

# 1

## Introduction

### 1.1 Importance of Heat Transfer

The subject of heat transfer is of fundamental importance in many branches of engineering. A *mechanical engineer* may be interested in knowing the mechanisms of heat transfer involved in the operation of equipment, for example boilers, condensers, air pre-heaters, economizers, and so on, in a thermal power plant in order to improve their performance. Nuclear power plants require precise information on heat transfer, as safe operation is an important factor in their design. Refrigeration and air-conditioning systems also involve heat-exchanging devices, which need careful design. *Electrical engineers* are keen to avoid material damage due to hot spots, developed by improper heat transfer design, in electric motors, generators and transformers. An *electronic engineer* is interested in knowing the efficient methods of heat dissipation from chips and semiconductor devices so that they can operate within safe operating temperatures. A *computer hardware engineer* is interested in knowing the cooling requirements of circuit boards, as the miniaturization of computing devices is advancing at a rapid rate. *Chemical engineers* are interested in heat transfer processes in various chemical reactions. A *metallurgical engineer* would be interested in knowing the rate of heat transfer required for a particular heat treatment process, for example, the rate of cooling in a casting process has a profound influence on the quality of the final product. *Aeronautical engineers* are interested in knowing the heat transfer rate in rocket nozzles and in heat shields used in re-entry vehicles. An *agricultural engineer* would be interested in the drying of food grains, food processing and preservation. A *civil engineer* would need to be aware of the thermal stresses developed in quick-setting concrete, the effect of heat and mass transfer on building and building materials and also the effect of heat on nuclear containment, and so on. An *environmental engineer* is concerned with the effect of heat on the dispersion of pollutants in air, diffusion of pollutants in soils, thermal pollution in lakes and seas and their impact on life. The global, thermal changes and associated problems caused by *El Nino* are very well known phenomena, in which energy transfer in the form of heat exists.

The previously-mentioned examples are only a sample of heat transfer applications to name but a few. The solar system and the associated energy transfer are the principal factors for existence of life on earth. It is not untrue to say that it is extremely difficult, often impossible, to avoid some form of heat transfer in any process on earth.

The study of heat transfer provides economical and efficient solutions for critical problems encountered in many engineering items of equipment. For example, we can consider the development of heat pipes that can transport heat at a much greater rate than copper or silver rods of the same dimensions, even at almost isothermal conditions. The development of present day gas turbine blades, in which the gas temperature exceeds the melting point of the material of the blade, is possible by providing efficient cooling systems and is another example of the success of heat transfer design methods. The design of computer chips, which encounter heat flux of the order occurring in re-entry vehicles, especially when the surface temperature of the chips is limited to less than  $100^{\circ}\text{C}$ , is again a success story for heat transfer analysis.

Although there are many successful heat transfer designs, further developments are still necessary in order to increase the life span and efficiency of the many devices discussed previously, which can lead to many more new inventions. Also, if we are to protect our environment, it is essential to understand the many heat transfer processes involved and, if necessary, to take appropriate action.

## 1.2 Heat Transfer Modes

Heat transfer is that section of engineering science that studies the energy transport between material bodies due to a temperature difference (Bejan 1993; Holman 1989; Incropera and Dewitt 1990; Sukhatme 1992). The three modes of heat transfer are

1. Conduction
2. Convection
3. Radiation.

The conduction mode of heat transport occurs either because of an exchange of energy from one molecule to another, without the actual motion of the molecules, or because of the motion of the free electrons if they are present. Therefore, this form of heat transport depends heavily on the properties of the medium and takes place in solids, liquids and gases if a difference in temperature exists.

Molecules present in liquids and gases have freedom of motion, and by moving from a hot to a cold region, they carry energy with them. The transfer of heat from one region to another, due to such macroscopic motion in a liquid or gas, added to the energy transfer by conduction within the fluid, is called *heat transfer* by convection. Convection may be free, forced or mixed. When fluid motion occurs because of a density variation caused by temperature differences, the situation is said to be a free, or natural, convection. When the fluid motion is caused by an external force, such as pumping or blowing, the state is

defined as being one of forced convection. A mixed convection state is one in which both natural and forced convections are present. Convection heat transfer also occurs in boiling and condensation processes.

All bodies emit thermal radiation at all temperatures. This is the only mode that does not require a material medium for heat transfer to occur. The nature of thermal radiation is such that a propagation of energy, carried by *electromagnetic waves*, is emitted from the surface of the body. When these electromagnetic waves strike other body surfaces, a part is reflected, a part is transmitted and the remaining part is absorbed.

All modes of heat transfer are generally present in varying degrees in a real physical problem. The important aspects in solving heat transfer problems are identifying the significant modes and deciding whether the heat transferred by other modes can be neglected.

### 1.3 The Laws of Heat Transfer

It is important to quantify the amount of energy being transferred per unit time and for that we require the use of rate equations.

For heat conduction, the rate equation is known as *Fourier's law*, which is expressed for one dimension as

$$q_x = -k \frac{dT}{dx} \quad (1.1)$$

where  $q_x$  is the heat flux in the  $x$  direction ( $\text{W}/\text{m}^2$ );  $k$  is the thermal conductivity ( $\text{W}/\text{mK}$ , a property of material, see Table 1.1) and  $dT/dx$  is the temperature gradient ( $\text{K}/\text{m}$ ).

For convective heat transfer, the rate equation is given by *Newton's law of cooling* as

$$q = h(T_w - T_a) \quad (1.2)$$

where  $q$  is the convective heat flux; ( $\text{W}/\text{m}^2$ );  $(T_w - T_a)$  is the temperature difference between the wall and the fluid and  $h$  is the convection heat transfer coefficient, ( $\text{W}/\text{m}^2\text{K}$ ) (film coefficient, see Table 1.2).

The convection heat transfer coefficient frequently appears as a boundary condition in the solution of heat conduction through solids. We assume  $h$  to be known in many such problems. In the analysis of thermal systems, one can again assume an appropriate  $h$  if not available (e.g., heat exchangers, combustion chambers, etc.). However, if required,  $h$  can be determined via suitable experiments, although this is a difficult option.

The maximum flux that can be emitted by radiation from a black surface is given by the *Stefan–Boltzmann Law*, that is,

$$q = \sigma T_w^4 \quad (1.3)$$

where  $q$  is the radiative heat flux, ( $\text{W}/\text{m}^2$ );  $\sigma$  is the Stefan–Boltzmann constant ( $5.669 \times 10^{-8}$ ), in  $\text{W}/\text{m}^2\text{K}^4$  and  $T_w$  is the surface temperature, (K).

The heat flux emitted by a real surface is less than that of a black surface and is given by

$$q = \epsilon \sigma T_w^4 \quad (1.4)$$

Table 1.1 Typical values of thermal conductivity of some materials in W/mK at 20 °C

| Material                        | Thermal conductivity            |
|---------------------------------|---------------------------------|
| <i>Metals:</i>                  |                                 |
| Pure silver                     | 410                             |
| Pure copper                     | 385                             |
| Pure aluminium                  | 200                             |
| Pure iron                       | 73                              |
| <i>Alloys:</i>                  |                                 |
| Stainless steel (18% Cr, 8% Ni) | 16                              |
| Aluminium alloy (4.5% Cr)       | 168                             |
| <i>Non metals:</i>              |                                 |
| Plastics                        | 0.6                             |
| Wood                            | 0.2                             |
| <i>Liquid:</i>                  |                                 |
| Water                           | 0.6                             |
| <i>Gases:</i>                   |                                 |
| Dry air                         | 0.025 (at atmospheric pressure) |

Table 1.2 Typical values of heat transfer coefficient in W/m<sup>2</sup>K

|                    |             |
|--------------------|-------------|
| Gases (stagnant)   | 15          |
| Gases (flowing)    | 15–250      |
| Liquids (stagnant) | 100         |
| Liquids (flowing)  | 100–2000    |
| Boiling liquids    | 2000–35,000 |
| Condensing vapours | 2000–25,000 |

where  $\epsilon$  is the radiative property of the surface and is referred to as the *emissivity*. The net radiant energy exchange between any two surfaces 1 and 2 is given by

$$Q = F_{\epsilon} F_G \sigma A_1 (T_1^4 - T_2^4) \quad (1.5)$$

where  $F_{\epsilon}$  is a factor that takes into account the nature of the two radiating surfaces;  $F_G$  is a factor that takes into account the geometric orientation of the two radiating surfaces and  $A_1$  is the area of surface 1.

When a heat transfer surface, at temperature  $T_1$ , is completely enclosed by a much larger surface at temperature  $T_2$ , the net radiant exchange can be calculated by

$$Q = qA_1 = \epsilon_1 \sigma A_1 (T_1^4 - T_2^4) \quad (1.6)$$

With respect to the laws of thermodynamics, only the first law is of interest in heat transfer problems. The increase of energy in a system is equal to the difference between the energy transfer by heat to the system and the energy transfer by work done on the surroundings by the system, that is,

$$dE = dQ - dW \quad (1.7)$$

where  $Q$  is the total heat entering the system and  $W$  is the work done on the surroundings. Since we are interested in the rate of energy transfer in heat transfer processes, we can restate the first law of thermodynamics as

‘The rate of increase of the energy of the system is equal to the difference between the rate at which energy enters the system and the rate at which the system does work on the surroundings’, that is,

$$\frac{dE}{dt} = \frac{dQ}{dt} - \frac{dW}{dt} \quad (1.8)$$

where  $t$  is the time.

## 1.4 Formulation of Heat Transfer Problems

In analysing a thermal system, the engineer should be able to identify the relevant heat transfer processes and only then can the system behaviour be properly quantified. In this section, some typical heat transfer problems are formulated by identifying appropriate heat transfer mechanisms.

### 1.4.1 Heat transfer from a plate exposed to solar heat flux

Consider a plate of size  $L \times B \times d$  exposed to a solar flux of intensity  $q_s$ , as shown in Figure 1.1. In many solar applications such as a solar water heater, solar cooker and so on, the temperature of the plate is a function of time. The plate loses heat by convection and radiation to the ambient air, which is at a temperature  $T_a$ . Some heat flows through the plate and is convected to the bottom side. We shall apply the law of conservation of energy to derive an equation, the solution of which gives the temperature distribution of the plate with respect to time.

*Heat entering the top surface of the plate:*

$$q_s A_T \quad (1.9)$$

*Heat loss from the plate to surroundings:*

Top surface:

$$hA_T(T - T_a) + \epsilon\sigma A_T(T^4 - T_a^4) \quad (1.10)$$

Side surface:

$$hA_S(T - T_a) + \epsilon\sigma A_S(T^4 - T_a^4) \quad (1.11)$$

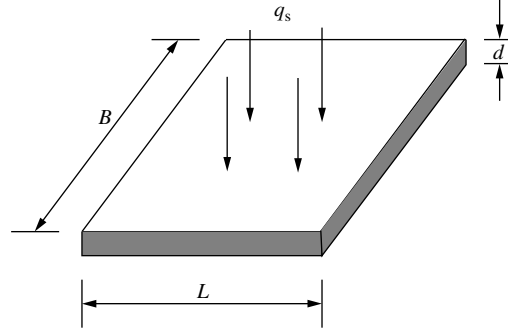


Figure 1.1 Heat transfer from a plate subjected to solar heat flux

Bottom surface:

$$hA_B(T - T_a) + \epsilon\sigma A_B(T^4 - T_a^4) \quad (1.12)$$

where the subscripts T, S and B are respectively the top, side and bottom surfaces. The subject of radiation exchange between a gas and a solid surface is not simple. Readers are referred to other appropriate texts for further details (Holman 1989; Siegel and Howell 1992). Under steady state conditions, the heat received by the plate is lost to the surroundings, thus

$$\begin{aligned} q_s A_T &= hA_T(T - T_a) + \epsilon\sigma A_T(T^4 - T_a^4) + hA_S(T - T_a) \\ &+ \epsilon\sigma A_S(T^4 - T_a^4) + hA_B(T - T_a) + \epsilon\sigma A_B(T^4 - T_a^4) \end{aligned} \quad (1.13)$$

This is a nonlinear algebraic equation (because of the presence of the  $T^4$  term). The solution of this equation gives the steady state temperature of the plate. If we want to calculate the temperature of the plate as a function of time,  $t$ , we have to consider the rate of rise in the internal energy of the plate, which is

$$(\text{Volume}) \rho c_p \frac{dT}{dt} = (LBd) \rho c_p \frac{dT}{dt} \quad (1.14)$$

where  $\rho$  is the density and  $c_p$  is the specific heat of the plate. Thus, at any instant of time, the difference between the heat received and lost by the plate will be equal to the heat stored (Equation 1.14). Thus,

$$\begin{aligned} (LBd) \rho c_p \frac{dT}{dt} &= q_s A_T - [hA_T(T - T_a) + \epsilon\sigma A_T(T^4 - T_a^4) + hA_S(T - T_a) \\ &+ \epsilon\sigma A_S(T^4 - T_a^4) + hA_B(T - T_a) + \epsilon\sigma A_B(T^4 - T_a^4)] \end{aligned} \quad (1.15)$$

This is a first-order nonlinear differential equation, which requires an initial condition, namely,

$$t = 0, \quad T = T_a \quad (1.16)$$

The solution is determined iteratively because of the nonlinearity of the problem. Equation 1.15 can be simplified by substituting relations for the surface areas. It should be noted, however, that this is a general equation that can be used for similar systems.

It is important to note that the spatial variation of temperature within the plate is neglected here. However, this variation can be included via Fourier's law of heat conduction, that is, Equation 1.1. Such a variation is necessary if the plate is not thin enough to reach equilibrium instantly.

### 1.4.2 Incandescent lamp

Figure 1.2 shows an idealized incandescent lamp. The filament is heated to a temperature of  $T_f$  by an electric current. Heat is convected to the surrounding gas and is radiated to the wall, which also receives heat from the gas by convection. The wall in turn convects and radiates heat to the ambient at  $T_a$ . A formulation of equations, based on energy balance, is necessary in order to determine the temperature of the gas and the wall with respect to time.

*Gas:*

Rise in internal energy of gas:

$$m_g c_{pg} \frac{dT_g}{dt} \quad (1.17)$$

Convection from filament to gas:

$$h_f A_f (T_f - T_g) \quad (1.18)$$

Convection from gas to wall:

$$h_g A_g (T_g - T_w) \quad (1.19)$$

Radiation from filament to gas:

$$\epsilon_f A_f \sigma (T_f^4 - T_g^4) \quad (1.20)$$

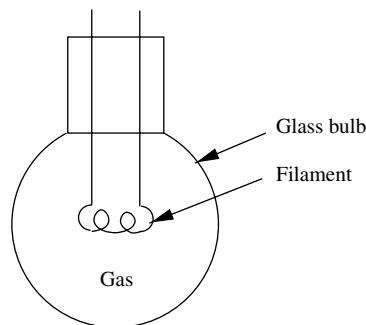


Figure 1.2 Energy balance in an incandescent light source

Now, the energy balance for gas gives

$$m_g c_{pg} \frac{dT_g}{dt} = h_f A_g (T_f - T_g) - h_g A_g (T_g - T_w) + \epsilon_f A_f \sigma (T_f^4 - T_g^4) \quad (1.21)$$

Wall:

Rise in internal energy of wall:

$$m_w c_{pw} \frac{dT_w}{dt} \quad (1.22)$$

Radiation from filament to wall:

$$\epsilon_f \sigma A_f (T_f^4 - T_w^4) \quad (1.23)$$

Convection from wall to ambient:

$$h_w A_w (T_w - T_a) \quad (1.24)$$

Radiation from wall to ambient:

$$\epsilon_w \sigma A_w (T_w^4 - T_a^4) \quad (1.25)$$

Energy balance for wall gives

$$m_w c_{pw} \frac{dT_w}{dt} = h_g A_g (T_g - T_w) + \epsilon_f \sigma A_f (T_f^4 - T_w^4) - h_w A_w (T_w - T_a) - \epsilon_w \sigma A_w (T_w^4 - T_a^4) \quad (1.26)$$

where  $m_g$  is the mass of the gas in the bulb;  $c_{pg}$ , the specific heat of the gas;  $m_w$ , the mass of the wall of the bulb;  $c_{pw}$ , the specific heat of the wall;  $h_f$ , the heat transfer coefficient between the filament and the gas;  $h_g$ , the heat transfer coefficient between the gas and wall;  $h_w$ , the heat transfer coefficient between the wall and ambient and  $\epsilon$  is the emissivity. The subscripts f, w, g and a respectively indicate filament, wall, gas and ambient.

Equations 1.21 and 1.26 are first-order nonlinear differential equations. The initial conditions required are as follows:

At  $t = 0$ ,

$$T_g = T_a \text{ and } T_w = T_a \quad (1.27)$$

The simultaneous solution of Equations 1.21 and 1.26, along with the above initial condition results in the temperatures of the gas and wall as a function of time.

### 1.4.3 Systems with a relative motion and internal heat generation

The extrusion of plastics, drawing of wires and artificial fibres (optical fibre), suspended electrical conductors of various shapes, continuous casting etc. can be treated alike.

In order to derive an energy balance for such a system, we consider a small differential control volume of length,  $\Delta x$ , as shown in Figure 1.3. In this problem, the heat lost to

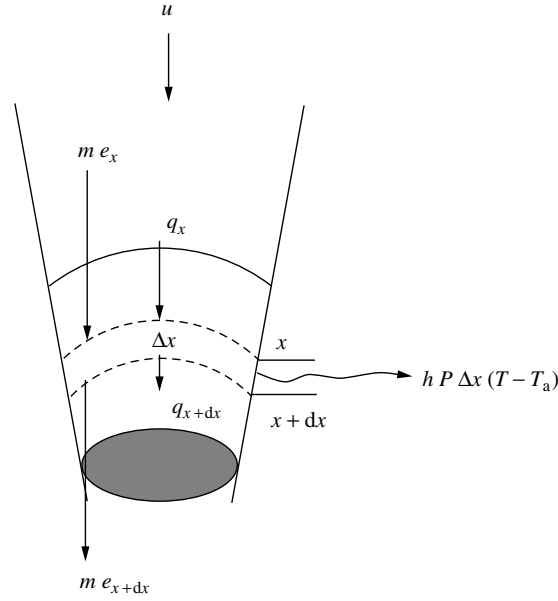


Figure 1.3 Conservation of energy in a moving body

the environment by radiation is assumed to be negligibly small. The energy is conducted, convected and transported with the material in motion. With reference to Figure 1.3, we can write the following equations of conservation of energy, that is,

$$q_x + m e_x + GA \Delta x = q_{x+dx} + m e_{x+dx} + h P \Delta x (T - T_a) \quad (1.28)$$

where  $m$  is the mass flow,  $\rho A u$  which is assumed to be constant;  $\rho$ , the density of the material;  $A$ , the cross-sectional area;  $P$ , the perimeter of the control volume;  $G$ , the heat generation per unit volume and  $u$ , the velocity at which the material is moving. Using a Taylor series expansion, we obtain

$$m(e_x - e_{x+dx}) = -m \frac{de_x}{dx} \Delta x = -m c_p \frac{dT}{dx} \Delta x \quad (1.29)$$

Note that  $de_x = c_p dT$  at constant pressure. Similarly, using Fourier's law (Equation 1.1),

$$q_x - q_{x+dx} = \frac{d}{dx} \left[ kA \frac{dT}{dx} \right] \quad (1.30)$$

Substituting Equations 1.29 and 1.30 into Equation 1.28, we obtain the following conservation equation:

$$\frac{d}{dx} \left[ kA \frac{dT}{dx} \right] - hP(T - T_a) - \rho c_p A u \frac{dT}{dx} + GA = 0 \quad (1.31)$$

In the above equation, the first term is derived from the heat diffusion (conduction) within the material, the second term is due to convection from the material surface to ambient, the third term represents the heat transport due to the motion of the material and finally the last term is added to account for heat generation within the body.

## 1.5 Heat Conduction Equation

The determination of temperature distribution in a medium (solid, liquid, gas or combination of phases) is the main objective of a conduction analysis, that is, to know the temperature in the medium as a function of space at steady state and as a function of time during the transient state. Once this temperature distribution is known, the heat flux at any point within the medium, or on its surface, may be computed from Fourier's law, Equation 1.1. A knowledge of the temperature distribution within a solid can be used to determine the structural integrity via a determination of the thermal stresses and distortion. The optimization of the thickness of an insulating material and the compatibility of any special coatings or adhesives used on the material can be studied by knowing the temperature distribution and the appropriate heat transfer characteristics.

We shall now derive the conduction equation in Cartesian coordinates by applying the energy conservation law to a differential control volume as shown in Figure 1.4. The solution of the resulting differential equation, with prescribed boundary conditions, gives the temperature distribution in the medium.

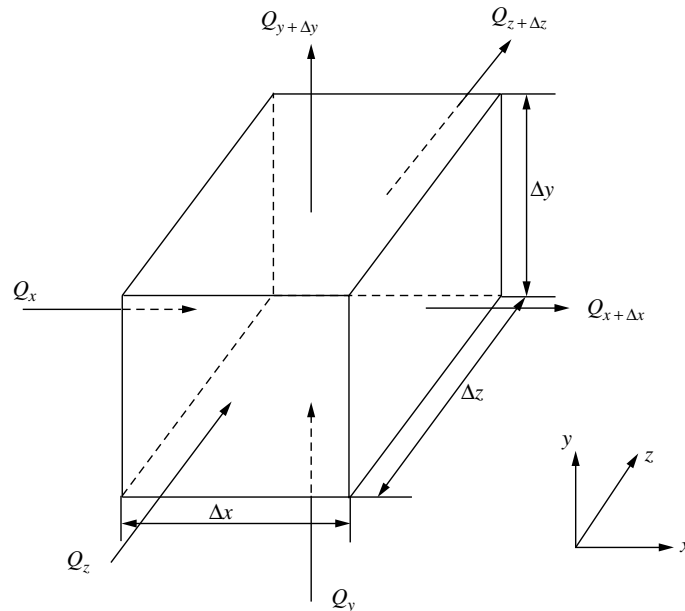


Figure 1.4 A differential control volume for heat conduction analysis

A Taylor series expansion results in

$$\begin{aligned} Q_{x+dx} &= Q_x + \frac{\partial Q_x}{\partial x} \Delta x \\ Q_{y+dy} &= Q_y + \frac{\partial Q_y}{\partial y} \Delta y \\ Q_{z+dz} &= Q_z + \frac{\partial Q_z}{\partial z} \Delta z \end{aligned} \quad (1.32)$$

Note that the second- and higher-order terms are neglected in the above equation. The heat generated in the control volume is  $G \Delta x \Delta y \Delta z$  and the rate of change in energy storage is given as

$$\rho \Delta x \Delta y \Delta z c_p \frac{\partial T}{\partial t} \quad (1.33)$$

Now, with reference to Figure 1.4, we can write the energy balance as

inlet energy + energy generated = energy stored + exit energy

that is,

$$G \Delta x \Delta y \Delta z + Q_x + Q_y + Q_z = \rho \Delta x \Delta y \Delta z \frac{\partial T}{\partial t} + Q_{x+dx} + Q_{y+dy} + Q_{z+dz} \quad (1.34)$$

Substituting Equation 1.32 into the above equation and rearranging results in

$$-\frac{\partial Q_x}{\partial x} \Delta x - \frac{\partial Q_y}{\partial y} \Delta y - \frac{\partial Q_z}{\partial z} \Delta z + G \Delta x \Delta y \Delta z = \rho c_p \Delta x \Delta y \Delta z \frac{\partial T}{\partial t} \quad (1.35)$$

The total heat transfer  $Q$  in each direction can be expressed as

$$\begin{aligned} Q_x &= \Delta y \Delta z q_x = -k_x \Delta y \Delta z \frac{\partial T}{\partial x} \\ Q_y &= \Delta x \Delta z q_y = -k_y \Delta x \Delta z \frac{\partial T}{\partial y} \\ Q_z &= \Delta x \Delta y q_z = -k_z \Delta x \Delta y \frac{\partial T}{\partial z} \end{aligned} \quad (1.36)$$

Substituting Equation 1.36 into Equation 1.35 and dividing by the volume,  $\Delta x \Delta y \Delta z$ , we get

$$\frac{\partial}{\partial x} \left[ k_x \frac{\partial T}{\partial x} \right] + \frac{\partial}{\partial y} \left[ k_y \frac{\partial T}{\partial y} \right] + \frac{\partial}{\partial z} \left[ k_z \frac{\partial T}{\partial z} \right] + G = \rho c_p \frac{\partial T}{\partial t} \quad (1.37)$$

Equation 1.37 is the transient heat conduction equation for a stationary system expressed in Cartesian coordinates. The thermal conductivity,  $k$ , in the above equation is a vector. In its most general form, the thermal conductivity can be expressed as a tensor, that is,

$$\mathbf{k} = \begin{bmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{yx} & k_{yy} & k_{yz} \\ k_{zx} & k_{zy} & k_{zz} \end{bmatrix} \quad (1.38)$$

The preceding equations, that is, 1.37 and 1.38 are valid for solving heat conduction problems in anisotropic materials with a directional variation in the thermal conductivities. In many situations, however, the thermal conductivity can be taken as a non-directional property, that is, isotropic. In such materials, the heat conduction equation is written as (constant thermal conductivity)

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{G}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad (1.39)$$

where  $\alpha = k/\rho c_p$  is the *thermal diffusivity*, which is an important parameter in transient heat conduction analysis.

If the analysis is restricted only to steady state heat conduction with no heat generation, the equation is reduced to

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0 \quad (1.40)$$

For a one-dimensional case, the steady state heat conduction equation is further reduced to

$$\frac{d}{dx} \left( k \frac{dT}{dx} \right) = 0 \quad (1.41)$$

The heat conduction equation for a cylindrical coordinate system is given by

$$\frac{1}{r} \frac{\partial}{\partial r} \left[ k_r r \frac{\partial T}{\partial r} \right] + \frac{1}{r^2} \frac{\partial}{\partial \phi} \left[ k_\phi \frac{\partial T}{\partial \phi} \right] + \frac{\partial}{\partial z} \left[ k_z \frac{\partial T}{\partial z} \right] + G = \rho c_p \frac{\partial T}{\partial t} \quad (1.42)$$

where the heat fluxes can be expressed as

$$\begin{aligned} q_r &= -k_r \frac{\partial T}{\partial r} \\ q_\phi &= -\frac{k_\phi}{r} \frac{\partial T}{\partial \phi} \\ q_z &= -k_z \frac{\partial T}{\partial z} \end{aligned} \quad (1.43)$$

The heat conduction equation for a spherical coordinate system is given by

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left[ k_r r^2 \frac{\partial T}{\partial r} \right] + \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \phi} \left[ k_\phi \frac{\partial T}{\partial \phi} \right] + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left[ k_\theta \sin \theta \frac{\partial T}{\partial \theta} \right] + G = \rho c_p \frac{\partial T}{\partial t} \quad (1.44)$$

where the heat fluxes can be expressed as

$$\begin{aligned} q_r &= -k_r \frac{\partial T}{\partial r} \\ q_\phi &= -\frac{k_\phi}{r \sin \theta} \frac{\partial T}{\partial \phi} \\ q_\theta &= -\frac{k_\theta}{r} \frac{\partial T}{\partial \theta} \end{aligned} \quad (1.45)$$

It should be noted that for both cylindrical and spherical coordinate systems, Equations 1.42 and 1.44 can be derived in a similar fashion as for Cartesian coordinates by considering the appropriate differential control volumes.

## 1.6 Boundary and Initial Conditions

The heat conduction equations, discussed in Section 1.5, will be complete for any problem only if the appropriate boundary and initial conditions are stated. With the necessary boundary and initial conditions, a solution to the heat conduction equations is possible. The boundary conditions for the conduction equation can be of two types or a combination of these—the *Dirichlet* condition, in which the temperature on the boundaries is known and/or the *Neumann* condition, in which the heat flux is imposed (see Figure 1.5):

*Dirichlet condition*

$$T = T_0 \text{ on } \Gamma_T \quad (1.46)$$

*Neumann condition*

$$q = -k \frac{\partial T}{\partial n} = C \text{ on } \Gamma_{qf} \quad (1.47)$$

In Equations 1.46 and 1.47,  $T_0$  is the prescribed temperature;  $\Gamma$  the boundary surface;  $n$  is the outward direction normal to the surface and  $C$  is the constant flux given. The insulated, or adiabatic, condition can be obtained by substituting  $C = 0$ . The convective heat transfer boundary condition also falls into the *Neumann* category and can be expressed as

$$-k \frac{\partial T}{\partial n} = h(T_w - T_a) \text{ on } \Gamma_{qc} \quad (1.48)$$

It should be observed that the heat conduction equation has second-order terms and hence requires two boundary conditions. Since time appears as a first-order term, only one initial value (i.e., at some instant of time all temperatures must be known) needs to be specified for the entire body, that is,

$$T = T_0 \text{ all over the domain } \Omega \text{ at } t = t_0 \quad (1.49)$$

where  $t_0$  is a reference time.

The constant, or variable temperature, conditions are generally easy to implement as temperature is a scalar. However, the implementation of surface fluxes is not as straight-

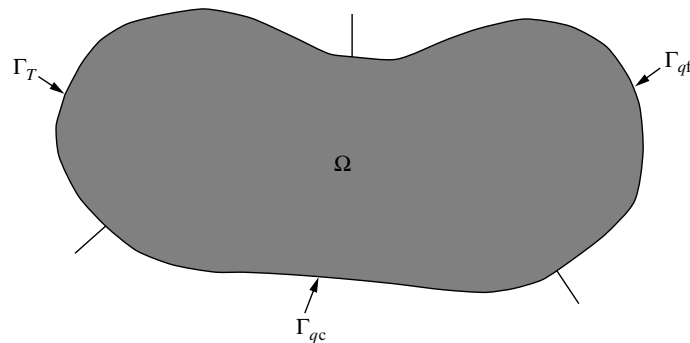


Figure 1.5 Boundary conditions

forward. Equation 1.47 can be rewritten with the direction cosines of the outward normals as

$$k_x \frac{\partial T}{\partial x} \tilde{l} + k_y \frac{\partial T}{\partial y} \tilde{m} + k_z \frac{\partial T}{\partial z} \tilde{n} = C \text{ on } \Gamma_{qf} \quad (1.50)$$

Similarly, Equation 1.48 can be rewritten as

$$k_x \frac{\partial T}{\partial x} \tilde{l} + k_y \frac{\partial T}{\partial y} \tilde{m} + k_z \frac{\partial T}{\partial z} \tilde{n} = h(T - T_a) \text{ on } \Gamma_{qc} \quad (1.51)$$

where  $\tilde{l}$ ,  $\tilde{m}$  and  $\tilde{n}$  are the direction cosines of the appropriate outward surface normals.

In many industrial applications, for example, wire drawing, crystal growth, continuous casting, and so on, the material will have a motion in space, and this motion may be restricted to one direction, as in the example (Section 1.4.3) cited previously. The general energy equation for heat conduction, taking into account the spatial motion of the body is given by

$$\frac{\partial}{\partial x} \left( k_x \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k_y \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k_z \frac{\partial T}{\partial z} \right) + G = \rho c_p \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right) \quad (1.52)$$

where  $u$ ,  $v$  and  $w$  are the components of the velocity in the three directions,  $x$ ,  $y$  and  $z$  respectively.

The governing equations for convection heat transfer are very similar to the above and will be discussed in Chapter 7.

## 1.7 Solution Methodology

Although a number of analytical solutions for conduction heat transfer problems are available (Carslaw and Jaeger 1959; Ozisik 1968), in many practical situations, the geometry and the boundary conditions are so complex that an analytical solution is not possible. Even if one could develop analytical relations for such complicated cases, these will invariably involve complex series solutions and would thus be practically difficult to implement. In such situations, conduction heat transfer problems do need a numerical solution. Some commonly employed numerical methods are the Finite Difference (Ozisik and Czisik 1994), Finite Volume (Patankar 1980), Finite Element and Boundary Elements (Ibanez and Power 2002) techniques. This text will address the issues related to the Finite Element Method (FEM) only (Comini *et al.* 1994; Huang and Usmani 1994; Lewis *et al.* 1996; Reddy and Gartling 2000).

In contrast to an analytical solution that allows for the temperature determination at any point in the medium, a numerical solution enables the determination of temperature only at discrete points. The first step in any numerical analysis must therefore be to select these points. This is done by dividing the region of interest into a number of smaller regions. These regions are bounded by points. These reference points are termed *nodal points* and their assembly results in a *grid* or *mesh*. It is important to note that each node represents a

certain region surrounding it, and its temperature is a measure of the temperature distribution in that region. The numerical accuracy of these calculations depends strongly on the number of designated nodal points, which control the number of elements generated. The accuracy approaches an exact value as the mesh size (region size) approaches zero.

Further details on the numerical methods, for example, accuracy and error will be discussed in later chapters.

## 1.8 Summary

In this chapter, the subject of heat transfer was introduced and various modes of heat transport were discussed. The fundamentals of energy conservation principles and the application of such principles to some selected problems were also presented. Finally, the general heat conduction equations in multi-dimensions were derived and the appropriate boundary and initial conditions were given. Although brief, we trust that this chapter gives the reader the essential fundamental concepts involved in heat transfer in general and some detailed understanding of conduction heat transfer in particular.

## 1.9 Exercise

**Exercise 1.9.1** *Extend the problem formulation of the plate subjected to a solar heat flux in Section 1.4.1 for a square plate. Assume the bottom surface of the plate is insulated.*

**Exercise 1.9.2** *Repeat the incandescent lamp problem of Section 1.4.2 but now assume that the light source is within an enclosure (room). Assume that the enclosure is also participating in conserving energy.*

**Exercise 1.9.3** *Derive the energy balance equations for a rectangular fin of variable cross section as shown in Figure 1.6. The fin is stationary and is attached to a hot heat source. (Hint: This is similar to the problem given in Section (1.4.3), but without relative motion).*

**Exercise 1.9.4** *Consider the respective control volumes in both cylindrical and spherical coordinates and derive the respective heat conduction equations. Verify these against Equations 1.42 and 1.44.*

**Exercise 1.9.5** *The inner body temperature of a healthy person remains constant at  $37^{\circ}\text{C}$ , while the temperature and humidity of the environment change. Explain, via heat transfer mechanisms between the human body and the environment, how the human body keeps itself cool in summer and warm in winter.*

**Exercise 1.9.6** *Discuss the modes of heat transfer that determine the equilibrium temperature of a space shuttle when it is in orbit. What happens when it re-enters the earth's atmosphere?*

**Exercise 1.9.7** *A closed plastic container, used to serve coffee in a seminar room, is made of two layers with an air gap placed between them. List all heat transfer processes associated*

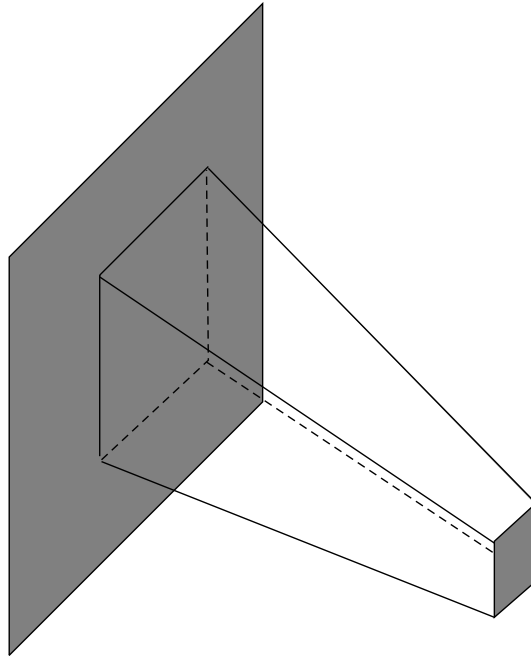


Figure 1.6 Rectangular fin

with the cooling of the coffee in the inner plastic vessel. What steps do you consider necessary for a better container design so as to reduce the heat loss to ambient.

**Exercise 1.9.8** A square chip of size 8 mm is mounted on a substrate with the top surface being exposed to a coolant flow at  $20^{\circ}\text{C}$ . All other surfaces of the chip are insulated. The chip temperature must not exceed  $80^{\circ}\text{C}$  in order for the chip to function properly. Determine the maximum allowable power that can be applied to the chip if the coolant is air with a heat transfer coefficient of  $250\text{ W/m}^2\text{K}$ . If the coolant is a dielectric liquid with a heat transfer coefficient of  $2500\text{ W/m}^2\text{K}$ , how much additional power can be dissipated as compared to air cooling?

**Exercise 1.9.9** Consider a person standing in a room that is at a temperature of  $21^{\circ}\text{C}$ . Determine the total heat rate from this person if the exposed surface area of the person is  $1.6\text{ m}^2$  and the average outer surface temperature of the person is  $30^{\circ}\text{C}$ . The convection coefficient from the surface of the person is  $5\text{ W/m}^2\text{ }^{\circ}\text{C}$ . What is the effect of radiation if the emissivity of the surface of the person is 0.90?

**Exercise 1.9.10** A thin metal plate has one large insulated surface and another large surface exposed to solar radiation at a rate of  $600\text{ W/m}^2$ . The surrounding air temperature is  $20^{\circ}\text{C}$ . Determine the equilibrium surface temperature of the plate if the convection heat transfer coefficient from the plate surface is  $20\text{ W/m}^2\text{K}$  and the emissivity of the top surface of the plate is 0.8.

**Exercise 1.9.11** A long, thin copper wire of radius  $r$  and length  $L$  has an electrical resistance of  $\rho$  per unit length. The wire is initially kept at a room temperature of  $T_a$  and subjected to an electric current flow of  $I$ . The heat generation due to the current flow is simultaneously lost to the ambient by convection. Set up an equation to determine the temperature of the wire as a function of time. Mention the assumptions made in the derivation of the equation.

**Exercise 1.9.12** In a continuous casting machine, the billet moves at a rate of  $u$  m/s. The hot billet is exposed to an ambient temperature of  $T_a$ . Set up an equation to find the temperature of the billet as a function of time in terms of the pertinent parameters. Assume that radiation also plays a role in the dissipation of heat to ambient.

**Exercise 1.9.13** In a double-pipe heat exchanger, hot fluid (mass flow  $M$  kg/s and specific heat  $c$  kJ/kg °C) flows inside a pipe and cold fluid (mass flow  $m$  kg/s and specific heat  $c$  kJ/kg °C) flows outside in the annular space. The hot fluid enters the heat exchanger at  $T_{h1}$  and leaves at  $T_{h2}$ , whereas the cold fluid enters at  $T_{c1}$  and leaves at  $T_{c2}$ . Set up the differential equation to determine the temperature variation (along the heat exchanger) for hot and cold fluids.

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