INTRODUCTION

The objective of life insurance is to provide financial security to policyholders and their families. Traditionally, this security has been provided by means of a lump sum payable contingent on the death or survival of the insured life. The sum insured would be fixed and guaranteed. The policyholder would pay one or more premiums during the term of the contract for the right to the sum insured. Traditional actuarial techniques have focused on the assessment and management of life-contingent risks: mortality and morbidity. The investment side of insurance generally has not been regarded as a source of major risk. This was (and still is) a reasonable assumption, where guaranteed benefits can be broadly matched or immunized with fixed-interest instruments.

But insurance markets around the world are changing. The public has become more aware of investment opportunities outside the insurance sector, particularly in mutual fund type investment media. Policyholders want to enjoy the benefits of equity investment in conjunction with mortality protection, and insurers around the world have developed equity-linked contracts to meet this challenge. Although some contract types (such as universal life in North America) pass most of the asset risk to the policyholder and involve little or no investment risk for the insurer, it was natural for insurers to incorporate payment guarantees in these new contracts—this is consistent with the traditional insurance philosophy.

In the United Kingdom, unit-linked insurance rose in popularity in the late 1960s through to the late 1970s, typically combining a guaranteed minimum payment on death or maturity with a mutual fund type investment. These contracts also spread to areas such as Australia and South Africa, where U.K. insurance companies were influential. In the United States, variable annuities and equity-indexed annuities offer different forms of equity-linking guarantees. In Canada, segregated fund contracts became popular in the late 1990s, often incorporating complex guaranteed values on
death or maturity. Germany recently introduced equity-linked endowment insurance. Similar contracts are also popular in many other jurisdictions. In this book the term *equity-linked insurance* is used to refer to any contract that incorporates guarantees dependent on the performance of a stock market indicator. We also use the term *separate account insurance* to refer to the group of products that includes variable annuities, segregated funds, and unit-linked insurance. For each of these products, some or all of the premium is invested in an equity fund that resembles a mutual fund. That fund is the separate account and forms the major part of the benefit to the policyholder. Separate account products are the source of some of the most important risk management challenges in modern insurance, and most of the examples in this book come from this class of insurance. The nature of the risk to the insurer tends to be low frequency in that the stock performance must be extremely poor for the investment guarantee to bite, and high severity in that, if the guarantee does bite, the potential liability is very large.

The assessment and management of financial risk is a very different proposition to the management of insurance risk. The management of insurance risk relies heavily on diversification. With many thousands of policies in force on lives that are largely independent, it is clear from the central limit theorem that there will be very little uncertainty about the total claims. Traditional actuarial techniques for pricing and reserving utilize deterministic methodology because the uncertainties involved are relatively minor. Deterministic techniques use “best estimate” values for interest rates, claim amounts, and (usually) claim numbers. Some allowance for uncertainty and random variation may be made implicitly, through an adjustment to the best estimate values. For example, we may use an interest rate that is 100 or 200 basis points less than the true best estimate. Using this rate will place a higher value on the liabilities than will using the best estimate as we assume lower investment income.

Investment guarantees require a different approach. There is generally only limited diversification amongst each cohort of policies. When a market indicator becomes unfavorable, it affects many policies at the same time. For the simplest contracts, either all policies in the cohort will generate claims or none will. We can no longer apply the central limit theorem. This kind of risk is referred to as *systematic, systemic, or nondiversifiable* risk. These terms are interchangeable.

Contrast a couple of simple examples:

- An insurer sells 10,000 term insurance contracts to independent lives, each having a probability of claim of 0.05 over the term of the contract. The expected number of claims is 500, and the standard deviation is 22 claims. The probability that more than, say, 600 claims arise is less than $10^{-5}$. If the insurer wants to be very cautious not to underprice
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or underreserve, assuming a mortality rate of 6 percent for each life instead of the best estimate mortality rate of 5 percent for each life will absorb virtually all mortality risk.

- The insurer also sells 10,000 pure endowment equity-linked insurance contracts. The benefit under the insurance is related to an underlying stock price index. If the index value at the end of the term is greater than the starting value, then no benefit is payable. If the stock price index value at the end of the contract term is less than its starting value, then the insurer must pay a benefit. The probability that the stock price index has a value at the end of the term less than its starting value is 5 percent.

The expected number of claims under the equity-linked insurance is the same as that under the term insurance—that is 500 claims. However, the nature of the risk is that there is a 5 percent chance that all 10,000 contracts will generate claims, and a 95 percent chance that none of them will. It is not possible to capture this risk by adding a margin to the claim probability of 5 percent.

This simple equity-linked example illustrates that, for this kind of risk, the mean value for the number (or amount) of claims is not very useful. We can also see that no simple adjustment to the mean will capture the true risk. We cannot assume that a traditional deterministic valuation with some margin in the assumptions will be adequate. Instead we must utilize a more direct, stochastic approach to the assessment of the risk. This stochastic approach is the subject of this book.

The risks associated with many equity-linked benefits, such as variable-annuity death and maturity guarantees, are inherently associated with fairly extreme stock price movements—that is, we are interested in the tail of the stock price distribution. Traditional deterministic actuarial methodology does not deal with tail risk. We cannot rely on a few deterministic stock return scenarios generally accepted as “feasible.” Our subjective assessment of feasibility is not scientific enough to be satisfactory, and experience—from the early 1970s or from October 1987, for example—shows us that those returns we might earlier have regarded as infeasible do, in fact, happen. A stochastic methodology is essential in understanding these contracts and in designing strategies for dealing with them.

In this chapter, we introduce the various types of investment guarantees commonly used in equity-linked insurance and describe some of the contracts that offer investment guarantees as part of the benefit package. We also introduce the two common methods for managing investment guarantees: the actuarial approach and the dynamic-hedging approach. The actuarial approach is commonly used for risk management of investment guarantees by insurance companies in North America and in the United Kingdom. The
dynamic-hedging approach is used by financial engineers in banks, in hedge funds, and (occasionally) in insurance companies. In later chapters we will develop both of these methods in relation to some of the major contract types described in the following sections.

**MAJOR BENEFIT TYPES**

**Equity Participation**

All equity-linked contracts offer some element of participation in an underlying index or fund or combination of funds, in conjunction with one or more guarantees. Without a guarantee, equity participation involves no risk to the insurer, which merely acts as a steward of the policyholders’ funds. It is the combination of equity participation and fixed-sum underpinning that provides the risk for the insurer. These fixed-sum risks generally fall into one of the following major categories.

**Guaranteed Minimum Maturity Benefit (GMMB)** The guaranteed minimum maturity benefit (GMMB) guarantees the policyholder a specific monetary amount at the maturity of the contract. This guarantee provides downside protection for the policyholder’s funds, with the upside being participation in the underlying stock index. A simple GMMB might be a guaranteed return of premium if the stock index falls over the term of the insurance (with an upside return of some proportion of the increase in the index if the index rises over the contract term). The guarantee may be fixed or subject to regular or equity-dependent increases.

**Guaranteed Minimum Death Benefit (GMDB)** The guaranteed minimum death benefit (GMDB) guarantees the policyholder a specific monetary sum upon death during the term of the contract. Again, the death benefit may simply be the original premium, or may increase at a fixed rate of interest. More complicated or generous death benefit formulae are popular ways of tweaking a policy benefit at relatively low cost.

**Guaranteed Minimum Accumulation Benefit (GMAB)** With the guaranteed minimum accumulation benefit (GMAB), the policyholder has the option to renew the contract at the end of the original term, at a new guarantee level appropriate to the maturity value of the maturing contract. It is a form of guaranteed lapse and reentry option.

**Guaranteed Minimum Surrender Benefit (GMSB)** The guaranteed minimum surrender benefit (GMSB) is a variation of the guaranteed minimum maturity benefit. Beyond some fixed date the cash value of the contract, payable
The guaranteed minimum income benefit (GMIB) ensures that the lump sum accumulated under a separate account contract may be converted to an annuity at a guaranteed rate. When the GMIB is connected with an equity-linked separate account, it has derivative features of both equities and bonds. In the United Kingdom, the guaranteed-annuity option is a form of GMIB. A GMIB is also commonly associated with variable-annuity contracts in the United States.

**CONTRACT TYPES**

**Introduction**

In this section some generic contract types are described. For each of these types, individual insurers’ product designs may differ in detail from the basic contract described below. The descriptions given here, however, give the main benefit details.

The first three are all separate account products, and have very similar risk management and modeling issues. These products form the basis of the analysis of Chapters 6 to 11. However, the techniques described in these chapters can be applied to other type of equity-linked insurance. The guaranteed annuity option is discussed in Chapter 12, and equity-indexed annuities are the topic of Chapter 13.

**Segregated Fund Contracts—Canada**

The segregated fund contract in Canada has proved an extremely popular alternative to mutual fund investment, with around $60 billion in assets in 1999, according to *Risk* magazine. Similar contracts are now issued by Canadian banks, although the regulatory requirements differ.

The basic segregated fund contract is a single premium policy, under which most of the premium is invested in one or more mutual funds on the policyholder’s behalf. Monthly administration fees are deducted from the fund. The contracts all offer a GMMB and a GMDB of at least 75 percent of the premium, and 100 percent of premium is common. Some contracts offer enhanced GMDB of more than the original premium. Many contracts offer a GMAB at 100 percent or 75 percent of the maturing value.

The rate-of-administration fee is commonly known as the _management expense ratio_ or _MER_. The MER differs by mutual fund type.

The name “segregated fund” refers to the fact that the premium, after deductions, is invested in a fund separate from the insurer’s funds. The management of the segregated funds is often independent of the insurer.
A policyholder may withdraw some or all of his or her segregated fund account at any time, though there may be a penalty on early withdrawals.

The insurer usually offers a range of funds, including fixed interest, balanced (a mixture of fixed interest and equity), broad-based equity, and perhaps a higher-risk or specialized equity fund. For policyholders who invest in several funds, the guarantee may apply to each fund separately (a fund-by-fund benefit) or may be based on the overall return (the family-of-funds approach).

**Variable Annuities—United States**

The U.S. variable-annuity (VA) contract is a separate account insurance, very similar to the Canadian segregated fund contract. The VA market is very large, with over $100 billion of annual sales each year in recent times.

Premiums net of any deductions are invested in subaccounts similar to the mutual funds offered under the segregated fund contracts. GMDBs are a standard contract feature; GMMBs were not standard a few years ago, but are beginning to become so. They are known as VAGLBs or variable-annuity guaranteed living benefits. Death benefit guarantees may be increased periodically.

**Unit-Linked Insurance—United Kingdom**

Unit-linked insurance resembles segregated funds, with the premium less deductions invested in a separate fund. In the 1960s and early 1970s, these contracts were typically sold with a GMMB of 100 percent of the premium. This benefit fell into disfavor, partly resulting from the equity crisis of 1973 to 1974, and most contracts currently issued offer only a GMDB.

Some unit-linked contracts associated with pensions policies carry a guaranteed annuity option, under which the fund at maturity may be converted to a life annuity at a guaranteed rate. This is a more complex option, of the GMIB variety. This option is discussed in Chapter 12.

**Equity-Indexed Annuities—United States**

The U.S. equity-indexed annuity (EIA) offers participation at some specified rate in an underlying index. A participation rate of, say, 80 percent of the specified price index means that if the index rises by 10 percent the interest credited to the policyholder will be 8 percent. The contract will offer a guaranteed minimum payment of the original premium accumulated at a fixed rate; a rate of 3 percent per year is common.

Fixed surrender values are a standard feature, with no equity linking. Other contract features vary widely by company. A form of GMAB may be offered in which the guarantee value is set by annual reset according to the participation rate.
Many features of the EIA are flexible at the insurer’s option. The MERs, participation rates, and floors may all be adjusted after an initial guarantee period.

The EIAs are not as popular as VA contracts, with less than $10 billion in sales per year. EIA contracts are discussed in more detail in Chapter 13.

**Equity-Linked Insurance—Germany**

These contracts resemble the U.S. EIAs, with a guaranteed minimum interest rate applied to the premiums, along with a percentage participation in a specified index performance. An unusual feature of the German product is that, for regulatory reasons, annual premium contracts are standard (Nonnemacher and Russ 1997).

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**EQUITY-LINKED INSURANCE AND OPTIONS**

**Call and Put Options**

Although the risks associated with equity-linked insurance are new to insurers, at least, relative to life-contingent risks, they are very familiar to practitioners and academics in the field of derivative securities. The payoffs under equity-linked insurance contracts can be expressed in terms of options.

There are many books on the theory of option pricing and risk management. In this book we will review the relevant fundamental results, but the development of the theory is not covered. It is crucially important for practitioners in equity-linked insurance to understand the theory underpinning option pricing. The book by Boyle et al. (1998) is specifically written with actuaries and actuarial applications in mind. For a general, readable introduction to derivatives without any technical details, Boyle and Boyle (2001) is highly recommended.

The simplest forms of option contracts are:

- A *European call option* on a stock gives the purchaser the right (but not the obligation) to purchase a specified quantity of the underlying stock at a fixed price, called the *strike price*, at a predetermined date, known as the *expiry* or *maturity date* of the contract.

- A *European put option* on a stock gives the purchaser the right to sell a specified quantity of the underlying stock at a fixed strike price at the expiry date.

*American options* are defined similarly, except that the option holder has the right to exercise the option at any time before expiry. *Asian options*
have a payoff based on an average of the stock price over a period, rather than on the final stock price.

To summarize the benefits under the option contracts, we introduce some notation. Let $K$ be the strike price of the option per unit of stock; let $S_t$ be the price of one unit of the underlying stock at time $t$; and let $T$ be the expiry date of the option. The payoff at time $T$ under the call option will be:

$$ (S_T - K)^+ = \max(S_T - K, 0) $$

and the payoff under the put option will be

$$ (K - S_T)^+ = \max(K - S_T, 0) $$

In subsequent chapters we shall see that it is natural to think of the investment guarantee benefits under separate account products as put options on the policyholder’s fund. On the other hand, it is more natural to use call options to value the benefits under an equity-indexed annuity.

We often use the terms in-the-money, at-the-money, and out-of-the-money in relation to options and to equity-linked insurance guarantees. A put option that is in-the-money at time $t < T$ has an underlying stock price $S_t < K$, so that if the stock price at maturity were to be the same as the current stock price, there would be a payment under the guarantee. For a call option, in-the-money means that $S_t > K$, and at-the-money means that the stock and strike prices are roughly equal. Out-of-the-money for a put option means $S_t > K$, and for a call option means $S_t < K$; in either case, if the stock price at maturity is the same as the current stock price, no payment would be required under the guarantee or option contract. We say a contract is deep out-of-the-money or in-the-money if the difference between the stock price and strike price is large, so that it is very likely that a deep out-of-the-money contract will remain out-of-the-money, and similarly for the deep in-the-money contract.

**The No-Arbitrage Principle**

The no-arbitrage principle states that, in well-functioning markets, two assets or portfolios having exactly the same payoffs must have exactly the same price. This concept is also known as the law of one price; it is a fundamental assumption of financial economics. The logic is that if prices differ by a fraction, it will be noticed by the market, and traders will move in to buy the cheaper portfolio and sell the more expensive, making an instant risk-free profit or arbitrage. This will pressure the price of the cheap portfolio back up, and the price of the expensive portfolio back down, until they return to equality. Therefore, any possible arbitrage opportunity will be eliminated in an instant. Many studies show consistently that the no-arbitrage assumption is empirically indisputable in major stock markets.
This simple and intuitive assumption is actually very powerful, particularly in the valuation of derivative securities. To value a derivative security such as an option, it is sufficient to find a portfolio, with known value, that precisely replicates the payoff of the option. If the option and the replicating portfolio do not have the same price, one could sell the more expensive and buy the cheaper, and make an arbitrage profit. Since this is assumed to be impossible, the value of the option and the value of the replicating portfolio must be identical under the no-arbitrage assumption.

**Put-Call Parity**

Using the no-arbitrage assumption allows us to derive an important connection between the put option and the call option on a stock.

Let $c_t$ denote the value at $t$ of a European call option on a unit of stock, and $p_t$ the value of a European put option on a unit of the same stock. Both options are assumed to mature at the same date $T > t$ with the same strike price, $K$. Assume the stock price at $t$ is $S_t$, then an investor who holds both a unit of stock and a put option on that unit of stock will have a portfolio at time $t$ with value $p_t + S_t$. The payoff at expiry of the portfolio will be

$$p_T + S_T = \max(K, S_T)$$

(1.3)

Similarly, consider an investor who holds a call option on a unit of stock together with a pure discount bond maturing at $T$ with face value $K$. We assume the pure discount bond earns a risk-free rate of interest of $r$ per year, continuously compounded, so that the value at time $t$ of the pure discount bond plus call option is $c_t + Ke^{-r(T-t)}$. The payoff at maturity of the portfolio of the pure discount bond plus call option will be

$$c_T + K = \max(K, S_T)$$

(1.4)

In other words, these two portfolios—“put plus stock” and “call plus bond”—have identical payoffs. The no-arbitrage assumption requires that two portfolios offering the same payoffs must have the same price. Hence we find the fundamental relationship between put and call options known as put-call parity, that is,

$$p_t + S_t = c_t + Ke^{-r(T-t)}$$

(1.5)

**Options and Equity-Linked Insurance**

Many benefits under equity-linked insurance contracts can be regarded as put or call options. For example, the liability under the maturity guarantee of a Canadian segregated fund contract can be naturally regarded as an embedded put option. That is, the policyholder who pays a single premium of $1000 with a 100 percent GMMB is guaranteed to receive at least
$K = 1000$ at maturity, even if the market value of her or his portfolio is less than $1000$ at that time. It is the responsibility of the insurer to pay $(1000 - S_T)^+$, the excess of the guaranteed amount over the market value of the assets, meaning that the insurer pays the payoff under a put option.

Therefore, the total segregated fund policy benefit is made up of the policyholder’s fund plus the payoff from a put option on the fund. From put-call parity we know that the same benefit can be provided using a bond plus a call option, but that route is not sensible when the contract is designed in the separate account format. Put-call parity also means that the U.S. EIA could either be regarded as a combination of fixed-interest security (meeting the minimum interest rate guarantee) and a call option on the underlying stock (meeting the equity participation rate benefit), or as a portfolio of the underlying stock (for equity participation) together with a put option (for the minimum benefit). In fact, the first method is a more convenient approach from the design of the contract.

The fundamental difference between the VA-type guarantee, which we value as a put option to add to the separate account proceeds, and the EIA guarantee, which we value as a call option added to the fixed-interest proceeds, arises from the withdrawal benefits. On withdrawal, the VA policyholder takes the proceeds of the separate account, without the put option payment. The EIA policyholder withdraws with their premium accumulated at some fixed rate, without the call-option payment.

American options may be relevant where equity participation and minimum accumulation guarantees are both offered on early surrender. Asian options are relevant for some EIA contracts where the equity participation can be based on an average of the underlying stock price rather than on the final value.

There is a substantial and rich body of theory on the pricing and financial management of options. Black and Scholes (1973) and Merton (1973) showed that it is possible, under certain assumptions, to set up a portfolio that consists of a long position in the underlying stock together with a short position in a pure discount bond and has an identical payoff to the call option. This is called the replicating portfolio. The theory of no-arbitrage means that the replicating portfolio must have the same value as the call option because they have the same payoff at the expiry date. Thus, the famous Black-Scholes option-pricing formula not only provides the price but also provides a risk management strategy for an option seller—hold the replicating portfolio to hedge the option payoff. A feature of the replicating portfolio is that it changes over time, so the theory also requires the balance of stocks and bonds to be rearranged at frequent intervals over the term of the contract.

The stock price, $S_t$, is the random variable in the payoff equations for the options (we assume that the risk-free rate of interest is fixed). The
Probability distribution of $S_t$ is known as the real-world measure, the physical measure, or the $P$-measure. The fundamental result of Black, Scholes, and Merton was that securities may be valued and the replicating portfolio derived by taking the expected value of the payoff, but under a different, artificial distribution known as the $Q$-measure (or risk-neutral measure). In Chapter 7 we discuss the relationship between these two measures.

There are some complications in applying this theory to the options embedded in equity-linked insurance. The major problem is the very long-term nature of the equity-linked options. The contract term for standard traded options might be a few weeks—an option with a term of more than six months would be considered long term. In contrast, the options implicit in equity-linked insurance commonly have terms of over 10 years, and some may be in force for 30 years or more. A challenge for actuaries managing equity-linked contracts is to adapt the methods of financial economics to the long time scales in which insurance companies work.

### Provision for Equity-Linked Liabilities

#### Reinsurance

An easy way for the insurer to manage the liability from options embedded in equity-linked contracts is to buy options, equivalent to those they have sold, from third parties. This is equivalent to reinsuring the entire risk; indeed, reinsurers have been involved in selling such options to insurers. As with reinsurance, the insurer is likely to pass on a substantial proportion of the expected profit on the contracts along with the risk. Also, (as with reinsurance) the insurer must be aware of the counterparty risk; that is, the risk that the option provider will not survive to the maturity date, which may be decades away.

For some markets, such as that for segregated fund contracts in Canada, reinsurers and other option providers are increasingly unwilling to provide the options at prices acceptable to the insurers.

#### Dynamic Hedging

As mentioned in the section on equity-linked insurance and options, the Black-Scholes analysis provides a risk management strategy for option providers; use the Black-Scholes equation to find the replicating portfolio. The portfolio will change continuously, so it is necessary to recalculate and adjust the portfolio frequently. Although the Black-Scholes equation contains some strong assumptions that cannot be realized in practice, the replicating portfolio still manages to provide a powerful method of hedging the liability. This method is explored in detail in Chapters 7 and 8.
Most of the academic literature relating to equity-linked insurance assumes a dynamic-hedging management strategy. See, for example, Boyle and Schwartz (1977), Brennan and Schwartz (1975, 1979), Bacinello and Ortu (1993), Ekern and Persson (1996), and Persson and Aase (1994); these papers appear in actuarial, finance, and business journals. Nevertheless, although the application by actuaries in practice of financial economic theory to the management of embedded options is growing, in many areas it is still not widely accepted.

The Actuarial Approach
In the mid 1970s the ground-breaking work of Black, Scholes, and Merton was relatively unknown in actuarial circles. In the United Kingdom, however, maturity guarantees of 100 percent of premium were a common feature of the unit-linked contracts, which were then proving very popular with consumers. The prolonged low stock market of 1973 to 1974 had awakened the actuaries to the possibility that this benefit, which had been treated as a relatively unimportant policy “tweak” with very little value or risk, constituted a serious potential liability. The then recent theory of Black and Scholes was considered to be too risky and unproven to be used for unit-linked guaranteed maturity benefits by the U.K. actuarial profession.¹

In 1980, the Maturity Guarantees Working Party (MGWP) suggested, instead, using stochastic simulation to determine an approximate distribution for the guarantee liabilities, and then using quantile reserving to convert the distribution into a usable capital requirement. The quantile reserve had already been used for many years, particularly in non-life insurance. To calculate the quantile reserve, the insurer assesses an appropriate quantile of the loss distribution, for example, 99 percent. The present value of the quantile is held in risk-free bonds, so that the office can be 99 percent certain that the liability will be met. This principle is identical to the value-at-risk (VaR) concept of finance, though generally applied over longer time periods by the insurance companies than by the banks.

The underlying principle of this method of calculating the capital requirements is that the capital is assumed to be invested in risk-free bonds. The use of the quantile of the distribution as a risk measure is not actually fundamental to this approach, and other risk measures may be preferable (this is discussed further in Chapter 9).

¹This was a decision that has had unfortunate consequences. If the actuarial profession had taken the opportunity to learn and apply option pricing theory and risk management at that time, then the design and management of embedded options in insurance contracts in the last 20 years would have been very different and actuaries would have been better placed to participate in the derivatives revolution.
This method of using stochastic simulation to project the liabilities, and then using the long-term fixed rate of interest to discount them, is referred to in this book (and elsewhere) as the “actuarial” approach. It is inherently different from the dynamic-hedging approach, in which assets are assumed to be invested in the replicating portfolio, not in the bonds. However, it should not be inferred that dynamic hedging is somehow not actuarial. Nor should it be assumed that the actuarial approach is incompatible with dynamic hedging. A synthesis of the two approaches may lead to better risk management than either provides separately.

The actuarial method is still popular (particularly with actuaries) and offers a valid alternative to the dynamic-hedging approach for some equity-linked contracts. The Canadian Institute of Actuaries’ Task Force on Segregated Funds (SFTF 2000) uses the actuarial approach as the underpinning methodology for determining capital requirements, although a combined hedging-actuarial approach is also accommodated. In Chapter 6, the actuarial approach to equity-linked liabilities is investigated.

The Ad Hoc Approach

There is a (diminishing) body of opinion amongst actuaries that the statistical analysis that forms the subject of this book is unnecessary or even irrelevant. Their approach to valuation and management of financial guarantees might be described as guesswork, or “actuarial judgment.” This is most common for the very low-frequency type options, where there is very little chance of any liability. An example might be a GMMB, which guarantees that the benefit after a 10-year investment will be no less than the original premium. There is very little chance that the separate account will fall to less than the original investment over the course of 10 years. Rather than model the risk statistically, it was common for actuaries to assume that there would never be a liability under the guarantee, so little or no provision was made. This view is uncommon now and tends to be unpopular with regulators.

For any actuary tempted by this approach, the Equitable Life (U.K.) story provides a clear demonstration of the risks of ignoring statistical methodology. Along with many U.K. insurers in the early 1980s, Equitable Life (U.K.) issued a large number of contracts carrying guaranteed-annuity options, under which the guarantee would move into the money only if interest rates fell below 6.5 percent. At the time the contracts were issued, interest rates were higher than 10 percent, and a cautious long-term view was that they might fall to 8 percent. Many actuaries, relying on their personal judgment, believed that these contracts would never move into the money, and therefore made little or no provision for the potential liability. This conclusion was made despite the fact that interest rates had been below 6.5 percent for decades up to the later 1960s. Of course, in the mid-1990s rates fell, the guarantees moved into the money, and the guarantee liabilities
were so large that Equitable Life (U.K.), a large mutual company more than 200 years old, was forced to close to new business. Many other companies were also hit hard and only substantial free surplus kept them trading. Yang (2001) has demonstrated that, had actuaries in the 1980s used the stochastic models and methods then available, it would have been clear that substantial provision would be required for this option.

**PRICING AND CAPITAL REQUIREMENTS**

There are several issues that are important for actuaries and risk managers involved in any area of policy design, marketing, valuation, or risk management of equity-linked insurance. The following are three main considerations:

1. What price should the policyholder be charged for the guarantee benefit?
2. How much capital should the insurer hold in respect of the benefit through the term of the contract?
3. How should this capital be invested?

Much work in equity-linked insurance has focused on pricing without very much consideration of the capital issues. But the three issues are crucially interrelated. For example, using the option approach for pricing maturity guarantees gives a price, but that price is only appropriate if it is suitably invested (in a dynamic-hedge portfolio, or by purchasing the options externally). Also, as we shall see in later chapters, different risk management strategies require different levels of capital (for the same level of risk), and therefore the implied price for the guarantee would vary.

The approach of this book is that all of these issues are really facets of the same issue. The first requirement for pricing or for determination of capital requirements is a credible estimate of the distribution of the liabilities, and that is the main focus of this book. Once this distribution is determined, it can be used for both pricing and capital requirement decisions. In addition, the liability issue is really an asset-liability issue, so the estimation of the liability distribution depends on the risk management decision.