Heterogeneous Regressions, ANCOVA, And Fixed-Effects Models

2.1 Introduction

Referring to various DVMs having interaction independent variables, namely the factors A, and B, and the time-period TP, then if we are considering one or more numerical variables as additional exogenous (cause, source, upstream, or independent) variable(s), we would be very confident, in a theoretical sense, that the numerical exogenous variable(s) should have differential effects on the corresponding endogenous (impact, down-stream or dependent) variable(s), either numerical, zero-one, or ordinal variable, between the cells or groups generated by A, B, and TP. For this reason, a heterogeneous regression model (HRM) should be considered as the best possible model to be applied.

This chapter would present alternative HRMs, starting with the simplest possible model in each specific group of models, by presenting their equation specifications (ESs). The other equation specifications are left for exercises.

2.2 HRMs having a Numerical Exogenous Variable

Following the DVMs presented in previous chapter, the HRMs considered would have the following general equation specification only.

\[ G(Y) \ F(X)@Expand(A,B,TP) \ Expand(A,B,TP) \]

For all possible HRMs by the factors A, B, and TP (time-period), have the following characteristics.
(i) $G(Y)$ represents a numerical endogenous variable, including proportion or percentage variable, $Y_{it}$, and its transformed variables such as $\log((Y_{it} - L)/\log(U - Y_{it}))$, $\log(Y_{it} - L)$, or $\log(U - Y_{it})$, where $L$ and $U$ are fixed lower and upper bounds of $Y_{it}$.

(ii) $F(X)$ represents any numerical exogenous variable $X_{it}$, including $Y_{i,t-p}$ and an environmental variable, namely $Z_{it}$, or $Z_{t-p}$, and the numerical time-$t$.

(iii) The other alternative functions of $X$ are the functions having no parameter, and without a constant, such as $\log(X_{it})$, for $X_{it} > 0$, $1/X_{it}$, and $X_{it}^\alpha$, for a fixed number $\alpha \neq 0$. Compare to the models (6.20), in the main book, using different type of functions, for instance $F_k(X) = C(k1) + C(k2) * X_{it}^{\alpha(k)}$, $\alpha(k) \neq 0$, in (6.20d), which has two parameters, with a constant parameter, because the models have independent dummy variables $Dij$, instead of using the function @Expand(*).

(iii) In addition note that $(aX+b)$, $b \neq 0$ cannot be inserted for the $F(X)$ in (2.1), because the design matrix will be perfectly singular. However, if you would like to use the function $F(X) = aX+b$, a new variable has to be generated, namely $X_{New} = aX+b$, then the variable $X_{New}$ can be used to replace $F(X)$ in (2.1).

(iv) On the other hand, the invers function $F(X) = 1/(ax+b)$ could be applied directly.

(v) However, in the following subsections, only some simple alternative HRMs would be presented.

The main objective of all HRMs in (2.1) are to study and to test the differential linear effects of $F(X)$ on $G(Y)$, between the cells generated by the cause or classification factors $A$, $B$, and the time-period, $TP$. For an illustration Table 2.1 presents the slope parameters or the linear effects of $F(X)$ on $G(Y)$ of a 2x2x2 HRM in (2.1), which are indicated by the parameters $C(1)$ up to $C(8)$.

<table>
<thead>
<tr>
<th>Table 2.1 Slope Parameters of a 2x2x2 factorial HRM in (2.1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A=1</td>
</tr>
<tr>
<td>TP(1-2)</td>
</tr>
</tbody>
</table>
Based on this table, hypotheses on the differential linear effects of \( F(X) \) on \( G(Y) \) between relevant cells/groups \((A=i,B=j,TP=k)\) can easily be defined, and then tested using the Wald test. Some of the hypotheses are as follows:

1. Conditional for \((A=i,B=j)\), the linear effect of \( F(X) \) on \( G(Y) \), depends on the factor \( TP \). In other words, \( F(X) \) has different linear effects on \( G(Y) \), between levels of the factor \( TP \), conditional for \((A=i,B=j)\). For an illustration, conditional for \((A=1,B=1)\), the statistical hypothesis is as follows:

\[
H_0: C(1) = C(5), \text{ versus } H_1: C(1) \neq C(5)
\]

2. Conditional for \((A=i)\), the linear effect of \( F(X) \) on \( G(Y) \), depends on the factors \( B \) and \( TP \). In other words, \( F(X) \) has different linear effects on \( G(Y) \), between the cells/groups generated by the factors \( B \) and \( TP \), conditional for \((A=i)\). For an illustration, conditional for \((A=1)\), the statistical hypothesis is as follows:

\[
H_0: C(1) = C(2) = C(5) = C(6), \text{ versus } H_1: \text{ Otherwise}
\]

3. The linear effect of \( F(X) \) on \( G(Y) \), depends on the factors \( A, B \) and \( TP \), with the following hypothesis.

\[
H_0: C(1) = C(2) = C(3) = C(4) = C(5) = C(6) = C(7) = C(8), \text{ versus } H_1: \text{ Otherwise}
\]

### 2.2.1 Heterogeneous Classical Growth Models (HCGMs)

It is recognized that there are three types of HCGMs can be considered, such as follows – refer to the classical growth models presented in Agung (2009a, and 2011b).

(i) HCGM for all firms or individuals has the following equation specification (ES). Note that if the data contains hundreds of firms, then this model is representing hundreds of classical growth models, for any positive endogenous variable \( Y_{it} \).

\[
\log(Y_{it}) \t \@ \text{Expand(Firm)} @ \text{Expand(Firm)}
\]

(ii) HCGMs by the firm groups/sectors or the cause/classification factors, namely \( A \) and \( B \), would have the following ES. Note that the researchers should be very confidence that all firms or all research objects within each group generated by \( A \) and \( B \) can be considered as a single homogenous group.

\[
\log(Y_{it}) \t \@ \text{Expand(A,B)} @ \text{Expand(A,B)}
\]
(iii) Peace-wise HCGMs by the cause or classification A and B, and the time-periods TP, have the following ES. Note that the peace-wise growths are indicated by the time-periods TP,

\[ \log(Y) \ast \text{Expand}(A,B,TP) \ast \text{Expand}(A,B,TP) \]  (2.4)

2.2.2 First-order Lagged Variable HRMs

First-order lagged variable HRMs, namely LV(1)_HRMs, have the following general ES.

\[ G(Y) \ast Y(-1) \ast \text{Expand}(A,B,TP) \ast \text{Expand}(A,B,TP) \]  (2.5)

2.2.3 First-order Autoregressive HRMs

First-order autoregressive HRMs, namely AR(1)_HRMs, have the following general ES.

\[ G(Y) \ast F(X) \ast \text{Expand}(A,B,TP) \ast \text{Expand}(A,B,TP) \ast \text{AR}(1) \]  (2.6)

2.3 HRMs having two Numerical Exogenous Variable

2.3.1 Hierarchical IxJxK Factorial HRMs

The HRMs of an endogenous variable Y, and two exogenous numerical variables X, X1 and X2, would have the following general equation specification:

\[ G(Y) \ast F_1(X1) \ast F_2(X2) \ast \text{Expand}(A,B,TP) \ast F_1(X1) \ast \text{Expand}(A,B,TP) \ast F_2(X2) \ast \text{Expand}(A,B,TP) \]  (2.7)

where \( F_1(X1), \ F_2(X2), \) and \( G(Y), \) respectively, can be any functions of the exogenous variables \( X1_{it}, \) and \( X2_{it}, \) and the endogenous variable \( Y_{it}, \) without a parameter. So that there would be a lot of possible HRMs could be proposed or defined by a researcher. Note that the functions \( F_1(X1), \) and \( F_2(X2), \) do not have constant numbers – refer to the notes for the function \( F(X) \) for the model in (2.1).

For an illustration, Table 2.2 presents the parameters of a 2x2x2 factorial HRM in (2.7). Based on the statistical results of this model and this table, the following findings, notes and comments are presented.
Table 2.2 Parameters of the 2x2x2 factorial HRMs in (2.7)

<table>
<thead>
<tr>
<th>Variable</th>
<th>A=1</th>
<th></th>
<th>A=2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B=1</td>
<td>B=2</td>
<td>B=1</td>
<td>B=2</td>
</tr>
<tr>
<td></td>
<td>TP=1</td>
<td>TP=2</td>
<td>TP=1</td>
<td>TP=2</td>
</tr>
<tr>
<td>$F_1(X1)*F_2(X2)$</td>
<td>(C(1))</td>
<td>(C(2))</td>
<td>(C(3))</td>
<td>(C(4))</td>
</tr>
<tr>
<td>$F_1(X1)$</td>
<td>(C(9))</td>
<td>(C(10))</td>
<td>(C(11))</td>
<td>(C(12))</td>
</tr>
<tr>
<td>$F_2(X2)$</td>
<td>(C(17))</td>
<td>(C(18))</td>
<td>(C(19))</td>
<td>(C(20))</td>
</tr>
<tr>
<td>Intercept</td>
<td>(C(25))</td>
<td>(C(26))</td>
<td>(C(27))</td>
<td>(C(28))</td>
</tr>
</tbody>
</table>

(1) The differential effects of the numerical interaction independent variables, namely $F_1(X1)*F_2(X2)$ on $G(Y)$ between cells/groups $(A=i,B=j,TP=k)$ for some relevant $i$, $j$, and $k$, could easily be defined and tested using the Wald test. The hypotheses would be similar to the three types/groups hypotheses presented in Section 2.2, such as follows:

1.1 Conditional for $(A=i,B=j)$, the adjusted effect of $F_1(X1)*F_2(X2)$ on $G(Y)$, depends on the factor $TP$. In other words, $F_1(X1)*F_2(X2)$ has different effects on $G(Y)$, between levels of the factor $TP$, conditional for $(A=i,B=j)$. For an illustration, conditional for $(A=1,B=1)$, the statistical hypothesis is as follows:

$$H_0: C(1) = C(2), \text{ versus } H_1: C(1) \neq C(2)$$

1.2 Conditional for $(A=i)$, the adjusted effect of $F_1(X1)*F_2(X2)$ on $G(Y)$, depends on the factors $B$ and $TP$. In other words, $F_1(X1)*F_2(X2)$ has different effects on $G(Y)$, between the cells/groups generated by the factors $B$ and $TP$, conditional for $(A=i)$. For an illustration, conditional for $(A=1)$, the statistical hypothesis is as follows:

$$H_0: C(1) = C(2) = C(3) = C(4), \text{ versus } H_1: \text{Otherwise}$$

(2) Within each of the cells $(A=i,B=j,TP=k)$, the effect of $F_1(X1)*F_2(X2)$ on $G(Y)$, adjusted for $F_1(X1)$ and $F_2(X2)$, can easily be tested using the test-statistic shown in the statistical results. However, for testing the effect of a numerical main independent variable, the following alternative hypotheses should be considered, because it is inappropriate to test the main effect, if its interaction is in the model.

(3) The hypothesis stated that the effect of $F_1(X1)$ on $G(Y)$, depends on $F_2(X2)$, within each of the cell $(A=i,B=j,TP=k)$. In other words, the hypothesis on the joint effects of $F_1(X1)$, and
For instance, within the cell \((A=1,B=1,TP=1)\), we have the statistical hypothesis as follows:

\[ H_0: C(1)=C(9)=0; \text{ versus } H_1: \text{ Otherwise} \]

(4) The hypothesis stated that \(F_1(X1)\), and \(F_1(X1)F_2(X2)\) have different joint effects on \(G(Y)\), within two or more relevant cells \((A=i,B=j,TP=k)\). For instance, as follows:

4.1 Between the two cells \((A=1,B=1,TP=1)\), and \((A=1,B=1,TP=2)\) we have the following statistical hypothesis.

\[ H_0: C(1)=C(2), C(9)=C(10); \text{ versus } H_1: \text{ Otherwise} \]

4.2 Between the four cells \((A=1,B=j,TP=k)\)‘s, and \((A=1,B=1,TP=2)\) we have the following statistical hypothesis.

\[ H_0: C(1)=C(2)=C(3)=C(4), C(9)=C(10)=C(11)=C(12); \text{ versus } H_1: \text{ Otherwise} \]

(5) If one or more numerical independent variables have large p-values, say Sig.(2-tailed > 0.30) within a cell of \((A=i,B=j,TP=k)\), then an acceptable reduced model would be explored. In this case, it is recommended to apply the manual stepwise selection method, as presented in the main book.

2.3.2 Nonhierarchical \(I \times J \times K\) Factorial HRMs

Note that all HRMs in (2.7) are hierarchical two-way interaction models within each of the cells/groups generated by the factors \(A\), \(B\), and \(TP\). In practice, however, either one of the following nonhierarchical reduced models could be a good fit model.

(1) Nonhierarchical two-way interaction models:

These HRMs have the following alternative equations specifications.

\[
\begin{align*}
G(Y) & \quad F_1(X1) \ast F_2(X2) \ast @Expand(A,B,TP) \\
F_1(X1) & \ast @Expand(A,B,TP) \quad @Expand(A,B,TP) \\
G(Y) & \quad F_1(X1) \ast F_2(X2) \ast @Expand(A,B,TP) \\
F_2(X1) & \ast @Expand(A,B,TP) \quad @Expand(A,B,TP)
\end{align*}
\]

(2.8)
(2) Additive models within each cell/group (A=i, B=j, TP=k):

These HRMs have the general equation specification as follows:

\[ G(Y) \times F_1(X_1) \times \text{@Expand}(A, B, TP) \times F_2(X_2) \times \text{@Expand}(A, B, TP) \times \text{@Expand}(A, B, TP) \]  \tag{2.11}

One of the HRMs, which has been widely applied is the translog linear model having the following ES, for positive variables \( X_1, X_2, \) and \( Y. \)

\[ \log(Y) \times \log(X_1) \times \text{@Expand}(A, B, TP) \times \log(X_2) \times \text{@Expand}(A, B, TP) \times \text{@Expand}(A, B, TP) \]  \tag{2.12}

This model could be extended to a bounded translog linear model having the following ES, where \( L \) and \( U \) are the fixed lower and upper bound of \( Y_{it}. \)

\[ \log((Y-L)/(U-Y)) \times \log(X_1) \times \text{@Expand}(A, B, TP) \times \log(X_2) \times \text{@Expand}(A, B, TP) \times \text{@Expand}(A, B, TP) \]  \tag{2.13}

On the other hand, for a comparison, the following ES is representing the worst additive models. Refer to Table 1.12, which shows the worst 2x2x2 factorial ANOVA model.

\[ \log(Y) \times \log(X_1)) \times \log(X_2) \times \text{@Expand}(A, @Droplast) \times \text{@Expand}(B, @Droplast) \times \text{@Expand}(TP, @Droplast) \]  \tag{2.14}

Referring to the simple models in (2.2) up to (2.4), then the exogenous variables \( X_1, \) and \( X_2, \) could be the time \( t \) as a numerical environmental variable, and \( Y(-1). \) In addition, either one of \( X_1, \) and \( X_2, \) or both can be replaced by \( X_1(-1), \) and \( X_2(-1). \) Some of the models are as follows:

2.3.1 Peace-wise HRMs with trends, having the general ES as follows:

\[ G(Y) \times F_1(X_1) \times \text{@Expand}(A, B, TP) \times t \times \text{@Expand}(A, B, TP) \times \text{@Expand}(A, B, TP) \]  \tag{2.15}

2.3.2 Peace-wise HRMs with the Time-Related-Effects (TRE), having the general ES as follows:
\[ G(Y) = t^*F_1(X_1)^*@Expand(A,B,TP) \ t^*@Expand(A,B,TP) \]
\[ F_1(X_1)^*@Expand(A,B,TP) \ @Expand(A,B,TP) \]
\[ F_2(Z)^*@Expand(A,B,TP) \ @Expand(A,B,TP) \]
\[ @Expand(A,B,TP) \]
\[ (2.16) \]

2.3.3 HRMS with an Environmental Variable, \( Z \), having the general ES as follows:
\[ G(Y) = F_1(X_1)^* F_2(Z)^*@Expand(A,B,TP) \ F_1(X_1)^*@Expand(A,B,TP) \]
\[ F_2(Z)^*@Expand(A,B,TP) \ @Expand(A,B,TP) \]
\[ (2.17) \]

2.3.4 LV1_ HRMs, having the general ES as follows:
\[ G(Y) = \ Y(-1)^*F_1(X_1)^*@Expand(A,B,TP) \ Y(-1)^*@Expand(A,B,TP) \]
\[ F_1(X_1)^*@Expand(A,B,TP) \ @Expand(A,B,TP) \]
\[ (2.18) \]

2.3.5 AR(1) _ HRMs, having the general ES as follows:
\[ G(Y) = F_1(X_1)^* F_2(X_2)^*@Expand(A,B,TP) \ F_1(X_1)^*@Expand(A,B,TP) \]
\[ F_2(X_2)^*@Expand(A,B,TP) \ @Expand(A,B,TP) \ AR(1) \]
\[ (2.19) \]

2.4 HRMs having three Numerical Exogenous Variables

2.4.1 General Equation

The HRMs of an endogenous variable \( Y \), and two exogenous numerical variables \( X_1, X_2, \) and \( X_3 \), would have the following general equation specification:
\[ G(Y) = F_1(X_1)^* F_2(X_2)^*F_3(X_3)^*@Expand(A,B,TP) \ F_1(X_1)^* F_2(X_2)^*@Expand(A,B,TP) \]
\[ F_1(X_1)^*F_3(X_3)^*@Expand(A,B,TP) \ F_2(X_2)^*F_3(X_3)^*@Expand(A,B,TP) \]
\[ F_1(X_1)^*@Expand(A,B,TP) \ F_2(X_2)^*@Expand(A,B,TP) \]
\[ F_3(X_3)^*@Expand(A,B,TP) \ @Expand(A,B,TP) \]
\[ (2.20) \]

where \( F_1(X_1), F_2(X_2), F_3(X_3) \), and \( G(Y) \), respectively, can be any functions of the exogenous variables \( X_{it}, X_{it}', \) and \( X_{it}'' \), and the endogenous variable \( Y_{it} \), without a parameter, and the functions \( F_1(X_1), F_2(X_2), \) and \( F_3(X_3) \), do not have constant numbers – refer to the notes for the various functions of \( F(X) \) in the model (2.1).
Table 2.3 Parameters of the 2x2x2 factorial HRMs in (2.17)

<table>
<thead>
<tr>
<th>Variable</th>
<th>A=1</th>
<th></th>
<th></th>
<th>A=2</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TP=1</td>
<td>TP=2</td>
<td>TP=1</td>
<td>TP=2</td>
<td>TP=1</td>
<td>TP=2</td>
<td></td>
</tr>
<tr>
<td>$F_1(X1)*F_2(X2)*F_3(X3)$</td>
<td>C(1)</td>
<td>C(2)</td>
<td>C(3)</td>
<td>C(4)</td>
<td>C(5)</td>
<td>C(6)</td>
<td>C(7)</td>
</tr>
<tr>
<td>$F_1(X1)*F_2(X2)$</td>
<td>C(9)</td>
<td>C(10)</td>
<td>C(11)</td>
<td>C(12)</td>
<td>C(13)</td>
<td>C(14)</td>
<td>C(15)</td>
</tr>
<tr>
<td>$F_1(X1)*F_3(X3)$</td>
<td>C(17)</td>
<td>C(18)</td>
<td>C(19)</td>
<td>C(20)</td>
<td>C(21)</td>
<td>C(22)</td>
<td>C(23)</td>
</tr>
<tr>
<td>$F_3(X1)$</td>
<td>C(33)</td>
<td>C(34)</td>
<td>C(35)</td>
<td>C(36)</td>
<td>C(37)</td>
<td>C(38)</td>
<td>C(39)</td>
</tr>
<tr>
<td>$F_3(X2)$</td>
<td>C(41)</td>
<td>C(42)</td>
<td>C(43)</td>
<td>C(44)</td>
<td>C(45)</td>
<td>C(46)</td>
<td>C(47)</td>
</tr>
<tr>
<td>$F_3(X3)$</td>
<td>C(49)</td>
<td>C(50)</td>
<td>C(51)</td>
<td>C(52)</td>
<td>C(53)</td>
<td>C(54)</td>
<td>C(55)</td>
</tr>
<tr>
<td><strong>Intercept</strong></td>
<td>C(57)</td>
<td>C(58)</td>
<td>C(59)</td>
<td>C(60)</td>
<td>C(61)</td>
<td>C(62)</td>
<td>C(63)</td>
</tr>
</tbody>
</table>

For an illustration, Table 2.3 presents the parameters of a 2x2x2 factorial HRM in (2.17). Based on the statistical results of this model and this table, the following findings, notes and comments are presented. Then, based on the statistical results of this model, and Table 2.17, various hypotheses similar to those based on Table 2.2 could be easily defined and tested. Do it for an exercise.

In addition, if a reduced model should be explored, then it is recommended to conduct the analysis using the manual stepwise selection method.

Note that all HRMs in (2.17) are hierarchical three-way interaction models within each of the cells/groups generated by the factors A, B, and TP. In practice, however, a good fit acceptable model would be nonhierarchical three-way interaction model, or hierarchical or nonhierarchical two-way interaction models, either one of the nonhierarchical reduced models could be a good fit model, and an additive model. Refer to the models in (2.7) to (2.11).

Furthermore, note that the exogenous variables $X_{1ib}$, $X_{2ib}$, and $X_{3ib}$, could be any numerical variables, as presented for the independent variable X of the models in (2.1). Refer to the models in (2.12) to (2.16).
2.4.2 Some Specific HRMs

Corresponding to all possible HRMs in (2.20), Figure 2.1 presents the path diagrams to present three specific up-and-down or seemingly causal relationships (SCMs), based on an endogenous variable $Y_{it}$, and two exogenous variables $X1_{it}$ and $X2_{it}$, within each of the cell or group generated by $A$, $B$, and $TP$. These path diagrams have the characteristics as follows:

![Figure 2.1](image)

**Figure 2.1** Alternative up-and-down relationships based on the variables LY, X1 and X

(1) The arrows with dashed lines are representing that the causal or up-and-down relationships between the corresponding pair of variables are not taken into account as dependent and independent variables, but they are taken into account in the models of $Y$, to indicate an interaction, either two- or three-way interactions, should be used as independent variables, as presented in the models (2.20), such as follows:

1.1 Figure 2.1(a) shows that the effect of $X1$ on $Y$, depends on $Y(-1)$, and $X2$. So that the interactions $Y(-1)*X1*X2$, $Y(-1)*X1$, and $X1*X2$, should be used as possible independent variables of the model, in a theoretical sense. In addition, the effect of $X2$ on $Y$, depends on $Y(-1)$, then the interaction $Y(-1)*X2$ also should be taken into account as an independent variable of the model. Finally, the three main variables $Y(-1)$, $X1$, and $X2$. Thence, the general model would have the ES as follows:

\[
Y \ Y(-1)*X1* X2*@Expand(A,B,TP) \ Y(-1)*X1*@Expand(A,B,TP) \\
Y(-1)* X2*@Expand(A,B,TP) \ X1* X2*@Expand(A,B,TP) \ Y(-1)*@Expand(A,B,TP) \\
X1*@Expand(A,B,TP) \ X2*@Expand(A,B,TP) \ Expand(A,B,TP) \quad (2.20a)
\]

However, since the seven exogenous variables are highly correlated, in general, then a good fit model within each cell or group would have only some of the seven numerical
independent variables which should be selected using the manual stepwise selection method. Refer to the Example 2.2.

1.2 Similarly, Figure 2.1(b) represents the model of \( Y \) on \( Y(-1) \cdot X1 \cdot X2(-1) \), \( Y(-1) \cdot X1 \), \( Y(-1) \cdot X2(-1) \), \( X1 \cdot X2(-1) \), \( Y(-1) \), \( X1 \), and \( X2(-1) \). 

1.3 Finally, Figure 2.1(c) represents the model of \( Y \) on \( Y(-1) \cdot X1(-1) \cdot X2(-1) \), \( Y(-1) \cdot X1(-1) \), \( Y(-1) \cdot X1(-1) \cdot X2(-1) \), \( X1(-1) \), \( X1(-1) \), and \( X2(-1) \).

(2) These models could be extended to the models with trends, the models with the time-related effects, and the models with environmental variable(s), similar to the models in (2.15), (2.16), and (2.17), respectively.

2.5 Advanced HRMs

2.5.1 Polynomial HRMs

As an extension of the simplest HRMs in (2.1), polynomial HRMs would be considered having the following general equation specification.

\[
G(Y) \cdot F(X) @ \text{Expand(} \text{Group,TP} \text{)} \ldots F(X)^k @ \text{Expand(} \text{Group,TP} \text{)} \\
\ldots F(X)^K @ \text{Expand(} \text{Group,TP} \text{)} \cdot \text{Expand(} \text{Group,TP} \text{)} 
\]

(2.22)

where the categorical variable \( \text{Group} \) can be generated by one or more variables, and \( \text{TP} \) is a time-period variable- refer to all possible functions of the variable \( X \), for the models in (2.1).

2.5.2 General Equation Specification

As the extension of the HRMs in (2.20), having three numerical independent variables, advanced HRMs of \( G(Y) \), would be presented using the following general equation specification.

\[
G(Y) \cdot V1 @ \text{Expand(} \text{Group,TP} \text{)} \ldots Vk @ \text{Expand(} \text{Group,TP} \text{)} \\
\ldots VK @ \text{Expand(} \text{Group,TP} \text{)} \cdot \text{Expand(} \text{Group,TP} \text{)} 
\]

(2.23)

where \( G(Y) \) is a function of an endogenous variable \( Y_{it} \), without a parameter, \( Vk \) can be any main factor or variable – refer to all possible choices of the variable \( X \) in ES (2.1), a two- or a three-way interaction of specifically selected main factors, the categorical variable \( \text{Group} \) can be generated by one or more variables, and \( \text{TP} \) is a time-period variable. Note that, instead of using the time-period \( \text{TP} \), the time-\( t \) (year or others), also could be used as the categorical independent variable, if and on if the time-\( t \) is not used as a numerical variables –refer to the models in (2.2), (2.3), and (2.4).
Note that these models can easily be modified to following HRMs.

2.5.2.1 HRMs by Individuals/Firm, and TP

The general equation of these HRMs are as follows:

$$G(Y) = C \times V_1 \times \text{Expand}(\text{Firm}, TP) \ldots V_k \times \text{Expand}(\text{Firm}, TP) \ldots V_K \times \text{Expand}(\text{Firm}, TP) \times \text{Expand}(\text{Firm}, TP, @Drop(*)$$

(2.23a)

Note this ES present a set of $N$ time series models, that is for all firms in the sample, by the time-periods $TP$.

2.5.2.2 HRMs by Cell-Factor(CF), and Time-T

The general equation of these HRMs are as follows:

$$G(Y) = C \times V_1 \times \text{Expand}(\text{CF}, T) \ldots V_k \times \text{Expand}(\text{CF}, T) \ldots V_K \times \text{Expand}(\text{CF}, T) \times \text{Expand}(\text{CF}, T, @Drop(*)$$

(2.23b)

where $CF$ is a cell-factor or a group variable, which should be invariant or constant over times, and it can be generated based one of more variables.

2.5.2.3 Selected Specific HRMs

Several specific alternative HRMs having the general ES (2.23), which need to be considered are as follows:

(1) Translog linear HRMs by the $Group$ and $TP$, with the ES as follows:

$$\log(Y) = \log(X_1) \times \text{Expand}(\text{Group}, TP) \ldots \log(X_k) \times \text{Expand}(\text{Group}, TP) \ldots \log(X_K) \times \text{Expand}(\text{Group}, TP) \times \text{Expand}(\text{Group}, TP)$$

(2.24)

where $Y$ and $X_k$, for all $k=1,\ldots,K$ are positive variables. These models are an extension of the Cobb-Douglas production functions by $Group$, and $TP$

(2) HRMs with trend by the $Group$, with the ES as follows:

$$G(Y) = V_1 \times \text{Expand}(\text{Group}) \ldots V_k \times \text{Expand}(\text{Group}) \times \text{Expand}(\text{Group})$$

(2.25)

(3) HRMs with the Time-Related-Effects by the Group, with the ES as follows:

$$G(Y) = V_1 \times \text{Expand}(\text{Group}) \ldots V_k \times \text{Expand}(\text{Group}) \times \text{Expand}(\text{Group}) \times \text{Expand}(\text{Group})$$

(2.26)
2.6 Various Alternatives HRMs

All equation specifications above can easily be applied for the following models. 
(1) The models having the numerical endogenous variable presented in subsection 1.3.1, such as the LS Regressions, Quantile Regressions, and Instrumental Variables Models. 
(2) The binary choice (probit, logit, and extreme value) models, for a dummy problem indicator, 
(3) The ordered choice (probit, logit, and extreme value) models, for an ordinal problem indicator, as presented in subsection 1.3.1.3, 
(4) The firm, or cross-section fixed-effects HRMs, and the time, or period fixed-effects HRMs, as the extension of FEMs presented in Section 1.4.3.

The data analyses are left for exercises. The following examples present the application of the manual stepwise selection method, which has not been known by most of the readers, and some selected specific models.

2.7 Application of the Manual Stepwise Selection Method

In order to obtain a reduced model, which is acceptable in both theoretical and statistical sense, it is recommended to apply the manual stepwise selection method (Agung, 2011), since by using the STEPLS – Stepwise Least Squares estimation method once for all possible independent variables, unexpected statistical results or the worst regression might be obtained. The important exogenous (cause, source or upstream) variable(s), the interaction variable(s) in particular, might not be selected as an independent variable(s).

2.7.1 Empirical Statistical Results

The following examples present two empirical statistical results of a common heterogeneous regressions model (HRM), and the simplest lagged variables model, respectively based on the data in CES.wf1.

Example 2.1 (A two-way interaction HRM by two dichotomous factors) Table 2.4 presents a summary of the statistical results of a HRM model of a numerical variable $LY=\log(Y)$ on $LK = \log(K)$, and $LL=\log(L)$ by two dichotomous factors $Group$, and $TP=1+1*(Year>14)$, in the data CES.wf1. The stages of analysis are as follows:
### Table 2.4 Summary of statistical results

<table>
<thead>
<tr>
<th>Variable</th>
<th>Stages-1,2, &amp; 3</th>
<th>Stage-4</th>
<th>Stage-5</th>
</tr>
</thead>
<tbody>
<tr>
<td>GROUP=1 AND TP=1</td>
<td>4.476</td>
<td>0.000</td>
<td>-1.695</td>
</tr>
<tr>
<td>GROUP=1 AND TP=2</td>
<td>1.747</td>
<td>0.000</td>
<td>-1.103</td>
</tr>
<tr>
<td>GROUP=2 AND TP=1</td>
<td>7.856</td>
<td>0.000</td>
<td>-2.354</td>
</tr>
<tr>
<td>GROUP=2 AND TP=2</td>
<td>4.527</td>
<td>0.000</td>
<td>-2.121</td>
</tr>
<tr>
<td>LK<em>LL</em>(GROUP=1 AND TP=1)</td>
<td>0.011</td>
<td>0.000</td>
<td>-0.005</td>
</tr>
<tr>
<td>LK<em>LL</em>(GROUP=1 AND TP=2)</td>
<td>0.005</td>
<td>0.000</td>
<td>-0.002</td>
</tr>
<tr>
<td>LK<em>LL</em>(GROUP=2 AND TP=1)</td>
<td>0.011</td>
<td>0.000</td>
<td>-0.016</td>
</tr>
<tr>
<td>LK<em>LL</em>(GROUP=2 AND TP=2)</td>
<td>0.007</td>
<td>0.000</td>
<td>-0.010</td>
</tr>
<tr>
<td>LK*(GROUP=1 AND TP=1)</td>
<td>0.621</td>
<td>0.000</td>
<td>0.865</td>
</tr>
<tr>
<td>LK*(GROUP=1 AND TP=2)</td>
<td>0.806</td>
<td>0.000</td>
<td>0.916</td>
</tr>
<tr>
<td>LK*(GROUP=2 AND TP=1)</td>
<td>0.450</td>
<td>0.000</td>
<td>0.865</td>
</tr>
<tr>
<td>LK*(GROUP=2 AND TP=2)</td>
<td>0.655</td>
<td>0.000</td>
<td>0.918</td>
</tr>
<tr>
<td>LL*(GROUP=1 AND TP=1)</td>
<td></td>
<td></td>
<td>0.407</td>
</tr>
<tr>
<td>LL*(GROUP=1 AND TP=2)</td>
<td></td>
<td></td>
<td>0.185</td>
</tr>
<tr>
<td>LL*(GROUP=2 AND TP=1)</td>
<td></td>
<td></td>
<td>0.681</td>
</tr>
<tr>
<td>LL*(GROUP=2 AND TP=2)</td>
<td></td>
<td></td>
<td>0.429</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.970</td>
<td></td>
<td>0.970</td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.969</td>
<td></td>
<td>0.970</td>
</tr>
<tr>
<td>S.E. of regression</td>
<td>0.336</td>
<td></td>
<td>0.333</td>
</tr>
<tr>
<td>Sum squared resid</td>
<td>258.125</td>
<td></td>
<td>252.648</td>
</tr>
<tr>
<td>Log likelihood</td>
<td>-748.953</td>
<td></td>
<td>-724.330</td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>0.032</td>
<td></td>
<td>0.034</td>
</tr>
</tbody>
</table>

The statistical results of the first three stages are presented in one column, since no reduced model should be developed at each stage of the data analysis. The equation specifications (ESs) used are as follows:

1. At Stage-1: the ES is: $LY \ @Expand(\text{Group}, TP)$. Which represents a two-way ANOVA model. In this stage, no reduced model should be developed even though one or more dummy variables do have large p-values.
1.1 At Stage-2: the ES is: \(LY @\text{Expand}(Group,TP)\) \(LK*LL*@\text{Expand}(Group,TP)\). In this stage, no reduced model should be developed, since all numerical independent variables have significant effects on \(LY\). Based on my own point of view, a numerical interaction independent variable(s) should be kept in the model if it has a \(p\)-value \(< 0.30\), because it would have either positive or negative significant effect on \(LY\), at the \(\alpha = 0.15\) level; of significance reduced model Lapin, 1973). For a comparison, Hosmer & Lemeshow (2000) propose to keep the independent variable if it has a \(p\)-value \(< 0.25\).

1.2 The final statistical results of the first three stages of the data analysis is using the ES as is: \(LY @\text{Expand}(Group,TP)\) \(LK*LL*@\text{Expand}(Group,TP)\) \(LK*@\text{Expand}(Group,TP)\).

1.3 The statistical results show that each independent variable \(LK*LL*(Group=i and TP=j)\) has significant effect. So that we do not have to reduce the model.

2) At Stage-4, the function \(LL*@\text{Expand}(Group,TP)\) is inserted as additional independent variables, where the statistical results show that the variable \(LK*LL*(Group=1 and TP=2)\) has the largest \(p\)-value \(= 0.693\). In statistical sense, a reduced model should be explored. But, this interaction independent variable should not be deleted from the model, because it is a more important independent variable, compare to the variable \(LL*(Group=1 and TP=2)\). So that, the new additional independent variable, namely \(LL*(Group=1 and TP=2)\), should be deleted from the model, even though it has a smaller \(p\)-value \(= 0.085\).

3) At Stage-5, the final results are obtained by using \(LL*@\text{Expand}(Group,TP, Drop(1,2))\) to replace the function \(LL*@\text{Expand}(Group,TP)\).

4) In addition, note that at the \(\alpha = 0.10\) level of significance, each of \(LK*LL*(Group=1 and TP=1)\), and \(LK*LL*(Group=2 and TP=2)\), has negative significant effect on \(LY\), with the \(p\)-values of \(0.198/2 = 0.099\), and \(0.114/2 = 0.072\), respectively. So that the final model should be considered as an acceptable model, in both theoretical and statistical sense, to present that the effect of \(LK\) (or \(LL\)) on \(LY\), is significantly dependent on \(LL\)(or \(LK\)).

5) Furthermore, note that the statistical results of the three models have small values of the Durbin-Watson statistic. It is recognized that the models can be improved by using the lag(s) of the dependent variable. The simplest lagged HRM would be obtained by inserting the independent variable \(LY(-1)*@\text{Expand}(Group,TP)\), in the final model, or at the second stage
of data analysis. Do it for exercise, and find special LV(1)_HRMs presented in the following example.

**Example 2.2 (Application of LV(1)_HRMs)** Referring to the path diagram in Figure 2.1(a), Figure 2.2 presents the statistical results of the final model of a special LV(1)_HRMs of $\log(Liability)$ on $X1 = \log(Sale)$, and $X2 = \log(Size)$, by two dichotomous factors $A$ and $B$, in Special_BPD.wf1. using the following equation specification.

\[
\begin{align*}
\text{ly} & \quad @\text{expand}(a,b) \quad \text{ly}(-1)\*\text{x1}\*\text{x2}@\text{expand}(a,b,\underline{\text{drop}}(1,1),\underline{\text{drop}}(2,1)) \\
\text{ly}(-1)\*\text{x1}@\text{expand}(a,b) & \quad \text{ly}(-1)\*\text{x2}@\text{expand}(a,b,\underline{\text{drop}}(2,2)) \\
\text{x1}\*\text{x2}@\text{expand}(a,b,\underline{\text{drop}}(2,2)) & \quad \text{ly}(-1)@\text{expand}(a,b,\underline{\text{drop}}(1,1),\underline{\text{drop}}(1,2)) \\
\text{x1}@\text{expand}(a,b,\underline{\text{drop}}(1,2),\underline{\text{drop}}(2,1)) & \quad \text{x2}@\text{expand}(a,b,\underline{\text{drop}}(1,1),\underline{\text{drop}}(2,2)) 
\end{align*}
\] (2.27)

Based on the ES in (2.27) and its statistical results presented in Figure 2.2, the following notes are presented

1. The ES in (2.27) shows eight groups of independent variables, so that there are eight stages of data analysis, as follows:
   1.1 At Stage-1, the SE applied is “$\log(Liability)@\text{Expand}(a,b)$”, which is an ANOVA model. In this stage, no reduced model should be developed, even though two of the means of $\log(Liability)$ have very large $p$-values. Because the final model should have four intercepts.
   1.2 At Stage-2, the SE applied is “$\log(Liability)@\text{Expand}(a,b)\log(Liability)\*\log(Sale)\*\text{Expand}(a,b)$”, but it is found that $\log(Liability)\*\log(Sale)\*(A=1 and B=1)$, and $\log(Liability)\*\log(Sale)\*(A=2 and B=1)$, have large $p$-values of 0.3305 and 0.3958, respectively, which are greater than 0.30. So I decide to reduce the model, by using $\log(Liability)\*\log(Sale)@\text{Expand}(a,b,\underline{\text{Drop}}(1,1),\underline{\text{Drop}}(2,2))$, as presented in the SE (2.24).
   1.3 Similar processes can easily be done by inserting each of the other functions, step by step, starting from $\log(Liability)\*\log(Sale)@\text{Expand}(a,b)$ up to $\log(Sale)@\text{Expand}(a,b)$, and deleting one or two of the new inserted independent variables, in order to keep the independent variables of previous model having $p$-values < 0.25 (or 0.30).
Dependent Variable: LY
Method: Panel Least Squares
Sample (adjusted): 2 8
Periods included: 7
Cross-sections included: 218
Total panel (unbalanced) observations: 1436

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coeff.</th>
<th>s.e.</th>
<th>t-Stat.</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A=1 AND B=1</td>
<td>1.711</td>
<td>0.139</td>
<td>12.286</td>
<td>0.000</td>
</tr>
<tr>
<td>A=1 AND B=2</td>
<td>2.778</td>
<td>0.156</td>
<td>17.853</td>
<td>0.000</td>
</tr>
<tr>
<td>A=2 AND B=1</td>
<td>6.008</td>
<td>0.370</td>
<td>16.219</td>
<td>0.000</td>
</tr>
<tr>
<td>A=2 AND B=2</td>
<td>6.196</td>
<td>0.810</td>
<td>7.645</td>
<td>0.000</td>
</tr>
<tr>
<td>LY(-1)<em>X1</em>X2*(A=1 AND B=2)</td>
<td>-0.022</td>
<td>0.014</td>
<td>-1.578</td>
<td>0.115</td>
</tr>
<tr>
<td>LY(-1)<em>X1</em>X2*(A=2 AND B=2)</td>
<td>0.002</td>
<td>0.001</td>
<td>3.387</td>
<td>0.001</td>
</tr>
<tr>
<td>LY(-1)<em>X1</em>(A=1 AND B=1)</td>
<td>0.061</td>
<td>0.008</td>
<td>7.408</td>
<td>0.000</td>
</tr>
<tr>
<td>LY(-1)<em>X1</em>(A=1 AND B=2)</td>
<td>0.076</td>
<td>0.006</td>
<td>13.675</td>
<td>0.000</td>
</tr>
<tr>
<td>LY(-1)<em>X1</em>(A=2 AND B=1)</td>
<td>0.054</td>
<td>0.006</td>
<td>8.587</td>
<td>0.000</td>
</tr>
<tr>
<td>LY(-1)<em>X1</em>(A=2 AND B=2)</td>
<td>0.068</td>
<td>0.013</td>
<td>5.081</td>
<td>0.000</td>
</tr>
<tr>
<td>LY(-1)<em>X2</em>(A=1 AND B=1)</td>
<td>-0.151</td>
<td>0.025</td>
<td>-5.954</td>
<td>0.000</td>
</tr>
<tr>
<td>LY(-1)<em>X2</em>(A=1 AND B=2)</td>
<td>0.299</td>
<td>0.074</td>
<td>4.024</td>
<td>0.000</td>
</tr>
<tr>
<td>LY(-1)<em>X2</em>(A=2 AND B=1)</td>
<td>-0.269</td>
<td>0.062</td>
<td>-4.333</td>
<td>0.000</td>
</tr>
<tr>
<td>X1<em>X2</em>(A=1 AND B=1)</td>
<td>0.204</td>
<td>0.021</td>
<td>9.800</td>
<td>0.000</td>
</tr>
<tr>
<td>X1<em>X2</em>(A=1 AND B=2)</td>
<td>-0.105</td>
<td>0.058</td>
<td>-1.815</td>
<td>0.070</td>
</tr>
<tr>
<td>X1<em>X2</em>(A=2 AND B=1)</td>
<td>0.140</td>
<td>0.046</td>
<td>3.062</td>
<td>0.002</td>
</tr>
<tr>
<td>X1<em>X2</em>(A=2 AND B=2)</td>
<td>-0.256</td>
<td>0.090</td>
<td>-2.853</td>
<td>0.004</td>
</tr>
<tr>
<td>LY(-1)*(A=2 AND B=1)</td>
<td>-0.276</td>
<td>0.114</td>
<td>-2.425</td>
<td>0.015</td>
</tr>
<tr>
<td>X1*(A=1 AND B=1)</td>
<td>0.283</td>
<td>0.057</td>
<td>5.005</td>
<td>0.000</td>
</tr>
<tr>
<td>X1*(A=2 AND B=2)</td>
<td>-0.073</td>
<td>0.101</td>
<td>-0.728</td>
<td>0.467</td>
</tr>
<tr>
<td>X2*(A=1 AND B=2)</td>
<td>-0.230</td>
<td>0.314</td>
<td>-0.734</td>
<td>0.463</td>
</tr>
<tr>
<td>X2*(A=2 AND B=1)</td>
<td>1.142</td>
<td>0.273</td>
<td>4.178</td>
<td>0.000</td>
</tr>
</tbody>
</table>

| R-squared                                     | 0.875  |      |         | 5.791 |
| Adjusted R-squared                            | 0.873  |      |         | 1.908 |
| S.E. of regression                            | 0.680  |      |         | 2.082 |
| Sum squared resid                             | 654.126|      |         | 2.163 |
| Log likelihood                                | -1473.020|      |         | 2.112 |

| Durbin-Watson stat                            | 1.472  |      |         |      |

**Figure 2.2** Statistical results of a LV(1)_HRM using the equation specification in (2.27)

(2) Note that the independent variables $X1*(A=2, B=2)$, and $X2*(A=1,B=2)$ do not have to be deleted from the final model, even though each has large $p$-value, because their interactions, namely $LY(-1)*X1*X2*(A=2,B=2)$, and $LY(-1)*X1*X2*(A=1,B=2)$, have significant effects.
on $LY$. However, if you wish to delete them, then an acceptable reduced model would be obtained.

(3) Since the model has a DW statistic of 1.472, then an AR(1) model is applied. It is obtained the model having a DW statistic of 2.0528, but some of the interaction independent variables have large $p$-values, such as $LY(-1)*X1*X2(A=1,B=2)$ has a $p$-value = 0.6052.. Thence a reduced model should be explored, which is left for the exercise.

(4) The causal or up-and-down stream relationships between the numerical variables $LY$, $X1$ and $X2$ in fact are defined based on the path diagram in Figure 2.1(a), which shows that $LY(-1)$ is an upstream variable of the endogenous variable $LY$, and both exogenous variables $X1$ and $X2$. In addition, $X2=\log(\text{Size})$ is an upstream variable of $X1=\log(\text{Sale})$. So that, in a theoretical sense, the effect of $LY(-1)$ on $LY$ should depend on $X1$ and $X2$, Thence, the interaction between the variables $LY(-1)$, $X1$, and $X2$ should be used as independent variables within each cell generated by the factors $A$, and $B$. This model could be extended to the models by the factors $A$, $B$, and $TP$, as well as to $LV(p)_\text{Models}$, for $p > 1$.

(5) On the other hand, the simplest reduced model is the translog linear model by the two factors $A$ and $B$, with the ES as follows, since $x1=\log(\text{Sale})$, and $x2=\log(\text{Size})$

$$ly @\text{expand}(a,b) ly(-1)*@\text{expand}(a,b) x1*@\text{expand}(a,b) x2*@\text{expand}(a,b) \quad (2.28)$$

2.7.2 Special Notes and Comments

Referring to the statistical results of the models presented in the above examples, it is recognized that the good fit models obtained are highly dependent on the ordering of the set of the numerical independent variables inserted step by step into the models. For this reason, it is important to present the following notes and comments.

(1) Referring to the full model in Example 2.1, the following notes are presented.

1.1 The sets of numerical variables $LK*LL*@\text{Expand}(\text{Group},TP)$, $LK*@\text{Expand}(\text{Group},TP)$, and $LL*@\text{Expand}(\text{Group},TP)$, could be inserted in $3!$ (= 3 factorial) = $1*2*3 = 6$ possible orderings or permutations. Do it as an exercise.

1.2 It happens the estimates obtained at Stage-4 are very simple, where only one of the independent variables, namely $LK*LL*(\text{GROUP}=1 \text{ AND } TP=2)$ has a large $p$-value > 0.30. So that a reduced model could be easily obtained. One of possible reduced model is presented at Stage-5, in Table 2.1.
(2) Now, referring to the full model of the reduced model in Example 2.2, the following notes are presented.

2.1 It is found, that the statistical results based on the full model of the reduced model in (2.27) would have many independent variables having large $p$-values, say $p$-values $> 0.30$. So that by inserting different ordering of the independent variables $LY(-1)*X1`*X2*@Expand(a,b)$ up to $X2*@Expand(a,b)$, several other good fit models would be obtained. In this case, we have $7! = 5,040$ possible ordering or permutation.

2.2 However, it is recommended to insert the highest order interaction numerical variables at the first stage, followed by the lower order interactions of selected main variables/factors which are considered as the most important upstream (cause, or source) variables, the most important main variables, and finally other additional upstream variable(s).

2.3 So that a researcher have to be using his/her knowledge and experience (judgment) to select the best possible ordering, in a theoretical sense, or some alternative orderings for a comparison results.

### 2.8 Cross Section Fixed-Effects Models

Fixed-effects models in fact are ANCOVA models. Based on cross-section data, Agung (2011, and 2006) has presented special notes that ANCOVA models are not recommended models, because the ANCOVA models have the assumption that the covariates have the same effects on the corresponding endogenous variable within all groups generated by the factors considered, and this assumption is not valid in general. On the other hand, an additive ANCOVA model is considered as the worst model among alternative ANCOVA models having the same set of numerical variables.

Corresponding to the ANCOVA Models, I would also consider that fixed-effects models, are commended models. However, fixed-effects models area acceptable models in a statistical sense, since fixed-effects models have been widely applied by many researchers, and presented in text books, such as Baltagi (2009), Gujaraty (2003), and Wooldridge (2002). This section presents special fixed-effects models, namely the cross-section fixed-effects models (CSFEMs), with special notes and comments, compare to other alternative models.
2.8.1 CSFEMs Based On a Variable $Y_{it}$

2.8.1.1 Classical Growth Model by Firms, and its CSFEM

As a modification of the HCGM in (2.2), the classical growth model of $Y_{it}$, $i=1, \ldots, N; \ t=1, \ldots, T$, by firms would be considered as the simplest heterogeneous regressions model, namely HCGM, by firms. The model would have the equation as follow:

$$\ln(Y_{it}) = \alpha_i + \beta_i t + \varepsilon_{it}, \ for \ i = 1, \ldots, N \tag{2.29}$$

where $\alpha_i$ and $\beta_i$, respectively, are the intercept and the exponential growth parameters of the model for the firm $i$, and $Y_{it}$ is a positive endogenous variable. Then the model (2.9) is representing the $N$ firms as having $N$ different growth rates, indicated by the parameters $\beta_i$, for $i=1,\ldots,N$. Furthermore, note that the $N$ regression functions of $\ln(Y)$ on the time-$t$, in fact are a set of $N$ time-series functions, and they can be graphically presented using $N$ heterogeneous regression lines in a two-dimensional coordinate system.

For the analysis using EViews, the following equation specification (ES) can be applied.

$$\log(Y) \ C \ t*@Expand(Firm) \ @Expand(Firm,@Dropfirst) \tag{2.30}$$

The most important reduced model of this model (2.30) is a cross-section fixed-effects model (CSFEM), or firm-fixed-effects model, having the ES as follows:

$$\log(Y) \ C \ t @Expand(Firm,@Dropfirst) \tag{2.31}$$

Note that this model can be considered as a one-way ANCOVA model of $\log(Y)$ by a single factor $FIRM$, with the time-$t$ as a covariate. Based on a panel data, a CSFEM can present hundreds or thousands of firms having the same growth rate, indicated by the parameter $C(2)$. For sure, this condition would never be observed in practice. Moreover, a single classical growth model (CGM) having the following equation.

$$\ln(Y_{it}) = \alpha + \beta t + \varepsilon_{it}, \tag{2.32}$$

where $\alpha$, and $\beta$, are fixed parameter for all firm-time $N*T$ observations. So that this model would be considered as the worst continuous panel data model. Similarly, for all continuous panel data models.

Example 2.3 (Application of CSFEM in (2.31)) Figure 2.3 presents a summary of the statistical results of the CSFEM in (2.31), based on the CES.wf1, and the testing on an omitted variables: $t*@Expand(Firm,@Dropfirst)$, for the following hypothesis.

$H_0: \ The \ CSFEM \ in \ (2.3), \ versus \ H_1: \ The \ HCGM \ by \ Firms \ in \ (2.2)$
Based on this summary, the following notes and comments are presented.

(1) The CSFEM is representing 82 parallel regression lines of $LY$ on the numerical time-$t$ for $t=1,…,28$. The model only has $81 = (82-1)$ firm-dummies, because the model contains the intercept parameter ‘C’. However, the list of their coefficients are not presented.

![Summary of the statistical results of CSFEM in (2.31), and an Omitted Variables test](image)

(2) It is found that $t^*@Expand(Firm, @Dropfirst)$ has a significant effect, based on the $F$-statistics of $F_0 = 65.412$, with $df = (81,2132)$, and $p$-value = 0.0000. In other words, the data supports the HCGM by Firms.

(3) It is also found that even for the first three firms, $t^*@Expand(Firm, @Dropfirst)$ has a significant effect, based on the $F$-statistics of $F_0 = 10.11448$, with $df = (2,78)$, and $p$-value = 0.0001. So that it can be said that the CSFEM is not acceptable model, in both theoretical and statistical sense, more over the continuous classical growth model in (2.4), which is the worst growth model for all panel data.

(4) Note that the model has a very small DW-statistic. Thence it is recommended to apply a lagged variable or autoregressive model. Fine the following example.

### 2.8.1.2 Lagged Variables Model by Firms and its CSFEM

The first-order lagged variable model of $Y_{it}$, by firms, would be considered as the simplest lagged variables heterogeneous regressions model, namely $LV(1)_{HRM}$, having the following equation specification.

$$ Y = C Y(-1)^*@Expand(Firm) @Expand(Firm, @Dropfirst) \quad (2.33) $$
with its two possible reduced models are as follows:

(i) A CSFEM having the following ES, which a One-way ANCOVA model of $Y$ on $Y(-1)$ with the $FIRM$ as a factor.

$$Y \ C \ Y(-1) \ \text{@Expand(Firm, @Dropfirst)} \quad (2.34)$$

(ii) A continuous LV(1) model, having the following ES, which is considered as the worst LV(1) panel data model.

$$Y \ C \ Y(-1) \quad (2.35)$$

### 2.8.1.3 LV(1) HRM with Trend, and its CSFEM
As an extension of the model (2.6), the LV(1)_HRM with Trend, would have the ES as follows:

$$Y \ C \ Y(-1) @Expand(Firm) \ t @Expand(Firm) \ @Expand(Firm, @Dropfirst) \quad (2.36)$$

with its two possible reduced models are as follows:

(i) A LV(1)_CSFEM with trend, having the ES as follows:

$$Y \ C \ Y(-1) \ t \ @Expand(Firm, @Dropfirst) \quad (2.37)$$

(ii) A continuous LV(1) model with trend, having the following ES, which is considered as the worst LV(1) panel data model with trend.

$$Y \ C \ Y(-1) \ t$$

\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \odo
Based on these results, the following findings and notes are presented.

1.1 Both outputs present exactly the same regression functions, indicated by the same values of 13 statistics presented at the bottom of the print-outs. The model in (2.39) is using the Firm=1 as a reference group, and the model in (2.40) is not using a reference group.

**Figure 2.4** Summary of the statistical results based on two LV(2)_CSFEMs, and their Omitted Variables Tests
In a statistical sense, a reduced model should be explored. It is recommended the reduced model should be developed based on the model in (2.40) by deleting $LY(-2)^{\text{ expanding(Firm=3)}}$ from the model, using the following ES.

$$LY \ t^{\text{ expanding(Firm)}} \ Ly(-1)^{\text{ expanding(Firm)}}$$

$$LY(-2)^{\text{ expanding(Firm, Drop(3))}} \text{ expanding(Firm, Dropfirst)}$$  \hspace{1cm} (2.41)

(2) Both LV(2)_CSFEMs models are additive models of LY on t, LY(-1), and LY(-2), and they have sufficient values of the DW-statistic, and each of the numerical variable has a significant effect. So that both models can be considered as acceptable models, but in a statistical sense.

(3) However, these models are using the hidden assumption that all numerical independent variables, namely t, LY(-1), and LY(-2), have the same effects on the dependent variable LY.
for the first three firms, and 82 firms, respectively, which is inappropriate, in a theoretical sense. Compare to the following models.

### 2.8.1.3 LV(1)_HRM with a Time-Related-Effects, and its CSFEM

As an extension of the model (2.36), the LV(1)_HRM with a Time-Related-Effect (TRE), would have the ES as follows.

\[
Y_i t = \alpha_i + \beta_i Y_{i,t-1} + \gamma_i t + \varepsilon_{it}
\]  
\tag{2.43}

Note that this model is presenting a set of N two-way interaction LV(1)_HRMs, having the following equation.

\[
Y_i t = \alpha_i + \beta_i Y_{i,t-1}^* + \delta_i Y_{i,t-1} + \gamma_i t + \varepsilon_{it}
\]  
\tag{2.42}

And all of the models above can be considered as its reduced models. Furthermore, two additional reduced models should be considered are as follows:

(i) A LV(1)_CSFEM with TRE, having the ES as follows:

\[
Y_i t = \alpha_i + \beta_i Y_{i,t-1} + \delta_i Y_{i,t-1} + \gamma_i t + \varepsilon_{it}
\]  
\tag{2.44}

(ii) A continuous LV(1) model with TRE, having the following ES, which is considered as the worst LV(1) panel data model with TRE.

\[
Y_i t = \alpha_i + \beta_i Y_{i,t-1} + \gamma_i t + \varepsilon_{it}
\]  
\tag{2.45}

### Example 2.5 (Application of LV(1)_HRM in (2.42), and LV(1)_CSFEM in (2.44))

Figure 2.6 presents the statistical results of the LV(1)_HRM in (2.42), and LV(1)_CSFEM in (2.44), for an endogenous variable $LY$, based on a subsample $\{FIRM < 4\}$. Based on these results, the following findings and notes are presented.

1. The LV(1)_HRM applied, since it is defined that each of the exogenous variables $LY(-1)^* t$, $LY(-1)$, and $t$ has different effects on $LY$ for the first three firms. On the other hand, the LV(1)_CSFEM applied under a special assumption that each exogenous variables has the same effects for the first three firms. So they have different prerequisites.

2. At the first stage, the choice for the LV(1)_CSFEM is completely depend on personal expert’s judgment, it does not depend on a testing hypothesis. If the assumption that the variables $LY(-1)^* t$, $LY(-1)$, and $t$ have the same effect for the first three firms can be
accepted, then LV(1)_CSFEM can be considered as a good fit model. However for a large number firms, such as hundreds or thousands of firms, the assumption would never be acceptable.

Figure 2.6 Statistical results based on the LV(1)_HRM in (2.42), and LV1_CSFM in (2.44)

Figure 2.7 Statistical results of two omitted variables tests

(3) For the illustration, Figure 2.7 presents the statistical results of the omitted variables tests based on the LV(1)_CSFEM, with the findings as follows:

3.1 At the $\alpha = 0.10$ level of significance, the variables $LY(-1)^*t*Expand(Firm, DROPFIRST)$ and $LY(-1)^*Expand(Firm, DROPFIRST)$ have an insignificant joint effects, based on the $F$-statistic of $F_0 = 1.624578$ with $df = (4.71)$, and a $p$-value $= 0.1776 > \alpha = 0.10$. However, the
variables $LY(-1) \times t \times @Expand(Firm, @Dropfirst)$ an insignificant joint effects, based on the $F$-statistic of $F_0 = 2.782379$, with $df = (2,73)$, and a $p$-value $= 0.0685 < \alpha = 0.10$. These findings indicate that at least the interaction $LY(-1) \times t \times @Expand(Firm, @Dropfirst)$, should be inserted as additional independent variables for the LV(1)_CSFEM.

3.2 For a comparison, Figure 2.8 presents the statistical results of a reduced model of the LV(1)_HRM in Figure 2.7, which is better than the LV(1)_CSFEM, in both theoretical and statistical senses.

3.3 Furthermore, for this model, it is found that the interactions $LY(-1) \times t \times @Expand(Firm)$, have a significant joint effects on $LY$, based on the $F$-statistic of $F_0 = 2.485546$, with $df = (3,71)$, and a $p$-value $= 0.0676 < \alpha = 0.10$.

2.8.2 Generalized CSFEMs

2.8.2.1 CSFEMs with Time-Related-Effects

As the extension of the CSFEM with TRE in (2.44), the CSFEMs with TRE considered would have the following general equation specifications.

$$G(Y) = C X1 \ldots Xk \ t \ t \times X1 \ldots t \times Xk \ @Expand(Firm, @Dropfirst)$$ (2.46)

where $G(Y) = Y$ or a transformed endogenous variable $Y$, and the exogenous variables $Xk$, $k=1,\ldots,K$, where $Xk$ can be a lag of the endogenous variable, an original numerical exogenous variable, a transformed variable, an environmental variable, namely $Z_t$, a dummy of defined time-period, and a two or higher interaction of the previous types of variables.

These models could be considered as One-Way ANCOVA models having a single factor, namely FIRM, with a set of covariates: $X1,\ldots, Xk, \ t, \ t \times X1, \ldots$, and $t \times Xk$, for a lot of possible $Xk$. So that a researcher could subjectively defined various alternative CSFEMs, under the assumption that all covariates have the same effect on the dependent variable, for all firms (the research objects). Even though the assumption might not be relevant or valid for a large number of the research objects, however fixed-effects models have been presented in the international
journals, such as the Journal of Finance, based on panel data sets having thousands or hundred-thousands of observations.

Note that all CSFEMs previously presented, as well as other CSFEMs, could be considered as the reduced models of this model, as well as the following reduced models.

2.8.2.2 CSFEMs with trend

They have the following general ES.

\[
G(Y) \ C \ X_{1} \ ... \ X_{k} \ ... \ t \ \ @\text{Expand}(\text{Firm},@\text{Dropfirst}) \tag{2.47}
\]

2.8.2.3 CSFEMs without the time-t

It is recognized that most of the papers in the international journals present the fixed-effects models without using the numerical time independent variable. So that those CSFEMs can be represented using the following general ES, which is a direct reduced model of (2.47). Find the following example

\[
G(Y) \ C \ X_{1} \ ... \ X_{k} \ ... \ @\text{Expand}(\text{Firm},@\text{Dropfirst}) \tag{2.48}
\]

Example 2.6 (An application of a CSFEM in (2.48)) Figure 2.9 presents two kinds of the statistical results of a LV(1)_CSFEM in (2.48), based on three variables \( LK, LL \) and \( LY \), in CES.wf1, using the following the ES, respectively

\[
ly \ ly(-1)*lk*ll \ ly(-1)*lk \ ly(-1)*ll \ lk*ll \ ly(-1) \ lk \ ll \ C \ @\text{Expand}(\text{Firm},@\text{Dropfirst}) \tag{2.49a}
\]

and

\[
ly \ ly(-1)*lk*ll \ ly(-1)*lk \ ly(-1)*ll \ lk*ll \ ly(-1) \ lk \ ll \ @\text{Expand}(\text{Firm}) \tag{2.49b}
\]

Based on these results, the following findings and notes are presented.

(1) Note that for each firm \( i \), the model is representing a three-way hierarchical time series model. Hence, various alternative time series models in Agung (2009) could be modified to CSFEMs.

(2) The ES in (2.49a) is applied with an objective to test the joint effects of all independent variables, namely 7 numerical variables, and 81 firm dummies, on the dependent variable. \( LY \). The result shows the 88 independent variables have a significant joint effects, based on the \( F \)-statistic of \( F_{0} = 32519.19 \) with a \( p \)-value = 0.0000.
(3) The LV(1)_CSFEM is a three-way interaction hierarchical model, which could be considered as an acceptable model, under the assumption that all independent variables have the same effect on \(LY\) for the 82 firms, because each numerical independent variables has a probability < 0.30, and only one of them has a probability > 0.25.

<table>
<thead>
<tr>
<th>Dependent Variable: LY</th>
<th>Model with an intercept</th>
<th>Model without an intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>LY(-1)<em>LK</em>LL</td>
<td>0.0009</td>
<td>0.0004</td>
</tr>
<tr>
<td>LY(-1)*LK</td>
<td>-0.0205</td>
<td>0.0072</td>
</tr>
<tr>
<td>LY(-1)*LL</td>
<td>-0.0131</td>
<td>0.0124</td>
</tr>
<tr>
<td>LK*LL</td>
<td>-0.0137</td>
<td>0.0111</td>
</tr>
<tr>
<td>LY(-1)</td>
<td>1.4110</td>
<td>0.2020</td>
</tr>
<tr>
<td>K</td>
<td>0.3844</td>
<td>0.1765</td>
</tr>
<tr>
<td>L</td>
<td>0.2714</td>
<td>0.2511</td>
</tr>
<tr>
<td>C</td>
<td>-7.0650</td>
<td>3.9658</td>
</tr>
<tr>
<td># Firm Dummies</td>
<td>(82-1)</td>
<td></td>
</tr>
<tr>
<td>R-squared</td>
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<td></td>
</tr>
<tr>
<td>Adjusted R-squared</td>
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</tr>
<tr>
<td>S.E. of regression</td>
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<td></td>
</tr>
<tr>
<td>Sum squared resid</td>
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<tr>
<td>Log likelihood</td>
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<tr>
<td>F-statistic</td>
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</tr>
<tr>
<td>Prob(F-statistic)</td>
<td>0.0000</td>
<td></td>
</tr>
</tbody>
</table>

Figure 2.9 Statistical results of the LV(1)_CSFEMs in (2.49a), and (2.49b)

(4) On the other hand, it is found that \(LY(-1)*LK*LL*@Expand(Firm, @Dropfirst)\) have a significant joint effects, based on the \(F\)-statistic of \(F_0 = 3.833972\) with \(df = (81,2044)\) and a \(p\)-value = 0.0000, using the omitted variables test. The other interactions also can easily be done. Based on this finding, the following notes should be very important to be considered.

4.1 Even though \(LY(-1)*LK*LL*@Expand(Firm, @Dropfirst)\) as a significant effect, it does not mean that all of the 81 variables presented by this function have significant effects. So that some of them should be deleted.
4.2 If the interaction between the numerical variables with the dummies would be used as additional independent variables, then the model will present 82 heterogeneous regressions by FIRM,. then it will be too many regressions should be presented in a paper, thesis or dissertation. For this reason it is recommended to applied a HRM by group of the firms, even though the variable GROUP should be subjectively defined or generated based on one or more variables. Refer to the HRMs presented in previous sections.

4.3 Furthermore the HRM might could be reduced to the ANCOVA model or group-fixed-effects model (GRFEM), which is acceptable in a statistical sense.

(5) The CSFEMs in (2.49a), and (2.49b) can easily be extended to the CSFEMs with trend, and CSFEMs with the time related-effects By inserting additional independent variables, it is recommended to apply the manual stepwise selection method, as presented in Example 2.2.

(6) In addition they could be extended to CSFEMs by inserting a defined time-period (TP) independent variable, either as a function @Expand(TP) (time-period dummies) or interaction NV*@Expand(TP) for at least one numerical variable (NV).

(7) Note that the LV(1) model is applied in order to have a sufficient value of the DW-statistic, which could be extended to LV(p) models, for $p > 1$

2.8.2.4 CSFEMs by Time-Period (TP)

It is recognized that all CSFEMs could be modified or extended to CSFEMs by a time-period (TP). So that based on the most general CSFEMs in (2.46), we may have two groups of CSFEMs, namely the heterogeneous CSFEMs by TP, and the bi-factorial ANCOVA models, having the following equations specifications. In addition, refer to all possible reduced models of the CSFEMs in (2.46), which can also be the reduced models of the CSFEMs by TP.

(i). Heterogeneous CSFEMs by TP.

These models can be represented using the following general equation specification (ES), which shows all numerical independent variables have different slopes between the time-periods considered.

$$G(Y) \ C \ ... \ Xk*@Expand(TP) \ ... \ t*Expand(TP)$$

$$\ ... \ t*Xk*Expand(TP) \ ... \ @Expand(Firm,@Dropfirst)$$

(2.50)


(ii) Bi-factorial ANCOVA Models.

These models can be represented using the following general ES, with the assumption that all numerical independent variables have the same slopes between all groups generated by the variable FIRM, and TP. There are two types of ANCOVA models should be considered, such as follows:

1. The interaction ANCOVA models having the following ES, which shows that the dummy independent variables are the dummy of the interactions of the firm dummies and the time-period dummies.

\[ G(Y) \ C \ X_1 \ldots X_k \ t \ t*X_1 \ldots t*X_k \ \@Expand(Firm,TP,\@Dropfirst) \]  

(2.51)

2. The additive ANCOVA models having the following ES, which shows that the dummy independent variables are the additive of the firm dummies and the time-period dummies. These models are considered as the worst ANCOVA model among the ANCOVA models having the same independent variables (Agung, 2011). Note that these models in fact are two-way FEMs having the firm-fixed effects, and the time-period-fixed-effects.

\[ G(Y) \ C \ X_1 \ldots X_k \ t \ t*X_1 \ldots t*X_k \ \@Expand(Firm,\@Dropfirst) \ \@Expand(TP,\@Dropfirst) \]  

(2.52)

2.9 Time or Period Fixed Effects Models

2.9.1 Generalized Period Fixed Effects Models

Similar to the generalized CSFEMs in (2.48), the period fixed/effects models also are One-way ANCOVA models of \( G(Y_{it}) \), but with the discrete time variable as a single factor, and a set of covariates, namely \( X_{kt} \), \( k=1,\ldots,K \), where \( X_k \) can be a lag of the endogenous variable, an original numerical exogenous variable, a transformed variable, an environmental variable, a dummy variable of the firm-groups, and a two or higher interaction of the previous types of variables.

Then the generalized time or period fixed-effects models, namely PEFEMs, can be represented using the following equation specification. Note that these models are representing a
set of \((T-p)\) cross-section models if the lagged variables models, namely \(LV(p)\) Models, are applied.

\[
G(Y) \ C \ X1 \ ... \ Xk... \ @Expand(Time,@Dropfirst) \quad (2.53)
\]

2.9.2 Some Specific PEFEMs

2.9.2.1 Lagged Variables PEFEMs based on a single variable \(Y_{it}\)

The first-order lagged variable PEFEMs of \(Y_{it}\), which is the simplest PEFEM, namely \(LV(1)\) PEFEM, having the following equation .

\[
G(Y_{i,t-1}) = \delta_i + \gamma*G(Y_{i,t-1}) + \varepsilon_{it}, \text{ for } t=2,...,T \quad (2.54)
\]

which is representing a set of \((T-1)\) homogenous regressions of \(Y_{it}\) on \(Y_{i,t-1}\) for all \(t=2,...,T\), indicated by the \((T-1)\) intercept parameters \(\delta_i\), and a constant slope parameter \(\gamma\).

For the analysis using EViews, the following equation specification would be applied.

\[
G(Y) \ C \ G(Y(-1)) \ @Expand(Time,@Dropfirst) \quad (2.55)
\]

This model can easily be extended to \(LV(p)\) PEFEMs, having the following equation specification, where the value of \(p\) should be highly dependent on the data set used.

\[
G(Y) \ C \ G(Y(-1)) \ ... \ G(Y(-p)) \ @Expand(Time,@Dropfirst) \quad (2.56)
\]

2.9.1.2 The First-Order Lagged Variable PEFEMs based on \((X_{it},Y_{it})\)

It is recognized that the effect of a numerical independent variable on the dependent variable depends on the other numerical independent variable. Since the \(LV(1)\) PEFEMs based on \((X_{it},Y_{it})\) would have at least two numerical independent variables, namely \(Y_{i,t-1}\) and \(X_{i,t}\), then it can be accepted, in a theoretical sense, that the effects of the exogenous variable \(X_{it}\) on the endogenous variable \(Y_{it}\) depends on \(Y_{i,t-1}\), or the effects of \(Y_{i,t-1}\) on \(Y_{it}\) depends on \(X_{i,t}\). So that the \(LV(1)\) PEFEMs can be represented the following general equation specification (ES).

\[
G(Y) \ G(Y(-1))*F(X) \ G(Y(-1)) \ F(X) \ C \ @Expand(Time,@Dropfirst) \quad (2.57)
\]

where \(G(Y) = Y\) and \(F(X) = X\), or the transformed variables of \(Y\) and \(X\), respectively. Thence there would be a lot of possible \(LV(1)\) PEFEMs could be defined by a researcher, based on a bivariate \((X_{it},Y_{it})\). Note that these models are two-way interaction hierarchical models. However, the good fit models obtained can be either nonhierarchical two-way interaction models or additive models, which are highly dependent on the data sets used.
Furthermore, each of those models could easily be extended to higher-order lagged variables models, and various polynomial models, such as the simplest polynomial model which as an additive LV(1)_PEFEM having the ES as follows

\[ G(Y) \ \ G(Y(-1)) \ \ X \ \ X^2 \ \ ... \ \ X^n \ \ C \ \ \text{@Expand}(Time, \text{@Dropfirst}) \]  

(2.58)

2.9.1.3 The First-Order Lagged Variable PEFEMs based on \((X_{1i}, X_{2i}, Y_{it})\)

As the extension of the LV(1)_PEFEMs in (2.57), based on the variables \((X_{1i}, X_{2i}, Y_{it})\) we would have the three-way interaction LV(1)_PEFEMs, having the following general equation.

\[ G(Y) \ \ G(Y(-1)) \ast F(X1) \ast F(X2) \ \ G(Y(-1)) \ast F(X1) \ \ G(Y(-1)) \ast F(X2) \]

\[ F(X1) \ast F(X2) \ \ G(Y(-1)) \ \ F(X1) \ \ F(X2) \ \ C \ \ \text{@Expand}(Time, \text{@Dropfirst}) \]  

(2.59)

where \(G(Y) = Y\), \(F(X1)=X1\), and \(F(X2) = X2\), or the transformed variables of \(Y\), \(X1\) and \(X2\), respectively. Thence there would be a lot of possible LV(1)_PEFEMs could be defined by a researcher, based on a bivariate \((X_{1i}, X_{2i}, Y_{it})\). Note that these models are three-way interaction hierarchical models. However, the good fit models obtained can be either nonhierarchical three-way interaction models, two-way hierarchical and nonhierarchical models, or additive models, which are highly dependent on the data sets used.

Example 2.6  (An application of PEFEM in (2.59)) As a modification of the LV(1)_CSFEM in (1.49), Figure 2.10 presents the statistical results of a LV(1)_PEFEM based on the variable \(LK\), \(LL\), and \(LY\), using the following ES, and its acceptable reduced model

\[ ly \ \ ly(-1) \ast lk \ast ll \ \ ly(-1) \ast lk \ \ ly(-1) \ast ll \ \ lk \ast ll \ \ ly(-1) \ \ lk \ \ ll \ \ \text{@Expand}(t) \]  

(2.60)

Based on these results, the following findings and notes are presented.

(1) For each time point \(t\), the models are presenting a cross-section model. Hence, all models based on cross-section data sets, presented in Agung (2011) could be modified to PEFEMs.

(2) Furthermore, the models can easily be extended to the PEFEMs by a variable: GROUP of the firms, either as a function \(@\text{Expand}(\text{Group})\), or an interaction \(NV \ast @\text{Expand}(\text{Group})\), for at least one numerical variable (NV).
Table 2.10 Statistical results of the LV(1)_PEFEM (2.60), and its reduced model

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>LY(-1)<em>LK</em>LL</td>
<td>-7.84E-05</td>
<td>0.0002</td>
<td>-0.4445</td>
<td>0.6567</td>
<td>0.0001</td>
<td>0.0000</td>
<td>4.5353</td>
<td>0.0000</td>
</tr>
<tr>
<td>LY(-1)*LK</td>
<td>-0.0009</td>
<td>0.0030</td>
<td>-0.3074</td>
<td>0.7586</td>
<td>-0.0039</td>
<td>0.0008</td>
<td>-4.9405</td>
<td>0.0000</td>
</tr>
<tr>
<td>LY(-1)*LL</td>
<td>0.0036</td>
<td>0.0053</td>
<td>0.6750</td>
<td>0.4998</td>
<td>-0.0060</td>
<td>0.0012</td>
<td>-4.8555</td>
<td>0.0000</td>
</tr>
<tr>
<td>LK*LL</td>
<td>0.0035</td>
<td>0.0046</td>
<td>0.7528</td>
<td>0.4517</td>
<td>0.0030</td>
<td>0.0007</td>
<td>4.6050</td>
<td>0.0000</td>
</tr>
<tr>
<td>LY(-1)</td>
<td>0.9997</td>
<td>0.0899</td>
<td>11.5051</td>
<td>0.0000</td>
<td>1.1475</td>
<td>0.0299</td>
<td>38.3379</td>
<td>0.0000</td>
</tr>
<tr>
<td>LK</td>
<td>-0.0070</td>
<td>0.0746</td>
<td>-0.0997</td>
<td>0.9254</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LL</td>
<td>-0.1243</td>
<td>0.1045</td>
<td>-1.1891</td>
<td>0.2345</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2.9.1.4 The PEFEMs by GROUP

As the modification or extension of all possible PEFEMs in (2.53), we may have the PEFEMs by a defined groups of the firms, say the variable GROUP, which can be generated or defined based on one or more variables. Note that the groups of the firms should be invariant or constant over times. There would be two types of PEFEMs should be considered, namely the heterogeneous PEFEMs by GROUP, and the bi-factorial ANCOVA models, having the following equations specifications. In addition, refer to all possible reduced models of the PEFEMs in (2.53), which can also be the reduced models of the PEFEMs by GROUP.

(i) Heterogeneous PEFEMs by GROUP

These models can be represented using the following general ES, which shows that the heterogeneous PEFEMs, since the effects of the numerical variables Xk’s have different effects on G(Y) between the levels of the categorical variable : GROUP.
\[ G(Y) \ C X1^{*}\text{@Expand(\text{Group})} \ ... \ Xk^{*}\text{@Expand(\text{Group})} \ ... \ \text{@Expand(\text{Time},\text{@Dropfirst})} \ (2.61) \]

(ii) Bi-factorial ANCOVA Models.

These models can be represented using the following general ES, with the assumption that all numerical independent variables have the same slopes between all groups generated by the variable \text{GROUP} and \text{TIME-PERIOD (TP)}. There are two types of ANCOVA models should be considered, such as follows:

1. The interaction ANCOVA models having the following ES, which shows that the dummy independent variables are the dummy of the interactions of the group dummies and the time-period dummies.

\[ G(Y) \ C X1 \ ... \ Xk \ t \ t*X1 \ ... \ t*Xk \ \text{@Expand(\text{Group,TP},\text{@Dropfirst})} \ (2.62) \]

2. The additive ANCOVA models having the following ES, which shows that the dummy independent variables are the additive of the group dummies and the time-period dummies. These models are considered as the worst ANCOVA model among the ANCOVA models having the same independent variables (Agung, 2011). Note that these models in fact are two-way FEMs having the group fixed effects, and the time-period fixed effects.

\[ G(Y) \ C X1 \ ... \ Xk \ t \ t*X1 \ ... \ t*Xk \ \text{@Expand(\text{Firm,\text{@Dropfirst}}) \ \text{@Expand(\text{TP,\text{@Dropfirst}}})} \ (2.63) \]

2.10 Firm-Year of Two-Way Fixed Effects Models

It is recognized that all period-fixed-effects models (PEFEMs) can easily be modified to the firm-year or two-way fixed-effects models (TWFEMs). So that based on the general equation specification (ES) of the PEFEMs in (2.49), we have the general ES of the TWFEMs, as follows:

\[ G(Y) \ C X1 \ ... \ Xk \ ... \ \text{@Expand(\text{Firm,\text{@Dropfirst}}) \ \text{@Expand(\text{Time,\text{@Dropfirst}}})} \ (2.64) \]

Note that this general ES could be representing all possible TWFEMs, as well as their reduced models, which are highly dependent on the data used. However, it has been found that
no one researcher and book in econometric present the specific characteristics of a TWFEM, moreover its limitations. For this reason, the following sections present special notes and comments on selected TWFEMs.

2.10.1 Limitations of the TWFEMs

Note that the models in (2.64) is presenting an additive bi-factorial ANCOVA models with the factors: $FIRM$, and $TIME$; and covariates $X_1 \ldots X_k \ldots$. If the panel data has $(N \times T)$ firm-time observations, then this model is representing a set of $(N \times T)$ homogeneous regressions (regressions having the same slopes) with special pattern of the $(N \times T)$ intercepts, or $N \times (T-1)$ intercepts if and only if the $LV(1)$ model is applied. Find the following simple illustration, to show the limitation of TWFEMs.

Example 2.7 (A TWFEM based on a small subsample) Figure 2.11 presents the statistical results of the model in (2.65), based on the subsample \{Firm < 4 and t < 5\}, with its table of parameters.

\[
ly \text{lkl lkc} \expansion\{firm,\text{dropfirst}\} \expansion\{t,\text{dropfirst}\} \quad (2.65)
\]
Based on this figure, the following findings and notes are presented.

(1) The subsample \{FIRM < 4 and $T < 5$\} generates a 3x4 cross tabulation having 12 cells with a single observation in each cell.

(2) Note that the model is representing $12 = 3 \times 4 = N \times T$ homogeneous multiple regressions having the same slopes, namely C(1), C(2), and C(3), with a special pattern of the intercepts, which would never be observed in practice. So that each regression contains only a single point of observations. So that this model would be considered as the worst panel data model. Similarly the model in the following example based on the whole sample in CES.wf1.

(3) Note that the regression has such a large R-squared, namely $R^2 = 1$, because regression contains all observations. For an additional illustration, a regression line $Y$ on $X$, based on only two observations, would have a $R^2 = 1$. The $R^2 = 1$ does not mean that the model is a best fit model. In this case, the model is the worst model, or inappropriate model.

(4) In addition, it is unexpected the results present such a very large $F$-statistic of 77896.58, and DW-statistic of 4.071138.
Example 2.7 (A TWFEM corresponding to the CSFEM in (2.49), and PEFEM in (2.60))

For an illustration, Figure 2.12 presents the statistical results of the TWFEM in (2.66), based on the subsample \( \{ T > 1 \} \) of CES.wf1..

\[
ly ly(-1)*lk*ll ly(-1)*lk ly(-1)*ll lk*ll ly(-1) lk ll
C @Expand(Firm,@Dropfirst) @Expand(t,@Dropfirst)
\] (2.66)

Based on these statistical results, the following notes and comments are presented.

1. Similar to the TWFEM in Figure 2.10, this model represents 2,214 (=82x27) homogenous regressions with a special pattern of intercepts. So that I would consider this model is the worst panel data model, in a theoretical sense, among all possible panel data models having the same set of numerical independent variables.

2. However, it is an acceptable model, in a statistical sense, since the statistical results show acceptable estimates of the parameters, including the coefficients of the 81 firm dummies, and 27 year dummies, which are not presented in Figure 2.11. However, some or many of the dummies might have large \( p \)-values.

3. It is recognized, that the lagged variables models would have large \( R \)-squared, as well the autoregressive models using the AR term(s).

4. For additional illustrations, the following section presents selected TWFEMs, presented in the international journal, with special notes,
### 2.10.2 Fixed Effects Models Presented in International Journals

#### 2.10.2.1 Four-Way FEMs presented by Jie, Jun, and Strahan (2012)

Jie, Jun, and Strahan (2012) present several types of fixed effects models, namely three sets of models using (i). Cohort-Year Fixed Effects, Issuer Fixed Effects, and Initial Rating Category Dummies, which are Four-Way FEMs, (ii). Cohort-Year Fixed Effects and Initial Rating Category Dummies, and (iii). Cohort-Year Fixed Effects, based on the data having thousands of observations. The models also have interaction independent variables. The models presented have a minimum $R^2 = 0.613$, and a maximum $R^2 = 0.730$.

For an illustration, the Four-Way FEMs can be presented using the following general ES.

$$ G(Y) \sim V_1 \ldots V_k \sim V_k \sim C \sim @Expand(Cohort, @Dropfirst) \sim @Expand(Year, @Dropfirst) \sim @Expand(Issuer, @Dropfirst) \sim @Expand(InitialRating, @Dropfirst) \quad (2.67) $$

---

**Figure 2.12** Statistical results of the TWFEM in (2.66), and one of its possible reduced models

<table>
<thead>
<tr>
<th>Variable</th>
<th>Full Model</th>
<th>Reduced Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coeff.</td>
<td>s.e.</td>
</tr>
<tr>
<td>$LY(-1)<em>LK</em>LL$</td>
<td>0.0008</td>
<td>0.0004</td>
</tr>
<tr>
<td>$LY(-1)*LK$</td>
<td>-0.0207</td>
<td>0.0071</td>
</tr>
<tr>
<td>$LY(-1)*LL$</td>
<td>-0.0152</td>
<td>0.0123</td>
</tr>
<tr>
<td>$LK*LL$</td>
<td>-0.0092</td>
<td>0.0109</td>
</tr>
<tr>
<td>$LY(-1)$</td>
<td>1.3848</td>
<td>0.1988</td>
</tr>
<tr>
<td>$LK$</td>
<td>0.2560</td>
<td>0.1736</td>
</tr>
<tr>
<td>$LL$</td>
<td>0.1021</td>
<td>0.2518</td>
</tr>
<tr>
<td>C</td>
<td>-5.1907</td>
<td>3.9245</td>
</tr>
<tr>
<td># Firm Dummies</td>
<td>(82-1)</td>
<td></td>
</tr>
<tr>
<td># Year Dummies</td>
<td>(28-2)</td>
<td></td>
</tr>
<tr>
<td>R-squared</td>
<td>0.9993</td>
<td></td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.9993</td>
<td></td>
</tr>
<tr>
<td>F-statistic</td>
<td>25986.43</td>
<td></td>
</tr>
<tr>
<td>Prob(F-statistic)</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>1.7121</td>
<td></td>
</tr>
</tbody>
</table>

---
However, note that these models present a set of thousands homogeneous multiple regressions of $G(Y)$ on $V_k$, $k=1,\ldots,K$, with a special set of intercepts. Refer to an additive DVM in (1.14) with the model parameters presented in Table 1.16, which would never be observed in practice.

### 2.10.2.2 Four-Way FEMs presented by Engelberg, and Parson (2011)

Engelberg, and Parson (2011) present several four-way fixed effects models using Industry, Paper, City, and Date fixed effects, based on the data sets having 265,928 and 273,999 observations, with a minimum adjusted $R^2$ of 0.049, and a maximum $R^2$ of 0.089.

### 2.10.2.3 Two-Way FEMs presented by Puri, and Zarutskie (2012)

Puri and Zarutskie (2012) present industry-year fixed effects models having interactions independent variables, in addition to the hundred-thousands of industry and year dummies, based on the data set having 105,031 observations. So that the models in fact are presenting hundred-thousands of multiple regressions having the same slopes, with a special pattern of the intercepts, by industry and year. Refer to the dummy variables models in (1.19). Since the models have a dummy independent variable, in addition to the fixed effects, then they have small $R^2$. One of the model has $R^2 = 0.063$. Compare the following models having very small $R^2$.

### 2.10.2.4 One-Way FEMs presented by Jotikasthira, Lundblad, and Ramadorai (2012)

Jotikasthira, Lundblad, and Ramadorai (2012) present country fixed effects model with all other independent variables are dummy variables, including selected their interactions. So that it is common the models should have very small R-Squares. The models presented have $R^2$ within the range of 0.000 and 0.060.

### 2.10.2.5 FEMs presented in Baltagi (2000a, and 2000b)

It is recognized that the four previous papers, and many other papers, do not present the models with their Durbin Watson statistics. So it cannot be identified whether or not the models have autocorrelation problems. On the other hand, Baltagi (2000b) presents examples of fixed effects models having the characteristics as follows:
(1) The models have various values of DW-statistics, such as a cross-section fixed effect model (p.19) having a small DW-statistic of 0.326578, and a two-way fixed effects models (p.59), and (p. 254), having a DW-statistics of 0.333512, and 1.433777, respectively. It has been well known, that the value of DW-statistic could easily be increased by using a lagged variable model or an autoregressive model. So that the models presented in Baltagi could easily be modified.

(2) Most of the fixed models are additive models, with one of the simple models is quoted from Grunfeld (1958) having the basic equation as follows:

\[ I_{it} = \alpha + \beta_1 F_{it} + \beta_2 C_{it} + u_{it} \]  

(2.68)

where \( I_{it} \) denotes real gross investment for firm \( i \) in year \( t \), \( F_{it} \) is the real value of the firm (shares outstanding), and \( C_{it} \) is the real value of the capital stock. Based on the three variables \( I, F, \) and \( C \), the data analysis can be done based on the CSFEM, PEFEM, and TWFEM, and random fixed effects models, which will be presented in the following chapter.

2.11 Groups or Time-Period Fixed Effects Models

Referring to the limitation of the TWFEMs presented in Figure 2.11, and Figure 2.12, and hundreds or thousands of homogeneous regressions presented by CSFEMs and PEFEMs, which would never be observed in practice. For this reason, it is recommended to apply modified fixed effect models, namely groups or time-period fixed effects models, under the precondition that the firms within each level of the variable GROUP can be considered as homogeneous. So that their scores for each variable can be represented using their mean. Even though, the models still should be using the assumption that each numerical independent variable of the mean has the same effect of the corresponding dependent variable within all cells generated by the GROUP, and TP. For these models, there would be two types of FEMs would be considered, such as follows:

2.11.1 Group×TimePeriod Fixed Effects Models

2.11.1.1 Group×TimePeriod Fixed Effects Models without Trend
Corresponding to the general HRMs presented by the equation specification (ES) in (2.23), the \( \text{Group} \times \text{TimePeriod} \) fixed effects models (G\( \times \text{TP}_\text{FEMs} \)), which in fact are the ANCOVA models by G\( \times \text{TP} \), without trend (the numerical time independent variable) could be presented using the ES as follows:

\[
G(Y) \ V1... \ V_k... \ V_K \ C \ @\text{Expand}(\text{Group},\text{TP},@\text{Dropfirst}) \tag{2.69}
\]

where \( V_{k,n} \) can be any variable numeric, a dummy variable, an environmental variable, say \( Z_t \), and two- or three-way interaction, including the interaction between the dummy variable(s) with the numerical variable(s), and the lags of the numerical dependent and numerical independent variable(s). So that there would be a lot of possible models can be defined, even based on a small number of variables, such as three up to five numerical exogenous variables. For instance, the following special lagged variable G\( \times \text{TP}_\text{FEMs} \).

Note that these models also are using the assumption that each \( V_k \) has the same effect on G(\( Y \)) within all cells generated by the categorical variables GROUP, and Time-Period (TP). Furthermore, note that a good of fit model obtained could be unexpected reduced model, which is highly dependent on the data used, specifically the unpredictable impacts of multicollinearity between the independent variables. See the following example 2.10.

**Example 2.8 (G\( \times \text{TP}_\text{FEMs} \) based on \( X_1, Y \))** A type of G\( \times \text{TP}_\text{FEMs} \) out of a lot of possible models based on a bivariate \((X_1, Y)\) would be considered, is the first-order lagged variable G\( *\text{TPFEMs} \), which can be presented using the following general equation specification.

\[
G(Y) \ G(Y(-1)) \ast F(X_1) \ G(Y(-1)) \ F(X_1) \ C \ @\text{Expand}(\text{Group},\text{TP},@\text{Dropfirst}) \tag{2.70}
\]

where \( F(X) \), and \( G(Y) \), respectively, can be any functions of the variable \( X \) and \( Y \), having no parameter.

**Example 2.9 (G\( *\text{TPFEMs} \) based on \( X_1, X_2, Y \))** A type of G\( *\text{TPFEMs} \), out of a lot of possible models defined or proposed based on a trivariate \((X_1, X_2, Y)\), is the first-order lagged variable G\( *\text{TPFEMs} \), which can be presented using the following general equation specification.
\[
G(Y) \ G(Y(-1)) * F_1(X1) * F_2(X2) \ G(Y(-1)) * F_1(X1) \ G(Y(-1)) * F_2(X2) \\
G(Y(-1)) \ F_1(X1) \ F_1(X1) \ C \ @\text{Expand(}\ Group,TP,\ @\text{Dropfirst})
\]

(2.71)

where \( F_1(X1), \ F_2(X2), \) and \( G(Y) \) respectively, can be any functions of the variables \( X1, \ X2, \) and \( Y, \) having no parameter.

### 2.11.1.2 \( G^* \)TPFEMs with Trend

As an extension of the \( G^* \)TPFEMs in (2.69), and referring the HRMs with trend in (2.25), we have the \( G^* \)TPFEMs with Trend, which could be represented using the general ES as follows.

\[
G(Y) \ V1... Vk... VK \ t \ C \ @\text{Expand(}\ Group,TP,\ @\text{Dropfirst})
\]

(2.72)

### 2.11.1.3 \( G^* \)TPFEMs with Time-Related-Effects

As an extension of the \( G^* \)TPFEMs with trend in (2.72), and referring the HRMs with the time-related-effects (TRE) in (2.26), we have the \( G^* \)TPFEMs with TRE, which could be represented using the general ES as follows.

\[
G(Y) \ V1... Vk... VK \ t \ t*V1... t*Vk... t*VK \ C \ @\text{Expand(}\ Group,TP,\ @\text{Dropfirst})
\]

(2.73)

### Example 2.10 (An unexpected reduced model)

Compare to the statistical results of the full CSFEM in Figure 2.9, the full PEFEM in Figure 2.10, and the full Firm-Year FEM or TWFEM in Figure 2.12, Figure 2.13 presents the statistical results based on a full \( G^* \)TPFEM, using exactly the same set of numerical variables, and one of its unexpected reduced model. Based on these results the following notes and comments are presented.

(1) The statistical results of the full model are obtained using the following ES, which is similar to the ES in (2.66). However, these results are representing a set of four homogeneous regressions of \( LY, \) compare to 2,214 homogeneous regressions based on the model in (2.66)

\[
ly \ ly(-1) * lk*ll \ ly(-1) * lk \ ly(-1) * ll \ lk*ll \ ly(-1) \ lk \ ll \ C \ @\text{Expand(}\ Group,TP,\ @\text{Dropfirst})
\]

(2.74)
(2) The levels of the dichotomous variable GROUP of the firms could be easily extended, depending on the characteristics of the sampled firms. On the other hand, the sampled firm could be considered as a single homogeneous group.

![Figure 2.13](image-url) Statistical results of the model in (2.74), and two of its possible reduced models

(3) The reduced model-1 is obtained by using the manual selection method. It is an unexpected reduced model, because the main variable $LY(-1)$ is deleted, even it has a very small probability of 0.0000. It is recognized that other acceptable reduced models, where each of the independent variables has a probability less than 0.25 (or 0.30).

(4) On the other hand, the reduced model-2 is obtained by deleting the main factor having the greatest probability, that is $LK$, and then $LL$. The interaction independent variable(s) should be kept in the model, since it is defined the effect of $LY(-1)$ on $LY$ depends on $LK$ and $LL$.

(5) Then there is a question, which reduced model would be considered as a better model!
(5) Note that based on the trivaraite \((LK, LL, LY)\) a lot of possible models could be proposed or defined by a researcher, including a simpler continuous \(LV(1)\) Model presented in the following example.

### 2.11.2 Group+TimePeriod Fixed Effects Models

Referring to the limitations of the TWFEMs as presented in Example 2.7, then it is recommended to apply *Group+TimePeriod Fixed Effects Models* \((G+TP\_FEMs)\), which in fact are *ANCOVA Models* by \((Group+TimePeriod)\), namely additive ANCOVA models. Refer to the limitation of additive ANOVA Models presented in previous chapter. Corresponding to the models in (2.69) up to (2.73), the equation specifications of the \(G+TP\_FEMs\) can easily obtained.

For instance, corresponding to the \(G \times TP\_FEMs\) in (2.69), the \(G+TP\_FEMs\) would have the general equation specification as follows:

\[
G(Y) \ V1... \ Vk... \ VK \ C \ @Expand(\text{Group},\text{Dropfirst}) \ @Expand(TP,\text{Dropfirst}) \quad (2.75)
\]

Similarly, corresponding to the \(G \times TP\_FEMs\) in (2.72), the \(G+TP\_FEMs\) with trend would have the general equation specification as follows:

\[
G(Y) \ V1... \ Vk... \ VK \ t \ C \ @Expand(\text{Group},\text{Dropfirst}) \ @Expand(TP,\text{Dropfirst}) \quad (2.76)
\]

And corresponding to the \(G \times TP\_FEMs\) in (2.73), the \(G+TP\_FEMs\) with TRE would have the general equation specification as follows:

\[
G(Y) \ V1... \ Vk... \ VK \ t \ t*V1... \ t*Vk... \ t*VK \\
C \ @Expand(\text{Group},\text{Dropfirst}) \ @Expand(TP,\text{Dropfirst}) \quad (2.77)
\]

### 2.12 Continuous Regression Model

In addition to the models presented above, another type of models can be considered are the continuous regression models, or the models without using a dummy independent variable. However, such a continuous regression model would be considered as an inappropriate model or the worst model, because all \((N*T)\) firm-time observations, is represented by a single regression function only, especially for a large number observations. The regression functions obtained can show that they are good fit models, in a statistical sense. See the following examples.
2.12.1 The Simplest Lagged Variable Model and Alternatives

Example 2.11 (The simplest lagged variable model and alternative) Figure 2.14 presents the statistical results based on three LV(1) Models of LY, namely Model-1 is the simplest lagged variable continuous model, Model-2 is a LV(1) G×TP_FEM, and Model-3 is a LV(1) HRM (Heterogeneous Regression Model). Based on these statistical results, the following notes and comments are presented.

![Table of statistical results](image)

Figure 2.14 Statistical results of the simplest LV(1) Models of LY, and alternatives

1. Model-1 is a continuous regression model of $LY$ on $LY(-1)$, which shows that $LY(-1)$ is a best predictor for $LY$, since it has a very large $R^2$ of 0.999. In a two-dimensional space, this regression is presenting a single line, based on 2214 firm-time observations.

2. Model-2 is a LV(1) G×TP_FEM, in a four dimensional space generated by the variables $LY$, $LY(-1)$, Group, and Time-Period. So that it is an abstract space. However, in a two-dimensional space of $LY$ and $LY(1)$, the model can be presented by four parallel regression lines of $LY$ on $LY(-1)$.
(3) Model-3 is a LV(1) HRM in a four dimensional space generated by the variables \(LY, LY(-1), Group,\) and \(Time-Period.\) So that it is an abstract space. However, in a two-dimensional space of \(LY\) and \(LY(1),\) the model can be presented by four heterogeneous regression lines of \(LY\) on \(LY(-1).\)

(4) Since the three models have very large \(R^2\) of 0.999, and each of the independent variables has a significant effect, then every researcher can argue about the best possible model. Based on my point of view, the best model is Model-3.

2.12.2 \(LV(1)\) Translog Linear Model and Alternatives

Example 2.12 (LV(1) Translog Linier Model, and Alternatives) Figure 2.15 presents the statistical results based on three LV(1) Translog Linear Models of \(LY\) on \(LY(-1), LK,\) and \(LL,\) namely Model-1 is continuous model, Model-2 is a LV(1) G*TPFEM, and Model-3 is an advanced LV(1) HRM. Based on these models, the following notes and comments are presented.

(1) Model-1 is a continuous translog linear model. Since \(LL\) has a very large probability, then a reduced model would be developed by deleting \(LL,\) in a statistical sense. However, the reduced model is not presented, because the model will be compared to Model-2 and Model-3 having exactly the same numerical independent variables.

(2) Model-2 is a homogeneous translog linear model, or a \(G \times TP\_FEM,\) which shows that \(LK\) and \(LL\) have very large probabilities. Since both variables \(LK\) and \(LL\) have insignificant joint effects on \(LY,\) then both variables can be deleted to obtain a reduced model, which is the Model-2 in Figure 2.14.

(3) Model-3 is a heterogeneous translog linear regressions of \(LY\) on \(LY(-1), LK\) and \(LL,\) by \(Group,\) and \(TP.\) It is unexpected, the statistical results show only one of the independent variables, namely \(LL^*(Group=2 \text{ and } TP=1)\) has a \(p\)-value \(\geq 0.30.\) Note that at the \(\alpha = 0.15\) level of significance, each of the variables \(LK^*(Group=1 \text{ and } TP=1), LK^*(Group=1 \text{ and } TP=2),\) and \(LL^*(Group=1 \text{ and } TP=2),\) respectively, has a positive significant level with a probability \(0.2902/2 = 0.1451, 0.1463/2 = 0.07315,\) and \(0.2422/2 = 0.1211.\)

(4) Model-3 should be the best fit model to study the differential effects of \(LY(-1), LK,\) and \(LL\) on \(LY,\) between the cells generated by the variables \(Group,\) and \(TP.\)
2.12.3 Three-Way Interaction Continuous Model

Example 2.13 (Three-Way Interaction LV(1) Translog LS Regression) Figure 2.16 presents the statistical results of a full PLS regression, by using the following ES, and two of its possible reduced models. Based on these results, the following notes and comments are presented.

\[
\begin{align*}
    ly & \ ly(-1)^*ll \ ly(-1)^*lk \ ly(-1)^*ll & lk^*ll & ly(-1) & lk & ll & C \\
\end{align*}
\]  

(2.78)
**Figure 2.16** Statistical results of the PLS Model in (2.78) and two of its possible reduced models

<table>
<thead>
<tr>
<th>Variable</th>
<th>Full Model</th>
<th>Reduced Model-1</th>
<th>Reduced Model-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>LY(-1)*LK,LL</td>
<td>-0.0002</td>
<td>-0.3525</td>
<td>0.3940</td>
</tr>
<tr>
<td>LY(-1)*LK</td>
<td>0.0003</td>
<td>0.0833</td>
<td>0.9217</td>
</tr>
<tr>
<td>LY(-1)*LL</td>
<td>0.0075</td>
<td>1.3866</td>
<td>0.1557</td>
</tr>
<tr>
<td>LK*LL</td>
<td>0.0035</td>
<td>0.7506</td>
<td>0.4530</td>
</tr>
<tr>
<td>LY(-1)</td>
<td>0.9444</td>
<td>10.6596</td>
<td>0.0000</td>
</tr>
<tr>
<td>LK</td>
<td>-0.0150</td>
<td>-0.1973</td>
<td>0.8436</td>
</tr>
<tr>
<td>LL</td>
<td>-0.1745</td>
<td>-1.6413</td>
<td>0.1009</td>
</tr>
<tr>
<td>C</td>
<td>1.5507</td>
<td>0.8933</td>
<td>0.3718</td>
</tr>
</tbody>
</table>

**R-squared** | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9992 |
| Adjusted R-squared | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9992 |
| F-statistic | 377325.9 | 418670.9 | 526745.1 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| Prob(F-statistic) | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| Durbin-Watson stat | 1.6000 | 1.5230 | 1.5905 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

---

**Figure 2.17** Statistical results of two translog linear models, and three interaction translog models

<table>
<thead>
<tr>
<th>Variable</th>
<th>Model-1</th>
<th>Model-2</th>
<th>Model-3</th>
<th>Model-4</th>
<th>Model-5</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.048</td>
<td>0.002</td>
<td>0.049</td>
<td>0.001</td>
<td>-0.185</td>
</tr>
<tr>
<td>LY(-1)</td>
<td>1.006</td>
<td>0.000</td>
<td>1.006</td>
<td>0.000</td>
<td>0.973</td>
</tr>
<tr>
<td>LK</td>
<td>-0.006</td>
<td>0.048</td>
<td>-0.006</td>
<td>0.040</td>
<td>0.044</td>
</tr>
<tr>
<td>LL</td>
<td>0.000</td>
<td>0.918</td>
<td>0.000</td>
<td>0.123</td>
<td>-0.004</td>
</tr>
<tr>
<td>LY(-1)*LK</td>
<td>0.000</td>
<td>0.123</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>LY(-1)*LL</td>
<td>0.000</td>
<td>0.285</td>
<td>0.000</td>
<td>0.926</td>
<td>0.000</td>
</tr>
<tr>
<td>LK*LL</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.999</td>
<td>0.999</td>
<td>0.999</td>
<td>0.999</td>
<td>0.999</td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.999</td>
<td>0.999</td>
<td>0.999</td>
<td>0.999</td>
<td>0.999</td>
</tr>
<tr>
<td>F-statistic</td>
<td>869010.2</td>
<td>1304235.0</td>
<td>521958.4</td>
<td>439875.6</td>
<td>529807.8</td>
</tr>
<tr>
<td>Prob(F-statistic)</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>1.567</td>
<td>1.568</td>
<td>1.573</td>
<td>1.599</td>
<td>1.599</td>
</tr>
</tbody>
</table>
Referring to the path diagram in Figure 2.1a, the effect of $LY(-1)$ on $LY$ is defined to be dependent on $LK$ and $LL$, then the reduced model should be explored by deleting the main variables, say $LY(-1)$, $LK$, or $LL$.

2.13 Various of FEMs and Continuous Models

All equation specifications presented above for the FEMs and Continuous Models can easily be applied for the following models. Do for the exercises using your own panel data sets.

1. The models having the numerical endogenous variable presented in subsection 1.3.1, such as the LS Regressions, Quantile Regressions, and Instrumental Variables Models.

2. The binary choice (probit, logit, and extreme value) models, for a dummy problem indicator,

3. The ordered choice (probit, logit, and extreme value) models, for any ordinal problem indicator, as presented in subsection 1.3.1.3,

4. The firm, or cross-section fixed-effects HRMs, and the time, or period fixed-effects HRMs, as the extension of FEMs presented in Section 1.4.3.
Reference
Jie (Jack) He, Jun (QJ) Qian, and Philip E. Strahan, 2012. Are All Rating Created Equal?