contain 6–14 dynodes with a potential difference of 50–100 V between successive dynodes, 10⁶–10⁸ electrons reach the anode for each electron liberated from the photocathode. The amplification of the signal is highly dependent on the potential difference between dynodes, and the high-voltage supply for dynodes must be very stable.

Electrons collected by the anode are converted to a voltage pulse with an amplitude of a few millivolt to volt.
1.6 Radiation Detectors

1.6.3.3.2 Thin-film Photodiodes

In X-ray projection and some nuclear imaging applications, the scintillators are coupled to a flat panel layer of thin-film photodiodes or transistors printed as an active matrix circuitry on a flat piece of glass. The technology comes from flat panel display technology used in the construction of flat panel monitors and television. The layer consists of millions of photodiodes, each responsible for an image pixel, capturing the light from the scintillator and converting it into electronic charge that is saved within the pixel capacitor. The charge is read by sequentially turning on the transistor switches of each pixel in a raster manner till the whole panel is read.

In a thin-film photodiode coupled to a typical 0.5-mm-thick CsI layer, about 0.04 light photons are generated for each electronvolt of deposited X-ray energy. The quantum efficiency of photodiodes is typically 0.7. Thus, a 50 keV photon captured by such a detector will generate 1400 deposited light photon in a photodiode element.

1.6.4 Liquid Scintillation Detectors

With a solid scintillation detector such as a NaI(Tl) crystal, the radioactive sample is positioned outside the detector. However, in liquid scintillation counting, the radioactive sample is mixed with the scintillating material, and in doing so, the attenuation by materials between the radioactive sample and the scintillating material is minimized [26, 27]. Consequently, the detection efficiency is high for radiations with very short range, including weak $\beta$ particles such as those from $^3$H ($E_{\text{max}} = 0.018 \text{ MeV}$), $^{14}$C ($E_{\text{max}} = 0.156 \text{ MeV}$), and $^{35}$S ($E_{\text{max}} = 0.168 \text{ MeV}$). Usually, light from the mixture of scintillator and radioactive sample is directed toward at least two photomultiplier tubes. The signal from each photomultiplier tube is then transmitted by a preamplifier and amplifier to a coincidence circuit (see Figure 1.64). The coincidence circuit transmits a voltage pulse to the scaler only if a pulse is received simultaneously from both photomultiplier tubes.

Except for very low-energy particles and photons, radiation emitted by the sample usually produces a signal in each photomultiplier tube and a pulse passes to the scaler for most disintegrations of the sample. However, spurious pulses generated by thermal noise in the photomultiplier tubes or preamplifiers are received by the coincidence circuit from one direction only and a pulse is not passed to the scaler. In this manner, the coincidence...
The scintillation solution, or "cocktail," consists of the radioactive sample, solvent, primary fluor or solute, and, if necessary, a secondary fluor. Solvents dissolve the sample and transfer energy from the sites of interaction of the radiation to the molecules of fluor, present at a low concentration (1% or less), in the scintillation mixture.

Molecules of the primary fluor release light upon receipt of energy from the solvent molecules. The wavelengths of light emitted by the scintillation cocktail must correspond to the spectral sensitivity of the photocathodes of the photomultiplier tubes. Some photomultiplier tubes with quartz windows are sensitive to the light emitted by the primary fluor. With other photomultiplier tubes, however, a secondary fluor must be added to the scintillation solution. The secondary fluor is called a wavelength shifter because light of a longer wavelength is emitted by the scintillation solution when the secondary fluor is present.

In a liquid scintillation counter, the size of the voltage pulse depends on the energy dissipated in the scintillation cocktail by a photon or particle emitted by the radioactive sample. The number of pulses of different sizes is shown in Figure 1.65 for \( ^3\)H and \( ^{14}\)C. By adjusting upper and lower discriminators, one isotope may be counted in the presence of the other. For a number of reasons, these spectra do not exactly correspond to the energy distribution of the emitted particles.

Interference with the production or transmission of light in a liquid scintillation solution is termed quenching. Quenching, always present in liquid scintillation counting, is caused by (i) the interference with the mechanism of energy transfer contributed by the sample or other components of the cocktail: chemical quenching, (ii) absorption of light by colored materials in the sample: color quenching, (iii) passive interference with the mechanism of energy transfer resulting from dilution of the scintillation mixture by the sample or other material: dilution quenching, and (iv) absorption of light by the scintillation vial, fingerprints on the vial, etc.: optical quenching. Quenching shifts the spectrum for any isotope toward pulses of smaller size. Figure 1.66 shows the spectra for a \( ^{14}\)C-labeled sample dissolved in a scintillation cocktail and quenched with different amounts of carbon tetrachloride, a chemical quench agent.

The count rate for a particular sample must be corrected for quenching before the actual disintegration rate of the sample can be determined. Three methods

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**Figure 1.64** A liquid scintillation counter with two photomultiplier tubes and a coincidence circuit to reduce the background count rate.

**Figure 1.65** Liquid scintillation spectra for \( \beta \) particles from \( ^3\)H \( (E_{\text{max}} = 0.018 \text{ MeV}) \) and \( ^{14}\)C \( (E_{\text{max}} = 0.156 \text{ MeV}) \). The dashed curves illustrate the effects of the amplifier gain upon the \( ^{14}\)C spectrum.
1.6 Radiation Detectors

1.6.5 Semiconductor Radiation Detectors

Semiconductor detectors constitute the third technology used for the detection of ionizing radiation. Semiconductor detectors can exhibit many desirable properties, including (i) a response that varies linearly with the energy deposited in the detector independent of the type of radiation that deposits the energy, (ii) a negligible absorption of energy in the entrance window of the detector, (iii) excellent energy resolution, (iv) the formation of pulses with fast rise times, and (v) small size.

The mechanism of response of a semiconductor detector resembles that for an ionization chamber with the distinction of gas being replaced with a semiconductor material: ionization produced within the sensitive volume of the detector is converted to a voltage pulse that is amplified and counted, and the size of the voltage pulse is proportional to the energy expended in the detector by the incident radiation. When compared with an ionization chamber, density of the absorption material is higher, thus higher detection efficiency. The applied voltage and the voltage pulse are also larger and more accurately reflect the energy deposited in the detector. Also, the rise time of the pulse is shorter because the ionization is collected more rapidly.

In most gases, an average energy of 30–40 eV is expended per ion pair produced. However, in semiconductor detectors, that value is smaller by an order of magnitude. When compared with an ionization chamber, therefore, many more ion pairs are produced in a semiconductor detector for a given amount of energy absorbed, resulting in improved accuracy and precision.

The efficiency of semiconductor detectors is nearly 100% for particulate radiations and lower for X- and γ-rays. As in the case of scintillating detectors, the detection efficiency for X- and γ-rays is dependent on the thickness, atomic number, and density of the absorption material. The typical material used in semiconductor detectors include amorphous selenium (Z = 34), germanium (Z = 32), and silicon (Z = 14). Newer materials include cadmium telluride and cadmium zinc telluride, both of which offer a higher atomic number and thus absorption efficiency.

Semiconductor detectors are used extensively in imaging applications. They are currently used in a range of mammography, radiography, and fluoroscopy systems. Because of their excellent energy resolution, semiconductor detectors are widely used for X- and γ-ray counting solutions. Techniques have been developed for counting insoluble samples such as filter paper, paper chromatograms, and Millipore filter disks. Scintillating beads are sometimes used when liquid or gaseous samples are counted by liquid scintillation.
spectrometry. These detectors have also been used for fluorescence scanning, where quantitative estimates of iodine in the thyroid are obtained by measurement of iodine X-rays released as the thyroid is exposed to low-energy X-ray or $\gamma$-radiation [29]. Some effort has also been directed toward the extension of semiconductor detectors to imaging applications in nuclear medicine. There are few products that offer transistor arrays coupled to CdZnTl photoconductors, thereby eliminating the light guide and photomultiplier tubes and reducing the size and weight of the scanner.

1.6.5.1 The Detection Process

A semiconductor radiation detector is similar to a transistor and is diagrammed in Figure 1.67. The p-type region is composed of a semiconducting element (e.g. germanium or silicon) doped with an electron-acceptor impurity with fewer valence electrons. For example, a p-type semiconductor may be obtained by doping tetravalent germanium with trivalent boron, indium, or gallium. The n-type region is composed of germanium doped with an electron donor impurity such as antimony or lithium. Electrons flow from the n-type region to the p-type region and establish an electric field across the junction between the two regions. The region in the vicinity of the junction is termed the depletion region and may be increased in width by applying a reverse bias across the junction (positive potential to the n-type region, negative potential to the p-type region). The width of the depletion region in germanium or silicon detector may also be increased by a process known as lithium drifting to produce a Ge(Li) or Si(Li) detector.

If a charged particle or an X- or $\gamma$-ray loses energy within the depletion region, electrons are raised from the valence band to the conduction band, where they can migrate to the positive electrode (the n-type region). In

![Figure 1.67 Diagram of a p–n junction semiconductor detector.](image-url)
the valence band, electrons move closer to the positive terminal by jumping to holes left by the released electrons and other electrons fill the holes left by the jumping electrons. In this manner, holes migrate toward the negative terminal as if they were positively charged particles. The migration of positive holes in the valence band constitutes a current similar to that provided by electrons moving in the conduction band to the positive terminal. In fact, electrons released from the valence band, together with the positive holes left behind, constitute the ion pairs for a semiconductor detector.

If a potential difference is applied across a pure semiconductor, a current is produced even if the semiconductor is not exposed to ionizing radiation. This current is the sum of (i) a bulk current that is dependent on the resistance of the semiconductor and the number of electron–hole pairs produced by thermal excitation and (ii) a current caused by charge leakage at the surface of the semiconductor. These currents interfere with the identification of signals produced as radiation interacts within the detector. The bulk current is reduced with the p–n junction described above, and this barrier to current flow is required in a semiconductor radiation detector. The p–n junction reduces the bulk current in silicon and selenium to an acceptable level at room temperature. Even with a p–n junction, however, the bulk current in a germanium detector is too great at room temperature. Consequently, germanium semiconductor detectors must be operated at reduced temperature. Germanium detectors are usually mounted in a cryostat and maintained at the temperature of liquid nitrogen (−190 °C). The surface-leakage current is reduced in germanium and silicon by special techniques for constructing the detectors.

The size $V$ of a voltage pulse from a semiconductor detector equals the charge $Q$ collected by the electrodes divided by the capacitance $C$ of the depletion region. Furthermore, the charge $Q$ equals $Ne$, where $N$ is the number of electron–hole pairs produced and $e$ is the charge of the electron. That is, the size of the voltage pulse is proportional to the energy lost in the detector by the incident radiation. The size of the pulse is not dependent on the specific ionization of the radiation because the ion pairs are swept away immediately and cannot recombine. Consequently, the response of the detector depends on the energy deposited in the detector but not on the type of radiation that deposits the energy.

As stated, the energy required to produce an ion pair in a semiconductor detector is only one tenth of the energy required in a gas and the voltage pulse from a semiconductor detector is about ten times larger than the pulse from a gas-filled ionization detector. This provides for a significantly higher energy resolution. For example, a 1-MeV α particle absorbed completely in the depletion region of a silicon semiconductor detector produces about $[10^6 \text{eV} / (3.5 \text{eV/ IP})] = 3 \times 10^5$ ion pairs. The same particle produces only about $[10^6 \text{eV} / (33.85 \text{eV/IP})] = 3 \times 10^4$ ion pairs in an air-filled ionization chamber. The resultant estimated percent standard deviation $\%$/N for the pulse is 0.18 for the pulse produced in the semiconductor detector and 0.58 for the gas-filled detector. Therefore, the range of pulse heights produced by the absorption of a given amount of energy is much narrower for a semiconductor detector than for a gas-filled ionization chamber, and consequently, the energy resolution of the semiconductor detector is much better. A similar analysis is applicable to the comparison of a semiconductor detector to a scintillation detector.

### 1.6.5.2 Charge Collection and Amplification

The charge collected from the photoconductor needs to be collected and amplified for processing. This data conditioning can influence the property of the detector signal; e.g. the maximum resolution obtainable with a semiconductor detector is limited by the preamplifier rather than by the detector. When used as an imaging sensor, the photoconductor is coupled to a thin-film transistor flat panel array consisting of multiple elements, each representing an image pixel. The charges collected by each element are read sequentially by turning on each transistor, one at a time, subsequent to the exposure.

### References


