1

Introduction

1.1 Wells and boreholes

Water wells have been a source of water for people, animals and crops since the earliest civilizations in Africa and Asia. In the Bible and Koran, for example, wells and springs feature prominently, sometimes as places for meeting and talking and often as metaphors for paradise. Since the first millennium BC, horizontal wells or *qanats* have been widely used for water supply and irrigation in the Middle East and western Asia, notably Iran, and continue to be used today (Figure 1.1). In Europe, the development of many towns and cities in the middle ages and on through the industrial period was aided considerably by the abstraction of relatively pure water supplies from wells and springs (Figure 1.2).

Wells continue to have an important role in society today. Over half the public water supplies in European Union countries come from groundwater, ranging from between 20 % and 30 % of drinking water supplied in Spain and the United Kingdom, to nearly 100 % in Austria, Lithuania and Denmark (Hiscock *et al*., 2002). The last 20 years have witnessed a huge increase in the use of wells for agricultural irrigation, especially in Asia (Figure 1.3): in India 53 % of irrigation water is supplied from groundwater while this proportion rises to 98 % in Saudi Arabia (Foster *et al*., 2000). In the USA groundwater pumping increased by 23 % between 1970 and 2000, with about 70 % of the daily withdrawal of 315 million cubic metres in 2000 being used for irrigated agriculture (McCray, 2004). There are also about 350 000 new wells constructed each year for domestic supplies in the USA.

Other uses of wells are many and diverse and include livestock watering (Figure 1.4), industrial supplies, geothermal or ground-source energy (Figure 1.5), construction dewatering, brine mining, water injection to oil reservoirs, aquifer clean up, river support and artificial recharge of aquifers. Wells and boreholes are also used extensively for monitoring water levels and groundwater quality.

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Figure 1.1 *Open section of falaj (qanat) running through a village in northern Oman. Photo by Bruce Misstear*

Figure 1.2 *Hand-dug well with ornamental canopy, Prague, Czech Republic. Photo by Bruce Misstear*

Figure 1.3 *A dual purpose irrigation and drainage well in the Indus valley, Pakistan. In this 'scavenger well' the outlet pipe in the foreground of the picture is discharging fresh groundwater from the upper part of the well, whereas the pipe to the right is discharging saline water from the lower section of the well, thus preventing the saline water from moving upwards and contaminating the good quality water. The good quality water is used for irrigation whilst the saline water is diverted to the drainage system. Photo by Bruce Misstear*

Figure 1.4 *Drilled well fitted with a windmill pump used for livestock watering, New South Wales, Australia. Photo by Bruce Misstear*

Figure 1.5 *Drilling rig being set up for constructing a well in a gravel aquifer used as a source of geothermal energy, Dublin, Ireland. Photo by Bruce Misstear*

Wells have long had a religious significance in many societies. Holy wells are an important feature of local culture throughout the Celtic lands in western Europe, for example, where there may be as many as 3000 holy wells in Ireland alone (Logan, 1980; Robins and Misstear, 2000). Many of these wells are still visited regularly and votive offerings such as rags, statues and coins are common (see Box 3.6 in Chapter 3).

Water wells have also been a source of conflict since Biblical times:

> *But when Isaac's servants dug in the valley and found there a well of springing water, the herdsmen of Gerar quarrelled with Isaac's herdsmen, saying 'This water is ours'.*
>
> Genesis 6, 19–20

They remain so today. A major point of contention in the Middle East is the control of the groundwater resources in the region.

Water wells come in many forms, orientations and sizes. Traditionally most water wells were excavated by hand as shallow, large diameter, shafts; nowadays, the majority are constructed from relatively small diameter boreholes drilled by machine, sometimes to great depths. Water wells are typically vertical but can be horizontal (infiltration gallery), a combination of vertical and horizontal well (radial collector well), or occasionally inclined (Figure 1.6). The water may be abstracted by hand-operated or motorized pumps, or it may flow to the surface naturally under positive upward pressure (artesian well; Figure 1.7) or by gravity drainage (*qanat* or *falaj*). This book deals mainly

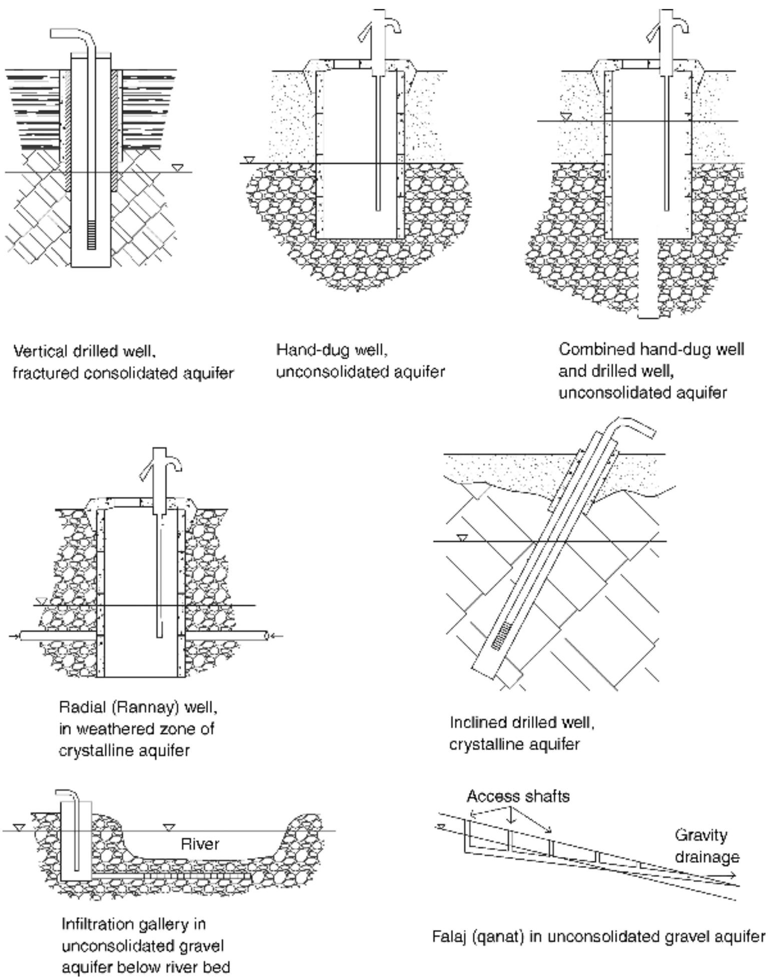

Figure 1.6 *Examples of different types of water well*

Figure 1.7 *Flowing artesian well, northern Myanmar. The well was drilled into a strongly confined sandstone aquifer. Children are enjoying the 'swimming pool' created by the discharge until such time as the well is capped. Photo by Bruce Misstear*

with drilled wells, since readers are likely to encounter these most often, but other types of wells and boreholes are also covered.

Water well terminology is not standard throughout the world, and different names are commonly applied to identical constructions. The terms used in this book are explained in Box 1.1. Further details of the different types of wells and boreholes, and their component parts, are included in Chapter 3.

Box 1.1 Well and borehole terminology

Water well	Any hole excavated in the ground that can be used to obtain a water supply
Drilled well	A water well constructed by drilling. Synonyms are tube-well, production well or production borehole. As drilled wells are the main focus of this book they will be referred to as wells for simplicity. Other types of water well will be distinguished, where necessary, using the terminology below
Hand-dug well	A large-diameter, usually shallow, water well constructed by manual labour. Synonyms are dug well or open well
Exploratory borehole	A borehole drilled for the specific purpose of obtaining information about the subsurface geology or groundwater. Synonyms are investigation borehole, exploration borehole or pilot borehole
Observation borehole	A borehole constructed to obtain information on variations in groundwater level or water quality. Also known as observation well
Piezometer	A small diameter borehole or tube constructed for the measurement of hydraulic head at a specific depth in an aquifer. In a piezometer, the section of the borehole (the screened section) in contact with the aquifer is usually very short
Test well	A borehole drilled to test an aquifer by means of pumping tests
Infiltration gallery	A shallow horizontal well usually constructed in the bed of a river or along a river bank in an alluvial aquifer
Radial collector well	A large diameter well with horizontal boreholes extending radially outwards into the aquifer. Also known as a Ranney well
Qanat	An infiltration gallery in which the water flows to the point of abstraction under gravity. There are many synonyms, including *falaj* (Oman), *karez* (Afghanistan) and *kariz* (Azerbaijan)

1.2 Groundwater occurrence

The remainder of this chapter provides the nonspecialist reader with a brief introduction to the occurrence of groundwater and the principles of groundwater flow, including radial flow to water wells. For a more comprehensive coverage of these topics the reader is referred to standard hydrogeology texts (Freeze and Cherry, 1979; Driscoll, 1986; Domenico and Schwartz, 1998; Fetter, 2001; Todd and Mays, 2004).

1.2.1 Aquifers, aquicludes and aquitards

Figure 1.8 illustrates some of the basic terminology used to describe groundwater and aquifers. While some authorities define *groundwater* as any water occurring in the subsurface – that is, water occurring in both the *unsaturated* and the *saturated* zones – we follow the tradition of defining groundwater as that portion of water in the subsurface that occurs in the saturated zone. A geological formation that is able to store and transmit groundwater in useful quantities is called an *aquifer*. Aquifer is thus a relative term, since a low permeability geological formation that would not be considered as an aquifer capable of meeting public water supply or irrigation water demands, may be able to supply 'useful quantities' of groundwater to a village or domestic well in regions where water is otherwise scarce. In this context, one can argue, for example, that low-permeability mudstones in parts of Africa are hugely valuable aquifers (MacDonald, 2003).

Aquifers are often described according to their water level or pressure head conditions (see Boxes 1.2 and 1.3 for explanations of groundwater head). An aquifer is said to be *unconfined* where its upper boundary consists of a free groundwater surface at which the pressure equals atmospheric. This free surface is known as the *water table* and unconfined aquifers are sometimes known as *water-table aquifers*. An aquifer is said

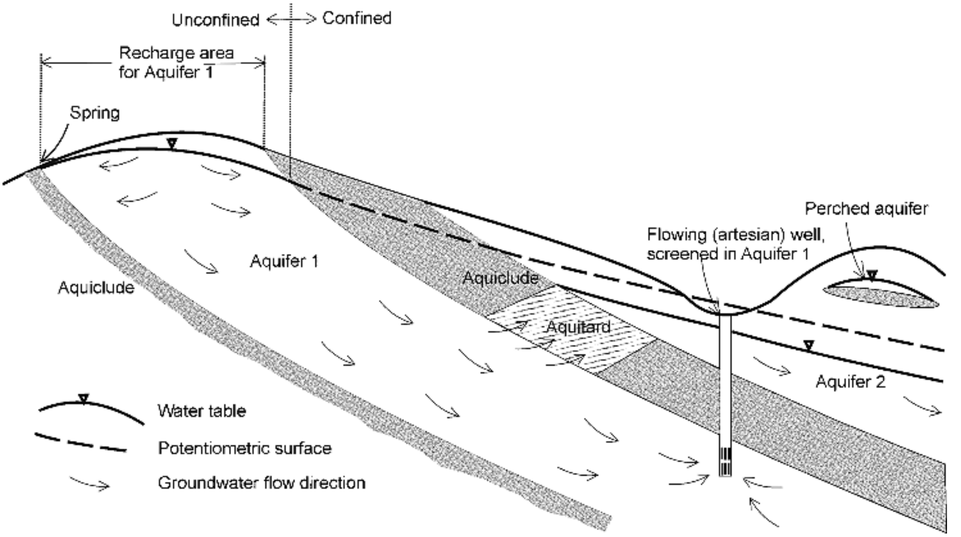

Figure 1.8 *Groundwater occurrence*

Box 1.2 What is groundwater head?

There is a common misconception that water always flows from high pressure to low pressure, but it does not. Consider two points, A and B, in the tank of water illustrated in Figure B1.2(i). The pressures (P) at points A and B are given by:

$$P = H\rho g$$

where H is the height of the column of water above the point (dimension [L]), ρ is the density of the water ($[M][L]^{-3} = c\ 1000\,\mathrm{kg\,m^{-3}}$) and g the acceleration due to gravity ($[L][T]^{-2} = 9.81\,\mathrm{m\,s^{-2}}$).

Thus, at point A, the water pressure is $14\,715\,\mathrm{N\,m^{-2}}$, and at point B it is $53\,955\,\mathrm{N\,m^{-2}}$. But water does not flow from B to A – the water in the tank is static. Clearly we need a more sophisticated concept. In fact, we can use the concept of *potential energy*: groundwater always flows from areas of high potential energy to low potential energy. *Groundwater head* (h) is a measure of the potential energy of a unit mass of groundwater at any particular point. This is the sum of potential energy due to elevation and that due to pressure.

$$\text{Potential energy} = \frac{P}{\rho} + zg \ \ (\text{in J kg}^{-1})$$

To obtain head (in metres), we divide by g (a constant).

$$h = \frac{P}{\rho g} + z$$

where z is the elevation above an arbitrary datum [L]. Returning to the tank of water example, the heads at A and B, relative to the base of the tank, are:

$$h_A = \frac{14\,715}{1000 \times 9.81} + 5 = 6.5\,\mathrm{m} \qquad h_B = \frac{53\,955}{1000 \times 9.81} + 1 = 6.5\,\mathrm{m}$$

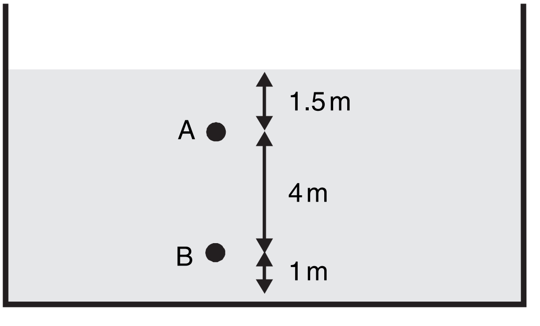

Figure B1.2(i) *Sketch of a water tank showing two points where pressure and head can be calculated*

In other words, they are identical and there is no tendency to flow between the two points. Note that we can compare heads in different locations relative to an arbitrary datum *only* if the density is constant (i.e. 1 m in elevation is equivalent in energy terms to the pressure exerted by a 1 m column of fluid). If we are considering groundwater systems of variable salinity (and density), it is easy to get into difficulties by applying simplistic concepts of head.

In an unconfined aquifer, the elevation of the water table represents groundwater head at that point in the aquifer. While it is often assumed that the water table represents the boundary between unsaturated and saturated aquifer material, this is not quite true, as there is a thin capillary fringe of saturated material above the water table. Strictly speaking, the water table is the surface at which the pressure is equal to atmospheric (i.e. the water pressure is zero).

For confined aquifers, we can imagine contours joining all locations of equal head. These contours then define a surface which is called the *piezometric surface* or *potentiometric surface*. The slope of this surface defines the hydraulic gradient, which in turn controls the direction of groundwater flow. Water will rise in a borehole sunk into the confined aquifer to a level corresponding to the potentiomentric surface.

Box 1.3 Groundwater head as a three-dimensional concept

The distribution of groundwater head in an aquifer can be imagined as a *three-dimensional scalar field*. Each point in the scalar field has a unique value of groundwater head $h(x,y,z)$. Points of equal head can be joined by groundwater head contours. Groundwater flow has a tendency to follow the maximum gradient of head; in other words, the groundwater flow vector ($\boldsymbol{Q}$) is proportional to $-\text{grad}(h)$. In vector-speak:

$$\boldsymbol{Q} \propto -\nabla h$$

Thus, if we construct groundwater head contours in a porous medium aquifer, the groundwater flow lines will be perpendicular to the head contours (in fractured aquifers, groundwater flow *may* not be perpendicular to the regional head contours, as the groundwater is constrained to flow along fracture pathways which may not exist parallel to the head gradient).

Figure 1.8 implies that artesian boreholes can occur in confined aquifers where the potentiometric surface is higher than ground level. However, artesian boreholes *can* also occur in unconfined aquifers. Consider the two aquifer sections below. Figure B1.3(i) shows a relatively high permeability aquifer. The water-table gradient is shallow and groundwater flow is predominantly horizontal. Thus, the head contours are approximately vertical and the head at any depth in the aquifer at a given horizontal (x,y) coordinate is approximately equal to the elevation of the water table. Hence wells exhibit similar static water levels, irrespective of depth [wells A and B in Figure B1.3(i)]. Groundwater flow thus approximately follows the gradient of the water table.

Consider, then, the second drawing [Figure B1.3(ii)], of groundwater flow in a low permeability aquifer in an area of high topography. Here, head is truly three-dimensional, varying with elevation (z) as well as horizontally (x,y). Head contours are complex and *not* necessarily vertical. Groundwater flow has upwards and downwards components. Typically, in recharge areas, head decreases with increasing depth, and groundwater flow has a downward component. A deep-drilled well here (well C) will have a lower static water level than a shallow one (well D). In discharge areas, head increases with increasing depth and groundwater flow has an upward component. A deep-drilled well here (well E) will have a static water level higher than a shallow

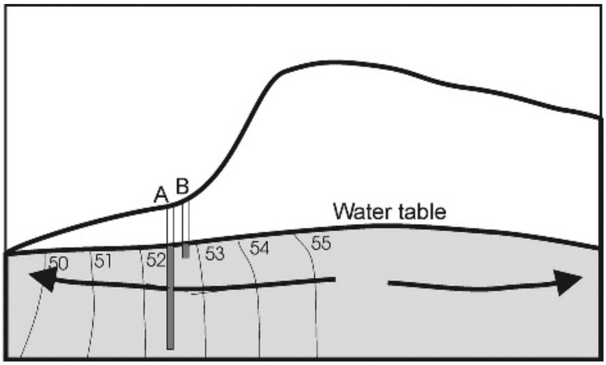

Figure B1.3(i) *Cross-section through a relatively permeable aquifer. The water table gradient is flat. Contours on piezometric head (numbered contours, in metres above sea level) are approximately vertical. Wells A and B have similar static water levels irrespective of depth*

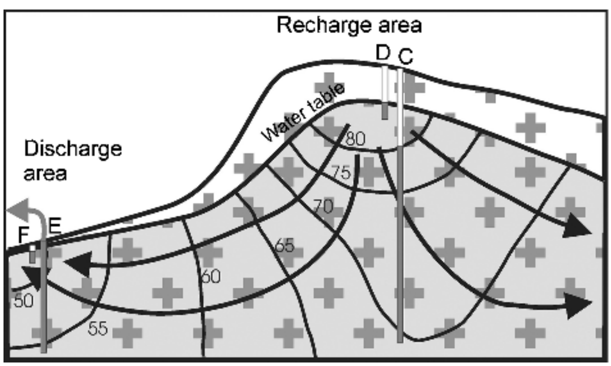

Figure B1.3(ii) *Cross-section through a relatively low permeability aquifer, such as granite. The water table gradient reflects topography. Contours on piezometric head (numbered contours, in metres above sea level) are strongly three-dimensional. Pairs of wells (C, D and E, F) have differing static water levels depending on well depth. Deep wells may even be artesian (overflowing) in discharge areas (well E)* one (well F). In extreme cases, deep wells in discharge areas in *unconfined* aquifers may even have artesian heads, and overflow at the ground surface [as shown by well E in Figure B1.3(ii)].

Aquifers with strongly three-dimensional head distributions will typically either have a strong topography or have relatively low permeability (or both). Erosionally resistant crystalline bedrock aquifers are typically of this type. Note that a two-dimensional network of observation boreholes with long well screens may be adequate to characterize the head distribution in aquifers of the type illustrated in Figure B1.3(i), but are inadequate to characterize three-dimensional head distributions of the type in Figure B1.3(ii). For the latter type, a three-dimensional network of piezometers to varying depths is required. Each piezometer will have a very short open section, and will give a reading of head (h) at a specific point (x, y, z).

to be *confined* when it is fully saturated and its *potentiometric surface* (hydraulic head) lies in an overlying, low-permeability confining layer. Very low permeability layers bounding aquifers are known as *aquicludes*. However, many low permeability formations can transmit quantities of groundwater that may be significant on a regional scale, and the term *aquitard* is used for such formations. Where an aquitard allows some leakage of water to or from an aquifer, the aquifer is often said to be *semi-confined* or *leaky*. In a system of aquifers separated by aquitards or aquicludes, each aquifer may have a different hydraulic head, as depicted in Figure 1.8, and may contain water of a different quality. A *perched aquifer* may occur where a shallow water table has developed locally on a low permeability layer that lies above the regional water table.

Aquifers can be divided into three broad classes: crystalline aquifers; consolidated aquifers; and unconsolidated aquifers. Crystalline aquifers are typified by the igneous and metamorphic rocks that underlie large areas of the world. They include the ancient granites and gneisses that form the 'basement complex' of sub-Saharan Africa and the younger volcanic rocks of the Deccan traps in southern India. Groundwater flow in crystalline aquifers takes place through discrete fractures, rather than through intergranular pore spaces.

Consolidated aquifers are composed of lithified (but not metamorphosed) sedimentary rocks, such as sandstones and limestones (the term consolidated is used here in its general meaning of any sediment that has been solidified into a rock, rather than in the geotechnical engineering sense of a fine-grained cohesive soil that has been compressed). Major consolidated aquifers are found in the Chalk of England and France, the Floridan limestones in southeast USA and the Nubian sandstone in north Africa. Groundwater flow in consolidated aquifers tends to take place through a combination of fractures and intergranular pore spaces.

Unconsolidated aquifers are typically formed of relatively young sediments laid down by water, wind or glaciers. Notable examples include the High Plains alluvial aquifer of the mid-west USA and the Indus valley alluvial aquifer system in Pakistan. Flow through such sediments is typically via intergranular pore spaces.

The main hydraulic properties of the three aquifer classes are described in the following sections. The threefold aquifer classification also forms the basis of the general introduction to drilled well design given in Chapter 3.

1.2.2 Porosity and aquifer storage

Porosity. The ability of a geological formation to store water is governed by its porosity (n), which is the ratio between the volume of voids and the total volume of geological material. *Primary porosity* is a characteristic of unconsolidated aquifers and some consolidated aquifers where the voids were formed at the same time as the geological material. In crystalline aquifers and in consolidated aquifers where the original pores have been infilled with cement, porosity results from openings formed at a later time due to fracturing and weathering. This is known as *secondary porosity* and typically comprises tectonic fractures and dissolution fissures. Secondary porosity is usually much smaller than primary porosity. In karst limestone aquifers, secondary porosity can develop into extensive cavern and conduit flow systems because of dissolution of soluble calcium carbonate minerals along the fractures (Figure 1.9). Groundwater flow rates of several hundred metres per hour can occur, comparable with surface water velocities (Drew and Daly, 1993; Banks *et al.*, 1995). Porosity values for a range of geological formations are given in Table 1.1. Figure 1.10 illustrates different types of porosity.

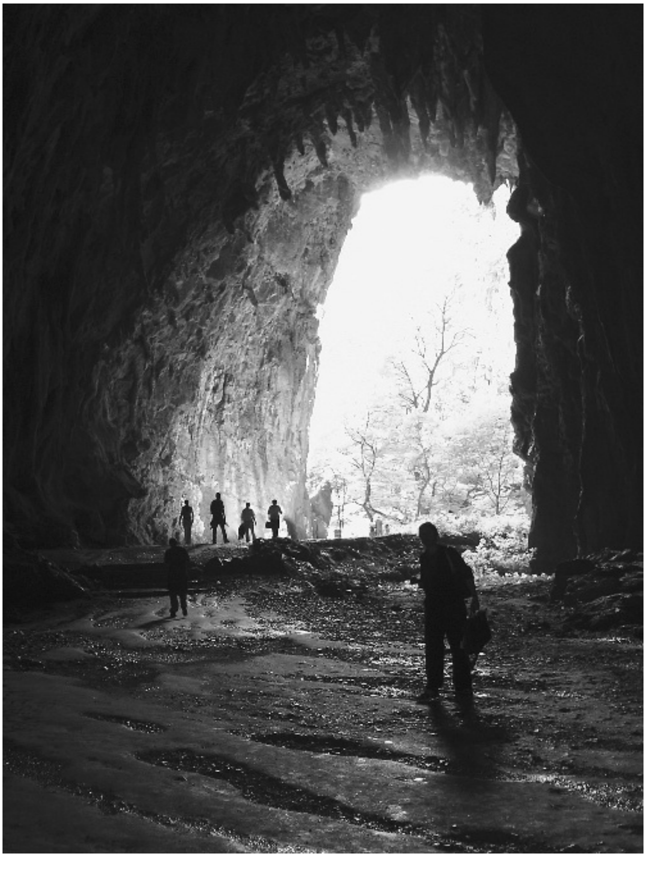

Figure 1.9 *Entrance to large limestone cave in Kras (karst) area of Slovenia. Photo by Bruce Misstear*

Table 1.1 *Typical values for hydraulic properties of geological formations*

Lithology	Dominant porosity type	Porosity (%)	Specific yield (%)	Hydraulic conductivity (m day^{-1})
Unconsolidated sediments				
Clay	P (S)	30–60	1–10	10^{-7}–10^{-3}
Silt	P	35–50	5–30	10^{-3}–1
Sand	P	25–50	10–30	1–100
Gravel	P	20–40	10–25	50–1000
Consolidated sediments				
Shale	S	<1–10	0.5–5	10^{-7}–10^{-3}
Sandstone	P/S	5–30	5–25	10^{-4}–10
Limestone	S/P	1–20	0.5–15	10^{-4}–1000
Crystalline rocks				
Granite	S	<1–2[a]	<1–2	10^{-8}–1
Basalt	S/P	<1–50	<1–30	10^{-8}–1000
Schist	S	<1–2[a]	<1–2	10^{-8}–10^{-1}
Weathered crystalline rocks				
Clayey saprolite	S*	—[b]		10^{-2}–10^{-3}
Sandy saprolite	S*	2–5		10^{-1}–10
Saprock	P/S*	<2		1–100

P, primary porosity; S, secondary porosity (fractures, vesicles, fissures); S*, intergranular secondary porosity due to weathering and disaggregation of crystals.
[a]The typical kinematic (effective) porosity of crystalline rock aquifers may be <0.05 % (Olofsson, 2002).
[b]Effective (not total) porosity of clayey saprolite 0.1–2 % (Rebouças, 1993).
Main sources: Freeze and Cherry (1979), Heath (1983), Open University (1995), Robins (1990), Rebouças (1993), US Environmental Protection Agency (1994), Todd and Mays (2004).

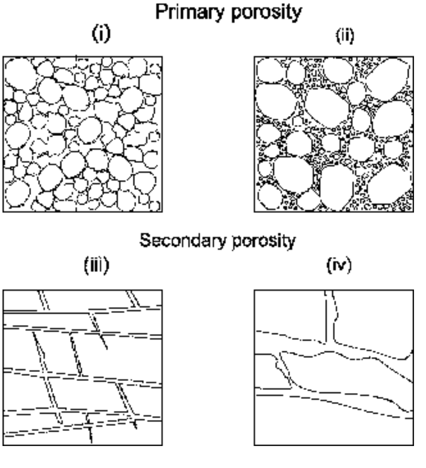

Figure 1.10 *Types of porosity: (i) primary porosity, well sorted unconsolidated formation; (ii) primary porosity, poorly sorted unconsolidated formation; (iii) secondary porosity, consolidated formation; (iv) secondary porosity, carbonate formation, illustrating enlargement of fractures by chemical dissolution*

Sometimes, active groundwater flow only occurs through a portion of an aquifer's total porosity (some of the pores may be 'blind' or too small to permit efficient flow). This porosity is often referred to as the *effective porosity* (n_e).

Aquifer storativity or coefficient of storage. While porosity gives an indication of the amount of water that can be held by a geological formation it does not indicate how much it will release. The amount of water that an aquifer will readily take up or release is determined by its *storativity* or *coefficient of storage*. Aquifer storativity is defined as the volume of water that an aquifer will absorb or release per unit surface area, for a unit change in head. It is a dimensionless quantity. Aquifer storativity has two facets (Figure 1.11): unconfined storage (*specific yield,* S_y) and confined storage (*specific storage*, S_s or *elastic storage*).

The *specific yield* of an unconfined aquifer is the volume of water that will drain from it by gravity alone, per unit area, when the water table falls by one unit. The quantity is dimensionless. The water that is unable to drain and which is retained in the pores is termed the *specific retention* (S_r). Specific yield and specific retention together equal the porosity. Fine-grained materials, such as clays and silts, have a high specific retention. Because of this, and because of their low permeability, they do not normally form good aquifers. Examples of specific yield values for different geological formations are included in Table 1.1.

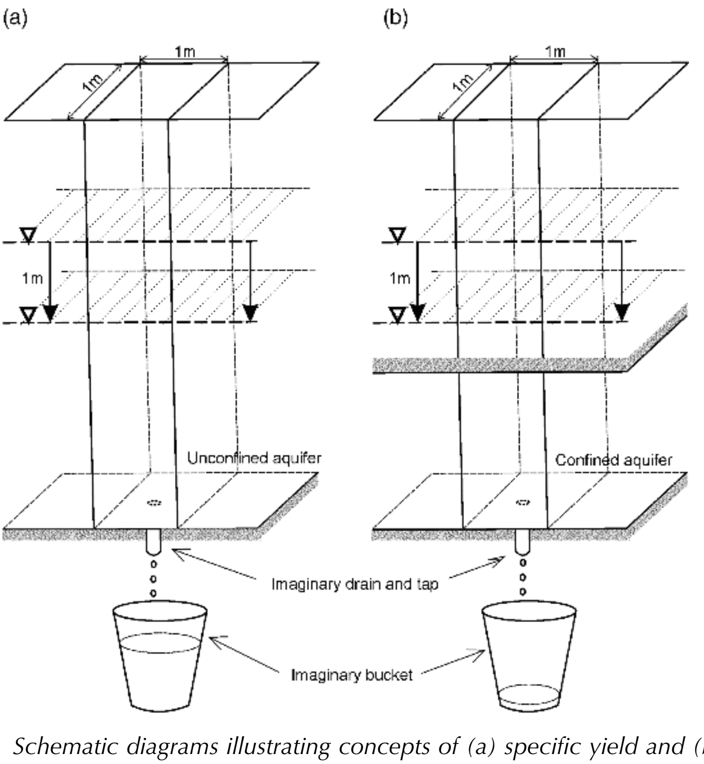

Figure 1.11 *Schematic diagrams illustrating concepts of (a) specific yield and (b) confined storage*

The *specific storage* of a confined aquifer is the volume of water released from storage per unit volume of aquifer per unit fall in head. The aquifer remains saturated: this storage is related to the elastic deformation (compressibility) of water and of the aquifer fabric.

$$S_s = \frac{\Delta Q}{V \Delta h} \tag{1.1}$$

where V is the volume of confined aquifer, ΔQ the amount of water released to or from storage and Δh the change in head. S_s has units of $[\mathrm{L}]^{-1}$; for example, m^{-1}.

The *storativity* of a confined aquifer (i.e. the volume of water released per unit change in head per unit *area* $[A]$) is given by the product of S_s and aquifer thickness (b), and is dimensionless:

$$S = \frac{\Delta Q}{A \Delta h} = S_s b \tag{1.2}$$

The elastic storativity of a confined aquifer is usually two or three orders of magnitude smaller than the specific yield of an unconfined aquifer of a similar lithology (Box 1.4).

It should also be noted an unconfined aquifer subject to a change in water table will release water both from drainable storage (specific yield S_y) and elastic storage (S_s). However, as specific yield is much greater in magnitude, we will often (but not always) neglect the latter term. Further discussion can be found in Chapter 7 on well testing.

Box 1.4 Calculations involving specific yield and coefficient of storage

An unconfined sand and gravel aquifer has a porosity of 0.32 and a specific retention of 0.06. If the water table is lowered by an average of 4 m over an area of 5 ha due to pumping, estimate the volume of water removed.

First, we need to calculate the specific yield. This equals the porosity minus the specific retention:

$$S_y = 0.32 - 0.06 = 0.26$$

Now we can calculate the volume of water removed. This equals the fall in water table over the area of aquifer affected, multiplied by the specific yield:

$$4\,\mathrm{m} \times 50\,000\,\mathrm{m}^2 \times 0.26 = 52\,000\,\mathrm{m}^3$$

The same aquifer has a coefficient of storage of 0.0002 where it is confined. Estimate the volume removed if the potentiometric surface also fell by 4 m over an area of 5 ha (assuming the aquifer remains confined).

The volume removed is:

$$4\,\mathrm{m} \times 50\,000\,\mathrm{m}^2 \times 0.0002 = 40\,\mathrm{m}^3$$

These simple calculations illustrate that coefficient of storage in a confined aquifer is much smaller than specific yield in an unconfined aquifer.

1.3 Groundwater flow

Groundwater under natural conditions flows from areas of recharge, normally the aquifer's outcrop area, to points of discharge at springs, rivers or in the sea. The driving force of groundwater flow is the *hydraulic gradient* – the difference in head between the recharge and discharge areas, divided by the length of the flow path. Hydraulic gradients vary vertically as well as laterally along the flow path: in the recharge area, the vertical component of the hydraulic gradient will be downwards whereas in the discharge area the gradient and therefore flow direction will be upwards (Box 1.3).

1.3.1 Darcy's equation

The flow of water through the saturated zone of an aquifer can be represented by the Darcy equation:

$$Q = -AKi \tag{1.3}$$

where Q is the groundwater flow rate (dimension $[L]^3[T]^{-1}$), A the cross-sectional area of flow ($[L]^2$), K the hydraulic conductivity ($[L][T]^{-1}$) and i the hydraulic gradient in the direction of flow (dimensionless). This most fundamental equation of groundwater flow is empirical: it is based on Darcy's experimental observations of flow through sand filters in the 1850s (Box 1.5). The negative sign indicates that flow takes place in the direction of negative (i.e. decreasing) hydraulic gradient, although in subsequent equations in this chapter it will be omitted as we are usually only concerned with the magnitude of flow (the direction being obvious).

The flow rate per unit cross-sectional area of saturated aquifer is given by the Darcy velocity (v), also known as the specific discharge:

$$v = \frac{Q}{A} = Ki \tag{1.4}$$

The cross-sectional area of aquifer includes the solid material as well as the pores. To obtain an estimate of the flow velocity through the pores it is necessary to divide the Darcy velocity by the effective porosity, n_e. This gives the linear seepage velocity v_s:

$$v_s = \frac{Ki}{n_e} \tag{1.5}$$

This equation represents only the *average* linear seepage velocity. The actual velocity of a water particle is affected by dispersion, which depends mainly on the tortuosity of the flow path. This is especially important in contaminant transport problems, as the first arrival of a contaminant at a well may be much faster than *average linear velocity* predicted by the above equation (Box 1.6).

Box 1.5 Henry Darcy (1803–1858)

Henry Philibert Gaspard Darcy was born in Dijon, France on 10 June 1803 (Brown, 2002). He was not a hydrogeologist (although he did assist on the development of Dijon's Saint Michel well, and also worked on the Blaizy tunnel, where he would have been able to observe water seepage), as that science had not yet been formally invented. He was a water engineer, educated at Paris's L'Ecole Polytechnique and, subsequently L'Ecole des Ponts et Chaussées (School of Roads and Bridges). Darcy was a practical, empirical researcher, rather than a pure theoretician. Most of his life he worked with public water supply for his home city of Dijon [Figure B1.5(i)], but was also employed later by the cities of Paris (as Chief Director for Water and Pavements) and Brussels (as a consultant). He developed among other things formulae for water velocity in various types of open channels, and formulae for estimating water flow in pipes (Darcy, 1857). Some of this work was published posthumously (Darcy and Bazin, 1865) by his protégé and collaborator, Henri Emile Bazin (1829–1917).

In 1855, Darcy's health deteriorated and he returned to Dijon to work and experiment on further hydraulic issues that presumably had long interested him. During 1855 and 1856, Darcy and his friend Charles Ritter empirically studied the flow of water through columns of sand in the laboratory. Ostensibly, this was to improve the design of the sand filters used for purification of surface water supplies (and still widely used today). Brown (2002) argues, however, that Darcy would also clearly have appreciated the importance of such studies for understanding groundwater flow. In 1856 Darcy published a report on the water supply of Dijon city, and a technical appendix to this report contained the results of his experimentation (Darcy, 1856). The famous appendix contended that the flow of water (Q) through a sand filter was proportional to the area

Figure B1.5(i) *The fountains and water feature in the Square Henry Darcy in Dijon, France, created in honour of the great French water engineer in recognition of his work in bringing about the first potable water supply reservoir for the town. Photo by Bruce Misstear*

(A) of the filter and the difference in water head across the filter, and that it was inversely proportional to the filter's thickness (L). In other words (Brown, 2002):

$$Q = KA \frac{(\varsigma_1 + z_1) - (\varsigma_2 + z_2)}{L}$$

where ς and z are pressure head and elevation at locations 1 and 2 on the flow path, respectively, and K is a coefficient of proportionality (hydraulic conductivity). Note that total head $h = \varsigma + z$. This is what we know today as Darcy's law [also expressed in the main text as Equation (1.3)]. Henry Darcy died of pneumonia, while on a trip to Paris, on 3 January 1858 (Tarbé de St-Hardouin, 1884).

The terms *hydraulic conductivity* and *coefficient of permeability* are often used interchangeably, especially in engineering texts. It is important to note, however, that some engineers and most petroleum geologists also use a quantity called *intrinsic permeability* (k). The term *hydraulic conductivity* assumes that the fluid under consideration is water (in our case groundwater). The *intrinsic permeability* of the porous medium is independent of the properties of the fluid involved, is a characteristic of the porous medium alone and is related to the hydraulic conductivity by the equation:

$$k = \frac{K\mu}{\rho g} = \frac{K\upsilon}{g} \tag{1.6}$$

where k is the intrinsic permeability (dimension $[\mathrm{L}]^2$), K the hydraulic conductivity ($[\mathrm{L}][\mathrm{T}]^{-1}$), υ the fluid kinematic viscosity ($[\mathrm{L}]^2[\mathrm{T}]^{-1}$), μ the dynamic viscosity ($[\mathrm{M}][\mathrm{L}]^{-1}[\mathrm{T}]^{-1}$), ρ the fluid density ($[\mathrm{M}][\mathrm{L}]^{-3}$) and g the gravitational acceleration ($[\mathrm{L}][\mathrm{T}]^{-2}$). Intrinsic permeability is a particularly useful parameter for the petroleum industry when dealing with multi-phase fluids with different kinematic viscosities. In hydrogeology, the kinematic fluid viscosity does not vary much over the normal temperature and density range of most groundwaters, and so hydraulic conductivity is the parameter of permeability most commonly used.

Typical hydraulic conductivity values for a range of geological formations are given in Table 1.1. The hydraulic conductivity of an intergranular aquifer depends on the grain size and sorting of the aquifer material and the degree of cementation. In fractured/fissured aquifers the intensity of fracturing and the aperture, continuity and connectivity of individual fractures control the hydraulic conductivity.

The Darcy equation only applies where the flow is laminar. In laminar flow the water particles move along streamlines that are approximately parallel to each other. This is normally the situation with groundwater flow in intergranular aquifers, where the flow velocities are very small. However, higher flow velocities can occur in fissured aquifers (notably where the fissures have been enlarged through karstification) and near wells in both fissured and intergranular aquifers. Flow in these situations may become turbulent, whereby the water particles move erratically in speed and direction. A dimensionless ratio, known as the Reynolds number, can be used to indicate whether the flow is likely to be laminar or nonlaminar:

$$R = \frac{vd}{\upsilon} \tag{1.7}$$

Box 1.6 The use and misuse of Darcy's law

Consider the multilayered aquifer system illustrated in Figure B1.6(i), subject to a hydraulic gradient of 0.01. Let us suppose we have carried out some kind of pumping test and determined that the transmissivity (T) of the sequence is 300 m^2 day^{-1}. We can thus calculate that the (average) hydraulic conductivity (K) of the sequence is 300 m^2 day^{-1}/9 m = 33 m day^{-1}.

The groundwater flux (Q) through the entire thickness (b) of the aquifer is given by:

$$Q = Ti = Kbi = 33\,\text{m day}^{-1} \times 0.01 \times 9\,\text{m} = 3\,\text{m}^3\text{day}^{-1}\text{per m aquifer width}$$

Now, let us consider a contamination incident, such that a volume of polluted groundwater starts migrating in the aquifer. We wish to find out how long it will take to travel 100 m to a protected spring area. If we make the reasonable assumption that the effective porosity (n_e) is 0.10, and apply Equation (1.5) to calculate linear flow velocity (v_s):

$$v_s = \frac{Ki}{n_e} = 33\,\text{m day}^{-1} \times 0.01/0.10 = 3.3\,\text{m day}^{-1}$$

We thus calculate that it will take 30 days for the pollution to migrate 100 m. We will return to our office and relax a little, imaging that we have about a month to try and come up with a remediation scheme. However, the telephone rings after only 12 days to tell us that the pollution has already arrived at the spring.

We have made the mistake of calculating the *average* linear flow velocity and assuming that the aquifer was homogeneous. It is not: the water in the coarse sand will be travelling at 80 m day^{-1} $\times$ 0.01/0.10 = 8 m day^{-1}, while that in the medium sand will only be travelling at 1 m day^{-1}.

Darcy's law is very robust when considering problems of bulk groundwater flux. We do not need to know too much about the detailed aquifer structure. However, when considering problems involving actual groundwater and contaminant flow velocity, it is very easy to make mistakes. Not only is the result very sensitive to the value of n_e selected, we also need to know how hydraulic conductivity and porosity are distributed throughout the aquifer.

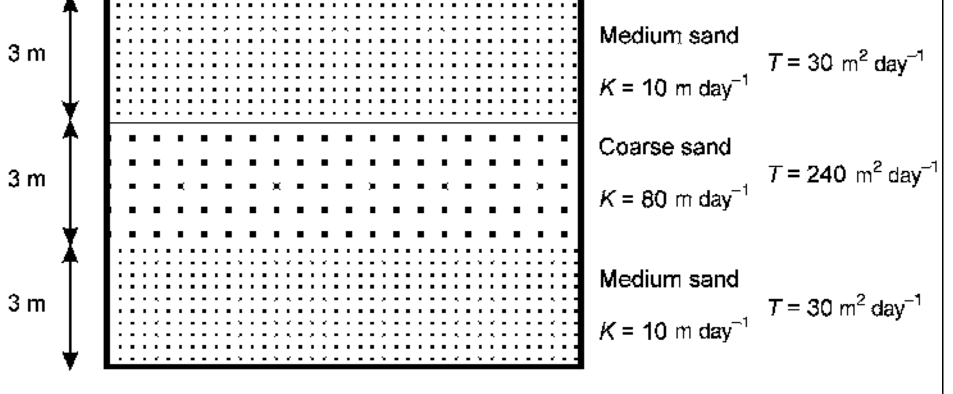

Figure B1.6(i) *Multilayered aquifer system* where R is the Reynolds number (dimensionless), v the Darcy velocity (dimension $[\mathrm{L}][\mathrm{T}]^{-1}$), d the average pore diameter ($[\mathrm{L}]$) and υ the fluid kinematic viscosity ($[\mathrm{L}]^2[\mathrm{T}]^{-1}$). Studies quoted in the standard hydrogeological literature indicate that laminar flow occurs where the Reynolds number is less than 1 and the breakdown of laminar flow may occur where R is in the range 1–10 (Freeze and Cherry, 1979; Fetter, 2001; Todd and Mays, 2004). In most natural groundwater flow situations the Reynolds number is less than 1.

Transmissivity is the rate at which water can pass through the full thickness of aquifer per unit width under a unit hydraulic gradient. The transmissivity in a uniform aquifer is the hydraulic conductivity multiplied by the saturated aquifer thickness. However, as uniform aquifers are uncommon in nature, the transmissivity (T) is usually derived by summing, over the entire aquifer, the transmissivities of individual horizons ($i = 1$ to n), where the transmissivity of each horizon is given by the product of the horizon's hydraulic conductivity (K_i) and its thickness (b_i):

$$T = \sum_{i=1}^{n} K_i b_i \tag{1.8}$$

An aquifer having the same properties in all directions from a point is referred to as *isotropic*. If the properties of the aquifer are also the same at all locations the aquifer is said to be *homogeneous*. Sedimentary aquifers can be relatively homogeneous but they are rarely isotropic. This is because the hydraulic conductivity along the direction of the bedding planes is usually greater than that at right angles to the bedding. Crystalline aquifers, and consolidated aquifers with secondary porosity are both heterogeneous and anisotropic.

The nature of hydraulic conductivity of fractured rock aquifers is considered in more detail in Box 1.7, whilst Box 1.8 compares groundwater flow velocity in fractured and porous aquifers.

1.3.2 General equations of groundwater flow

Groundwater flow in a confined aquifer. The general equations for groundwater flow in porous media are based on the Darcy equation and on the principles of conservation of energy and mass. For transient flow in a confined aquifer, the general equation of groundwater flow is:

$$K_x \frac{\partial^2 h}{\partial x^2} + K_y \frac{\partial^2 h}{\partial y^2} + K_z \frac{\partial^2 h}{\partial z^2} = \frac{S}{b}\frac{\partial h}{\partial t} = S_s \frac{\partial h}{\partial t} \tag{1.9}$$

where K_x, K_y and K_z are the hydraulic conductivities in the principal directions x, y and z; h is the hydraulic head, S is the dimensionless aquifer coefficient of storage, S_s the specific storage coefficient, b the aquifer thickness and t the time. In a homogeneous and isotropic aquifer where $K_x = K_y = K_z$, the equation becomes:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = \frac{S}{Kb}\frac{\partial h}{\partial t} = \frac{S}{T}\frac{\partial h}{\partial t} \tag{1.10}$$

Box 1.7 The hydraulic conductivity of fractured aquifers

Imagine a single horizontal fracture in an impermeable rock mass. If the fracture's aperture is b_a, and if its sides are smooth, planar and parallel, then its *fracture transmissivity* (T_f) is given by (Snow, 1969; Walsh, 1981):

$$T_f = \frac{\rho g b_a^3}{12\mu} \approx 629\,000 b_a^3$$

where T_f is in $m^2\ s^{-1}$ and b_a is in m, ρ is the density of water (c. $1000\ kg\ m^{-3}$), g the acceleration due to gravity ($9.81\ m\ s^{-2}$) and μ is the dynamic viscosity of water (c. $0.0013\ kg\ s^{-1}\ m^{-1}$). We see that transmissivity (the ability of the fracture to transmit water) is proportional to the cube of the aperture. Thus, one ideal plane-parallel fracture of aperture 1 mm is hydraulically equivalent to 1000 fractures of aperture 0.1 mm. The implication of this in a real aquifer is that the bulk of the groundwater is transported through fractures of large aperture. As fracture apertures in natural geological media are typically approximately log-normally distributed (Long *et al.*, 1982), these will be relatively few and far between. In real boreholes in crystalline rocks, the entire well yield typically comes from only a few major fractures.

From the above equation, and changing to units of $m^2\ day^{-1}$, we can see that an idealized fracture of aperture 0.1 mm thus has a transmissivity of $0.05\ m^2\ day^{-1}$, while one of aperture 0.5 mm has a T_f of $7\ m^2\ day^{-1}$.

The *hydraulic conductivity* of fractured rocks can be defined as the total transmissivity within a given interval divided by the thickness of that interval (B). Thus, for the total interval of 5 m in Figure B1.7(i), the total transmissivity of the two fractures is $7.05\ m^2\ day^{-1}$, and the hydraulic conductivity is around $1.4\ m\ day^{-1}$ (equivalent to that, say, for a fine sand). However, if we now consider only the 1 m interval containing the larger fracture, we would calculate a hydraulic conductivity of $7\ m\ day^{-1}$. The calculated hydraulic conductivity thus depends heavily on the interval of measurement and is said to be *scale-dependent*. This scale dependence can be significantly reduced by choosing a large enough interval, which can be referred to as a *representative element*.

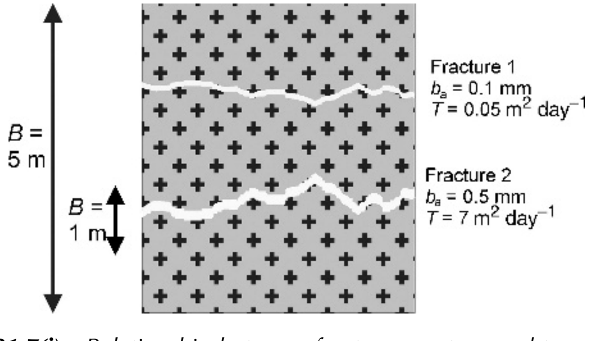

Figure B1.7(i) *Relationship between fracture aperture and transmissivity*

In reality, fractures are not smooth, planar or parallel. Rather, flow within a fracture plane may be canalized; indeed, in some limestone aquifers, the flow features penetrated by wells are distinctly cylindrical and pipe-like in appearance [see Figure 6.26(b) in Chapter 6]. The Frenchman, Jean Louis Poiseuille (1799–1869), conducted experiments on laminar (nonturbulent) fluid flow in cylindrical tubes and found that the rate of flow (Q) was proportional to the *fourth* power of the pipe's diameter. More formally:

$$Q = \frac{\pi r^4 \Delta P}{8\mu \Delta l} \text{ or, in terms of head, } Q = \frac{\pi \rho g r^4 \Delta h}{8\mu \Delta l}$$

where Δl is the length of the tube (dimension [L]), r the radius of the tube ([L]), ΔP the difference in pressure between the two ends of the tube ([M][L]$^{-1}$[T]$^{-2}$), Δh the head difference between the two ends of the pipe ([L]), μ the dynamic viscosity of the fluid ([M][L]$^{-1}$[T]$^{-1}$), ρ the fluid density ([M][L]$^{-3}$), and g is the acceleration due to gravity ([L][T]$^{-2}$).

Rearranging in terms of head loss per unit length of pipe:

$$\frac{\Delta h}{\Delta l} = \frac{8 v \mu}{\rho g r^2}$$

where v is average flow velocity $= Q/\pi r^2$. This is, in fact, identical to the Darcy–Weisbach equation [named after Henry Darcy – see Box 1.5 – and the Saxonian Julius Weisbach (1806–1871)]:

$$\frac{\Delta h}{\Delta l} = \frac{f v^2}{4 r g}$$

where f is a friction loss factor. For laminar flow $f = 64/R$, where R is the Reynolds number, named after Osborne Reynolds (1842–1912) of the University of Manchester. For circular pipes, R is given by:

$$R = \frac{2 r \rho v}{\mu} = \frac{2 \rho Q}{\pi r \mu}$$

If R is low (<2000, for circular pipes), flow is typically laminar. When R exceeds this figure, flow gradually becomes turbulent and the Poiseuille equation is not valid.

While these concepts and equations help our understanding of flow in fissures, they can also be useful in well design, for example, in estimating head losses during flow within the well casing (Section 4.5).

Box 1.8 Groundwater flow velocity in fractured and porous aquifers

Consider the two 5 m thick aquifers shown in Figure B1.8(i). One is a crystalline granite containing a single ideal plane parallel fracture of aperture 0.5 mm. We saw, from Box 1.7, that its transmissivity would be around 7 m^2 day^{-1}, and thus the bulk hydraulic conductivity of the aquifer is $7/5\,\text{m day}^{-1} = 1.4\,\text{m day}^{-1}$. The second aquifer is a homogeneous fine sand aquifer with a hydraulic conductivity also equal to 1.4 m day^{-1}.

The hydraulic conductivities and transmissivities of the aquifers are the same and Darcy's law [Equation (1.3)] thus states that they will transmit the same flow of groundwater (Q) under a hydraulic gradient of 0.01, namely:

$$Q = KBi = 1.4\,\text{m day}^{-1} \times 0.01 \times 5\,\text{m} = 0.07\,\text{m}^3\,\text{day}^{-1} \text{ per m of aquifer width}$$

However, if we are investigating a contamination incident and wish to know the velocity (v_s) at which the contamination is flowing towards a well, then Equation (1.5) should be used:

$$v_s = \frac{Ki}{n_e}$$

For the sand, the effective porosity (n_e) might be, say, 0.17, resulting in a derived v_s of 0.08 m day^{-1}. This is the average linear velocity. Dispersion effects will mean that some contaminant travels somewhat faster than this and some slightly slower.

In the fractured rock aquifer, the effective porosity is probably no more than 0.0005 m/5 m = 0.0001, yielding a transport velocity of some 140 m day^{-1}.

From this, we can learn two valuable lessons:

(1) Groundwater flows much faster in fractured and fissured aquifers than in equivalent porous aquifers.
(2) Groundwater flow velocity depends very heavily on the value selected for effective porosity (and in many aquifers, especially a low permeability one, this is very difficult to derive).

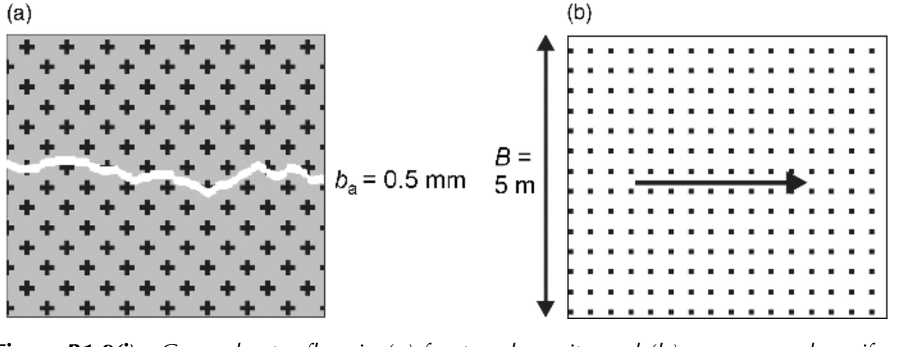

Figure B1.8(i) *Groundwater flow in (a) fractured granite and (b) porous sand aquifers*

The quantity T/S (or K/S_s) is called the *hydraulic diffusivity* ($[L]^2[T]^{-1}$). For steady state flow, head in the aquifer does not change with time ($\partial h/\partial t = 0$) and the equation reduces to:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0 \tag{1.11}$$

This is known as the Laplace equation. The assumptions underlying this equation are:

- the aquifer is confined;
- the aquifer is homogeneous and isotropic;
- the fluid is incompressible (a density term can be introduced into the equation for compressible fluids);
- groundwater flow is in steady-state;
- all the flow comes from water stored within the aquifer (that is, there is no leakage into the aquifer from overlying or underlying layers).

The solution to Equation (1.11) describes hydraulic head in terms of the x, y and z coordinates. The solution to Equation (1.10) describes the hydraulic head at any point in the three-dimensional flow system at any time t. These equations are often reduced to two dimensions – and sometimes to one dimension – to facilitate their solution by graphical, analytical or numerical methods.

Groundwater flow in an unconfined aquifer. Whereas Darcy's equation (1.3) can be applied to simple one-dimensional flow problems in a confined aquifer (under steady-state conditions), the problem is more complex for the situation of the unconfined aquifer in Figure 1.12 because the flow is not horizontal; indeed, the water table represents a flow line whose shape is both governed by, and plays a role in governing, flow in the remainder of the aquifer (Todd and Mays, 2004). Darcy's law strictly speaking states that flow (q) is proportional to hydraulic gradient along the direction of flow; i.e. $q \propto \mathrm{d}h/\mathrm{d}s$, where s is a distance coordinate along a flow line. If flow is horizontal then

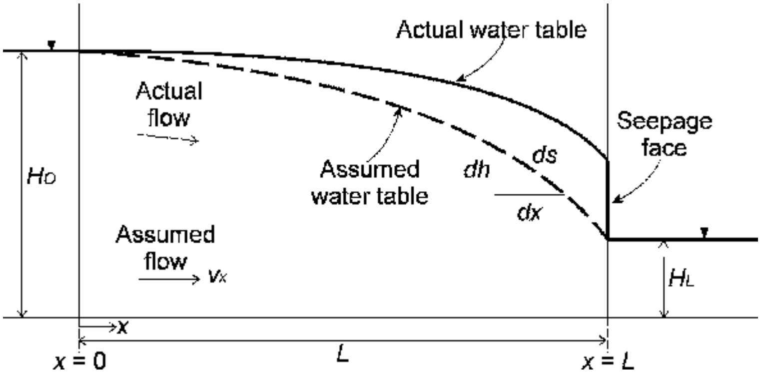

Figure 1.12 *Dupuit–Forchheimer assumptions for groundwater flow in an unconfined aquifer*

$dh/dx = dh/ds$, where x is the horizontal distance coordinate. In an unconfined aquifer this is not strictly the case. A solution to this problem was proposed by Dupuit (Dupuit, 1863; see Box 7.4 in Chapter 7) and developed by Forchheimer (Forchheimer, 1930). The Dupuit–Forchheimer solution allows this two-dimensional flow problem to be reduced to one dimension by assuming (a) that flow is horizontal and uniform throughout the vertical section, and (b) that $q \propto dh/dx = dh/ds$. Applying Darcy's equation (1.3) and the Dupuit–Forchheimer assumptions, the flow per unit width (q) through the vertical section of aquifer in Figure 1.12 is given by:

$$q = -KH\frac{dH}{dx} \tag{1.12}$$

where K is the hydraulic conductivity, H the elevation of the water table (hydraulic head) *relative to the impervious base of the aquifer* and dH/dx the hydraulic gradient (along a horizontal axis). Integrating gives:

$$qx = -\frac{KH^2}{2} + c \tag{1.13}$$

where c is the coefficient of integration. For the boundary conditions $H = H_0$ at $x = 0$, and $H = H_L$ at $x = L$, then:

$$q = \frac{K}{2L}(H_0^2 - H_L^2) \tag{1.14}$$

This is known as the Dupuit equation or the Dupuit–Forchheimer discharge equation. In Figure 1.12, we see that the hydraulic gradient increases downstream. This is because the saturated thickness (and hence the transmissivity) of the aquifer decreases as the water table falls. As the hydraulic gradient steepens, the Dupuit–Forchheimer assumptions become increasingly violated and the calculated water table departs increasingly from the actual water table. Indeed, in actuality a *seepage* face develops at the downstream end of the aquifer block. Nevertheless, the Dupuit–Forchheimer assumptions are useful in a variety of situations: they can be applied to the estimation of recharge (Section 2.6.3) and to radial flow to a well in an unconfined aquifer (Chapter 7).

If the thickness of an unconfined aquifer is great relative to variations in the water table level and if water table gradients are relatively low, we can assume that transmissivity does not vary greatly nor depend on water table level. It thus becomes possible to apply equations derived for confined aquifers to unconfined aquifer situations.

1.3.3 Radial flow to wells

The natural flow conditions in an aquifer are disturbed when a well is pumped. The action of pumping water from the well lowers the level of groundwater and creates a hydraulic head difference between the water in the well and that in the aquifer. This head difference causes water to flow in to the well and so lowers the hydraulic head in the aquifer around

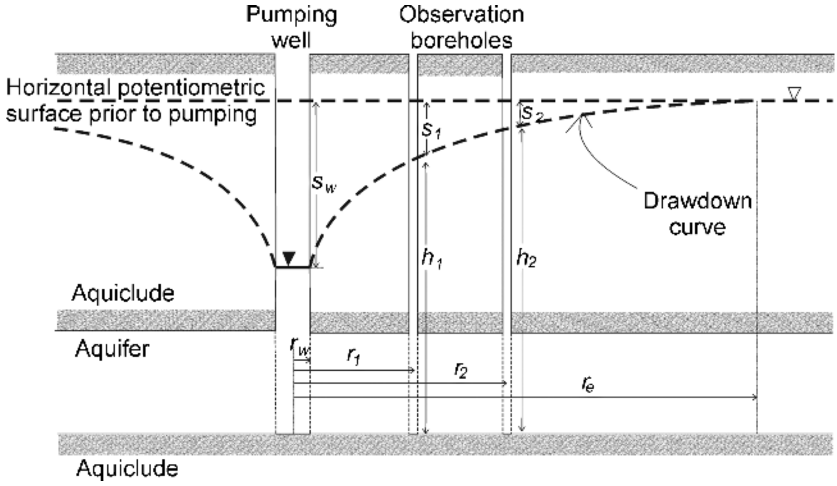

Figure 1.13 *Cone of depression of potentiometric surface around a pumping well*

the well. The effects of pumping spread radially through the aquifer. The lowering of the water table or potentiometric surface forms a *cone of depression* around the pumping well. This cone can be seen and measured in observation wells (Figure 1.13).

Radial flow to a well in a confined aquifer. If it is assumed that flow is horizontal, Equations (1.10) and (1.11) can be reduced to the following expressions for two-dimensional flow in a confined aquifer:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = \frac{S}{Kb}\frac{\partial h}{\partial t} = \frac{S}{T}\frac{\partial h}{\partial t} \quad (1.15)$$

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0 \text{ for steady-state conditions} \quad (1.16)$$

In analysing groundwater flow to water wells we must convert Equations (1.15) and (1.16) into radial coordinates, with $r = (x^2 + y^2)^{1/2}$:

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} = \frac{S}{Kb}\frac{\partial h}{\partial t} = \frac{S}{T}\frac{\partial h}{\partial t} \quad (1.17)$$

and

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} = 0 \quad \text{for steady-state flow} \quad (1.18)$$

The first solution to Equation (1.17) for *transient* (i.e. nonsteady-state) flow to a well in a confined aquifer was proposed by Theis (1935). The Theis solution enables the estimation of aquifer transmissivity and storage coefficient from a well pumping test by a curve-matching technique. This and other methods for analysing pumping test data are described in Chapter 7.

Simple equilibrium equations for steady-state radial flow to a well have applications in well siting and design (Chapters 2–4), and so are introduced here rather than in Chapter 7. Thiem (Thiem, 1906; see Box 7.4 in Chapter 7) developed a solution for radial flow to a well in a confined aquifer by applying Darcy's equation to a cylindrical flow section. Different boundary conditions are illustrated in Figure 1.13. For the case of two observation wells located near a pumping well, the Thiem equation can be written as:

$$h_2 - h_1 = \frac{Q}{2\pi T} \ln \frac{r_2}{r_1} \tag{1.19}$$

where h_1 and h_2 are the hydraulic heads in the observation wells located at distances r_1 and r_2 from the pumping well, respectively; Q is the discharge rate in the pumping well and T the transmissivity.

The hydraulic head values can be replaced by the drawdown values ($h_2 - h_1 = s_1 - s_2$) to give:

$$s_1 - s_2 = \frac{Q}{2\pi T} \ln \frac{r_2}{r_1} \tag{1.20}$$

For the case where s_2 is zero – which occurs at a distance from the pumping well known as the *radius of influence* – and where the first observation well is replaced by measurements in the pumping well itself, Equation (1.20) becomes:

$$s_w = \frac{Q}{2\pi T} \ln \frac{r_e}{r_w} \tag{1.21}$$

where s_w is the drawdown in the pumping well, r_e the radius of influence of the pumping well and r_w the well radius. It should be added that the 'radius of influence' at equilibrium is really a somewhat flawed concept, since theory dictates that the cone of depression will continue to expand with continued pumping unless there is a source of recharge to the aquifer (this inconsistency provided much of the motivation for C.V. Theis's work – see Box 7.5 in Chapter 7).

Rearranging in terms of transmissivity, Equation (1.21) becomes:

$$T = \frac{Q}{2\pi s_w} \ln \frac{r_e}{r_w} \tag{1.22}$$

Equations (1.19)–(1.22) apply to a single pumping well that penetrates the entire thickness of an infinite, homogeneous and isotropic aquifer of uniform thickness, which is confined, and where the potentiometric surface is horizontal prior to pumping. These underlying assumptions are never met fully in reality, but they are more likely to be at least partly satisfied in extensive unconsolidated and consolidated aquifers characterized by primary porosity than in consolidated or crystalline aquifers with fracture-porosity. The equilibrium equations can be applied in the case of several wells in a well field in order to calculate the interference drawdowns between the wells, using the principle of superposition (Section 2.9).

A number of simplifications of the Thiem equilibrium equation have been proposed (Misstear, 2001), including that by Logan (1964). In Equation (1.22) the ratio r_e/r_w cannot be determined accurately during a pumping test unless we have data from several observation boreholes. Although this ratio may vary significantly, the log term is relatively insensitive to these variations. Logan proposed a value of 3.32 as typical for the $\log_{10}$ ratio (equal to 7.65 for the ln ratio), and thus reduced Equation (1.22) to the following approximation:

$$T = \frac{1.22Q}{s_w} \tag{1.23}$$

Similar approximations have been proposed elsewhere. For example, the relationship:

$$T = \frac{1.32Q}{s_w} \tag{1.24}$$

was obtained from a large number of well pumping tests in alluvial aquifers in the Indus valley of Pakistan (Bakiewicz *et al.*, 1985). This type of equilibrium approximation equation can be used to calculate the required length of screen when designing a well in a thick, uniform aquifer (Section 3.1.4).

Radial flow to a well in an unconfined aquifer. The equations for a confined aquifer can be approximately applied also to an unconfined aquifer, provided that the aquifer thickness is relatively large compared with the amount of drawdown (i.e. that transmissivity does not vary significantly with drawdown). If this is not the case, we can derive a modified, unconfined variant of the Thiem equation by supposing a well pumping at rate Q, and then imagining that this induces a flow Q (at steady state) through an imagined cylinder at a distance r around the well. Applying Darcy's law and the Dupuit assumptions:

$$Q = -2\pi KH\frac{\mathrm{d}H}{\mathrm{d}r} \tag{1.25}$$

where H is the height of the water table (head) relative to the impermeable base of the aquifer. Integration between two radii r_1 and r_2 yields:

$$Q = \pi K\frac{(H_2^2 - H_1^2)}{\ln(r_2/r_1)} \tag{1.26}$$

This is sometimes known as the Dupuit equation (Kruseman *et al*., 1990). Note that, if the aquifer thickness H is very much greater than the drawdown:

$$Q = \pi K \frac{(H_2^2 - H_1^2)}{\ln(r_2/r_1)} = \pi K \frac{(H_2 - H_1)(H_2 + H_1)}{\ln(r_2/r_1)} \approx 2\pi KH \frac{(H_2 - H_1)}{\ln(r_2/r_1)} \qquad (1.27)$$

which is identical to Thiem's equation for confined aquifers [i.e. to Equation (1.19) when expressed in terms of Q].