Supplemental Document 1 - Single-Asset P&L Distribution

Sections / Topics Covered

- Focus on building and then using the P&L distribution for a single asset
- Based around Figure 8.7 from chapter 8 (p. 209)
- Builds P&L distribution by variance-covariance (VCV) or delta-normal method
- Builds P&L distribution by combining security DV01 and Risk Factor distribution
- There may be a single or multiple RFs. For multiple RFs the separate RF distributions must be combined into the overall P&L. The details of how RF distributions are combined into an overall portfolio distribution is discussed in the Document 2
- For using the P&L distribution we focus on the volatility and VaR.
- Focus on the VaR as a tail measure
- Talk about the problem of estimating and using the tails of a distribution
  - We can see it clearly by looking at the distribution - most of the mass is not in the tail, so most of the history is not relevant when focusing on the tails
  - Discuss alternative functional forms - Student t-distribution and mixture of normals.
  - The central part of the distributions are roughly similar. Only far out in the tails do the differences appear and become important.

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Parameters, Initialization, Functions

Estimating Portfolio P&L Distribution

Estimating the P&L distribution, whether for a single security or a complete portfolio, conforms to the following steps in one way or another.

1. Asset / Risk Factor Mapping - Calculate transformation from individual assets to risk factors
2. Risk Factor Distributions - Estimate the range of possible levels and changes in market risk factors
3. Generate P&L Distribution - Generate risk factor P&L and sum to produce the portfolio P&L distribution
4. Calculate Risk Measures - Estimate the VaR, volatility, or other desired characteristics of the P&L distribution

In this document we will focus on the simplest case - estimating the P&L distribution for a single security.

Graphical Representation of Building P&L Distribution

Figure 1 lays out the four steps graphically. It builds on Figure 8.7 in section 8.3 of the text but for now we focus on a single asset, $20 million of a 10-year U.S. Treasury. Figure 1 shows parametric (delta-normal) estimation of the portfolio P&L distribution.

Figure 1: Methodology for Estimating Characteristics of the P&L Distribution
In detail, the steps in Figure 1 are:

1. **Asset / Risk Factor Mapping** - Transform from the asset holdings to the risk factor delta-equivalents. For the 10-year Treasury bond the risk factor is the 10-year bond yield - in this case the transformation is a simple one-to-one transformation from bond price to yield. For the delta-normal approach we use the DV01 or delta (also called the dollar duration). The bond itself is the 3.75% of 15-Nov-2018. With a settle date of 27-Jan-2009 and a yield of 2.53% the DV01 is $914.59 per 1bp in yield for a $1 million position and so $18,291.9 per 1bp for a $20 million position.

2. **Risk Factor Distributions** - We assume that this is normally-distributed. Figure 1 shows the volatility. For the US 10-year yield the volatility is 7.148 bp/day. We are not concerned for now with how we estimate the risk factor distributions. They are estimated from history, but for now we take these as given. The current focus is on how we combine historical risk factor information with current portfolio positions to generate the estimated portfolio P&L distribution.

3. **Generate P&L Distribution** - this entails combining the market risk factor distribution with the position delta to produce a P&L distribution for each risk factor. When there are multiple risk factors then these separate distributions are combined into the security P&L distribution. (For the US Treasury bond there is only one risk factor. Other securities, such as the 10-year UK Gilt, will have multiple risk factors.)

For the parametric or delta-normal approach the asset P&L is assumed to depend linearly on the risk factor. So for the 10-year Treasury:

\[
P & L(\text{10 year Treasury}) = \text{sensitivity} \times \text{change in yield} = 18291.9 \times \Delta \text{yield}
\]

Each risk factor is assumed normally distributed, and since a linear function of a normal variable is still normal, the P&L for the 10-year Treasury will be normal with volatility given by:

\[
\text{Volatility(10 year Treasury)} = 18291.9 \times 7.148 = 130750
\]

For this case we only have a single risk factor and so the risk factor P&L is the security P&L; we do not have multiple risk factors to combine into a single overall P&L. As we will see shortly there could be multiple risk factors for a single security and then we would have
to combine them. But we will defer discussion of how we combine separate distributions into one overall P&L distribution until supplementary document 2.

4. Calculate Risk Measures - The portfolio P&L will be normal with a known volatility. For the single risk factor here (10-year US yields) the volatility is $130,800. Now calculating risk measures is simple. We have the volatility, and calculating the VaR is a matter of looking up the appropriate multiplicative factor from a table for the normal distribution. For the 5%/95% VaR the factor is roughly 1.645.

P&L Distribution and Risk Measures - Volatility and VaR

I have stressed repeatedly that, in measuring risk, we are interested in the P&L distribution. When we know the P&L distribution we know virtually everything. But as a practical matter we look at “risk measures” such as the volatility or VaR. These are simply summary measures that tell us the dispersion of the distribution, because it is the dispersion of possible P&L outcomes that matters, how much we might gain or lose.

Figure 2 shows the P&L distribution from step 4 of Figure 1, drawn on its own and showing the volatility and VaR. The volatility (also known as the standard deviation) is the most commonly-used measure for dispersion of a distribution. It is extremely useful.

**Figure 2: P&L Distribution for $20 million of 10-year US Treasury, Showing Volatility and VaR**

**Interactive Section**

Figure 1 shows the steps for estimating the P&L for $20 million of the 10-year Treasury. Figure 3 extends this to allow the user to choose both the amount (in local currency) and the security itself, out of the following list:

- 5yr UST
- 10yr UST
- 5yr Bond Opt
- 10yr Swap
- 5yr UKG
- 10yr UKG
- CAC Index Futures
- FTE CDS
- 5yr Bund
- 10yr Bund
- FTE Equity
- GBP Cash
- EUR Cash

The portfolio is assumed dollar-based so that the US securities (the bonds, option, and swap) have no currency exposure. The 5-year and 10-year US Treasuries have a single risk factor - the underlying yield. Turning to the 5-year UK Gilt, however, we see that there are two risk factors - the yield and the FX rate. The 10-year UK Gilt will now have two deltas or sensitivities - first the DV01 with respect to the 10-year UK yield (GBP Yield 10yr) and then the amount of the bond holding. The yield sensitivity or DV01 is $22,843.8 per bp. (The DV01 is roughly £8.16 per 100bp change in yields for £100 nominal of the bond. For £20 million pounds nominal this gives £16,300 for 1bp change in yield, or $22,800 for 1bp change in yields.) The bond holding is £20 million nominal, roughly £22.4 million market price, which translates to roughly £31.4 million. Figure 3 shows these figures when the security is changed to the 10-year UK Gilt and the amount set at 20 (£20 million).

Figure 3 then shows the risk factor volatilities. These are combined with the risk factor sensitivities or deltas to give the position volatilities and thus (since we are assuming normality) the risk factor P&L distributions. These are combined (as discussed in document 2) to give the overall security P&L.

**Figure 3: Interactive Calculation of the P&L Distribution**
1. ASSET / RISK FACTOR MAPPING

SECURITY 10yr UKG £20mn

Security Risk Factors {GBPYld10yr, GBPFX}
Delta [22,843.8, 3.1383 × 10^7]

2. RISK FACTOR DIST’NS

Portfolio Risk Factors
GBPYld10yr
GBPFX

RF Volatility 6.56975 0.00811012

3. GENERATE P&L DIST’NS and COMBINE THEM

Risk Factor Delta 22,843.8 3.1383 × 10^7
Pos’n Volatility 150,100 254,500

4. CALCULATE RISK MEASURES

OVERAL PORTFOLIO 5%/95% VaR−527,200 Volatility 20,500

The overall security P&L tells us how likely we are to see large gains or large losses. We will almost always summarize the distribution by the volatility or the VaR. These are shown in Figure 3, and expanded in Figure 4. In Figure 4 we can choose not only the security and amount, but also the probability level for the VaR and the assumed functional form for the overall portfolio distribution. Figure 4 shows two sets of graphs. The first is shown on an absolute scale so that changing the amount visibly changes the spread of the distribution. The second is drawn to fill the space - the graph does not appear to change but note that the values for the volatility and VaR do change when the security or amount are altered.

Figure 4: Interactive Calculation of the P&L Distribution
There are a few things we can examine by manipulating Figure 4. First, remember that the probability level for the VaR is chosen by the user. It is set initially at 0.05 (5%) so that there is a 5 percent probability that losses will be worse and a 95% chance losses will be better than the 5%/95% VaR. (For $20 million of the 10-year US Treasury the 5%/95% VaR is $215,100 (Figure 2) while for £20 million of the 10-year UK Gilt the 5%/95% VaR is $376,600.) A 5 percent probability level is not very low as far as these things go, yet we see that this already takes us reasonably far out in the tail. Changing the probability level will move the VaR - higher probability moves it to the right and lower to the left.

A probability level of 16 percent (16%/84% VaR) is close to the volatility (for the normal distribution). In this sense the volatility can be thought
of as the 16%/84% VaR. For the Student t-distribution (6 degrees of freedom) the probability is closer to 14 percent, while for the mixture of normals (1.25 percent probability of a high-vol regime with volatility 2.5-times higher) the probability is roughly 15%.

A probability level of 1 percent (1%/99% VaR) is pretty far to the left. When we focus on the 1%/99% VaR we are ignoring most of the distribution. This is appropriate in one sense since when using VaR we are invariably focusing on large losses (and this ignoring the bulk of standard trading days). But we do have to remember that we are ignoring most of the distribution and thus ignoring most of the history or other information we have from trading and operations. The consequence is that we will not be able to estimate the 1%/99% VaR very accurately. This is unavoidable - focusing on extreme events means we are doing exactly that - focusing on rare and unusual events. We cannot hope to have very much confidence in any VaR estimate for small probability values.