CHAPTER 1

Antennas

1.1 INTRODUCTION

An antenna is defined by Webster’s Dictionary as “a usually metallic device (as a rod or wire) for radiating or receiving radio waves.” The IEEE Standard Definitions of Terms for Antennas (IEEE Std 145–1983)” defines the antenna or aerial as “a means for radiating or receiving radio waves.” In other words the antenna is the transitional structure between free-space and a guiding device, as shown in Figure 1.1. The guiding device or transmission line may take the form of a coaxial line or a hollow pipe (waveguide), and it is used to transport electromagnetic energy from the transmitting source to the antenna, or from the antenna to the receiver. In the former case, we have a transmitting antenna and in the latter a receiving antenna.

A transmission-line Thevenin equivalent of the antenna system of Figure 1.1 in the transmitting mode is shown in Figure 1.2 where the source is represented by an ideal generator, the transmission line is represented by a line with characteristic impedance $Z_c$, and the antenna is represented by a load $Z_A = (R_L + R_r) + jX_A$ connected to the transmission line. The Thevenin and Norton circuit equivalents of the antenna are also shown in Figure 2.27. The load resistance $R_L$ is used to represent the conduction and dielectric losses associated with the antenna structure while $R_r$, referred to as the radiation resistance, is used to represent radiation by the antenna. The reactance $X_A$ is used to represent the imaginary part of the impedance associated with radiation by the antenna. This is discussed more in detail in Sections 2.13 and 2.14. Under ideal conditions, energy generated by the source should be totally transferred to the radiation resistance $R_r$, which is used to represent radiation by the antenna. However, in a practical system there are conduction-dielectric losses due to the lossy nature of the transmission line and the antenna, as well as those due to reflections (mismatch) losses at the interface between the line and the antenna. Taking into account the internal impedance of the source and neglecting line and reflection (mismatch) losses, maximum power is delivered to the antenna under conjugate matching. This is discussed in Section 2.13.

The reflected waves from the interface create, along with the traveling waves from the source toward the antenna, constructive and destructive interference patterns, referred to as standing waves, inside the transmission line which represent pockets of energy concentrations and storage, typical of resonant devices. A typical standing wave pattern is shown dashed in Figure 1.2, while another is exhibited in Figure 1.15. If the antenna system is not properly designed, the transmission line


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could act to a large degree as an energy storage element instead of as a wave guiding and energy transporting device. If the maximum field intensities of the standing wave are sufficiently large, they can cause arching inside the transmission lines.

The losses due to the line, antenna, and the standing waves are undesirable. The losses due to the line can be minimized by selecting low-loss lines while those of the antenna can be decreased by
Figure 2.2 Two-dimensional normalized field pattern (linear scale), power pattern (linear scale), and power pattern (in dB) of a 10-element linear array with a spacing of $d = 0.25$.

A radiation lobe is a “portion of the radiation pattern bounded by regions of relatively weak radiation intensity.” Figure 2.3(a) demonstrates a symmetrical three-dimensional polar pattern with a number of radiation lobes. Some are of greater radiation intensity than others, but all are classified as lobes. Figure 2.3(b) illustrates a linear two-dimensional pattern [one plane of Figure 2.3(a)] where the same pattern characteristics are indicated.
Figure 2.11  Three- and two-dimensional power patterns (in linear scale) of $U(\theta) = \cos^2(\theta) \cos^2(3\theta)$.
Analogous to the power transferred to the load and the power scattered by the antenna in the receiving mode. In Figure 2.27 it is assumed that the generator is directly connected to the antenna. If there is a transmission line between the two, which is usually the case, then $Z_g$ represents the equivalent impedance of the generator transferred to the input terminals of the antenna using the impedance transfer equation. If, in addition, the transmission line is lossy, then the available power to be radiated by the antenna will be reduced by the losses of the transmission line. Figure 2.27(c) illustrates the Norton equivalent of the antenna and its source in the transmitting mode.

The use of the antenna in the receiving mode is shown in Figure 2.28(a). The incident wave impinges upon the antenna, and it induces a voltage $V_T$ which is analogous to $V_a$ of the transmitting mode. The Thevenin equivalent circuit of the antenna and its load is shown in Figure 2.28(b) and the Norton equivalent in Figure 2.28(c). The discussion for the antenna and its load in the receiving mode parallels that for the transmitting mode, and it will not be repeated here in detail. Some of the results will be summarized in order to discuss some subtle points. Following a procedure similar to that for the antenna in the transmitting mode, it can be shown using Figure 2.28 that in the receiving mode under conjugate matching ($R_r + R_L = R_T$ and $X_A = -X_T$) the powers delivered to $R_T$, $R_r$, and
If \( \mathbf{J} \) and \( \mathbf{M} \) represent linear densities (m\(^{-1}\)), (3-49) and (3-50) reduce to surface integrals, or

\[
\mathbf{A} = \frac{\mu}{4\pi} \int_S \mathbf{J}_s(x', y', z') \frac{e^{-jkR}}{R} \, ds' \tag{3-51}
\]

\[
\mathbf{F} = \frac{\varepsilon}{4\pi} \int_S \mathbf{M}_s(x', y', z') \frac{e^{-jkR}}{R} \, ds' \tag{3-52}
\]

For electric and magnetic currents \( \mathbf{I}_e \) and \( \mathbf{I}_m \), (3-51) and (3-52) reduce to line integrals of the form

\[
\mathbf{A} = \frac{\mu}{4\pi} \int_C \mathbf{I}_e(x', y', z') \frac{e^{-jkR}}{R} \, dl' \tag{3-53}
\]

\[
\mathbf{F} = \frac{\varepsilon}{4\pi} \int_C \mathbf{I}_m(x', y', z') \frac{e^{-jkR}}{R} \, dl' \tag{3-54}
\]

### 3.6 FAR-FIELD RADIATION

The fields radiated by antennas of finite dimensions are spherical waves. For these radiators, a general solution to the vector wave equation of (3-14) in spherical components, each as a function of \( r, \theta, \phi \), takes the general form of

\[
\mathbf{A} = \hat{\alpha}_r A_r(r, \theta, \phi) + \hat{\alpha}_\theta A_\theta(r, \theta, \phi) + \hat{\alpha}_\phi A_\phi(r, \theta, \phi) \tag{3-55}
\]

The amplitude variations of \( r \) in each component of (3-55) are of the form \( 1/r^n, n = 1, 2, \ldots \) [1], [2]. Neglecting higher order terms of \( 1/r^n(1/r^2 = 0, n = 2, 3, \ldots) \) reduces (3-55) to

\[
\mathbf{A} \approx \left[ \hat{\alpha}_r A'_r(\theta, \phi) + \hat{\alpha}_\theta A'_\theta(\theta, \phi) + \hat{\alpha}_\phi A'_\phi(\theta, \phi) \right] e^{-jkR/r}, \quad r \to \infty \tag{3-56}
\]

The \( r \) variations are separable from those of \( \theta \) and \( \phi \). This will be demonstrated in the chapters that follow by many examples.

Substituting (3-56) into (3-15) reduces it to

\[
\mathbf{E} = \frac{1}{r} \left\{ -j\omega e^{-jkR} [\hat{\alpha}_r(0) + \hat{\alpha}_\theta A'_\theta(\theta, \phi) + \hat{\alpha}_\phi A'_\phi(\theta, \phi)] + \frac{1}{r^2} \{ \cdots \} + \cdots \right\} \tag{3-57}
\]

The radial \( \mathbf{E} \)-field component has no \( 1/r \) terms, because its contributions from the first and second terms of (3-15) cancel each other.

Similarly, by using (3-56), we can write (3-2a) as

\[
\mathbf{H} = \frac{1}{r} \left\{ -\frac{j\omega}{\eta} e^{-jkR} [\hat{\alpha}_r(0) + \hat{\alpha}_\theta A'_\theta(\theta, \phi) - \hat{\alpha}_\phi A'_\phi(\theta, \phi)] \right\} + \frac{1}{r^2} \{ \cdots \} + \cdots \tag{3-57a}
\]

where \( \eta = \sqrt{\mu/\varepsilon} \) is the intrinsic impedance of the medium.
\( E_{\psi} = \begin{cases} \varepsilon_0 \mu_0 \delta_0 e^{-jkr} \\ -j \frac{\omega \mu_0 \delta_0 e^{-jkr}}{4\pi r} \end{cases} = \text{constant (isotropic)} \) (4-116c)

These components match those of Example 4.5 that follows, which are derived based on the vector potential approach. Also, based on (4-116a)–(4-116c), it is easier to decide on the shape of the amplitude pattern and ascertain the polarization of the wave in the three principal planes.

**Example 4.5**

Using the vector potential \( \mathbf{A} \) and the procedure outlined in Section 3.6 of Chapter 3, derive the far-zone spherical electric and magnetic field components of a horizontal infinitesimal dipole placed at the origin of the coordinate system of Figure 4.1.

**Solution:** Using (4-4), but for a horizontal infinitesimal dipole of uniform current directed along the \( y \)-axis, the corresponding vector potential can be written as

\[
\mathbf{A} = \hat{\mathbf{a}}_y \frac{\mu_0 I_0 e^{-jkx}}{4\pi r}
\]

with the corresponding spherical components, using the rectangular to spherical components transformation of (4-5), expressed as

\[
A_\theta = A_y \cos \theta \sin \phi = \frac{\mu_0 I_0 e^{-jkx}}{4\pi r} \cos \theta \sin \phi
\]

\[
A_\phi = A_y \cos \phi = \frac{\mu_0 I_0 e^{-jkx}}{4\pi r} \cos \phi
\]

Using (3-58a) and (3-58b), we can write the corresponding far-zone electric and magnetic field components as

\[
E_\theta \approx -j \omega A_\theta = -j \frac{\omega \mu_0 I_0 e^{-jkx}}{4\pi r} \cos \theta \sin \phi
\]

\[
E_\phi \approx -j \omega A_\phi = -j \frac{\omega \mu_0 I_0 e^{-jkx}}{4\pi r} \cos \phi
\]

\[
H_\theta \approx -\frac{E_\phi}{\eta} = j \frac{\omega \mu_0 I_0 e^{-jkx}}{4\pi \eta r} \cos \phi
\]

\[
H_\phi \approx +\frac{E_\theta}{\eta} = j \frac{\omega \mu_0 I_0 e^{-jkx}}{4\pi \eta r} \cos \theta \sin \phi
\]

Although the electric-field components, and thus the magnetic field components, take a different analytical form than (4-111), the patterns are the same.

To examine the variations of the total field as a function of the element height above the ground plane, the two-dimensional elevation plane patterns (normalized to 0 dB) for \( \phi = 90^\circ \) (\( y-z \) plane) when \( h = 0, \lambda/8, \lambda/4, 3\lambda/8, \lambda/2 \), and \( \lambda \) are plotted in Figure 4.29. Since this antenna system is not symmetric with respect to the \( z \) axis, the azimuthal plane (\( x-y \) plane) pattern is not isotropic.
the antenna [30]. A similar correction for the loop would result in a better agreement between the computed and measured susceptances. Computations for a half-loop above a ground plane were also performed by J. E. Jones [31] using the Moment Method.

The radiation resistance and maximum directivity of a loop antenna with a cosinusoidal current distribution \( I_\phi(\phi) = I_0 \cos \phi \) was derived in [2], [16] and evaluated by integrating in far-zone fields. Doing this, the values are plotted, respectively, in Figures 5.16(a,b) where they are compared with those based on a uniform and nonuniform current distribution.

**Figure 5.16** Radiation resistance \((R_r)\) and maximum directivity \((D_\theta)\) of a circular loop with uniform, cosinusoidal and nonuniform current distributions.

d. Matlab computer program, designated Circular_Loop_Uniform, for computing the radiation characteristics of a loop. A description of the program is found in the README file of the corresponding program in the publisher's website for this book.

e. The Matlab computer program Circular_Loop_Nonuniform can be used to compute the radiation characteristics (current distribution, input impedance, amplitude pattern, and directivity pattern) of a circular loop with uniform sinusoidal and Fourier series current distributions.

f. Power Point (PPT) viewgraphs, in multicolor.
(c) Radiation efficiency of the loop (in %).
(d) Maximum gain (dimensionless and in dB) of the loop.

5.13. Design a lossless resonant circular loop operating at 10 MHz so that its single-turn radiation resistance is 0.73 ohms. The resonant loop is to be connected to a matched load through a balanced “twin-lead” 300-ohm transmission line.
(a) Determine the radius of the loop (in meters and wavelengths).
(b) To minimize the matching reflections between the resonant loop and the 300-ohm transmission line, determine the closest number of integer turns the loop must have.
(c) For the loop of part b, determine the maximum power that can be expected to be delivered to a receiver matched load if the incident wave is polarization matched to the lossless resonant loop. The power density of the incident wave is $10^{-6}$ watts/m².

5.14. A resonant six-turn loop of closely spaced turns is operating at 50 MHz. The radius of the loop is $\lambda /30$, and the loop is connected to a 50-ohm transmission line. The radius of the wire is $\lambda /300$, its conductivity is $\sigma = 5.7 \times 10^7$ S/m, and the spacing between the turns is $\lambda /100$. Determine the
(a) directivity of the antenna (in dB)
(b) radiation efficiency taking into account the proximity effects of the turns
(c) reflection efficiency
(d) gain of the antenna (in dB)

5.15. A horizontal, lossless, one-turn circular loop of circumference $C = \lambda$, with a nonuniform current distribution, is radiating in free space. The far-field pattern of the antenna can be approximated by

$$E_{\phi} \approx C_0 \cos^2 \theta \frac{e^{-jkr}}{r} \begin{cases} \quad 0^\circ \leq \theta \leq 90^\circ \\
\quad 0^\circ \leq \phi \leq 360^\circ 
\end{cases}$$

where $C_0$ is a constant and $\theta$ is measured from the normal to the plane/area of the loop. Determine the
(a) Maximum exact directivity (dimensionless and in dB) of the antenna.
(b) Approximate input impedance of the loop.
(c) Input reflection coefficient when the antenna is connected to a balanced “twin-lead” transmission line with a characteristic impedance of 300 ohms.
(d) Maximum gain of the loop (dimensionless and in dB).
(e) Maximum absolute gain of the loop (dimensionless and in dB).

5.16. Find the radiation efficiency (in percent) of an eight-turn circular-loop antenna operating at 30 MHz. The radius of each turn is $a = 15$ cm, the radius of the wire is $b = 1$ mm, and the spacing between turns is $2c = 3.6$ mm. Assume the wire is copper ($\sigma = 5.7 \times 10^7$ S/m), and the antenna is radiating into free-space. Account for the proximity effect.

5.17. A very small circular loop of radius $a(a < \lambda /6\pi)$ and constant current $I_0$ is symmetrically placed about the origin at $x = 0$ and with the plane of its area parallel to the $y$-$z$ plane. Find the
(a) spherical E- and H-field components radiated by the loop in the far zone
(b) directivity of the antenna

5.18. Repeat Problem 5.17 when the plane of the loop is parallel to the $x$-$z$ plane at $y = 0$.

5.19. Using the computer program of this chapter, compute the radiation resistance and the directivity of a circular loop of constant current with a radius of
(a) $a = \lambda /50$  
(b) $a = \lambda /10$  
(c) $a = \lambda /4$  
(d) $a = \lambda /2$
There are a plethora of antenna arrays used for personal, commercial, and military applications utilizing different elements including dipoles, loops, apertures, microstrips, horns, reflectors, and so on. Arrays of dipoles are shown in Figures 4.26, 10.19, and 11.15. The one in Figure 4.26 is an array that is widely used as a base-station antenna for mobile communication. It is a triangular array consisting of twelve dipoles, with four dipoles on each side of the triangle. Each four-element array, on each side of the triangle, is basically used to cover an angular sector of 120° forming what is usually referred to as a sectoral array. The one in Figure 10.19 is a classic array of dipoles, referred to as the Yagi-Uda array, and it is primarily used for TV and amateur radio applications. The array of Figure 11.17 is also an array of dipoles, which is referred to as the log-periodic antenna, which is primarily used for TV reception and has wider bandwidth than the Yagi-Uda array but slightly smaller directivity. An array of loops is shown in Figure 5.1 and one utilizing microstrips as elements is displayed in Figure 14.35. An advanced array design of slots, used in the AWACS, is shown in Figure 6.29.

The simplest and one of the most practical arrays is formed by placing the elements along a line. To simplify the presentation and give a better physical interpretation of the techniques, a two-element array will first be considered. The analysis of an N-element array will then follow. Two-dimensional analysis will be the subject at first. In latter sections, three-dimensional techniques will be introduced.

### 6.2 TWO-ELEMENT ARRAY

Let us assume that the antenna under investigation is an array of two infinitesimal horizontal dipoles positioned along the z-axis, as shown in Figure 6.1(a). The total field radiated by the two elements, assuming no coupling between the elements, is equal to the sum of the two and in the y-z plane it is given by

\[
E_i = E_1 + E_2 = \hat{\alpha}_{i\eta} \frac{k l_0 d e^{-jkr}}{4\pi} \left\{ \frac{e^{-jkr_1 - (\beta/2)}}{r_1} \cos \theta_1 + \frac{e^{-jkr_2 + (\beta/2)}}{r_2} \cos \theta_2 \right\} \tag{6-1}
\]

where \( \beta \) is the difference in phase excitation between the elements. The magnitude excitation of the radiators is identical. Assuming far-field observations and referring to Figure 6.1(b),

\[
\begin{align*}
\theta_1 &\simeq \theta_2 \simeq \theta \\
r_1 &\simeq r - \frac{d}{2} \cos \theta \\
r_2 &\simeq r + \frac{d}{2} \cos \theta \\
r_1 &\simeq r_2 \simeq r
\end{align*}
\tag{6-2a}
\]

for phase variations

\[
\begin{align*}
\theta_1 &\simeq \theta_2 \simeq \theta \\
r_1 &\simeq r - \frac{d}{2} \cos \theta \\
r_2 &\simeq r + \frac{d}{2} \cos \theta \\
r_1 &\simeq r_2 \simeq r
\end{align*}
\tag{6-2b}
\]

for amplitude variations

Equation 6-1 reduces to

\[
E_i = \hat{\alpha}_{i\eta} \frac{k l_0 d e^{-jkr}}{4\pi r} \cos \theta \left[ e^{+j(kd \cos \theta + \beta)/2} + e^{-j(kd \cos \theta + \beta)/2} \right]
\]

\[
E_i = \hat{\alpha}_{i\eta} \frac{k l_0 d e^{-jkr}}{4\pi r} \cos \theta \left\{ 2 \cos \left[ \frac{1}{2} (kd \cos \theta + \beta) \right] \right\} \tag{6-3}
\]

It is apparent from (6-3) that the total field of the array is equal to the field of a single element positioned at the origin multiplied by a factor which is widely referred to as the array factor. Thus
elements of a very long array should be

\[
\beta = - \left( kd + \frac{2.92}{N} \right) \right\} \Rightarrow \text{for maximum in } \theta_0 = 0^\circ \\
\beta = + \left( kd + \frac{2.92}{N} \right) \right\} \Rightarrow \text{for maximum in } \theta_0 = 180^\circ
\]

These requirements are known today as the Hansen-Woodyard conditions for end-fire radiation. They lead to a larger directivity than the conditions given by (6-20a) and (6-20b). It should be pointed out, however, that these conditions do not necessarily yield the maximum possible directivity. In fact, the maximum may not even occur at \( \theta_0 = 0^\circ \) or \( 180^\circ \); its value found using (6-10c) or (6-10d) may not be unity, and the side lobe level may not be \(-13.46\) dB. Both of them, maxima and side lobe levels, depend on the number of array elements, as will be illustrated.

To realize the increase in directivity as a result of the Hansen-Woodyard conditions, it is necessary that, in addition to the conditions of (6-23a) and (6-23b), \(|\psi|\) assumes values of

For maximum radiation along \( \theta_0 = 0^\circ \)

\[
|\psi| = |kd \cos \theta + \beta|_{\theta=0^\circ} = \frac{\pi}{N} \quad \text{and} \quad |\psi| = |kd \cos \theta + \beta|_{\theta=180^\circ} \approx \pi
\]

\(1\) In principle, the Hansen-Woodyard condition was derived for an infinitely long antenna with continuous distribution. It thus gives good results for very long, finite length discrete arrays with closely spaced elements.
given by [14]

\[ \gamma = \cosh \left(\frac{1}{N-1} \ln \left( R + \sqrt{R^2 - 1} \right) \right) \]  

(6-76c)

where \( R \) represents the side lobe level (as a voltage ratio).

Design curves for \( d_{\text{max}} = d_{\text{min}} \) (from \(-10 \) dB to \(-100 \) dB), for a broadside Dolph-Tschebyscheff array with elements \( N = 10, 20, 30, \) and \( 40 \), are displayed in Figure 6.24(b). As expected, the maximum/optimum element separation, for each array with fixed number of elements, it gets smaller as the side lobe level decreases. Also the design curves for \( d_{\text{max}} = d_{\text{min}} \) as a function of the number of elements, \( N = 3 - 20 \) for side lobe levels \(-30, -40, -50, \) and \(-60 \) dB, are displayed in Figure 6.24(b). As expected, for each of the side lobe levels, the maximum/optimum element separation increases as the number of elements increase.

The excitation coefficients of a Dolph-Tschebyscheff array can be derived using various documented techniques [11]–[13] and others. One method, whose results are suitable for computer calculations, is that by Barbiere [11]. The coefficients using this method can be obtained using

\[
ad_n = \begin{cases} 
\sum_{q=m}^{M} (-1)^{M-q} (z_0)^{2q-1} \frac{(q + M - 2)!(2M - 1)!}{(q-n)!(q+n-1)!(M-q)!} & \text{for even } 2M \text{ elements} \\
\sum_{q=n}^{M+1} (-1)^{M-q+1} (z_0)^{2(q-1)} \frac{(q + M - 2)!2M}{\epsilon_n(q-n)!(q+n-2)!(M-q+1)!} & \text{for odd } 2M+1 \text{ elements} 
\end{cases} 
\]  

(6-77a)

where \( \epsilon_n = \begin{cases} 
2 & n = 1 \\
1 & n \neq 1 
\end{cases} \)

\[(6-77b)\]

C. Beamwidth and Directivity

For large Dolph-Tschebyscheff arrays scanned not too close to end-fire and with side lobes in the range from \(-20 \) to \(-60 \) dB, the half-power beamwidth and directivity can be found by introducing a beam broadening factor given approximately by [4]

\[ f = 1 + 0.636 \left( \frac{2}{R_0} \cosh \left( \frac{1}{\sqrt{\ln(\cosh^{-1} R_0)^2 - \pi^2}} \right) \right)^2 \]  

(6-78)

where \( R_0 \) is the major-to-side lobe voltage ratio. The beam broadening factor is plotted in Figure 6.25(a) as a function of side lobe level (in dB).

The half-power beamwidth of a Dolph-Tschebyscheff array can be determined by

1. calculating the beamwidth of a uniform array (of the same number of elements and spacing using (6-22a) or reading it off Figure 6.12
2. multiplying the beamwidth of part (1) by the appropriate beam broadening factor \( f \) computed using (6-78) or reading it off Figure 6.25(a)

The same procedure can be used to determine the beamwidth of arrays with a cosine-on-a-pedestal distribution [4].
where \( z'_n \) indicates the position of the \( n \)th element (element in question) symmetrically placed about the geometrical center of the array.

**Example 7.5**

Repeat the design of Example 7.4 for a linear array of 10 elements using the Woodward-Lawson method with odd samples and an element spacing of \( d = \lambda/2 \).

**Solution:** According to (7-19), (7-19b), (7-22) and (7-23a), the excitation coefficients of the array at the sampling points are the same as those of the line-source. Using the values of \( b_m \) as listed in Example 7.4, the computed array factor pattern using (7-21) is shown in Figure 7.8(a). A good synthesis of the desired pattern is displayed. The sidelobe level, relative to the pattern value at \( \theta = 90^\circ \), is 0.221 (−13.1 dB). The agreement between the line-source and the linear array Woodward-Lawson designs are also good.

The normalized pattern of the symmetrical discrete array can also be generated using the array factor of (6-61a) or (6-61b), where the normalized excitation coefficients \( a_n \)'s of the array elements are obtained using (7-24). For this example, the excitation coefficients of the 10-element array, along with their symmetrical position, are listed below. To achieve the normalized amplitude pattern of unity at \( \theta = 90^\circ \) in Figure 7.8(a), the array factor of (6-61a) must be multiplied by \( 1/2a_0 = 1/0.4482 = 2.2312 \).

<table>
<thead>
<tr>
<th>Element Number</th>
<th>Element Position</th>
<th>Excitation Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pm 1 )</td>
<td>( \pm 0.25\lambda )</td>
<td>0.6089</td>
</tr>
<tr>
<td>( \pm 2 )</td>
<td>( \pm 0.75\lambda )</td>
<td>-0.1996</td>
</tr>
<tr>
<td>( \pm 3 )</td>
<td>( \pm 1.25\lambda )</td>
<td>-0.0000</td>
</tr>
<tr>
<td>( \pm 4 )</td>
<td>( \pm 1.75\lambda )</td>
<td>0.8730</td>
</tr>
<tr>
<td>( \pm 5 )</td>
<td>( \pm 2.25\lambda )</td>
<td>-0.0003</td>
</tr>
</tbody>
</table>

In general, the Fourier transform synthesis method yields reconstructed patterns whose mean-square error (or deviation) from the desired pattern is a minimum. However, the Woodward-Lawson synthesis method reconstructs patterns whose values at the sampled points are identical to the ones of the desired pattern; it does not have any control of the pattern between the sample points, and it does not yield a pattern with least-mean-square deviation.

Ruze [9] points out that the least-mean-square error design is not necessarily the best. The particular application will dictate the preference between the two. However, the Fourier transform method is best suited for reconstruction of desired patterns which are analytically simple and which allow the integrations to be performed in closed form. Today, with the advancements in high-speed computers, this is not a major restriction since the integration can be performed (with high efficiency) numerically. In contrast, the Woodward-Lawson method is more flexible, and it can be used to synthesize any desired pattern. In fact, it can even be used to reconstruct patterns which, because of their complicated nature, cannot be expressed analytically. Measured patterns, either of analog or digital form, can also be synthesized using the Woodward-Lawson method.

### 7.6 TAYLOR LINE-SOURCE (TSCHEBYSCHEFF-ERROR)

In Chapter 6 we discussed the classic Dolph-Tschebyscheff array design which yields, for a given sidelobe level, the smallest possible first-null beamwidth (or the smallest possible sidelobe level for a given first-null beamwidth). Another classic design that is closely related to it, but is more applicable
(ii) Using the computer program Directivity and (10-35):
   a. ordinary end-fire \( p = 0.8337 \): \( D_o = 12.678 \) (dimensionless)
      \[ = 11.03 \text{ dB} \]
   b. H-W end-fire \( p = 0.8012 \): \( D_o = 26.36 \) (dimensionless) \( = 14.21 \text{ dB} \)
   c. The axial ratio according to (10-34) is:
      \[ AR = \frac{2N+1}{2N} = \frac{20+1}{20} = 1.05 \text{ (dimensionless)} = 0.45 \text{ dB} \]

2. The three-dimensional linear power patterns for the two end-fire designs, ordinary and Hansen-Woodyard, are shown in Figure 10.16.

D. Feed Design
The nominal impedance of a helical antenna operating in the axial mode, computed using (10-30), is 100–200 ohms. However, many practical transmission lines (such as a coax) have characteristic impedance of about 50 ohms. In order to provide a better match, the input impedance of the helix must be reduced to near that value. There may be a number of ways by which this can be accomplished. One way to effectively control the input impedance of the helix is to properly design the first 1/4 turn of the helix which is next to the feed [8], [12]. To bring the input impedance of the helix from nearly 150 ohms down to 50 ohms, the wire of the first 1/4 turn should be flat in the form of a strip and the transition into a helix should be very gradual. This is accomplished by making the wire from the feed, at the beginning of the formation of the helix, in the form of a strip of width \( w \) by flattening it and nearly touching the ground plane which is covered with a dielectric slab of height \( h \)

\[ h = \frac{w}{\sqrt{\varepsilon_r Z_0}} - 2 \]  

(10-41)

where

\( w = \) width of strip conductor of the helix starting at the feed
\( \varepsilon_r = \) dielectric constant of the dielectric slab covering the ground plane
\( Z_0 = \) characteristic impedance of the input transmission line

Typically the strip configuration of the helix transitions from the strip to the regular circular wire and the designed pitch angle of the helix very gradually within the first 1/4–1/2 turn.

This modification decreases the characteristic impedance of the conductor-ground plane effective transmission line, and it provides a lower impedance over a substantial but reduced bandwidth. For example, a 50-ohm helix has a VSWR of less than 2:1 over a 40% bandwidth compared to a 70% bandwidth for a 140-ohm helix. In addition, the 50-ohm helix has a VSWR of less than 1.2:1 over a 12% bandwidth as contrasted to a 20% bandwidth for one of 140 ohms.

A simple and effective way of increasing the thickness of the conductor near the feed point will be to bond a thin metal strip to the helix conductor [12]. For example, a metal strip 70-mm wide was used to provide a 50-ohm impedance in a helix whose conducting wire was 13-mm in diameter and it was operating at 230.77 MHz.

A commercially available helix with a cupped ground plane is shown in Figure 10.17. It is right-hand circularly-polarized (RHCP) operating between 100–160 MHz with a gain of about 6 dB at 100 MHz and 12.8 dB at 160 MHz. The right-hand winding of the wire is clearly shown in the
transmission line cease to look like a “point” (usually about $\lambda_3/8$ where $\lambda_3$ is the wavelength at the highest desirable frequency). Practical bandwidths are on the order of about 40:1. Even higher ratios (i.e., 1,000:1) can be achieved in antenna design but they are not necessary, since they would far exceed the bandwidths of receivers and transmitters.

Even though the shape of the biconical antenna can be completely specified by angles, the current on its structure does not diminish with distance away from the input terminals, and its pattern does not have a limiting form with frequency. This can be seen by examining the current distribution as given by (9-11). It is evident that there are phase but no amplitude variations with the radial distance $r$. Thus the biconical structure cannot be truncated to form a frequency independent antenna. In practice, however, antenna shapes exist which satisfy the general shape equation, as proposed by Rumsey [1], to have frequency independent characteristics in pattern, impedance, polarization, and so forth, and with current distribution which diminishes rapidly.

Rumsey’s general equation will first be developed, and it will be used as the unifying concept to link the major forms of frequency independent antennas. Classical shapes of such antennas include the equiangular geometries of planar and conical spiral structures [2]–[4], and the logarithmically periodic structures [5], [6].

Fundamental limitations in electrically small antennas will be discussed in Section 11.5. These will be derived using spherical mode theory, with the antenna enclosed in a virtual sphere. Minimum Q curves, which place limits on the achievable bandwidth, will be included. Fractal antennas, discussed in Section 11.5, is one class whose design is based on this fundamental principle.

### 11.2 THEORY

The analytical treatment of frequency independent antennas presented here parallels that introduced by Rumsey [1] and simplified by Elliott [6] for three-dimensional configurations.

We begin by assuming that an antenna, whose geometry is best described by the spherical coordinates $(r, \theta, \phi)$, has both terminals infinitely close to the origin and each is symmetrically disposed along the $\theta = 0, \pi$-axes. It is assumed that the antenna is perfectly conducting, it is surrounded by an infinite homogeneous and isotropic medium, and its surface or an edge on its surface is described by a curve

$$r = F(\theta, \phi)$$

where $r$ represents the distance along the surface or edge. If the antenna is to be scaled to a frequency that is $K$ times lower than the original frequency, the antenna’s physical surface must be made $K$ times greater to maintain the same electrical dimensions. Thus the new surface is described by

$$r' = KF(\theta, \phi)$$

The new and old surfaces are identical; that is, not only are they similar but they are also congruent (if both surfaces are infinite). Congruence can be established only by rotation in $\phi$. Translation is not allowed because the terminals of both surfaces are at the origin. Rotation in $\theta$ is prohibited because both terminals are symmetrically disposed along the $\theta = 0, \pi$-axes.

For the second antenna to achieve congruence with the first, it must be rotated by an angle $C$ so that

$$KF(\theta, \phi) = F(\theta, \phi + C)$$

The angle of rotation $C$ depends on $K$ but neither depends on $\theta$ or $\phi$. Physical congruence implies that the original antenna electrically would behave the same at both frequencies. However the radiation
formed by the two arms, and it is linearly polarized. Although the patterns of this and other log-periodic structures are not completely frequency independent, the amplitude variations of certain designs are very slight. Thus, practically, they are frequency independent.

Log-periodic wire antennas were introduced by DuHamel [2]. While investigating the current distribution on log-periodic surface structures of the form shown in Figure 11.6(a), he discovered that the fields on the conductors attenuated very sharply with distance. This suggested that perhaps

Figure 11.6  Planar and wire logarithmically periodic antennas.
Using the mathematical formulation introduced by Chu [18], the source or current distribution of the antenna system inside the sphere is not uniquely determined by the field distribution outside the sphere. Since it is possible to determine an infinite number of different source or current distributions inside the sphere, for a given field configuration outside the sphere, Chu [18] confined his interest to the most favorable source distribution and its corresponding antenna structure that could exist within the sphere. This approach was taken to minimize the details and to simplify the task of identifying the antenna structure. It was also assumed that the desired current or source distribution minimizes the amount of energy stored inside the sphere so that the input impedance at a given frequency is resistive.

Because the spherical wave modes outside the sphere are orthogonal, the total energy (electric or magnetic) outside the sphere and the complex power transmitted across the closed spherical surface are equal, respectively, to the sum of the energies and complex powers associated with each corresponding spherical mode. Therefore there is no coupling, in energy or power, between any two modes outside the sphere. As a result, the space outside the sphere can be replaced by a number of independent equivalent circuits as shown in Figure 11.16(b). The number of equivalent circuits is equal to the number of spherical wave modes outside the sphere, plus one. The terminals of each equivalent circuit are connected to a box which represents the inside of the sphere, and from inside the box a pair of terminals are drawn to represent the input terminals. Using this procedure, the antenna space problem has been reduced to one of equivalent circuits.

The radiated power of the antenna is calculated from the propagating modes while all modes contribute to the reactive power. When the sphere (which encloses the antenna element) becomes

very small, there exist no propagating modes. Therefore the $Q$ of the system becomes very large since all modes are evanescent (below cutoff) and contribute very little power. However, unlike closed waveguides, each evanescent mode here has a real part (even though it is very small).

For a lossless antenna (radiation efficiency $e_{rd} = 100\%$), the equivalent circuit of each spherical mode is a single network section with a series $C$ and a shunt $L$. The total circuit is a ladder network of $L - C$ sections (one for each mode) with a final shunt resistive load, as shown in Figure 11.16(c). The resistive load is used to represent the normalized antenna radiation resistance.

From this circuit structure, the input impedance is found. The $Q$ of each mode is formed by the ratio of its stored to its radiated energy. When several modes are supported, the $Q$ is formed from the contributions of all the modes.

It has been shown that the higher order modes within a sphere of radius $a$ become evanescent when $ka < 1$. Therefore the $Q$ of the system, for the lowest order TM$_{0n}$ mode, reduces to [16]-[18]

$$Q = \frac{1 + 2(ka)^2}{(ka)^2[1 + (ka)^2]} \approx \frac{1}{(ka)^2} \quad (11-35a)$$

or for the lowest TM$_{01}$ mode, according to [16], to

$$Q = \frac{1}{(ka)^3} + \frac{1}{ka} = \frac{1 + (ka)^2}{(ka)^3} \approx \frac{1}{(ka)^2} \quad (11-35b)$$

Both (11-35a) and (11-35b) lead to the same results when $ka \ll 1$.

When two modes are excited, one TE and the other TM, the values of $Q$ are halved. Equations (11-35a) and (11-35b), which relate the lowest achievable $Q$ to the largest linear dimension of an electrically small antenna, are independent of the geometrical configuration of the antenna within the sphere of radius $a$. The shape of the radiating element within the bounds of the sphere only determines whether TE, TM, or TE and TM modes are excited. Therefore (11-35a) and (11-35b) represent the fundamental limit on the electrical size of an antenna. In practice, this limit is only approached but is never exceeded or even equaled.

The losses of an antenna can be taken into account by including a loss resistance in series with the radiation resistance, as shown by the equivalent circuits of Figures 2.27(b) and 2.28(b). This influences the $Q$ of the system and the antenna radiation efficiency as given by (2-90).

Computed values of $Q$ versus $kr$ for idealized antennas enclosed within a sphere of radius $a$, and with radiation efficiencies of $e_{rd} = 100$, 50, 10, and 5, are shown plotted in Figure 11.17. These curves represent the minimum values of $Q$ that can be obtained from an antenna whose structure can be enclosed within a sphere of radius $a$ and whose radiated field, outside the sphere, can be represented by a single spherical wave mode.

For antennas with equivalent circuits of fixed values, the fractional bandwidth is related to the $Q$ of the system by

$$\text{fractional bandwidth} = \text{FBW} = \frac{\Delta f}{f_0} = \frac{1}{Q} \quad (11-36)$$

where

$f_0 =$ center frequency

$\Delta f =$ bandwidth

The relationship of (11-36) is valid for $Q \gg 1$ since the equivalent resonant circuit with fixed values is a good approximation for an antenna. For values of $Q < 2$, (11-36) is not accurate.
obtained, as shown in Figure 11.25(b). Sierpinski gaskets can be used as elements in monopoles and dipoles having geometries whose peripheries are similar to the cross section of conical monopoles and biconical dipoles. The Sierpinski gaskets exhibit favorable radiation characteristics in terms of resonance, impedance, directivity, pattern, and so on, just like the other fractals.

Fractals antennas exhibit space-filling properties that can be used to miniaturize classic antenna elements, such as dipoles and loops, and overcome some of the limitations of small antennas. The line that is used to represent the fractal can meander in such a way as to effectively fill the available space, leading to curves that are electrically long but compacted into a small physical space. This is part of the fundamental limit of small antennas, discussed in Section 11.5 and represented by (11-35a) and (11-35b) (see Figure 11.17), which leads to smaller Qs/larger bandwidths. It also results in antenna elements that, although are compacted in small space, can be resonate and exhibit input resistances that are much greater than the classic geometries of dipoles, loops, etc.

To demonstrate this, let us consider two classic geometries: a circular loop and a linear dipole. Both of these are discussed in Chapters 5 and 4, respectively, and both, when they are small electrically, exhibit small input resistances and must be large in order to resonate. For an ideal circular loop, as discussed in detail in Chapter 5, the input impedance is very small (usually around 1 ohm) if the loop is electrically small. This can be overcome by using a Koch loop of Figure 11.23(b). In fact, in Figure 11.26 we display a Koch loop of four iterations, shown also in Figure 11.23(b), circumscribed by a circular loop of equal radius. In Figure 11.55, we compare the input impedance below resonance of these two geometries for a circumference ranging from 0.15λ to 0.27λ (for the circular loop) and 0.39λ to 0.7λ for the Koch loop. While the input impedance of a small circular loop is very small (around 1.33 ohms at about 0.265λ in circumference for the circular loop), the input resistance of the Koch fractal loop of equal radius (radius ≈ 0.04218λ) is 35 ohms. Both antennas are below resonance and would require matching for the reactive part.

For an ideal linear dipole, the first resonance occurs when the overall length is λ/2 (as shown in Figure 8.17), which for some frequencies can be physically large. The length can be miniaturized using fractal dipoles, such as the Koch dipole and other similar geometries. To generate the Koch dipole, we apply the iterative generating procedure using the generator of Figure 11.23(b), as shown on the top part of Figure 11.28. This procedure can be extended to generate quasi-fractal tree dipoles and 3D quasi-fractal tree dipoles, as also illustrated in the second and third parts of Figure 11.28, respectively.

In Figure 11.29, we exhibit the resonant frequency for the first five iterations of each fractal dipole of Figure 11.28. It is apparent that the higher iterative geometries exhibit lower resonant frequencies, as if the overall length of the dipole was large electrically. The resonant frequencies plotted in
The concepts discussed above for loops and dipoles can be extended to other antenna elements, including microstrip/patch antennas [55]. The reader is directed to the references.

11.8 MULTIMEDIA

In the publisher’s website for this book the following multimedia resources are included for the review, understanding, and visualization of the material of this chapter:

a. Java-based interactive questionnaire, with answers.

b. Matlab and Fortran computer program, designated log_perd, for computing and displaying the radiation characteristics of a log-periodic linear dipole array design.

c. Power Point (PPT) viewgraphs, in multicolor.

REFERENCES

Using (12-5a) and (12-5b), (3-27) and (3-28) can be written as

\[
A = \frac{\mu}{4\pi} \iint_S J_s e^{-jkR} ds' \simeq \frac{\mu e^{-jkR}}{4\pi R} N
\]  

(12-6)

\[
N = \iint_S J_s e^{jkR'} \cos \psi \ ds'
\]  

(12-6a)

\[
F = \frac{\varepsilon}{4\pi} \iint_S M_s e^{-jkR} ds' \simeq \frac{\varepsilon e^{-jkR}}{4\pi R} L
\]  

(12-7)

\[
L = \iint_S M_s e^{jkR'} \cos \psi \ ds'
\]  

(12-7a)

In Section 3.6 it was shown that in the far-field only the \( \theta \) and \( \phi \) components of the \( \mathbf{E} \)- and \( \mathbf{H} \)-fields are dominant. Although the radial components are not necessarily zero, they are negligible compared to the \( \theta \) and \( \phi \) components. Using (3-58a)–(3-59b), the \( \mathbf{E}_A \) of (3-29) and \( \mathbf{H}_F \) of (3-30) can be written as

\[
(E_A)_\theta \simeq -j\omega A_\theta
\]  

(12-8a)

\[
(E_A)_\phi \simeq -j\omega A_\phi
\]  

(12-8b)

\[
(H_F)_\theta \simeq -j\omega F_\theta
\]  

(12-8c)

\[
(H_F)_\phi \simeq -j\omega F_\phi
\]  

(12-8d)

and the \( \mathbf{E}_F \) of (3-29) and \( \mathbf{H}_A \) of (3-30), with the aid of (12-8a)–(12-8d), as

\[
(E_F)_\theta \simeq +\eta (H_F)_\phi = -j\omega F_\phi
\]  

(12-9a)

\[
(E_F)_\phi \simeq -\eta (H_F)_\theta = +j\omega F_\theta
\]  

(12-9b)

\[
(H_A)_\theta \simeq -\frac{(E_A)_\phi}{\eta} = +j\omega A_\theta
\]  

(12-9c)

\[
(H_A)_\phi \simeq +\frac{(E_A)_\theta}{\eta} = -j\omega A_\phi
\]  

(12-9d)

Combining (12-8a)–(12-8d) with (12-9a)–(12-9d), and making use of (12-6)–(12-7a) the total \( \mathbf{E} \)- and \( \mathbf{H} \)-fields can be written as

\[
E_r \simeq 0
\]  

(12-10a)

\[
E_\theta \simeq \frac{jke^{-jkR}}{4\pi R} \left( L_\phi + \eta N_\phi \right)
\]  

(12-10b)

\[
E_\phi \simeq \frac{jke^{-jkR}}{4\pi R} \left( L_\theta - \eta N_\theta \right)
\]  

(12-10c)

\[
H_r \simeq 0
\]  

(12-10d)

\[
H_\theta \simeq \frac{jke^{-jkR}}{4\pi R} \left( N_\phi + \frac{L_\theta}{\eta} \right)
\]  

(12-10e)

\[
H_\phi \simeq -\frac{jke^{-jkR}}{4\pi R} \left( N_\theta + \frac{L_\phi}{\eta} \right)
\]  

(12-10f)
This is the **wrong** Fig. 13.13, in the 4th Edition. Should be replaced, see next figure that follows.
This is, and should be, Fig. 13.13 in the 4th edition.

It was Fig. 13.15 in the 3rd Edition.
14.32. Repeat Problem 14.29 for the annular microstrip patch antenna whose geometry is shown in Figure P14.32.

14.33. Repeat Problem 14.29 for the annular sector microstrip patch antenna whose geometry is shown in Figure P14.33.

14.34. Repeat the design of Problem 14.8 for a circular microstrip patch antenna operating in the dominant $TM_{110}^z$ mode. Use $\sigma = 10^7$ S/m and $\tan \delta = 0.0018$.

14.35. Repeat the design of Problem 14.9 for a circular microstrip patch antenna operating in the dominant $TM_{110}^z$ mode. Use $\sigma = 10^7$ S/m and $\tan \delta = 0.0018$.

14.36. For ground-based cellular telephony, the desired pattern coverage is omnidirectional and similar to that of a monopole (with a null toward zenith, $\theta = 0^\circ$). This can be accomplished using a circular microstrip patch antenna operating in a higher order mode, such as the $TM_{210}^z$. Assuming the desired resonant frequency is 900 MHz, design a circular microstrip patch antenna operating in the $TM_{210}^z$ mode. Assuming a substrate with a dielectric constant of 10.2 and a height of 0.127 cm:
   (a) Derive an expression for the resonant frequency of the $TM_{210}^z$ mode;
   (b) Determine the radius of the circular patch (in cm). Neglect fringing.

14.37. Microstrip (patch) antennas are usually designed so that the maximum of the amplitude radiation pattern is perpendicular to the patch. However for ground-based cellular telephony, the pattern of the antenna should usually match that of a vertical monopole with a null towards zenith ($\theta = 0^\circ$). This can be accomplished if a circular patch is selected and it is excited at a higher-order mode, such as the $TM_{210}^z$ mode.
   Assuming a $TM_{210}^z$ mode (NOT dominant $TM_{210}^z$ mode), the desired operating frequency, without taking into account fringing, is 1.9 GHz, the substrate has a dielectric constant of 10.2 and its height is 0.127 cm. Determine the:
   (a) Physical radius of the circular patch (in cm). Neglect fringing.
TABLE 15.1  Aperture Efficiency and Field Strength at the Edge of Reflector, Relative to That at the Vertex, due to the Feed Pattern (Feed) and Path Length (Path) between the Edge and the Vertex

<table>
<thead>
<tr>
<th>n</th>
<th>ε_{ap}</th>
<th>θ₀ (deg)</th>
<th>f/d</th>
<th>Feed (dB)</th>
<th>Path (dB)</th>
<th>Total (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.829</td>
<td>66</td>
<td>0.385</td>
<td>-7.8137</td>
<td>-3.056</td>
<td>-10.87</td>
</tr>
<tr>
<td>4</td>
<td>0.8196</td>
<td>53.6</td>
<td>0.496</td>
<td>-8.8215</td>
<td>-1.959</td>
<td>-10.942</td>
</tr>
<tr>
<td>6</td>
<td>0.8171</td>
<td>46.2</td>
<td>0.5861</td>
<td>-9.4937</td>
<td>-1.439</td>
<td>-10.93</td>
</tr>
<tr>
<td>8</td>
<td>0.8161</td>
<td>41.2</td>
<td>0.6651</td>
<td>-9.7776</td>
<td>1.137</td>
<td>-10.914</td>
</tr>
</tbody>
</table>

Another parameter to examine for the patterns of (15-56), when used with reflectors that lead to optimum efficiency, is the amplitude taper or illumination of the main aperture of the reflector which is defined as the ratio of the field strength at the edge of the reflector surface to that at the vertex. The aperture illumination is a function of the feed pattern and the f/d ratio of the reflector. To obtain that, the ratio of the angular variation of the pattern toward the two points \( [G_f(\theta' = 0)/G_f(\theta' = \theta_0)] \) is multiplied by the space attenuation factor \( (r_0/f)^2 \), where \( f \) is the focal distance of the reflector and \( r_0 \) is the distance from the focal point to the edge of the reflector. For each of the patterns, the reflector edge illumination for maximum efficiency is nearly 11 dB down from that at the vertex. The details for 2 ≤ n ≤ 8 are shown in Table 15.1.

The results obtained with the idealized patterns of (15-56) should only be taken as typical, because it was assumed that

1. the field intensity for \( \theta' > 90^\circ \) was zero
2. the feed was placed at the phase center of the system
3. the patterns were symmetrical
4. there were no cross-polarized field components
5. there was no blockage
6. there were no random errors at the surface of the reflector

Each factor can have a significant effect on the efficiency, and each has received much attention which is well documented in the open literature [1].

In practice, maximum reflector efficiencies are in the 65–80% range. To demonstrate that, paraboloidal reflector efficiencies for square corrugated horns feeds were computed, and they are shown plotted in Figure 15.24. The corresponding amplitude taper and spillover efficiencies for the aperture efficiencies of Figures 15.20(a) and 15.24 are displayed, respectively, in Figures 15.20(b) and 15.25. For the data of Figures 15.24 and 15.25, each horn had aperture dimensions of 8λ × 8λ, their patterns were assumed to be symmetrical (by averaging the E- and H-planes), and they were computed using the techniques of Section 13.6. From the plotted data, it is apparent that the maximum aperture efficiency for each feed pattern is in the range of 74–79%, and that the product of the taper and spillover efficiencies is approximately equal to the total aperture efficiency.

We would be remiss if we left the discussion of this section without reporting the gain of some of the largest reflectors that exist around the world [23]. The gains are shown in Figure 15.26 and include the 1,000-ft (305-m) diameter spherical reflector [12] at Arecibo, Puerto Rico, the 100-m radio telescope [15] at Effelsberg, West Germany, the 64-m reflector [16] at Goldstone, California, the 22-m reflector at Krim, USSR, and the 12-m telescope at Kitt Peak, Arizona. The dashed portions of the curves indicate extrapolated values. For the Arecibo reflector, two curves are shown. The 215-m diameter curve is for a reduced aperture of the large reflector (305-m) for which a line feed at 1,415 MHz was designed [12].
Example 15.3
A 10-m diameter reflector, with an $f/d$ ratio of 0.5, is operating at $f = 3$ GHz. The reflector is fed with an antenna whose primary pattern is symmetrical and which can be approximated by $G_p(\theta') = 6 \cos^2 \theta'$. Find its

(a) aperture efficiency
(b) overall directivity
(c) spillover and taper efficiencies
(d) directivity when the maximum aperture phase deviation is $\pi/8$ rad

Solution: Using (15-24), half of the subtended angle of the reflector is equal to

$$\theta_0 = \tan^{-1} \left[ \frac{0.5(0.5)}{(0.5)^2 - \frac{1}{16}} \right] = 53.13^\circ$$

(a) The aperture efficiency is obtained using (15-59a). Thus

$$\epsilon_{ap} = 24 \left( \sin^2(26.57^\circ) + \ln[\cos(26.57^\circ)] \right) \cot^2(26.57^\circ)$$

$$= 0.75 = 75\%$$

which agrees with the data of Figure 15.20.

(b) The overall directivity is obtained by (15-54), or

$$D = 0.75[\pi(100)]^2 = 74.022.03 = 48.69 \text{ dB}$$

(c) The spillover efficiency is computed using (15-61) where the upper limit of the integral in the denominator has been replaced by $\pi/2$. Thus

$$\epsilon_s = \int_0^{53.13^\circ} \frac{\cos^2 \theta' \sin \theta' d\theta'}{\int_0^{90^\circ} \cos^2 \theta' \sin \theta' d\theta'} = \frac{2 \cos^3 \theta' |_{0}^{53.13^\circ}}{2 \cos^3 \theta' |_{0}^{90^\circ}} = 0.784 = 78.4\%$$

In a similar manner, the taper efficiency is computed using (15-62). Since the numerator in (15-62) is identical in form to the aperture efficiency of (15-55), the taper efficiency can be found by multiplying (15-59a) by 2 and dividing by the denominator of (15-62). Thus

$$\epsilon_t = \frac{2(0.75)}{1.568} = 0.9566 = 95.66\%$$

The product of $\epsilon_s$ and $\epsilon_t$ is equal to

$$\epsilon_s \epsilon_t = 0.784(0.9566) = 0.75$$

and it is identical to the total aperture efficiency computed above.
or

\[ \theta = \cos^{-1}\left( \frac{d}{v_0 \Delta t} \right) = \cos^{-1}\left( \frac{d}{v_0 (t_1 - t_2)} \right) \]

This clearly demonstrates that the angle of incidence \( \theta \) (direction of arrival) can be determined knowing the time delay between the two elements \( \Delta t = t_1 - t_2 \), and the geometry of the antenna array (in this case a linear array of two elements with a spacing \( d \) between the elements).

Figure 16.24  Incoming signal on a two-element array.

16.8.2 Adaptive Beamforming

As depicted in Figure 16.10, the information supplied by the DOA algorithm is processed by means of an adaptive algorithm to ideally steer the maximum radiation of the antenna pattern toward the SOI and place nulls in the pattern toward the SNOIs. This is only necessary for DOA-based adaptive beamforming algorithms. However, for reference (or training) based adaptive beamforming algorithms, like the Least Mean Square (LMS) [51], [52] that is used in this chapter, the adaptive beamforming algorithm does not need the DOA information but instead uses the reference signal, or training sequence, to adjust the magnitudes and phases of each weight to match the time delays created by the impinging signals into the array. In essence, this requires solving a linear system of normal equations. The main reason why it is generally undesirable to solve the normal equations directly is because the signal environment is constantly changing. Before reviewing the most common adaptive algorithm used in smart antennas, an example is given, based on [53], to illustrate the basic concept of how the weights are computed to satisfy certain requirements of the pattern, especially the formation of nulls.

Example 16.2

Determine the complex weights of a two-element linear array, half-wavelength apart, to receive a desired signal of certain magnitude (unity) at \( \theta_0 = 0^\circ \) while tuning out an interferer (SNOI) at \( \theta_1 = 30^\circ \), as shown in Figure 16.25. The elements of the array in Figure 16.25 are assumed to be, for simplicity, isotropic and the impinging signals are sinusoids.