Chapter 1

The Basics

Collections of Numbers

It is important to feel comfortable with some terms, symbols, and operations before you review basic math and pre-algebra. Basic math involves many different collections of numbers. Understanding these will make it easier for you to understand basic math.

- ❑ **Natural or counting numbers:** 1, 2, 3, 4, 5, ...
- ❑ **Whole numbers:** 0, 1, 2, 3, 4, ...
- ❑ **Odd numbers:** Whole numbers not divisible by 2: 1, 3, 5, 7, 9, ...
- ❑ **Even numbers:** Whole numbers divisible by 2: 0, 2, 4, 6, 8, ...
- ❑ **Integers:** ... −2, −1, 0, 1, 2, ...
- ❑ **Negative integers:** ... −5, −4, −3, −2, −1
- ❑ **Rational numbers:** All fractions, such as: $-\frac{5}{6}, -\frac{7}{3}, \frac{3}{4}, \frac{9}{8}$. Every integer is a rational number (for example, the integer 2 can be written as $\frac{2}{1}$). All rational numbers can be written in the general form $\frac{a}{b}$, where a can be any integer and b can be any natural number. Rational numbers also include repeating decimals (such as 0.66...) and terminating decimals (such as 0.4), because they can be written in fraction form.
- ❑ **Irrational numbers:** Numbers that cannot be written as fractions, such as $\sqrt{2}$ and π (the Greek letter pi).

Ways to Show Multiplication and Division

We assume that you know the basics of multiplying and dividing single-digit numbers and have some idea of what these operations mean (multiplying 5 by 3 means if you had five groups of three things, you would have 15 things altogether; dividing 15 by 5 means if you had 15 things and divided them equally among 5 people, each person would get 3 things).

You can write these operations in many different ways, and it is important to recognize these variations. Some of the ways of indicating the multiplication of two numbers include the following:

- ❑ **Multiplication sign:** $2 \times 3 = 6$
- ❑ **Multiplication dot:** $2 \cdot 3 = 6$
- ❑ **An asterisk** (especially with computers and calculators): $2 * 3 = 6$

- ❑ **One set of parentheses:** 2(3) = 6 or (2)3 = 6
- ❑ **Two sets of parentheses:** (2)(3) = 6
- ❑ **A variable** (letter) next to a number: 2*a* means 2 times *a*.
- ❑ **Two variables** (letters) next to each other: *ab* means *a* times *b*.

Some of the ways of indicating the division of two numbers include the following:

- ❑ **Division sign:** $6 \div 3 = 2$
- ❑ **Fraction bar:** 6/3 = 2, $^6\!/_3 = 2$, or $\frac{6}{3} = 2$

Multiplying and Dividing Using Zero

Any number multiplied by zero equals zero (because several groups each having zero items in them means a total of zero items).

$$0 \times 3 = 0$$
$$8 * 9 * 0 * 4 = 0$$
$$(0)(10) = 0$$
$$2a \cdot 0 = 0$$

Zero divided by any number except zero is zero (if you have nothing, dividing it among several people still leaves nothing for each person).

$$0 \div 5 = 0$$
$$0/3 = 0$$

Any number divided by zero is undefined (if you have several things, it doesn't make any sense to divide them among zero people).

$$4 \div 0 \text{ is undefined}$$
$$\frac{0}{0} \text{ is undefined}$$

Symbols and Terminology

The following are commonly used symbols in basic math and algebra. It is important to know what each symbol represents.

$=$ is equal to
$\neq$ is not equal to
$<$ is less than
$\nless$ is not less than
$>$ is greater than
$\ngtr$ is not greater than
$\leq$ is less than or equal to
$\nleq$ is not less than or equal to
$\geq$ is greater than or equal to
$\ngeq$ is not greater than or equal to
$\approx$ is approximately equal to

Some Fundamental Properties

The four operations of addition, subtraction, multiplication, and division follow certain rules. Knowing these rules, and having names for them, is important before moving on.

Some Properties (Axioms) of Addition

- **Closure** is when all answers fall into the original set. When two even numbers are added, the answer will be an even number ($4 + 6 = 10$); thus, the set of even numbers has closure under addition. When two odd numbers are added, the answer is not an odd number ($1 + 3 = 4$); therefore, the set of odd numbers does not have closure under addition.
- The **commutative property of addition** means that the order of the numbers added does not matter.

$$3 + 4 = 4 + 3$$
$$7 + 10 = 10 + 7$$
$$a + b = b + a$$

Note: The commutative property does not hold true for subtraction.

$$5 - 3 \neq 3 - 5$$
$$a - b \neq b - a$$

- The **associative property of addition** means that the way the numbers are grouped does not matter.

$$(1 + 2) + 3 = 1 + (2 + 3)$$
$$(3 + 5) + 7 = 3 + (5 + 7)$$
$$a + (b + c) = (a + b) + c$$

The parentheses have moved, but both sides of the equation are still equal.

Note: The associative property does not hold true for subtraction.

$$(7 - 5) - 3 \neq 7 - (5 - 3)$$
$$(3 - 2) - 1 \neq 3 - (2 - 1)$$
$$(a - b) - c \neq a - (b - c)$$

- The **identity element** for addition is 0. When 0 is added to any number, it gives the original number.

$$5 + 0 = 5$$
$$0 + 3 = 3$$
$$a + 0 = a$$

- The **additive inverse** is the opposite or negative of the number. The sum of any number and its additive inverse is 0 (the identity).

$2 + (-2) = 0$; thus, 2 and -2 are additive inverses.

$\frac{5}{3} + \frac{-5}{3} = 0$; thus, $\frac{5}{3}$ and $-\frac{5}{3}$ are additive inverses.

$b + (-b) = 0$; thus, b and $-b$ are additive inverses.

Some Properties (Axioms) of Multiplication

- **Closure** is when all answers fall into the original set. When two even numbers are multiplied, they produce an even number ($2 \times 4 = 8$); thus, the set of even numbers has closure under multiplication. When two odd numbers are multiplied, the answer is an odd number ($3 \times 5 = 15$); thus, the set of odd numbers has closure under multiplication.
- **The commutative property of multiplication** means that the order of the numbers multiplied does not matter.

$$2 \times 3 = 3 \times 2$$
$$5(6) = (6)5$$
$$ab = ba$$

Note: The commutative property does not hold true for division.

$$6 \div 3 \neq 3 \div 6$$

- **The associative property of multiplication** means that the way the numbers are grouped does not matter.

$$(3 \times 4) \times 5 = 3 \times (4 \times 5)$$
$$(5 \times 7) \times 9 = 5 \times (7 \times 9)$$
$$(a \times b) \times c = a \times (b \times c)$$

The parentheses have moved, but both sides of the equation are still equal.

Note: The associative property does not hold true for division.

$$(12 \div 2) \div 3 \neq 12 \div (2 \div 3)$$

- The **identity element** for multiplication is 1. Any number multiplied by 1 gives the original number.

$$6 \times 1 = 6$$
$$9 \times 1 = 9$$
$$b \times 1 = b$$

- The **multiplicative inverse** is the **reciprocal** of the number, which means one divided by the number. When a number is multiplied by its reciprocal, the answer is 1.

$3 \times \frac{1}{3} = 1$; thus, 3 and $\frac{1}{3}$ are multiplicative inverses. $\frac{2}{3} \times \frac{3}{2} = 1$; thus, $\frac{2}{3}$ and $\frac{3}{2}$ are multiplicative inverses.

$a \times \frac{1}{a} = 1$; thus, a and $\frac{1}{a}$ are multiplicative inverses, as long as $a \neq 0$.

A Property of Two Operations

The **distributive property** is when the number on the outside of the parentheses is distributed to each term on the inside of the parentheses. Multiplication distributes over addition or subtraction.

$$3(4 + 5) = 3(4) + 3(5)$$
$$6(4 - 2) = 6(4) - 6(2)$$
$$a(b + c) = a(b) + a(c)$$

Note: The distributive property cannot be used with other combinations of operations.

$$2(3 \times 4 \times 5) \neq 2(3) \times 2(4) \times 2(5)$$
$$7 + (2 - 1) \neq (7 + 2) - (7 + 1)$$

Grouping Symbols and Order of Operations

Numbers or variables commonly are grouped using three types of symbols: parentheses ( ), brackets [ ], and braces { }. Of these three, parentheses are used most often. Operations inside grouping symbols are the first to be worked out; they must be performed before all other operations.

Example Problems

These problems show the answers and solutions.

1. Simplify $2(3 + 4)$.

   answer: 14

   $2(3 + 4) = 2(7) = 14$

2. Simplify $4(5 - 3)$.

   answer: 8

   $4(5 - 3) = 4(2) = 8$

3. Simplify $(1 + 4)(2 + 3)$.

   answer: 25

   $(1 + 4)(2+ 3) = (5)(5) = 25$

4. Simplify $(9 - 7)(8 - 4)$.

   answer: 8

   $(9 - 7)(8 - 4) = (2)(4) = 8$

   Brackets and braces are used less often than parentheses. In the order of operations, parentheses should be used first, then brackets, and then braces: { [( )] }. Larger parentheses sometimes are used in place of brackets and braces.

5. Simplify $[(5 - 3) \times 7]$.

   answer: 14

   We work from the inside out:

$$\begin{aligned}[(5 - 3) \times 7] &= [(2) \times 7] \\ &= 14\end{aligned}$$

6. Simplify 3{4 + [2(1 + 3) + 5]}.

answer: 51

$$\begin{aligned} 3\{4 + [2(1 + 3) + 5]\} &= 3\{4 + [2(4) + 5]\} \\ &= 3\{4 + [8 + 5]\} \\ &= 3\{4 + [13]\} \\ &= 3\{17\} \\ &= 51 \end{aligned}$$

7. Simplify $2\Big(4\big(2(5-3)\big)+4\Big)$.

answer: 40

$$\begin{aligned} 2(4(2(5 - 3))+4) &= 2(4(2(2)) + 4) \\ &= 2(4(4) + 4) \\ &= 2(16 + 4) \\ &= 2(20) \\ &= 40 \end{aligned}$$

8. Simplify 2{10 − [3(1 + 4) − 6]}.

answer: 2

$$\begin{aligned} 2\{10 - [3(1 + 4) - 6]\} &= 2\{10 - [3(5) - 6]\} \\ &= 2\{10 - [15 - 6]\} \\ &= 2\{10 - [9]\} \\ &= 2\{1\} \\ &= 2 \end{aligned}$$

Order of Operations

The order of operations is important if multiplication, division, exponents, addition, subtraction, parentheses, and so on, are all in the same problem. The order in which these operations should be carried out is as follows

1. Parentheses
2. Exponents
3. Multiplication and division, from left to right
4. Addition and subtraction, from left to right

Example Problems

These problems show the answers and solutions.

1. Simplify $3 \times 4 + 2$.

answer: 14

First do the multiplication:

$$3 \times 4 + 2 = 12 + 2$$

Then the addition:

$$12 + 2 = 14$$

2. Simplify $7 - 4 \div 2$.

answer: 5

Division comes first:

$$7 - 4 \div 2 = 7 - 2$$

Then the subtraction:

$$7 - 2 = 5$$

3. Simplify $12 - 4 + 2 \times 10^2 + (5 + 3) - 3$.

answer: 213

First do the operation inside the parentheses:

$$12 - 4 + 2 \times 10^2 + (5 + 3) - 3 = 12 - 4 + 2 \times 10^2 + (8) - 3$$

Then apply the exponent:

$$12 - 4 + 2 \times 10^2 + 8 - 3 = 12 - 4 + 2 \times 100 + 8 - 3$$

Next multiply:

$$12 - 4 + 2 \times 100 + 8 - 3 = 12 - 4 + 200 + 8 - 3$$

Finally, carry out the addition and subtraction starting on the left:

$$\begin{aligned} 12 - 4 + 200 + 8 - 3 &= 8 + 200 + 8 - 3 \\ &= 208 + 8 - 3 \\ &= 216 - 3 \\ &= 213 \end{aligned}$$

4. Simplify $100 - 2[1 + 5(3 + 2^2)]$.

answer: 28

First do the work inside the parentheses, but because there is an exponent inside the parentheses, you must apply the exponent before adding what is inside the parentheses:

$$\begin{aligned} 100 - 2[1 + 5(3 + 2^2)] &= 100 - 2[1 + 5(3 + 4)] \\ &= 100 - 2[1 + 5(7)] \end{aligned}$$

Then do the multiplication inside the brackets:

$$100 - 2[1 + 5(7)] = 100 - 2[1 + 35]$$

Next finish the operation inside the brackets:

$$100 - 2[1 + 35] = 100 - 2[36]$$

Then do the multiplication:

$$= 100 - 72$$

And, finally, finish with the subtraction:

$$100 - 72 = 28$$

Note: A handy phrase some people like for remembering the order of operations is: **P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally, where the first letters are reminders to work in the order **P**arentheses, **E**xponents, **M**ultiplication/**D**ivision, **A**ddition/**S**ubtraction).