We are rapidly moving into a *digital era*. Until as recently as some 10 years ago, clock speeds and data rates of digital systems were in the hundreds of megahertz (MHz) range, with rise and fall times of the pulses in the nanosecond (1 ns = 10\(^{-9}\) s) range. Prior to that time, the *lands* (conductors of rectangular cross section) that interconnect the electronic modules on printed circuit boards (PCBs) had little or no impact on the proper functioning of those electronic circuits. Today, the clock and data speeds have moved rapidly into the low-gigahertz (GHz) range. The rise and fall times of those digital waveforms have decreased into the picosecond (1 ps = 10\(^{-12}\) s) range. For example, a 1-GHz digital clock signal consists of trapezoidal-shaped pulses having rise and fall times on the order of 100 ps or less. A digital clock waveform is illustrated in Fig. 1.1.

The period of the periodic waveform \(T\) is the reciprocal of the clock fundamental frequency \(f_0\). The rise and fall times are denoted as \(\tau_r\) and \(\tau_f\), respectively, and the pulse width (between 50% levels) is denoted as \(\tau\). Digital data waveforms are similar except that the period starts immediately after the previous pulse [i.e. \(T = \tau + (\tau_r + \tau_f)/2\)], and the occurrence of a pulse during the adjacent time intervals is random. As the frequencies of the clocks increase, the period \(T\) decreases and hence the rise and fall times of the pulses must be reduced commensurately in order that pulses resemble a trapezoidal...
shape rather than a sawtooth waveform, thereby giving adequate setup and hold time intervals. Reducing the pulse rise and fall times has had the consequence of increasing the spectral content of the waveshape according to the Fourier series of the waveform. Typically, this spectral content is significant up to the inverse of the rise and fall times, $1/\tau_r$, as we will see.

For example, a 1-GHz digital clock signal having rise and fall times of 100 ps has significant spectral content at multiples (harmonics) of the basic clock frequency (1 GHz, 2 GHz, 3 GHz, ...) up to around 10 GHz. As the demand for faster data processing continues to escalate, these speeds will no doubt continue to increase into the gigahertz frequency range. The pulse rise and fall times will be reduced commensurately, thereby increasing the spectral content further into the gigahertz frequency range. This also applies to mixed-signal systems containing both digital and analog signals.

Although the physical lengths of the lands that interconnect the electronic modules on the PCBs have not changed significantly over these intervening years, their electrical lengths (in wavelengths) have increased because of the increased spectral content of the signals that the lands carry. Today these interconnects can have a significant effect on the signals they are carrying, so that just getting the systems to work properly has become a major design problem. Remember that it does no good to write sophisticated software if the hardware cannot execute those instructions faithfully. This has generated a new design problem, referred to as signal integrity. Good signal integrity means that the interconnect conductors (the lands) should not adversely affect the operation of the modules that the conductors interconnect.

Prior to some 10 years ago, these interconnects could be modeled reliably with lumped-circuit models that are easily analyzed using Kirchhoff’s voltage and current laws and lumped-circuit analysis methods, or could be ignored altogether. Because these interconnects are becoming “electrically long,” lumped-circuit modeling of them is becoming inadequate and gives erroneous answers. The interconnect conductors must now be treated as distributed-circuit transmission lines. The interaction of the electric and magnetic fields
between two adjacent transmission lines also causes portions of the voltage and current waveforms on one line to appear inadvertently at the ends of the adjacent line, thereby creating potential interference problems in the electronic devices that the adjacent line interconnects. This is called crosstalk and is also rapidly becoming a significant problem in high-speed digital electronics.

This book is intended to be a thorough but concise description of the analysis of transmission lines with respect to signal integrity and crosstalk in modern high-speed digital and high-frequency analog systems. This chapter covers some important basic skills and concepts that facilitate an understanding of the behavior of those transmission lines as well as demonstrating when the interconnects need to be considered as transmission lines.

1.1 UNITS AND UNIT CONVERSION

The internationally accepted system of units is the International System, or SI system, where the primary units are the meter, kilogram, second, and ampere, thus termed the MKSA system. All quantities in any formula or law must be in these units. For example, Coulomb’s law for the force between two point charges that are separated a distance \( R \) is

\[
F = \frac{Q_1 Q_2}{4\pi \varepsilon_0 R^2} \quad \text{N(newtons)}
\]

The SI units of the charges \( Q_1 \) and \( Q_2 \) are coulombs (C), and the distance between the two charges, \( R \), is in meters (m). The SI units of force are newtons (N). The constant in the denominator is the permittivity of free space (essentially air):

\[
\varepsilon_0 \approx \frac{1}{36\pi} \times 10^{-9} \text{F/m}
\]

We will see in the other electromagnetic laws a similar constant, known as the permeability of free space:

\[
\mu_0 = 4\pi \times 10^{-7} \text{H/m}
\]

These two constants have the units of a capacitance in farads (F) per unit of length and an inductance in henrys (H) per unit of length. These important constants appear throughout the laws of electromagnetics and in transmission-
line applications and should be committed to memory. The speed of light in a
vacuum (and essentially in air) is

\[
v_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}
\]
\[
= 2.99792458 \times 10^8
\]
\[
\cong 3 \times 10^8 \text{ m/s}
\]

Throughout science we must deal with numbers spanning many orders of
magnitude. Table 1.1 gives the common unit multipliers along with their
abbreviations, which every engineer should commit to memory through
frequent use.

Finally, there is an important distinction between radian frequency \(\omega\) and
cyclic frequency \(f\):

\[
\omega = 2\pi f \quad \text{rad/s}
\]

where the units of cyclic frequency \(f\) are hertz (Hz) (previously called cycles/
s). For example, the cyclic frequency of commercial electric voltage and
current in the United States is 60 Hz, which, in radian frequency, is 377 rad/s.
The reader should never make the serious mistake of using cyclic frequency
when radian frequency is required (i.e., \(\omega \neq f\)).

Although the SI system of units is accepted as the standard throughout the
world, a few countries (including the United States) have not converted to SI
units. Although there has been a strong attempt in the United States to convert
to SI units, non-SI units are still in widespread use. Hence we have no choice
but to learn to convert between the two systems of units. In the United States

<table>
<thead>
<tr>
<th>TABLE 1.1. Unit Multipliers</th>
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<tr>
<td>Prefix</td>
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<tr>
<td>--------</td>
</tr>
<tr>
<td>giga</td>
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<tr>
<td>mega</td>
</tr>
<tr>
<td>kilo</td>
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<td>micro</td>
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<tr>
<td>nano</td>
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<td>pico</td>
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the prevalent system of units is the *English system*, where the units of length are inches (in), feet (ft), yards (yd), and statute miles (mi). The important conversion of inches to centimeters:

\[
1 \text{ in} = 2.54 \text{ cm}
\]

allows the conversion between units of length in the SI and English systems. The remaining units of length in the English system are:

- 1 foot (ft) = 12 inches
- 1 yard (yd) = 3 feet
- 1 mile (mi) = 5280 ft

To convert flawlessly between units that are in the different systems, simply multiply by unit ratios: for examples \(1 \text{ mi} = 5280 \text{ ft}, \ 1 \text{ ft} = 12 \text{ in}, \ 1 \text{ in} = 2.54 \text{ cm}, \ 1 \text{ m} = 100 \text{ cm}, \ 1 \text{ km} = 1000 \text{ m}\). For example, to convert 100 miles to kilometers, we write

\[
100 \text{ mi} \times \frac{5280 \text{ ft}}{1 \text{ mi}} \times \frac{12 \text{ in}}{1 \text{ ft}} \times \frac{2.54 \text{ cm}}{1 \text{ in}} \times \frac{1 \text{ m}}{100 \text{ cm}} \times \frac{1 \text{ km}}{1000 \text{ m}} = 160.93 \text{ km}
\]

Cancellation of the unit names in this conversion avoids the improper multiplication (division) by a unity ratio when division (multiplication) should be used. There is one other dimensional unit that is used very frequently in the United States to give the dimensions of lands and PCBs: the unit of mils. One mil is one-one thousandth of an inch:

\[
1000 \text{ mils} = 1 \text{ in}
\]

To convert a dimension given in mils to the equivalent dimension in meters, we multiply by \(2.54 \times 10^{-5} \text{ m/mil}\) or \(1 \text{ mil} = 25.4 \text{ micrometers (microns)}\). For example, a common width of PCB lands is 10 mils, which is equivalent to 0.254 mm. A common thickness of printed circuit boards is 64 mils, which is equivalent to 1.63 mm. It is also common to state the per-unit-length internal inductance of a round wire in nanohenrys per inch. The exact value in SI units is \(0.5 \times 10^{-7} \text{ H/m}\). This is converted as

\[
0.5 \times 10^{-7} \frac{\text{H}}{\text{m}} \times \frac{1 \text{ m}}{100 \text{ cm}} \times \frac{2.54 \text{ cm}}{1 \text{ in}} \times \frac{10^9 \text{nH}}{1 \text{ H}} = 1.27 \text{nH/in}
\]
1.2 WAVES, TIME DELAY, PHASE SHIFT, WAVELENGTH, AND ELECTRICAL DIMENSIONS

In the analysis of electric circuits using Kirchhoff’s voltage and current laws and lumped-circuit models, we ignored the connection leads attached to the lumped elements. When is this permissible? Consider the lumped-circuit element having attachment leads of total length $L$ shown in Fig. 1.2. Single-frequency sinusoidal currents along the attachment leads are, in fact, waves, which can be written in terms of position $z$ along the leads and time $t$ as

$$i(z, t) = I \cos(\omega t - \beta z)$$  \hspace{1cm} (1.1)

where the radian frequency $\omega$ is written in terms of cyclic frequency $f$ as $\omega = 2\pi f \text{ rad/s}$ and $\beta$ is the phase constant in units of rad/m. (Note that the argument of the cosine must be in radians and not degrees.) To observe the movement of these current waves along the connection leads, we observe and track the movement of a point on the wave in the same way as we observe the movement of an ocean wave at the seashore. Hence the argument of the cosine in (1.1) must remain constant in order to track the movement of a point on the wave so that $\omega t - \beta z = C$, where $C$ is a constant. Rearranging this as $z = (\omega/\beta)t - C/\beta$ and differentiating with respect to time gives the velocity

\[ \frac{dz}{dt} = \frac{\omega}{\beta}, \]

FIGURE 1.2. Current waves on connection leads of lumped-circuit elements.
of propagation of the wave as

$$v = \frac{\omega}{\beta} \text{ m/s} \quad (1.2)$$

Since the argument of the cosine, $\omega t - \beta z$, in (1.1) must remain a constant in order to track the movement of a point on the wave, as time $t$ increases so must the position $z$. Hence the form of the current wave in (1.1) is said to be a forward-traveling wave since it must be traveling in the $+z$ direction in order to keep the argument of the cosine constant for increasing time. Similarly, a backward-traveling wave traveling in the $-z$ direction would be of the form $i(z, t) = I\cos(\omega t + \beta z)$ since as time $t$ increases, position $z$ must decrease in order to keep the argument of the cosine constant and thereby track the movement of a point on the waveform. Since the current is a traveling wave, the current entering the leads, $i_1(z, t)$, and the current exiting the leads, $i_2(z, t)$, are separated in time by a time delay of

$$T_D = \frac{\phi}{v} \text{ s} \quad (1.3)$$

as illustrated in Fig. 1.2. These single-frequency waves suffer a phase shift of $\phi = \beta z$ radians as they propagate along the leads. Substituting (1.2) for $\beta = \omega/v$ into the equation of the wave in (1.1) gives an equivalent form of the wave as

$$i(z, t) = I\cos \left( \omega \left( t - \frac{z}{v} \right) \right) \quad (1.4)$$

which indicates that phase shift is equivalent to a time delay.

Figure 1.2 plots the current waves versus time. Figure 1.3 plots the current wave versus position in space at fixed times. As we will see, the critical property of a traveling wave is its wavelength, denoted as $\lambda$. A wavelength is the distance the wave must travel in order to shift its phase by $2\pi$ radians or $360^\circ$. Hence $\beta \lambda = 2\pi$, or

$$\lambda = \frac{2\pi}{\beta} \text{ m} \quad (1.5)$$
Substituting the result in (1.2) for $\beta$ in terms of the wave velocity of propagation $v$ gives an alternative result for computing the wavelength:

$$\lambda = \frac{v}{f} \text{ m}$$  \hspace{1cm} (1.6)

Table 1.2 gives the wavelengths of single-frequency sinusoidal waves in free space (essentially, air), where $v_0 \approx 3 \times 10^8$. (The velocities of propagation of current waves on the lands of a PCB are less than in free space, which is due to the interaction of the electric fields with the board material. Hence wavelengths on a PCB are shorter than they are in free space.) Observe that a wave of frequency 300 MHz has a wavelength of 1 m. Wavelengths scale linearly with frequency. As frequency decreases, the wavelength increases,

![Waves in space, and wavelength.](image)

**FIGURE 1.3.** Waves in space, and wavelength.
and vice versa. For example, the wavelength of a 7 MHz wave is easily computed as

$$\lambda_{@7 \text{ MHz}} = \frac{300 \text{ MHz}}{7 \text{ MHz}} \times 1 \text{ m} = 42.86 \text{ m}$$

Similarly, the wavelength of a 2-GHz cell phone wave is 15 cm, which is approximately 6 in.

Now we turn to the important criterion of physical dimensions in terms of wavelengths: that is, electrical dimensions. To determine a physical dimension, $L$, in terms of wavelengths (its electrical dimension), we write $L = k\lambda$ and determine the length in wavelengths as

$$k = \frac{L}{\lambda} = \left(\frac{L}{v}\right)f$$

where we have substituted the wavelength in terms of the frequency and velocity of propagation as $\lambda = v/f$. Hence we obtain an important relation for the electrical length in terms of frequency and time delay:

$$\frac{L}{\lambda} = f \frac{L}{v} = fT_D$$

Hence a dimension is one wavelength, $L = \lambda$, at a frequency that is the inverse of the time delay:

$$f |_{\lambda = \lambda} = \frac{1}{T_D}$$
A single-frequency sinusoidal wave shifts phase as it travels a distance \( L \) of

\[
\phi = \beta L = 2\pi \frac{L}{\lambda} \text{ rad} = \frac{L}{\lambda} \times 360^\circ \text{ deg} \tag{1.9}
\]

Hence if a wave travels a distance of one wavelength, \( L = \lambda \), it shifts phase by \( \phi = 360^\circ \). If the wave travels a distance of one-half wavelength, \( L = \frac{\lambda}{2} \), it shifts phase by \( \phi = 180^\circ \). This can provide for cancellation: for example, when two antennas that are separated by a distance of one-half wavelength transmit the same frequency signal. Along a line containing the two antennas, the two radiating waves being of opposite phase cancel each other, giving a result of zero. This is the essential reason why antennas have “patterns” where a null is produced in one direction, whereas a maximum is produced in another direction. Phased-array radars electronically “steer” their beams using this principle rather than by rotating the antennas mechanically. Next, consider a wave that travels a distance of one-tenth of a wavelength, \( L = \frac{\lambda}{10} \). The phase shift incurred in doing so is only \( \phi = 36^\circ \), and a wave that travels one-one-hundredth of a wavelength, \( L = \frac{\lambda}{100} \), incurs a phase shift of \( \phi = 3.6^\circ \). Hence we say that

For any distance less than, say, \( L < \frac{\lambda}{10} \), the phase shift is said to be negligible and the distance is said to be electrically short.

For electric circuits whose maximum physical dimension is electrically short, \( L < \frac{\lambda}{10} \), Kirchhoff’s voltage and current laws and other lumped-circuit analysis solution methods work very well. For physical dimensions that are not electrically short, Kirchhoff’s laws and lumped-circuit analysis methods give erroneous answers! For example, consider an electric circuit that is driven by a 10-kHz sinusoidal source. The wavelength at 10 kHz is 30 km (18.641 mi)! Hence at this frequency any circuit having a maximum dimension of less than 3 km (1.86 mi) can be analyzed successfully using Kirchhoff’s laws and lumped-circuit analysis methods. Electric power distribution systems operating at 60 Hz can be analyzed using Kirchhoff’s laws and lumped-circuit analysis principles as long as their physical dimensions, such as the transmission-line length, are less than some 310 mi! Similarly, a circuit driven by a 1-MHz sinusoidal source can be analyzed successfully using lumped-circuit analysis methods if its maximum physical dimension is less than 30 m! On the other hand, cell phone electronic circuits operating at a
frequency of around 2 GHz cannot be analyzed using lumped-circuit analysis methods unless the maximum dimension is less than around 1.5 cm, about 0.6 in! We can alternatively determine the frequency where a dimension is electrically short in terms of the time delay from (1.7):

$$f|_{\lambda=(1/10)l} = \frac{1}{10T_D}$$  \hspace{1cm} (1.10)

Substituting $\lambda f = v$ into the time-delay expression in (1.3) gives the time delay as a portion of the period of the sinusoid, $T$:

$$T_D = \frac{\mathcal{L}}{v} = \frac{\mathcal{L}}{\lambda f} = \frac{\mathcal{L}}{\lambda T}$$  \hspace{1cm} (1.11)

where the period of the sinusoidal wave is $T = 1/f$. This shows that if we plot the current waves in Fig. 1.2 that enter and leave the connection leads versus time $t$ on the same time plot, they will be displaced in time by a fraction of the period, $\mathcal{L}/\lambda$. If the length of the connection leads $\mathcal{L}$ is electrically short at this frequency, the two current waves will be displaced from each other in time by an inconsequential amount, $T/10$, and may be considered to be coincident in time. This is the reason that Kirchhoff’s laws and lumped-circuit analysis methods work well only for circuits whose maximum physical dimension is “electrically small.”

Waves propagated along transmission lines and radiated from antennas are of the same mathematical form as the currents on the connection leads of an element shown in (1.1). These are said to be plane waves where the electric and magnetic field vectors lie in a plane transverse or perpendicular to the direction of propagation of the wave, as shown in Fig. 1.4. These are termed Transverse ElectroMagnetic (TEM) waves.

### 1.3 THE TIME DOMAIN VS. THE FREQUENCY DOMAIN

Perhaps the most important concept for all engineers is that of the time and frequency domains. All engineers are used to being able to think of a problem in either domain with considerable flexibility. A problem may be more easily understood and solved in one domain than the other. For example, electric
filters are more easily designed in the frequency domain. Essentially, the time domain is the actual problem where voltages, currents, and so on, are viewed in terms of their time variations. The frequency domain views these time-domain variations in terms of their sinusoidal frequency components via the Fourier series or transform.

### 1.3.1 Spectra of Digital Signals

A time-domain waveform \( x(t) \) is periodic if \( x(t \pm mT) = x(t) \), where \( T \) is the period of the waveform and \( m \) is an integer. In other words, a periodic waveform repeats itself in intervals of length \( T \). The Fourier series allows us to view a periodic waveform alternatively as being composed of the sum of a constant (dc) term and an infinite sum of sinusoids having frequencies (harmonics) that are integer multiples of the fundamental frequency, which is the inverse of the period, \( f_0 = 1/T \). Hence the Fourier series allows us, alternatively, to represent a periodic waveform as being composed of sinusoidal components:

\[
x(t) = c_0 + c_1 \cos(\omega_0 t + \theta_1) + c_2 \cos(2\omega_0 t + \theta_2) + c_3 \cos(3\omega_0 t + \theta_3) + \cdots \\
= c_0 + \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t + \theta_n)
\]

(1.12)
where \( \omega_0 = 2\pi f_0 \). The constant (dc component) \( c_0 \) is the average value of the waveform over one period:

\[
c_0 = \frac{1}{T} \int_0^T x(t) \, dt \tag{1.13a}
\]

The magnitudes and angles of the sinusoidal components are computed from

\[
c_n / \theta_n = \frac{2}{T} \int_0^T x(t) e^{-j\omega_0 t} \, dt \tag{1.13b}
\]

where \( j = \sqrt{-1} \) and \( e^{-j\omega_0 t} = \cos \omega_0 t - jsin \omega_0 t \). For a single pulse or, equivalently, as \( T \to \infty \), these discrete frequency components merge into a continuous spectrum which is called the Fourier transform of the single pulse.

For the digital clock spectrum shown in Fig. 1.1, a general expression can be obtained for the magnitudes and angles of the Fourier components in (1.13), but the result is somewhat complicated and little insight is gained from it. However, if we restrict the result to trapezoidal pulses having equal rise and fall times, \( t_r = t_f \) (which digital clock and data waveforms tend to approximate), we can obtain a simple and informative result. For the case of equal rise and fall times, we obtain

\[
c_0 = A \frac{\tau}{T} \\
c_n / \theta_n = 2A \frac{\tau}{T} \frac{\sin(n\pi \tau / T)}{(n\pi \tau / T)} \frac{\sin(n\pi \tau_r / T)}{n\pi \tau_r / T} \quad n\pi \frac{\tau + \tau_r}{T} = n\pi \frac{\tau_f}{T} \\
\quad \tau_r = \tau_f
\tag{1.14}
\]

This result is in the form of the product of two \( \sin(x)/x \) expressions, with the first depending on the ratio of the pulse width to the period, \( \tau / T \) (also called the duty cycle of the waveform \( D = \tau / T \)) and the second depending on the ratio of the pulse rise and fall times to the period, \( \tau_r / T \). [The magnitude of the coefficient, denoted \( c_n \), must be a positive number. Hence there may be an additional \( \pm 180^\circ \) added to the angle shown in (1.14), depending on the signs of each \( \sin(x) \) term.] If, in addition to the rise and fall times being equal, the duty cycle is 50%, that is, the pulse is “on” for half the period and “off” for the other half of the period (which digital waveforms also tend to approximate), \( \tau = \frac{1}{2} T \), the result for the coefficients given in (1.14) simplifies to
\[ c_0 = \frac{A}{2} \]
\[ c_n/\theta_n = A \frac{\sin(n\pi/2)}{n\pi/2} \frac{\sin(n\pi \tau_r/T)}{n\pi \tau_r/T} / - n\pi \left( \frac{1}{2} + \frac{\tau_r}{T} \right) \quad \tau_r = \tau_f, \quad \tau = T/2 \]

Note that the first \( \sin(x)/x \) function is zero for \( n \) even, so that for equal rise and fall times and a 50% duty cycle, the even harmonics are zero and the spectrum consists only of odd harmonics. By replacing \( n/T \) with the smooth frequency variable \( f \), \( n/T \rightarrow f \), we obtain the envelope of the magnitudes of these discrete frequencies as

\[
c_n = 2A \frac{\tau}{T} \left| \sin(\pi f \tau) \right| \left| \sin(\pi f \tau_r) \right| \frac{\tau_r = \tau_f}{n \rightarrow f} \quad (1.15)
\]

In doing so, remember that the spectral components occur only at the discrete frequencies \( f_0, 2f_0, 3f_0, \ldots \).

Observe some important properties of the \( \sin(x)/x \) function:

\[
\lim_{x \to 0} \frac{\sin(x)}{x} = 1
\]

which relies on the property that \( \sin(x) \approx x \) for small \( x \) (or using l’Hôpital’s rule) and

\[
\left| \frac{\sin(x)}{x} \right| \leq \begin{cases} 
1 & x \leq 1 \\
\frac{1}{x} & x \geq 1
\end{cases}
\]

The second property allows us to obtain a bound on the magnitudes of the \( c_n \) coefficients and relies on the fact that \( |\sin(x)| \leq 1 \) for all \( x \).

A square wave is the trapezoidal waveform where the rise and fall times are zero:

\[
c_0 = A \frac{\tau}{T} \]
\[
c_n/\theta_n = 2A \frac{\tau}{T} \frac{\sin(n\pi \tau/T)}{n\pi \tau/T} / - n\pi \frac{\tau}{T} \quad \tau_r = \tau_f = 0
\]

If the duty cycle of the square wave is 50%, this result simplifies to
Figure 1.5 shows a plot of the magnitudes of the $c_n$ coefficients for a square wave where the rise and fall times are zero, $\tau_r = \tau_f = 0$. The spectral components appear only at discrete frequencies, $f_0, 2f_0, 3f_0, \ldots$. The envelope is shown by a dashed line. Observe that the envelope goes to zero where the argument of $\sin(\pi f \tau)$ becomes a multiple of $\pi$ at $f = 1/\tau, 2/\tau, \ldots$

A more useful way of plotting the envelope of the magnitudes of the spectral coefficients is by plotting the horizontal frequency axis logarithmically and similarly plotting the magnitudes of the coefficients along the vertical axis in decibels as $|c_n|_{\text{dB}} = 20\log_{10}|c_n|$. The envelope as well as the bounds of the magnitudes of the $\sin(x)/x$ function are shown in Fig. 1.6. Observe that the actual result is bounded by 1 for $x \leq 1$ and decreases at a rate of $-20$ dB/decade for $x \geq 1$. This rate is equivalent to a $1/x$ decrease. Also note that the magnitudes of the actual spectral components go to zero where the argument of $\sin(x)$ goes to a multiple of $\pi$ or $x = \pi, 2\pi, 3\pi, \ldots$.

The amplitudes of the spectral components of a trapezoidal waveform where $\tau_r = \tau_f$ given in (1.14) are the product of two $\sin(x)/x$ functions: $\sin(x_1)/x_1 \times \sin(x_2)/x_2$. When log-log axes are used, this gives the result for the bounds on the amplitudes of the spectral coefficients shown in Fig. 1.7. Note that the bounds are constant (0 dB/decade) out to the first breakpoint of

$$c_0 = \frac{A}{2}$$

$$c_n/\theta_n = \begin{cases} 
\frac{2A}{n\pi} - \frac{\pi}{2} & n \text{ odd} \\
0 & n \text{ even}
\end{cases} \quad \tau_r = \tau_f = 0, \quad \tau = T/2$$
The duty cycle is $D = \frac{\tau}{T} = \frac{\tau_0}{T}$. Above this they decrease at a rate of $-20$ dB/decade out to a second breakpoint of $f_2 = \frac{1}{\pi \tau_r}$, and decrease at a rate of $-40$ dB/decade above that. This plot shows the important result that the high-frequency spectral content of the trapezoidal pulse train has been reduced.

**FIGURE 1.6.** The envelope and bounds of the $\sin(x)/x$ function are plotted with logarithmic axes.

$$f_1 = \frac{1}{\pi \tau} = \frac{f_0}{\pi D},$$

where the duty cycle is $D = \frac{\tau}{T} = \frac{\tau_0}{T}$. Above this they decrease at a rate of $-20$ dB/decade out to a second breakpoint of $f_2 = \frac{1}{\pi \tau_r}$, and decrease at a rate of $-40$ dB/decade above that. This plot shows the important result that the high-frequency spectral content of the trapezoidal pulse train has been reduced.

**FIGURE 1.7.** Bounds on the spectral coefficients of the trapezoidal pulse train for equal rise and fall times, $\tau_r = \tau_f$. 

$$\frac{1}{\pi \tau} = f_0 = \frac{\pi D}{A_D},$$

$$\frac{1}{\pi \tau_r} = \frac{1}{\pi \tau_r} = A_D = \frac{1}{\pi \tau_r}.$$
clock waveform is determined by the pulse rise and fall times. Longer rise and fall times push the second breakpoint lower in frequency, thereby reducing the high-frequency spectral content. Shorter rise and fall times push the second breakpoint higher in frequency, thereby increasing the high-frequency spectral content.

1.3.2 Bandwidth of Digital Signals

Although the Fourier series in (1.12) shows that a time-domain waveform can be thought of equivalently as being composed of the sum of sinusoidal frequency components whose frequencies are integer multiples of the fundamental frequency \( f_0 = 1/T \), the sum must, ideally, include an infinite number of harmonic frequency components. It is, of course, not possible to include an infinite number of harmonics, so a question arises as to how many frequency components should be included in order to give a reasonable reconstruction of the waveform. Hence we must construct a finite-term approximation to \( x(t) \) using the dc component and the first \( NH \) harmonics of the Fourier series as

\[
\tilde{x}(t) = c_0 + \sum_{n=1}^{NH} c_n \cos(n\omega_0 t + \theta_n)
\] (1.16)

where \( NH \) represents the number of harmonics. To judge how well the finite-term approximation \( \tilde{x}(t) \) in (1.16) approximates the actual \( x(t) \), we examine in time the pointwise errors between the actual waveform and the finite-term approximation: \( x(t) - \tilde{x}(t) \). One way of quantifying the approximation error is with the mean-square error criterion (MSE):

\[
\text{MSE} = \frac{1}{T} \int_0^T [x(t) - \tilde{x}(t)]^2 dt
\] (1.17)

By squaring the pointwise errors we weight a negative pointwise error, \( x(t) < \tilde{x}(t) \), and a positive pointwise error, \( x(t) > \tilde{x}(t) \), equally, as we should, since either is equally detrimental. The MSE adds the pointwise errors squared over a period and averages that over the period. It can be shown that this criterion is equivalent to giving the difference in the average power in the actual waveform:

\[
P_{AV} = \frac{1}{T} \int_0^T x^2(t) dt
\] (1.18)

and the average power in the finite-term approximation:
\[ \tilde{P}_{AV} = \frac{1}{T} \int_{0}^{T} \tilde{x}^2(t) dt \]  
\hspace{1cm} (1.19)

as

\[ \text{MSE} = P_{AV} - \tilde{P}_{AV} \]  
\hspace{1cm} (1.20)

For the trapezoidal clock waveform in Fig. 1.1 having equal rise and fall times, \( \tau_r = \tau_f \), and a 50% duty cycle, the average power in the waveform is

\[ P_{AV} = A^2 \left( \frac{1}{2} - \frac{1}{3} \frac{\tau_r}{T} \right) \]  
\hspace{1cm} (1.21)

The average power in the finite-term approximation can be shown to be

\[ \tilde{P}_{AV} = c_0^2 + \sum_{n=1}^{NH} \frac{c_n^2}{2} \]  
\hspace{1cm} (1.22)

Hence the average power in the finite-term approximation is the sum of the average powers in the dc component and the NH sinusoidal harmonic components, which is Parseval’s theorem. The choice of the Fourier coefficients in (1.13) causes the Fourier series to converge uniformly, meaning that adding terms successively causes the mean-square error between the actual waveform and the finite-term approximation to decrease uniformly. Successively adding harmonic components uniformly gives a better approximation to the original waveform. Note that the units of the MSE are watts (W).

The MSE criterion in (1.17) adds the squares of the pointwise errors in order to give equal weight to negative and positive pointwise errors. A more logical error criterion is to sum the absolute values of the pointwise errors, giving the \textit{mean absolute error criterion} (MAE):

\[ \text{MAE} = \frac{1}{T} \int_{0}^{T} |x(t) - \tilde{x}(t)| dt \]  
\hspace{1cm} (1.23a)

Substituting (1.12) and (1.16) gives an alternative expression for the MAE:

\[ \text{MAE} = \frac{1}{T} \int_{0}^{T} \left| \sum_{n=NH+1}^{\infty} c_n \cos(n\omega_0 t + \theta_n) \right| dt \]  
\hspace{1cm} (1.23b)

Hence the pointwise error is simply the remainder terms of the Fourier series for \( x(t) \) for \( n > NH \) in (1.12):

\[ x(t) - \tilde{x}(t) = \sum_{n=NH+1}^{\infty} c_n \cos(n\omega_0 t + \theta_n) \]
which makes sense. Since the higher-order coefficients tend to decrease with increasing $n$, this makes sense. Note that if the units of $x(t)$ are volts (V), the units of the MAE are also volts (V). Hence the MAE gives a more appropriate measure of the pointwise approximation error $x(t) - \tilde{x}(t)$. This error criterion again weights equally the positive and negative pointwise errors but generally cannot be integrated in closed form. This is the reason that the MSE is usually chosen over the MAE.

The important question now is: How do we define the bandwidth (BW) of the periodic digital clock waveform $x(t)$? A sensible criterion for choosing this is that the BW should contain the significant spectral content of the waveform. In other words, the BW should logically be defined as the minimum number of harmonic terms required to reconstruct the original periodic waveform such that adding more harmonics gives a negligible reduction in the pointwise error, whereas using fewer harmonics gives an excessive pointwise reconstruction error.

We investigate the answer to the question of how we choose the number of harmonics (NH) in a finite-term approximation in order to give a reasonable approximation to the actual waveform by using an example of a 1-GHz clock signal waveform (period of $T = 1$ ns) having rise and fall times of 100 ps, a 50% duty cycle, and an amplitude of $A = 5$ V. Table 1.3 shows the magnitudes and angles of the first 13 harmonics computed from (1.14), along with the wavelengths in free space, $\lambda_0 = v_0/f$, of each component. The velocities of propagation of current waves on the lands of a PCB are less than in free space, which is due to the interaction of the electric fields with the board material. Hence wavelengths on a PCB are shorter than they are in free space. Note that because of the 50% duty cycle, the even harmonics are zero. The dc component of the waveform, is the average value of the waveform; which is $A/2 = 2.5$ V.

Observe from Table 1.3 that the ninth harmonic of 9 GHz has a wavelength of 3.33 cm. To use Kirchhoff’s voltage and current laws and lumped-circuit analysis principles to analyze a circuit driven by this frequency would require

<table>
<thead>
<tr>
<th>Harmonic</th>
<th>Frequency (GHz)</th>
<th>Wavelength, ($\lambda_0$ cm)</th>
<th>Level (V)</th>
<th>Angle (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>30</td>
<td>3.131</td>
<td>-108</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>10</td>
<td>0.9108</td>
<td>-144</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>6</td>
<td>0.4053</td>
<td>-180</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>4.29</td>
<td>0.1673</td>
<td>144</td>
</tr>
<tr>
<td>9</td>
<td>9</td>
<td>3.33</td>
<td>0.0387</td>
<td>108</td>
</tr>
<tr>
<td>11</td>
<td>11</td>
<td>2.73</td>
<td>0.0259</td>
<td>-108</td>
</tr>
<tr>
<td>13</td>
<td>13</td>
<td>2.31</td>
<td>0.0485</td>
<td>-144</td>
</tr>
</tbody>
</table>
that the largest dimension of the circuit be less that 3.33 mm (0.131 in)!
Similarly, to analyze a circuit that is driven by the fundamental frequency of
1 GHz, whose wavelength is 30 cm, using Kirchhoff’s laws and lumped-circuit
analysis methods would restrict the maximum circuit dimensions to being less
than 3 cm or about 1 in (2.54 cm). This shows clearly that use of lumped-circuit
analysis methods to analyze a circuit having a physical dimension of, say, 1 in
that is driven by this clock waveform would result in erroneous results for all but
perhaps the fundamental frequency of the waveform!

Figure 1.8(a), (b), and (c) show the approximation to the clock waveform
achieved by adding the dc component and the first three harmonics, the first
five harmonics, and the first nine harmonics, respectively. The spectral levels
and angles of the harmonic components in Table 1.3 were computed with
MATLAB using the m file trapgen.m that is contained in the CD included with
this book. Figure 1.8 was also generated using MATLAB m file trapgen.m.

Combining the dc term and the fundamental and third harmonics as shown
in Fig. 1.8(a) gives poor convergence. Combining the dc term, the funda-
mental, and the third and fifth harmonics as shown in Fig. 1.8(b) while giving a
better approximation still has some troublesome oscillations. Finally, com-
bining the dc term, the fundamental, and the third, fifth, seventh, and ninth

![Trapezoidal pulse reconstructed using the first 3 harmonics](image)

FIGURE 1.8. (a) Approximating the clock waveform with (a) the first three harmonics,
(b) the first five harmonics, and (c) the first nine harmonics.
FIGURE 1.8. (Continued)
harmonics as shown in Fig. 1.8(c) provides an excellent reproduction of the trapezoidal waveform. It is particularly important that the approximation during the steady-state time between the rise and fall times should be A, thereby giving adequate setup and hold time intervals. Any large oscillations during this time interval may result in logic errors.

Figure 1.9(a), (b), (c), and (d) give plots of the absolute pointwise error, \( |x(t) - \tilde{x}(t)| \) for \( NH = 3, 5, 7, \) and 9, respectively, for the 5-V 1-GHz clock waveform having a 50% duty cycle and rise and fall times of 0.1 ns. The absolute error plots in figure were plotted with MATLAB using the file plotMAE.m that is contained in the CD included with this book.

To give a more quantitative measure of how well a finite-term approximation reproduces the original waveform, we investigate the MSE and MAE error criteria for this waveform, given in (1.17) and (1.23), respectively. The MSE error criterion in (1.17) amounts to giving the difference in the average powers of the actual waveform and the finite-term approximation. The average power in the actual waveform \( x(t) \) is given in (1.21) and for the example waveform is \( P_{AV} = 11.667 \) W. The average powers of the dc and harmonic components are given in (1.22). We can compute these from Table 1.3. The average power in the dc component is 6.25 W, and the average powers in the first 13 harmonics are 4.9 W, 0.415 W, 82.1 mW, 14 mW, 0.75 mW, 0.335 mW, and 1.18 mW. The total average power contained in the dc component and the first 13 harmonic components is 11.663 W, which is 99.97% of the total average power in the waveform! However, note that 96% of the total average power in the waveform is contained in the dc component and the first harmonic! The MSE for \( NH = 1, 3, 5, 7, \) and 9 are 0.5151, 0.1003, 0.0182, 0.0042, and 0.0035 W, respectively.

The MAE error criterion in (1.23) gives a more meaningful quantitative measure of the approximation error \( x(t) - \tilde{x}(t) \). The MAE in (1.23) cannot be integrated in closed form, so we obtain a numerical value for the MAE for the example waveform using a trapezoidal numerical integration routine. Table 1.4 gives the MAE for the 1-GHz clock waveform for various numbers of harmonics. The MAEs in the table were computed using the MATLAB m file plotMAE.m that is contained in the CD included with this book.

The results in Table 1.4 are plotted in Fig. 1.10. Note that the MAE reaches a somewhat minimum level after \( NH = 9 \) harmonics are used. Hence adding more harmonics gives little additional reduction in the reconstruction error, whereas using fewer than nine harmonics gives an increasingly larger reconstruction error.

How do we quantitatively determine the bandwidth of a periodic clock waveform that has different parameters than those of the example above? Recall that the logical definition of the bandwidth (BW) of the waveform is that the BW should be the significant spectral content of the waveform. In other words, the BW should be the minimum number of harmonic terms required to reconstruct the original periodic waveform such that adding more
FIGURE 1.9. Plot of the pointwise absolute error for (a) NH = 3, (b) NH = 5, (c) NH = 7, and (d) NH = 9 for the 5-V 1-GHz clock waveform having a 50% duty cycle and rise and fall times of 0.1 ns.
FIGURE 1.9. (Continued)
TABLE 1.4. Mean Absolute Error for Various Numbers of
Harmonics for a 5-V, 1-GHz, 100-ps Rise/Fall Time, 50%
Duty Cycle Trapezoidal Waveform

<table>
<thead>
<tr>
<th>Number of Harmonics</th>
<th>Mean Absolute Error (mV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>600.3</td>
</tr>
<tr>
<td>3</td>
<td>265.2</td>
</tr>
<tr>
<td>5</td>
<td>110.1</td>
</tr>
<tr>
<td>7</td>
<td>50.2</td>
</tr>
<tr>
<td>9</td>
<td>35.0</td>
</tr>
<tr>
<td>11</td>
<td>39.3</td>
</tr>
<tr>
<td>13</td>
<td>33.0</td>
</tr>
<tr>
<td>15</td>
<td>22.1</td>
</tr>
<tr>
<td>17</td>
<td>15.5</td>
</tr>
<tr>
<td>19</td>
<td>12.2</td>
</tr>
<tr>
<td>21</td>
<td>13.9</td>
</tr>
<tr>
<td>23</td>
<td>12.8</td>
</tr>
<tr>
<td>25</td>
<td>9.9</td>
</tr>
</tbody>
</table>

harmonics gives a negligible reduction in the pointwise error, whereas using fewer harmonics gives an excessive pointwise reconstruction error. If we look at the plot of the bounds on the magnitude spectrum shown in Fig. 1.7, we see that above the second breakpoint, \( f_2 = 1/\pi \tau_r \), the levels of the harmonics are rolling off at a rate of \(-40 \text{ dB/decade}\). If we go past this second breakpoint by

FIGURE 1.10. Plot of the MAE for a 1-GHz trapezoidal clock waveform having equal rise and fall times of \( \tau_r = \tau_f = 100 \text{ ps} \) and a 50% duty cycle for various numbers of harmonics used to reconstruct it.
a factor of about 3 to a frequency that is the inverse of the rise and fall time, 
\[ f = \frac{1}{\tau_r}, \]
the levels of the component at the second breakpoint will have been reduced further, by approximately 20 dB. Hence above this frequency the remaining frequency components are probably of such small magnitude that they do not provide any substantial contribution to the shape of the resulting waveform. Hence we might define the \textit{bandwidth} of the trapezoidal clock waveform (and other data waveforms of similar shape) to be

\[ BW \approx \frac{1}{\tau_r} \]  

(1.24)

For the 1-GHz clock waveform above having a 50% duty cycle and 100-ps rise and fall times, by this criterion the bandwidth is 10 GHz. Hence for this waveform, the first nine harmonics contain the \textit{significant spectral content of the waveform}. This correlates with the data in Fig. 1.10. Figure 1.11 shows a plot of the \textit{envelope} of the spectrum of the 1-GHz waveform for \( A = 1 \) V and a 50% duty cycle with rise and fall times of 0.1 ns along with the bounds shown in Fig. 1.7. Remember that the actual spectral components occur at the discrete frequencies 1, 3, 5, 7, 9, \ldots GHz. The spectrum for \( A = 5 \) V can be obtained from this by adding 20 log\(_{10}\)(5) = 13.98 dB. Because of the 50% duty cycle, the first breakpoint in Fig. 1.7, \( f_1 = f_0 / \pi D = 636.6 \) MHz, occurs below the fundamental frequency of 1 GHz and hence is not shown on this plot.

**FIGURE 1.11.** Plot of the spectrum of a 1-V 1-GHz clock waveform having a 50% duty cycle and rise and fall times of 0.1 ns.
The second breakpoint of $f_2 = 1/\pi \tau_r = 3.183 \text{ GHz}$ marks the frequency where the bounds change from $-20 \text{ dB/decade}$ to $-40 \text{ dB/decade}$. From Table 1.3 the ninth harmonic level of $0.0387 \text{ V}$ has a level of $-28.25 \text{ dB}$. Subtracting $20 \log_{10}(5) = 13.98 \text{ dB}$ gives the plotted level of $-42.23 \text{ dB}$.

The BW criterion in (1.24) clearly does not apply to a square wave where $\tau_r = \tau_f = 0$, since it would imply that $\text{BW} = \infty$. So a corresponding BW criterion must be derived for a square wave, although an ideal square wave cannot be generated in practice.

The bandwidth criterion in (1.24) is not meant to be exact but is intended only to give an indication of how many frequency components should be applied to the digital system to ascertain accurately how that system would have processed the actual waveform. If the system through which the clock waveform is applied is linear (such as a transmission line having linear terminations), the principle of superposition supports this concept, as we show next.

1.3.3 Computing the Time-Domain Response of Transmission Lines Having Linear Terminations Using Fourier Methods and Superposition

Consider a linear system having an input $x(t)$ and an output of $y(t)$, as shown in Fig. 1.12. A linear system is one for which the principle of superposition applies. In other words, the system is linear if $x_1(t) \rightarrow y_1(t)$ and $x_2(t) \rightarrow y_2(t)$; then (1) $(x_1(t) + x_2(t)) \rightarrow (y_1(t) + y_2(t))$ and (2) $kx(t) \rightarrow ky(t)$.

We can decompose a periodic waveform into its Fourier components according to

$$x(t) = c_0 + c_1 \cos(\omega_0 t + \theta_1) + c_2 \cos(2\omega_0 t + \theta_2) + c_3 \cos(3\omega_0 t + \theta_3) + \cdots$$

(1.12)

If we pass each component of the waveform through the linear system individually and sum, in time, the responses to these components, we will have determined, indirectly, the response of the system to the entire waveform being passed through it, as indicated in Fig. 1.13. It is important to note that
this technique gives only the **steady-state response**. In other words, any transient portion of the response to the original signal is not included.

As an example of this powerful technique, consider an RC circuit that is driven by a periodic square-wave voltage source as shown in Fig. 1.14. The square wave has an amplitude of 1 V, a period of 2 s, and a pulse width of 1 s (50% duty cycle). The RC circuit consisting of the series connection of $R = 1 \, \Omega$ and $C = 1 \, \text{F}$ has a time constant of $RC = 1 \, \text{s}$, and the voltage across the capacitor is the desired output voltage of this linear “system.” The nodes of the circuit are numbered in preparation for using the SPICE circuit analysis program (or the personal computer version, PSPICE) to analyze it and plot the exact solution. The Fourier series of the input, $v_S(t)$, using only the first seven harmonics, is ($\omega_0 = 2\pi/T = \pi$)

\[
v_S(t) = c_0 + c_1 \cos(\omega_0 t + \theta_1) + c_2 \cos(2\omega_0 t + \theta_2) + \cdots + c_n \cos(n\omega_0 t + \theta_n)
\]

FIGURE 1.13. Using superposition to determine the (steady-state) response of a linear system to a waveform by passing the individual Fourier components through the system and summing their responses at the output.

FIGURE 1.14. Example of using superposition of the Fourier components of a signal in obtaining the (steady-state) response to that signal.
The phasor (sinusoidal steady-state) transfer function of this linear system is

\[
\hat{H}(j\omega_0) = \frac{\hat{V}}{\hat{V}_s} = \frac{1}{1 + jn\omega_0 RC} = \frac{1}{1 + jn\pi} = \frac{1}{\sqrt{1 + (n\pi)^2}} \tan^{-1} n\pi = H_n \angle \phi_n
\]

The phasor (sinusoidal steady state) voltages and currents will be denoted with carets and are complex-valued having a magnitude and an angle: \(\hat{V} = V \angle \theta_V\) and \(\hat{I} = I \angle \theta_I\). The output of this “linear system” is the voltage across the capacitor, \(v(t)\), whose Fourier coefficients are obtained as \(c_n \hat{H}_n \angle (\theta_n + \phi_n)\), giving the Fourier series of the time-domain output waveform as

\[
v(t) = 0.5 + 0.1931\cos(\pi t - 162.34^\circ) + 0.0224\cos(3\pi t - 173.94^\circ) + 0.0081\cos(5\pi t - 176.36^\circ) + 0.0041\cos(7\pi t - 177.4^\circ)
\]

Figure 1.15 shows the approximation to the output waveform for \(v(t)\) obtained by \textit{summing in time} the steady-state responses to only the dc component and the first seven harmonics of \(v_S(t)\).

The exact result for \(v(t)\) is obtained with PSPICE and shown in Fig. 1.16. Note that there is an initial transient part of the solution over the first 2 or 3 s due to the capacitor being charged up to its steady-state voltage. These results make sense because as the square-wave transitions to 1 V, the voltage across the capacitor increases according to \(1 - e^{-t/RC}\). Since the time constant is \(RC = 1\) s, the voltage has not reached steady state (which requires about five time constants to have elapsed) when the square wave turns off at \(t = 1s\). Then the capacitor voltage begins to discharge. But when the square wave turns on
again at $t = 2\, \text{s}$, the capacitor has not fully discharged and begins recharging. This process and the resulting output voltage waveform repeats with a period of 2 s. The transitions in the exact waveform of the output voltage in Fig. 1.16 are sharper than the corresponding transitions in the approximate waveform in Fig. 1.15 obtained by summing the responses to the first seven harmonics of the Fourier series of the square wave.

FIGURE 1.15. Voltage waveform across the capacitor of Fig. 1.14 obtained by adding the (steady-state) responses of the dc component and the first seven harmonics of the Fourier series of the square wave.

FIGURE 1.16. PSPICE solution for $v(t)$ for the circuit in Fig. 1.14.
responses to the high-frequency components of the input waveform and is a general property.

This example shows a powerful method of using superposition and the Fourier series indirectly to determine the response of a linear system to a complicated time-domain waveform. Quite often it is simpler to take this route rather than confronting the solution to the problem directly in the time domain by solving the differential equation relating $v(t)$ to $v_S(t)$. We will find this technique to be quite useful in determining the time-domain response of a transmission line and at the same time including frequency-dependent line losses that would make a direct time-domain solution very tedious and involved.

1.4 THE BASIC TRANSMISSION-LINE PROBLEM

A transmission line connects a source to a load as shown in Fig. 1.17(a). The objective will be to determine the time-domain response waveform of the output voltage of the line, $V_L(t)$, given the termination impedances, $R_S$ and $R_L$, the source voltage waveform $V_S(t)$, and the properties of the transmission line. If the source and termination impedances are linear, we may, alternatively, view the transmission-line problem as a linear system having an input $V_S(t)$ and an output $V_L(t)$ by embedding the terminations in one system, as shown in

**FIGURE 1.17.** Basic transmission-line problem.
Fig. 1.17(b). Hence the time-domain solution for $V_L(t)$ can be obtained in an approximate fashion using superposition and summing, in time, the responses to the components of the Fourier series for $V_S(t)$. We will develop methods for direct sketching of the waveform of $V_L(t)$ by hand when the termination impedances are linear resistors. We will use PSPICE to solve this problem when the termination impedances are linear capacitors or inductors or a combination of these elements.

The essential questions to be answered in this book are whether the two parallel conductors that interconnect the source and the load and comprise the transmission line have any significant effect on the signal transmitted to the load, and if so, how we calculate that effect. Lumped-circuit analysis principles would suggest that the line conductors can be ignored and the load voltage can be computed by voltage division as

$$V_L(t) = \frac{R_L}{R_S + R_L} V_S(t)$$

But this result assumes that the length of the transmission line, $L$, is very short, electrically (i.e. $L \ll \lambda$). However, if the line length is not electrically short at all the significant frequencies of the source voltage (within its bandwidth), the transmission line cannot be ignored and will have the possibility of affecting adversely the signal transmitted to the load.

### 1.4.1 Two-Conductor Transmission Lines and Signal Integrity

*Signal integrity* means that the two interconnect conductors (the interconnect line) connecting two electronic modules should not appreciably affect the signal transmitted along the interconnect line to its load. In other words, for the digital modules connected by these conductors to perform reliably, we expect (hope!) that the interconnect conductors have no appreciable effect on those transmitted signals other than imposing the inevitable time delay. We study this in Part I.

Figure 1.18 is an example of an interconnecting set of conductors (lands on a PCB) causing severe logic errors, resulting in poor signal integrity. Two CMOS inverters (buffers) are connected by 2 in of lands ($L = 2 \text{ in} = 0.0508 \text{ m}$) on a PCB. The output of the left inverter is shown as a Thévenin equivalent circuit having a low source resistance of 10 $\Omega$. This is fairly typical of CMOS devices except that the output resistance is somewhat nonlinear. The load on the line is the input to the other CMOS inverter, which is represented as a 5-pF capacitor, which is also typical of the input to CMOS devices. We are interested in determining the voltage at the output of the interconnect line, $V_L(t)$, which is the voltage at the input to the second CMOS inverter.
For the cross-sectional dimensions of the line shown in Fig. 1.19, the characteristic impedance is \( Z_C = 124 \, \Omega \), and the velocity of propagation is \( v = 1.7 \times 10^8 \, \text{m/s} \). These computations are explained later in the book. This gives a one-way time delay of

\[
T_D = \frac{\ell}{v} = \frac{0.0508 \, \text{m}}{1.7 \times 10^8 \, \text{m/s}} = 0.3 \, \text{ns}
\]

The source voltage is a 5 V, 50-MHz \((T = 20 \, \text{ns})\) clock waveform having a 50% duty cycle and rise and fall times of 0.5 ns as shown in Fig. 1.20. The bandwidth of this waveform is

\[
\text{BW} = \frac{1}{\tau_r} = \frac{1}{0.5 \times 10^{-9}} = 2 \, \text{GHz}
\]

The line length of 2 in is electrically small at

\[
f\big|_{\ell=(1/10)\lambda} = \frac{1}{10T_D} = \frac{1}{3 \times 10^{-9}} = 333 \, \text{MHz}
\]

The line length is electrically short for only the first seven harmonics. However, the line is not electrically short for a substantial portion of the
spectrum of the input waveform $V_S(t)$ (from 333 MHz to 2 GHz, or 33 harmonics). Hence the interconnect line should be modeled using the transmission-line model in order to determine the load voltage $V_L(t)$ correctly.

Figure 1.21(a) shows the ideal load voltage, $V_L(t)$, but separated from the source voltage, $V_S(t)$, by the inevitable one-way time delay of $T_D = 0.3$ ns. This represents the ideal signal integrity solution that we wish to achieve in order for the system to work properly. Figure 1.21(b) shows the actual response for the load voltage computed with PSPICE using the exact transmission-line model contained in PSPICE. Typical thresholds for CMOS circuits are around halfway between the logic 1 and logic 0 levels, which in this case are 5 V and 0 V. Observe that there is severe “ringing” in the response and the response drops below the 2.5 V high level and rises above the 2.5 V low level thereby producing false logic triggering. Hence signal integrity is not achieved here.

If we examine this problem in the frequency domain, we obtain additional insight into why the load voltage has such severe ringing. Table 1.5 shows the Fourier components for the 5-V 50-MHz source voltage $V_S(t)$, which has a 50% duty cycle waveform and rise and fall times of 0.5 ns. Figure 1.22 shows the magnitude and angle of the (phasor) frequency response, $\hat{V}_L/\hat{V}_S$ from 1 MHz to 10 GHz, which is also computed using PSPICE and representing the interconnect line with the transmission-line model.

Observe that there is a peak in the magnitude of the frequency response of the transfer function in Fig. 1.22(a) occurring at 340 MHz of level 17.45 dB, which is also the frequency of the ringing in Fig. 1.21. This peak is due to the 5-pF load capacitor. The remaining resonances are due to the transmission line, which is one wavelength at $f \mid_{f=1} = 1/T_D = 3.333$ GHz and occur at multiples of a half wavelength. Hence the load capacitance causes a peak in the frequency response of 17.45 dB (a ratio of 7.46) at 340 MHz, which multiplies the level of the seventh harmonic of 350 MHz (0.4322 V) by a factor of about 7.46, thereby enhancing the ringing at that frequency on the time-domain waveform. Observe that the frequency response above some 500 MHz (where
FIGURE 1.21. Load voltage, $V_L(t)$, compared to the source voltage, $V_S(t)$, for (a) the ideal case where signal integrity is achieved, and (b) the actual case indicating logic errors and false switching of the terminal inverter.
the interconnect line is electrically long) decreases at a rate of $-20\,\text{dB/decade}$, which is due to the load capacitor. In this case, the higher-frequency harmonics of the source voltage which are rolling off in amplitude at a rate of $-40\,\text{dB/decade}$ above $f_2 = 1/\pi \tau_r = 636\,\text{MHz}$ (see Fig. 1.7) are rolling off in the load voltage $V_L(t)$ at a rate of $-60\,\text{dB/decade}$ at frequencies where the line is electrically long and hence are not of much consequence. Hence a lumped-circuit model of the line shown in Fig. 1.23 may suffice in this case.

The construction of this lumped equivalent circuit is explained later in the book. We can calculate the ringing frequency approximately from this lumped equivalent circuit of the interconnect line as

$$f_{\text{ringing}} = \frac{1}{2\pi \sqrt{LC}}$$

$$= \frac{1}{2\pi \sqrt{(37.05\,\text{nH})(1.205\,\text{pF} + 5\,\text{pF})}}$$

$$= 332\,\text{MHz}$$

which is approximately the ringing frequency in Fig. 1.21. Although the lumped equivalent circuit in Fig. 1.23 is an approximate representation of the line, an insight such as this is one of the advantages of using it to approximately represent the line.

Figure 1.24 shows a comparison of the time-domain results obtained with the transmission-line model and the lumped-circuit model of Fig. 1.23. The predictions of the lumped-circuit model in Fig. 1.23 correlate reasonably well with those of the transmission-line model, although there is a slight time shift between them.

The magnitudes of the frequency-domain transfer functions computed by the transmission-line model and the lumped-circuit model of Fig. 1.23 are compared in Fig. 1.25. Note that the predictions of the lumped-circuit model of the line in Fig. 1.23 compare well with those of the transmission-line model.
FIGURE 1.22. (a) Magnitude of the transfer function, $\tilde{V}_L/\tilde{V}_S$, and (b) angle $\angle \frac{\tilde{V}_L}{\tilde{V}_S}$ of the linear system in Fig. 1.18.
for frequencies where the line is electrically short (i.e., below around 350 MHz). If we increase the rise and fall times to 5 ns and keep all other parameters unchanged, the Fourier coefficients of \( V_S(t) \) are as shown in Table 1.6. Observe that the magnitudes of the Fourier coefficients are reduced considerably from those in Table 1.5, where the rise and fall times were 0.5 ns. For rise and fall times of 5 ns, the bandwidth of \( V_S(t) \) is \( \text{BW} = 1/\tau_r = 200 \text{ MHz} \). Hence the majority of the significant frequency components fall below the resonance of 340 MHz shown in the frequency-domain transfer function in Figs. 1.22 and 1.25, and the ringing should be reduced substantially.

Figure 1.26 shows the load voltage for a 5-ns rise and fall time, with all other parameters remaining the same. Note that the ringing is reduced substantially. This is due to the fact that the magnitude of the seventh harmonic (350 MHz) is reduced from 0.4322 V for \( \tau_r = 0.5 \text{ ns} \) to 0.0585 V for \( \tau_r = 5 \text{ ns} \), a factor of
7.39. Figure 1.27 shows a comparison of the predictions of the transmission-line model and the lumped-circuit model of Fig. 1.23 for the case of 5-ns rise and fall times, confirming the adequacy of both models for this case.

The key to achieving signal integrity is the relationship between the bandwidth (BW) of the signal that is carried by the interconnect conductors and the bandwidth of the interconnect transmission line. These are two different bandwidths.

If the frequency domain transfer function is constant (magnitude and phase) from $f = 0$ to $f = \text{BW}$, then the load and source voltage will be identical in

**TABLE 1.6. Spectral (Frequency) Components of a 5-V, 50-MHz, 50% Duty Cycle, 5-ns Rise/Fall-Time Digital Clock Signal**

<table>
<thead>
<tr>
<th>Harmonic</th>
<th>Frequency (MHz)</th>
<th>Level (V)</th>
<th>Angle (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
<td>2.8658</td>
<td>$-135$</td>
</tr>
<tr>
<td>3</td>
<td>150</td>
<td>0.3184</td>
<td>135</td>
</tr>
<tr>
<td>5</td>
<td>250</td>
<td>0.1146</td>
<td>$-135$</td>
</tr>
<tr>
<td>7</td>
<td>350</td>
<td>0.0585</td>
<td>135</td>
</tr>
<tr>
<td>9</td>
<td>450</td>
<td>0.0354</td>
<td>$-135$</td>
</tr>
<tr>
<td>11</td>
<td>550</td>
<td>0.0237</td>
<td>135</td>
</tr>
<tr>
<td>13</td>
<td>650</td>
<td>0.0170</td>
<td>$-135$</td>
</tr>
</tbody>
</table>
FIGURE 1.26. Time-domain load voltage for the circuit in Fig. 1.18 with the rise and fall times of $V_S(t)$ increased to 5 ns.

FIGURE 1.27. Comparison of the predictions of the transmission-line model and the lumped-circuit model of Fig. 1.23 for the case of 5-ns rise and fall times.
shape, and signal integrity will be achieved. For example, see the transfer function in Fig. 1.22. Essentially, the transfer function is unity from dc to around 100 MHz. If the load capacitor is replaced with a resistor, \( R_L \), then the transfer function will be, by voltage division, at these lower frequencies, 
\[
\frac{R_L}{R_S + R_L}
\]
which gives the load voltage as 
\[
V_L(t) = \frac{R_L}{R_S + R_L} V_S(t)
\]
which also achieves signal integrity.

### 1.4.2 Multiconductor Transmission Lines and Crosstalk

Crosstalk is the unintended coupling of a signal on a pair of conductors onto an adjacent pair of conductors, thereby possibly causing interference in the electronic devices interconnected by the adjacent pair of conductors. This has become a serious design problem in today’s high-speed digital systems. We study crosstalk in Part II.

We illustrate this problem of crosstalk by adding to the PCB of the previous problem a second land of width 10 mils. The two lands are separated edge to edge by 10 mils, as shown in Fig. 1.28. The total length of the lands is again \( L = 2 \text{ in} = 0.0508 \text{ m} \). The lands are terminated as shown in Fig. 1.28 with 50-\( \Omega \) resistors simulating the source impedance of a commercial pulse generator and the input impedances of an oscilloscope that may be used to measure the resulting voltages at the ends of the lands. Each land with the ground plane (beneath the substrate) forms a circuit. The generator circuit is driven at the left end by a pulse source having a source impedance of 50 \( \Omega \) and is terminated by 50 \( \Omega \) at the right end. The voltage source, \( V_S(t) \), again simulates a 5-V trapezoidal waveform of frequency 50 MHz and a 50% duty cycle as shown in Fig. 1.20 but having rise and fall times of 0.1 ns. The receptor circuit is terminated at both ends by 50 \( \Omega \) resistors. The near-end crosstalk voltage, \( V_{NE} \), is at the left end of the receptor circuit, and the far-end

![Diagram of PCB and crosstalk](image)
crosstalk voltage, $V_{FE}$, is at the right end of the receptor circuit. The following predictions of the near-end and far-end crosstalk voltages are those of the transmission-line model and are obtained with PSPICE using a crosstalk subcircuit model that we derive in Part II.

**FIGURE 1.29.** Near-end crosstalk voltage for $\tau_r = \tau_f = 0.1$ ns computed using the transmission-line model.

**FIGURE 1.30.** Far-end crosstalk voltage for $\tau_r = \tau_f = 0.1$ ns computed using the transmission-line model.
The bandwidth of the signal for a rise and fall time of 0.1 ns is \( BW \cong \frac{1}{\tau_r} = 10 \text{ GHz} \). The one-way time delay is approximately \( T_D \cong \left( \frac{L}{v_0} \right) \sqrt{\epsilon'_r} = 0.286 \text{ ns} \), where \( \epsilon'_r \cong \frac{1 + 4.7}{2} = 2.85 \) is an “effective” relative permittivity as though the surrounding medium were homogeneous and represents the average of the relative permittivity of air, \( \epsilon_r = 1 \), and the relative permittivity of the PCB board, \( \epsilon_r = 4.7 \). This is discussed in more depth in Part II. Hence the line is electrically short for frequencies below about

![Lumped equivalent-circuit model](image1)

**FIGURE 1.31.** Lumped equivalent-circuit model of the problem in Fig. 1.28.

![Near-end crosstalk voltage](image2)

**FIGURE 1.32.** Near-end crosstalk voltage for \( \tau_r = \tau_f = 5 \text{ ns} \) computed using the transmission-line and lumped-circuit models of Fig. 1.31.
Hence, for rise and fall times of 0.1 ns, there are a substantial number of spectral components of \( V_S(t) \), where the transmission line is electrically long. Therefore, for these short rise and fall times we cannot model the line with a lumped-circuit model and must use the distributed-

FIGURE 1.33. Far-end crosstalk voltage for \( \tau_r = \tau_f = 5 \) ns computed using the transmission-line and lumped-circuit models of Fig. 1.31.

\[
\frac{1}{10T_D} = 350 \text{ MHz. Hence, for rise and fall times of 0.1 ns, there are a substantial number of spectral components of } V_S(t), \text{ where the transmission line is electrically long. Therefore, for these short rise and fall times we cannot model the line with a lumped-circuit model and must use the distributed-}
\]

FIGURE 1.34. Comparison of the predictions of the magnitudes of the frequency-domain transfer function \( |\tilde{V}_{NE}/\tilde{V}_S| \) by the transmission-line and lumped-circuit models of Fig. 1.31.
parameter transmission-line model. Figure 1.29 shows the near-end crosstalk voltages, and Fig. 1.30 shows the far-end crosstalk voltages computed using the exact PSPICE subcircuit model that we discuss in Part II. Observe that these crosstalk voltages occur during rise and fall times of $V_S(t)$. In Part II we find this to be a general result.

If we slow the rise/fall times of $V_S(t)$ to 5 ns, this signal will have a bandwidth of only $BW = 1/\tau_r = 200$ MHz. Since the line is electrically short for frequencies that are less than $1/10T_D = 350$ MHz, a lumped-circuit model of the line shown in Fig. 1.31 should give accurate predictions of the crosstalk voltages. The exact predictions of the transmission-line model and the predictions of the lumped-circuit model in Fig. 1.31 are compared in Figs. 1.32 and 1.33. Observe that the exact transmission-line model and the lumped-circuit model of Fig. 1.31 give virtually identical predictions of the crosstalk voltages, as expected. Note further that the crosstalk voltages appear as rectangular pulses occurring during the rise and fall times of $V_S(t)$. We confirm this observation in Part II and determine the values of these crosstalk pulses using a much simpler model for electrically short lines.

Plots of the magnitude of the frequency-domain transfer functions $|\hat{V}_{NE}/V_S|$ in Fig. 1.34 and $|\hat{V}_{FE}/V_S|$ in Fig. 1.35 show, as expected, that the lumped-circuit model of Fig. 1.31 and the transmission-line model give virtually identical predictions for frequencies where the line is electrically short (i.e., below around 350 MHz). Hence the close predictions of the two models for $\tau_r = \tau_f = 5$ ns in Figs 1.32 and 1.33 are to be expected.

![Graph of Transfer Function VFE/VS](image)

**FIGURE 1.35.** Comparison of the predictions of the magnitude of the frequency-domain transfer function $|\hat{V}_{FE}/V_S|$ by the transmission-line and lumped-circuit models of Fig. 1.31.
PROBLEMS

1.1 Express the following values of resistance, capacitance, and inductance in terms of the multipliers in Table 1.1.

(a) \(25 \times 10^4 \, \Omega \) [250 k\(\Omega\)]
(b) \(0.035 \times 10^4 \, \Omega \) [350 \(\Omega\)]
(c) \(0.00045 \, \text{F} \) [450 \(\mu\text{F}\)]
(d) \(0.003 \times 10^{-7} \, \text{F} \) [0.3n\(\text{F}\)]
(e) \(0.005 \times 10^{-2} \, \text{H} \) [50 \(\mu\text{H}\)]

1.2 Convert the following dimensions to those indicated.

(a) \(30 \, \text{mi} \) to \(\text{km} \) \([48.3 \, \text{km}]\)
(b) \(1 \, \text{ft} \) to \(\text{mils} \) \([12,000 \, \text{mils}]\)
(c) \(100 \, \text{yd} \) (length of a U.S. football field) to \(\text{m} \) \([91.44 \, \text{m}]\)
(d) \(5 \, \text{mm} \) to \(\text{mils} \) \([196.85 \, \text{mils}]\)
(e) \(20 \, \mu\text{m} \) to \(\text{mils} \) \([0.7874 \, \text{mils}]\)
(f) \(880 \, \text{yd} \) (race distance) to \(\text{m} \) \([804.67 \, \text{m}]\)

1.3 A sinusoidal current wave is described below. Determine the velocity of propagation and the wavelength. If the wave travels a distance \(d\), determine the time delay and phase shift. Determine the frequency where the distance is one wavelength and the distance in terms of a wavelength.

(a) \(i(t, z) = I_0 \sin(2\pi \times 10^6 t - 2.2 \times 10^{-2} z), \ d = 3 \, \text{km}\)

\[
\begin{align*}
[\nu \ &= 2.856 \times 10^8 \, \text{m/s}, \ \lambda = 285.6 \, \text{m}, \ T_D = 10.5 \, \mu\text{s}, \ \phi \\
&= 3781.5^\circ = 181.5^\circ, \ f\big|_{d=1\lambda} = 95.2381 \, \text{kHz}, \ d = 10.5\lambda]
\end{align*}
\]

(b) \(i(t, z) = I_0 \sin(6\pi \times 10^9 t - 75.4z), \ d = 4 \, \text{in}\)

\[
\begin{align*}
[\nu \ &= 2.5 \times 10^8 \, \text{m/s}, \ \lambda = 83.3 \, \text{mm}, \ T_D \\
&= 0.41 \, \text{ns}, \ \phi = 438.92^\circ = 78.92^\circ, \ f\big|_{d=1\lambda} = 2.461 \, \text{GHz}, \ d = 1.219\lambda]
\end{align*}
\]

(c) \(i(t, z) = I_0 \sin(30\pi \times 10^7 t - 3.15z), \ d = 20 \, \text{ft}\)

\[
\begin{align*}
[\nu \ &= 2.99 \times 10^8 \, \text{m/s}, \ \lambda = 1.995 \, \text{m}, \ T_D = 20.4 \, \text{ns}, \ \phi \\
&= 1100.2^\circ = 20.2^\circ, \ f\big|_{d=1\lambda} = 49.08 \, \text{MHz}, \ d = 3.06\lambda]
\end{align*}
\]

(d) \(i(t, z) = I_0 \sin(6\pi \times 10^3 t - 0.126 \times 10^{-3} z), \ d = 50 \, \text{mil}\)

\[
\begin{align*}
[\nu \ &= 1.5 \times 10^8 \, \text{m/s}, \ \lambda = 50 \, \text{km}, \ T_D = 0.54 \, \text{ms}, \ \phi = 580.9^\circ \\
&= 220.9^\circ, \ f\big|_{d=1\lambda} = 1859 \, \text{kHz}, \ d = 1.61\lambda]
\end{align*}
\]
1.4 Determine the wavelength at the following frequencies in SI and in English units.

(a) Loran C long-range navigation at 90 Hz [3,333.3 km, 2,071.2 mil]
(b) Submarine communication at 1 kHz [300 km, 186.41 mil]
(c) Automatic direction finder in aircraft at 350 kHz [857.14 m, 0.533 mil]
(d) AM radio transmission at 1.2 MHz [250 m, 820.2 ft]
(e) Amateur radio at 35 MHz [8.57 m, 28.12 ft]
(f) FM radio transmission at 110 MHz [2.73 m, 8.95 ft]
(g) Instrument landing system at 335 MHz [89.55 cm, 2.94 ft]
(h) Satellite at 6 GHz [5 cm, 1.97 in]
(i) Remote sensing at 45 GHz [6.67 mm, 262.5 mils]

1.5 Determine the following physical dimensions in wavelengths (i.e., their electrical dimension).

(a) A 50-mil length of a 60-Hz power transmission line [1/62\(\lambda\)]
(b) A 500-ft AM broadcast antenna broadcasting at 500 kHz [0.254\(\lambda\)]
(c) A 4.5-ft FM broadcast antenna broadcasting at 110 MHz [0.5\(\lambda\)]
(d) A 2-in land on a printed circuit board (assume a velocity of propagation of 1.5 \times 10^8 \text{ m/s}) at 2 GHz [0.677\(\lambda\)]

1.6 Determine the dc and first seven components of a 200-MHz clock waveform having an amplitude of 5 V, a 50% duty cycle, and rise and fall times of 0.3 ns. Determine the average power of the actual waveform and the MSE using the constant and the first seven harmonics.

\[
\begin{align*}
\frac{1}{2}V & : 5V \\
2V & : 9572\, \text{V} \\
3V & : 8097\, \text{V} \\
4V & : 3723\, \text{V} \\
5V & : 8097\, \text{V} \\
6V & : 3723\, \text{V} \\
7V & : 9572\, \text{V} \\
\text{Power} & : 10.75 \, \text{W}, 0.0013 \, \text{W}
\end{align*}
\]

1.7 Determine the dc and first seven components of a 70-MHz clock waveform having an amplitude of 5 V, a 50% duty cycle, and rise and fall times of 3 ns. Determine the average power of the actual waveform and the MSE using the constant and the first seven harmonics.

\[
\begin{align*}
\frac{1}{2}V & : 5V \\
2V & : 2.9572\, \text{V} \\
3V & : 1.8097\, \text{V} \\
4V & : 0.3723\, \text{V} \\
5V & : 0.0417\, \text{V} \\
6V & : 0.0502\, \text{V} \\
\text{Power} & : 3.96 \, \text{W}, 0.0011 \, \text{W}
\end{align*}
\]

1.8 Determine the dc and first seven components of a 600-MHz clock waveform having an amplitude of 3 V, a 50% duty cycle, and rise and fall times of 0.3 ns. Determine the average power of the actual waveform and the MSE using the constant and the first seven harmonics.

\[
\begin{align*}
\frac{1}{2}V & : 3V \\
2V & : 1.8097\, \text{V} \\
3V & : 0.3723\, \text{V} \\
4V & : 0.0417\, \text{V} \\
5V & : 0.0502\, \text{V} \\
\text{Power} & : 3.96 \, \text{W}, 0.0011 \, \text{W}
\end{align*}
\]

1.9 Compute the values of the bounds on the magnitudes of the coefficients for the waveform in Problem 1.6 using the asymptotic plot in Fig. 1.7,
and compare to the exact values. [3.1831 V, 1.06103 V, 0.63662 V, 0.34463 V]

1.10 Compute the values of the bounds on the magnitudes of the coefficients for the waveform in Problem 1.7 using the asymptotic plot in Fig. 1.7 and compare to the exact values. [3.1831 V, 0.5361 V, 0.193 V, 0.0985 V]

1.11 Compute the values of the bounds on the magnitudes of the coefficients for the waveform in Problem 1.8 using the asymptotic plot in Fig. 1.7 and compare to the exact values. [1.90986 V, 0.37526 V, 0.135 V, 0.06893 V]

1.12 Determine the bandwidths of the clock waveforms in Problems 1.6, 1.7, and 1.8. [3.33 GHz, 333.3 MHz, 3.33 GHz]

1.13 Plot the exact and approximate waveforms using the dc component and first seven harmonics for the clock waveform in Problem 1.6 using, for example, MATLAB.

1.14 Plot the exact and approximate waveforms using the dc component and first seven harmonics for the clock waveform in Problem 1.7 using, for example, MATLAB.

1.15 Plot the exact and approximate waveforms using the dc component and first seven harmonics for the clock waveform in Problem 1.8 using, for example, MATLAB.