Correction for Exercise 7.2

7.2. (Ajax Lights)

Ajax Lights plc sells LED lights and believes that the demand and price are connected linearly. Previously the price was $10 per bulb and total sales were 50,000 per month. There has been a technical advance making the cost to produce these bulbs cheaper at $5 per bulb. There is a 3 month time horizon before the market changes radically with a new supplier entering. The firm imports and packs the bulbs but these are obtained from a supplier, who will supply a fixed amount per month with a one month lead time. Ajax has 100,000 LED lights in stock, and has already set its price at $8 per bulb for the next month. The first decision will be the total amount to be ordered for use during the period of months 2 and 3 which we call $Y$, and this decision must be made straight away. After discovering the sales in month 1 Ajax will then deduce the two parameters of the demand function (intercept and slope) and set its price for months 2 and 3 so that its entire stock (purchased amount plus remaining stock) is used up by the end of month 3.

Writing $Y$ for the amount ordered and $S$ for the uncertain demand in month 1, show that the problem can be formulated as

$$
\max_Y (-500,000 - 5Y + ES(Q(Y, S)))
$$

where

$$
Q(Y, S) = (100,000 - S + Y) \frac{11S - 500,000 - Y}{S - 50,000} + 8S.
$$

Hence show that Ajax should make an order

$$
Y = \frac{7}{2} (S - 50,000)
$$

if $S$ is known in advance. Find an expression for the right order size if $S$ takes the value $S_1$ with probability $q$ and $S_2$ with probability $1 - q$.

Answer:

The firm believes that the monthly demand is $K - \alpha p$ where $p$ is the price per bulb. So $K - 10\alpha = 50,000$. And if $S$ are sold next month at $8$ then this will demonstrate that $K - 8\alpha = S$. Solving these two equations gives $\alpha = (S/2) - 25,000$ and $K = 5S - 200,000$. 

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Given \( Y \) and sales of \( S \) then the amount left to sell after 1 month is \( 100\,000 - S + Y \). The sales per month need to be half this and must match the demand per month. Thus (using the formulae for \( K \) and \( \alpha \) that we have derived)

\[ 50\,000 - \frac{(S/2) + (Y/2)}{2} = (5S - 200\,000) - ((S/2) - 25\,000)p. \]

Thus

\[ ((S/2) - 25\,000)p = \frac{11S - 500\,000 - Y}{2}, \]

and hence

\[ p = \frac{11S - 500\,000 - Y}{S - 50\,000}. \]

The stage 1 costs of the order (allowing for the previous cost of the existing stock assumed at $5 per bulb) are

\[ C_1(Y) = 500\,000 + 5Y. \]

Then write \( Q(Y, S) \) for the total revenue arising from all sales (price times volume), so

\[ Q(Y, S) = (100\,000 - S + Y) \frac{11S - 500\,000 - Y}{S - 50\,000} + 8S. \]

Now suppose that \( S = S_i \) with probability \( q_i \) (where \( \sum_{i=1}^{N} q_i = 1 \)) then the expected profit to be maximized is

\[ \Pi(Y) = -C_1(Y) + \sum_{i=1}^{N} q_i Q(Y, S_i). \]

We start with the general case. Taking derivatives, the optimal \( Y \) is given by

\[ -5 + \sum_{i=1}^{N} \frac{q_i}{S_i - 50\,000} \left[-(100\,000 - S_i + Y) + 11S_i - 500\,000 - Y\right] = 0. \]

Using the fact that \( \sum_{i=1}^{N} q_i = 1 \), this simplifies to

\[ Y \sum_{i=1}^{N} q_i \frac{1}{S_i - 50\,000} = \frac{7}{2}. \]

When \( N = 2 \) and (in the notation of the question) we set \( q_1 = q \) and \( q_2 = 1 - q \) we obtain

\[ Y = \frac{7}{2} \left( \frac{q}{S_1 - 50\,000} + \frac{1 - q}{S_2 - 50\,000} \right)^{-1}. \]

Setting \( q = 1 \) and \( S_1 = S \) for the case without uncertainty, this becomes

\[ Y = \frac{7}{2} (S - 50\,000). \]