

CONTENTS

LIST OF SYMBOLS	xi
1 DELTA FUNCTIONS AND DISTRIBUTIONS	1
1.1 The δ -function	1
1.2 Test functions and distributions	3
1.3 Simple examples	4
1.4 Three-dimensional delta-functions	6
1.5 Delta-functions on lines and surfaces	7
1.6 Multiplication of distributions	10
1.7 Change of variables	10
1.8 The derivative of a distribution	12
1.9 Properties of the derivative	13
1.10 Partial derivatives of scalar functions	15
1.11 Derivatives of $\delta(x)$	17
1.12 Partial derivatives of δ -functions	19
1.13 Piecewise continuous scalar functions	21
1.14 Vector operators	22
1.15 Piecewise continuous vector functions	24
References	26
2 CONCENTRATED SOURCES	28
2.1 Scalar sources	28
2.2 Application to the electrostatic potential	30
2.3 Multipole expansion of an integral: the first term	32
2.4 Multipole expansion of an integral: the second term	33
2.5 Multipole expansion of an integral: the third term	34
2.6 Application to the magnetostatic vector potential	35
2.7 Radiation fields	37
2.8 Hierarchy of terms in the multipole expansion	38
2.9 Multipole expansion of the current	40
2.10 More examples of concentrated sources	41
2.11 Application to waveguides	44
2.12 Equivalence of electric and magnetic currents	46
2.13 The curl of a surface field	49
2.14 The equivalence of electric and magnetic surface sources	52
2.15 Boundary conditions	53
2.16 Multipole expansion for magnetic currents	55
2.17 Waveguide excitation by magnetic currents	56
References	58
3 GREEN'S DYADICS	59
3.1 Volume potential integrals	59
3.2 Single-layer potential	60

3.3	Double-layer potential	63
3.4	Extension to scalar radiation problems	65
3.5	Two-dimensional potential problems	66
3.6	First derivatives of the volume potential	68
3.7	First derivatives of the single- and double-layer potentials	69
3.8	First derivatives of the vector potential	71
3.9	Second derivatives of the volume potential	72
3.10	The \mathbf{L} dyadic	74
3.11	Green's dyadic for the electric field: exterior points	76
3.12	Interpretation of the elements of \mathbf{G}_e^0	79
3.13	Green's dyadic for the electric field: interior points	80
3.14	The full electric Green's dyadic \mathbf{G}_e	82
3.15	More on the evaluation of the electric field	83
3.16	The source term for \mathbf{G}_e	85
3.17	The Green's dyadic as a limit	88
3.18	Fields in material media	89
3.19	Green's dyadic for the magnetic field	91
3.20	Helmholtz' theorem in unbounded space	93
3.21	Green's dyadic for a cavity	94
3.22	Helmholtz' theorem in a bounded volume	97
3.23	Helmholtz' theorem in eigenvector form	98
3.24	Magnetic field generated by electric currents in a waveguide	100
3.25	Electric field generated by electric currents in a waveguide	103
3.26	Splitting the electric dyadic	105
3.27	Magnetic currents in a waveguide	107
3.28	The importance of the ψ_0 term	109
3.29	Derivation of the dyadics from a differential equation	110
	References	112
4	SINGULARITIES AT AN EDGE	116
4.1	The importance of understanding field behaviour at an edge	116
4.2	The singularity exponent near a perfectly conducting wedge	118
4.3	Fields, currents, and charges associated with a perfectly conducting wedge	121
4.4	On the possible absence of singularities	122
4.5	Further comments on the edge condition	124
4.6	Scattering by a semi-infinite, perfectly conducting plane	126
4.7	More confirmations of the edge behaviour	131
4.8	Uniqueness and the edge condition	137
4.9	Enforcing the edge condition: one-dimensional series	140
4.10	Enforcing the edge condition: one-dimensional subdomain functions	143
4.11	Enforcing the edge condition: two-dimensional subdomain functions	144
4.12	Enforcing the edge condition: two-dimensional subdomain functions	146
4.13	Singularities at a dielectric wedge. First symmetry	149
4.14	Singularities at a dielectric wedge. Second symmetry	152
4.15	Singularities at a magnetic wedge	153
4.16	Composite wedges	155
	References	160

5 SINGULARITIES AT THE TIP OF A CONE	163
5.1 Perfectly conducting circular cone. Electric singularity	163
5.2 Perfectly conducting circular cone. Magnetic singularity	167
5.3 Dielectric cone of circular cross-section	171
5.4 Metallic cone of arbitrary cross-section	175
5.5 Numerical procedure	178
5.6 The elliptic cone	179
5.7 The flat sector. Singularity of the electric field	183
5.8 The flat sector. Singularity of the magnetic field	190
5.9 The metallic pyramid	195
5.10 The sharp metallic cone	199
5.11 Dielectric cone of arbitrary cross-section	200
References	203
APPENDIX A DOUBLE LAYERS	206
A.1 Double layer of charge	206
A.2 Double layer of current	208
A.3 Boundary conditions at double current sheets	210
APPENDIX B DYADIC ANALYSIS	213
B.1 The dyadic notation	213
B.2 The transpose dyadic	214
B.3 Products	214
B.4 The identity dyadic	215
B.5 Operators	215
B.6 Integral formulas	216
B.7 Green's dyadics	216
APPENDIX C MODAL EXPANSION IN A CAVITY	218
APPENDIX D MODAL EXPANSION IN A WAVEGUIDE	222
APPENDIX E BASIS FUNCTIONS	226
APPENDIX F VARIATIONAL PRINCIPLES	229
AUTHOR INDEX	233
SUBJECT INDEX	235