

CONTENTS

PREFACE	xix
ACKNOWLEDGMENTS	xxiii
1 INTRODUCTION	1
1.1 Introduction / 1	
1.2 Direct and Indirect Measurements / 2	
1.3 Measurement Error Sources / 2	
1.4 Definitions / 3	
1.5 Precision versus Accuracy / 4	
1.6 Redundant Measurements in Surveying and Their Adjustment / 7	
1.7 Advantages of Least Squares Adjustment / 8	
1.8 Overview of the Book / 10	
Problems / 10	
2 OBSERVATIONS AND THEIR ANALYSIS	12
2.1 Introduction / 12	
2.2 Sample versus Population / 12	
2.3 Range and Median / 13	
2.4 Graphical Representation of Data / 14	
2.5 Numerical Methods of Describing Data / 17	
2.6 Measures of Central Tendency / 17	
2.7 Additional Definitions / 18	

- 2.8 Alternative Formula for Determining Variance / 21
- 2.9 Numerical Examples / 23
- 2.10 Derivation of the Sample Variance (Bessel's Correction) / 28
- 2.11 Programming / 29
- Problems / 30

3 RANDOM ERROR THEORY 33

- 3.1 Introduction / 33
- 3.2 Theory of Probability / 33
- 3.3 Properties of the Normal Distribution Curve / 36
- 3.4 Standard Normal Distribution Function / 38
- 3.5 Probability of the Standard Error / 41
 - 3.5.1 50% Probable Error / 42
 - 3.5.2 95% Probable Error / 42
 - 3.5.3 Other Percent Probable Errors / 43
- 3.6 Uses for Percent Errors / 43
- 3.7 Practical Examples / 44
- Problems / 47

4 CONFIDENCE INTERVALS 50

- 4.1 Introduction / 50
- 4.2 Distributions Used in Sampling Theory / 52
 - 4.2.1 χ^2 Distribution / 52
 - 4.2.2 t (Student) Distribution / 54
 - 4.2.3 F Distribution / 55
- 4.3 Confidence Interval for the Mean: t Statistic / 56
- 4.4 Testing the Validity of the Confidence Interval / 59
- 4.5 Selecting a Sample Size / 60
- 4.6 Confidence Interval for a Population Variance / 61
- 4.7 Confidence Interval for the Ratio of Two Population Variances / 63
- Problems / 65

5 STATISTICAL TESTING 68

- 5.1 Hypothesis Testing / 68
- 5.2 Systematic Development of a Test / 71
- 5.3 Test of Hypothesis for the Population Mean / 72
- 5.4 Test of Hypothesis for the Population Variance / 74

5.5	Test of Hypothesis for the Ratio of Two Population Variances / 77	
	Problems / 81	
6	PROPAGATION OF RANDOM ERRORS IN INDIRECTLY MEASURED QUANTITIES	84
6.1	Basic Error Propagation Equation / 84	
	6.1.1 Generic Example / 88	
6.2	Frequently Encountered Specific Functions / 88	
	6.2.1 Standard Deviation of a Sum / 88	
	6.2.2 Standard Deviation in a Series / 89	
	6.2.3 Standard Deviation of the Mean / 89	
6.3	Numerical Examples / 89	
6.4	Conclusions / 94	
	Problems / 95	
7	ERROR PROPAGATION IN ANGLE AND DISTANCE OBSERVATIONS	99
7.1	Introduction / 99	
7.2	Error Sources in Horizontal Angles / 99	
7.3	Reading Errors / 100	
	7.3.1 Angles Observed by the Repetition Method / 100	
	7.3.2 Angles Observed by the Directional Method / 101	
7.4	Pointing Errors / 102	
7.5	Estimated Pointing and Reading Errors with Total Stations / 103	
7.6	Target Centering Errors / 104	
7.7	Instrument Centering Errors / 106	
7.8	Effects of Leveling Errors in Angle Observations / 110	
7.9	Numerical Example of Combined Error Propagation in a Single Horizontal Angle / 112	
7.10	Use of Estimated Errors to Check Angular Misclosure in a Traverse / 114	
7.11	Errors in Astronomical Observations for an Azimuth / 116	
7.12	Errors in Electronic Distance Observations / 121	
7.13	Use of Computational Software / 123	
	Problems / 123	

8	ERROR PROPAGATION IN TRAVERSE SURVEYS	127
8.1	Introduction / 127	
8.2	Derivation of Estimated Error in Latitude and Departure / 128	
8.3	Derivation of Estimated Standard Errors in Course Azimuths / 129	
8.4	Computing and Analyzing Polygon Traverse Misclosure Errors / 130	
8.5	Computing and Analyzing Link Traverse Misclosure Errors / 135	
8.6	Conclusions / 140	
	Problems / 140	
9	ERROR PROPAGATION IN ELEVATION DETERMINATION	144
9.1	Introduction / 144	
9.2	Systematic Errors in Differential Leveling / 144	
9.2.1	Collimation Error / 144	
9.2.2	Earth Curvature and Refraction / 146	
9.2.3	Combined Effects of Systematic Errors on Elevation Differences / 147	
9.3	Random Errors in Differential Leveling / 148	
9.3.1	Reading Errors / 148	
9.3.2	Instrument Leveling Errors / 148	
9.3.3	Rod Plumbing Error / 148	
9.3.4	Estimated Errors in Differential Leveling / 150	
9.4	Error Propagation in Trigonometric Leveling / 152	
	Problems / 156	
10	WEIGHTS OF OBSERVATIONS	159
10.1	Introduction / 159	
10.2	Weighted Mean / 161	
10.3	Relation between Weights and Standard Errors / 163	
10.4	Statistics of Weighted Observations / 164	
10.4.1	Standard Deviation / 164	
10.4.2	Standard Error of Weight w and Standard Error of the Weighted Mean / 164	
10.5	Weights in Angle Observations / 165	
10.6	Weights in Differential Leveling / 166	

10.7 Practical Examples / 167
 Problems / 170

11 PRINCIPLES OF LEAST SQUARES 173

- 11.1 Introduction / 173
 - 11.2 Fundamental Principle of Least Squares / 174
 - 11.3 Fundamental Principle of Weighted Least Squares / 176
 - 11.4 Stochastic Model / 177
 - 11.5 Functional Model / 177
 - 11.6 Observation Equations / 179
 - 11.6.1 Elementary Example of Observation Equation Adjustment / 179
 - 11.7 Systematic Formulation of the Normal Equations / 181
 - 11.7.1 Equal-Weight Case / 181
 - 11.7.2 Weighted Case / 183
 - 11.7.3 Advantages of the Systematic Approach / 184
 - 11.8 Tabular Formation of the Normal Equations / 184
 - 11.9 Using Matrices to Form the Normal Equations / 185
 - 11.9.1 Equal-Weight Case / 185
 - 11.9.2 Weighted Case / 187
 - 11.10 Least Squares Solution of Nonlinear Systems / 188
 - 11.11 Least Squares Fit of Points to a Line or Curve / 191
 - 11.11.1 Fitting Data to a Straight Line / 192
 - 11.11.2 Fitting Data to a Parabola / 194
 - 11.12 Calibration of an EDM Instrument / 195
 - 11.13 Least Squares Adjustment Using Conditional Equations / 196
 - 11.14 Example 11.5 Using Observation Equations / 198
- Problems / 200

12 ADJUSTMENT OF LEVEL NETS 205

- 12.1 Introduction / 205
- 12.2 Observation Equation / 205
- 12.3 Unweighted Example / 206
- 12.4 Weighted Example / 209
- 12.5 Reference Standard Deviation / 211
 - 12.5.1 Unweighted Example / 212
 - 12.5.2 Weighted Example / 213

12.6 Another Weighted Adjustment / 213
Problems / 216

13 PRECISION OF INDIRECTLY DETERMINED QUANTITIES 221

13.1 Introduction / 221
13.2 Development of the Covariance Matrix / 221
13.3 Numerical Examples / 225
13.4 Standard Deviations of Computed Quantities / 226
Problems / 229

14 ADJUSTMENT OF HORIZONTAL SURVEYS: TRILATERATION 233

14.1 Introduction / 233
14.2 Distance Observation Equation / 235
14.3 Trilateration Adjustment Example / 237
14.4 Formulation of a Generalized Coefficient Matrix for a
More Complex Network / 243
14.5 Computer Solution of a Trilaterated Quadrilateral / 244
14.6 Iteration Termination / 248
 14.6.1 Method of Maximum Iterations / 249
 14.6.2 Maximum Correction / 249
 14.6.3 Monitoring the Adjustment's Reference
 Variance / 249
Problems / 250

15 ADJUSTMENT OF HORIZONTAL SURVEYS: TRIANGULATION 255

15.1 Introduction / 255
15.2 Azimuth Observation Equation / 255
 15.2.1 Linearization of the Azimuth Observation
 Equation / 256
15.3 Angle Observation Equation / 258
15.4 Adjustment of Intersections / 260
15.5 Adjustment of Resections / 265
 15.5.1 Computing Initial Approximations in the
 Resection Problem / 266
15.6 Adjustment of Triangulated Quadrilaterals / 271
Problems / 275

16	ADJUSTMENT OF HORIZONTAL SURVEYS: TRAVERSES AND NETWORKS	283
16.1	Introduction to Traverse Adjustments / 283	
16.2	Observation Equations / 283	
16.3	Redundant Equations / 284	
16.4	Numerical Example / 285	
16.5	Minimum Amount of Control / 291	
16.6	Adjustment of Networks / 291	
16.7	χ^2 Test: Goodness of Fit / 300	
	Problems / 301	
17	ADJUSTMENT OF GPS NETWORKS	310
17.1	Introduction / 310	
17.2	GPS Observations / 311	
17.3	GPS Errors and the Need for Adjustment / 314	
17.4	Reference Coordinate Systems for GPS Observations / 314	
17.5	Converting between the Terrestrial and Geodetic Coordinate Systems / 316	
17.6	Application of Least Squares in Processing GPS Data / 321	
17.7	Network Preadjustment Data Analysis / 322	
	17.7.1 Analysis of Fixed Baseline Measurements / 322	
	17.7.2 Analysis of Repeat Baseline Measurements / 324	
	17.7.3 Analysis of Loop Closures / 325	
	17.7.4 Minimally Constrained Adjustment / 326	
17.8	Least Squares Adjustment of GPS Networks / 327	
	Problems / 332	
18	COORDINATE TRANSFORMATIONS	345
18.1	Introduction / 345	
18.2	Two-Dimensional Conformal Coordinate Transformation / 345	
18.3	Equation Development / 346	
18.4	Application of Least Squares / 348	
18.5	Two-Dimensional Affine Coordinate Transformation / 350	
18.6	Two-Dimensional Projective Coordinate Transformation / 353	

- 18.7 Three-Dimensional Conformal Coordinate Transformation / 356
- 18.8 Statistically Valid Parameters / 362
- Problems / 364

19 ERROR ELLIPSE 369

- 19.1 Introduction / 369
- 19.2 Computation of Ellipse Orientation and Semiaxes / 371
- 19.3 Example Problem of Standard Error Ellipse Calculations / 376
 - 19.3.1 Error Ellipse for Station Wisconsin / 376
 - 19.3.2 Error Ellipse for Station Campus / 377
 - 19.3.3 Drawing the Standard Error Ellipse / 378
- 19.4 Another Example Problem / 378
- 19.5 Error Ellipse Confidence Level / 379
- 19.6 Error Ellipse Advantages / 381
 - 19.6.1 Survey Network Design / 381
 - 19.6.2 Example Network / 383
- 19.7 Other Measures of Station Uncertainty / 385
- Problems / 386

20 CONSTRAINT EQUATIONS 388

- 20.1 Introduction / 388
- 20.2 Adjustment of Control Station Coordinates / 388
- 20.3 Holding Control Station Coordinates and Directions of Lines Fixed in a Trilateration Adjustment / 394
 - 20.3.1 Holding the Direction of a Line Fixed by Elimination of Constraints / 395
- 20.4 Helmert's Method / 398
- 20.5 Redundancies in a Constrained Adjustment / 403
- 20.6 Enforcing Constraints through Weighting / 403
- Problems / 406

21 BLUNDER DETECTION IN HORIZONTAL NETWORKS 409

- 21.1 Introduction / 409
- 21.2 A Priori Methods for Detecting Blunders in Observations / 410
 - 21.2.1 Use of the K Matrix / 410
 - 21.2.2 Traverse Closure Checks / 411

- 21.3 A Posteriori Blunder Detection / 412
- 21.4 Development of the Covariance Matrix for the Residuals / 414
- 21.5 Detection of Outliers in Observations / 416
- 21.6 Techniques Used in Adjusting Control / 418
- 21.7 Data Set with Blunders / 420
- 21.8 Some Further Considerations / 428
 - 21.8.1 Internal Reliability / 429
 - 21.8.2 External Reliability / 429
- 21.9 Survey Design / 430
- Problems / 432

22 GENERAL LEAST SQUARES METHOD AND ITS APPLICATION TO CURVE FITTING AND COORDINATE TRANSFORMATIONS 437

- 22.1 Introduction to General Least Squares / 437
- 22.2 General Least Squares Equations for Fitting a Straight Line / 437
- 22.3 General Least Squares Solution / 439
- 22.4 Two-Dimensional Coordinate Transformation by General Least Squares / 443
 - 22.4.1 Two-Dimensional Conformal Coordinate Transformation / 444
 - 22.4.2 Two-Dimensional Affine Coordinate Transformation / 447
 - 22.4.3 Two-Dimensional Projective Transformation / 448
- 22.5 Three-Dimensional Conformal Coordinate Transformation by General Least Squares / 449
- Problems / 451

23 THREE-DIMENSIONAL GEODETIC NETWORK ADJUSTMENT 454

- 23.1 Introduction / 454
- 23.2 Linearization of Equations / 456
 - 23.2.1 Slant Distance Observations / 457
 - 23.2.2 Azimuth Observations / 457
 - 23.2.3 Vertical Angle Observations / 459
 - 23.2.4 Horizontal Angle Observations / 459
 - 23.2.5 Differential Leveling Observations / 460
 - 23.2.6 Horizontal Distance Observations / 460

- 23.3 Minimum Number of Constraints / 462
- 23.4 Example Adjustment / 462
 - 23.4.1 Addition of Slant Distances / 464
 - 23.4.2 Addition of Horizontal Angles / 465
 - 23.4.3 Addition of Zenith Angles / 466
 - 23.4.4 Addition of Observed Azimuths / 467
 - 23.4.5 Addition of Elevation Differences / 467
 - 23.4.6 Adjustment of Control Stations / 468
 - 23.4.7 Results of Adjustment / 469
 - 23.4.8 Updating Geodetic Coordinates / 469
- 23.5 Building an Adjustment / 471
- 23.6 Comments on Systematic Errors / 471
- Problems / 474

24 COMBINING GPS AND TERRESTRIAL OBSERVATIONS 478

- 24.1 Introduction / 478
- 24.2 Helmert Transformation / 480
- 24.3 Rotations between Coordinate Systems / 484
- 24.4 Combining GPS Baseline Vectors with Traditional Observations / 484
- 24.5 Other Considerations / 489
- Problems / 489

25 ANALYSIS OF ADJUSTMENTS 492

- 25.1 Introduction / 492
- 25.2 Basic Concepts, Residuals, and the Normal Distribution / 492
- 25.3 Goodness-of-Fit Test / 496
- 25.4 Comparison of Residual Plots / 499
- 25.5 Use of Statistical Blunder Detection / 501
- Problems / 502

26 COMPUTER OPTIMIZATION 504

- 26.1 Introduction / 504
- 26.2 Storage Optimization / 504
- 26.3 Direct Formation of the Normal Equations / 507
- 26.4 Cholesky Decomposition / 508
- 26.5 Forward and Back Solutions / 511

- 26.6 Using the Cholesky Factor to Find the Inverse of the Normal Matrix / 512
- 26.7 Sparseness and Optimization of the Normal Matrix / 513
Problems / 518

APPENDIX A INTRODUCTION TO MATRICES 520

- A.1 Introduction / 520
- A.2 Definition of a Matrix / 520
- A.3 Size or Dimensions of a Matrix / 521
- A.4 Types of Matrices / 522
- A.5 Matrix Equality / 523
- A.6 Addition or Subtraction of Matrices / 524
- A.7 Scalar Multiplication of a Matrix / 524
- A.8 Matrix Multiplication / 525
- A.9 Computer Algorithms for Matrix Operations / 528
- A.9.1 Addition or Subtraction of Two Matrices / 528
- A.9.2 Matrix Multiplication / 529
- A.10 Use of the MATRIX Software / 531
Problems / 531

APPENDIX B SOLUTION OF EQUATIONS BY MATRIX METHODS 534

- B.1 Introduction / 534
- B.2 Inverse Matrix / 534
- B.3 Inverse of a 2×2 Matrix / 535
- B.4 Inverses by Adjoins / 537
- B.5 Inverses by Row Transformations / 538
- B.6 Example Problem / 542
Problems / 543

APPENDIX C NONLINEAR EQUATIONS AND TAYLOR'S THEOREM 546

- C.1 Introduction / 546
- C.2 Taylor Series Linearization of Nonlinear Equations / 546
- C.3 Numerical Example / 547
- C.4 Using Matrices to Solve Nonlinear Equations / 549

C.5 Simple Matrix Example / 550

C.6 Practical Example / 551

Problems / 554

APPENDIX D NORMAL ERROR DISTRIBUTION CURVE AND OTHER STATISTICAL TABLES 556

D.1 Development of the Normal Distribution Curve Equation / 556

D.2 Other Statistical Tables / 564

D.2.1 χ^2 Distribution / 564

D.2.2 t Distribution / 566

D.2.3 F Distribution / 568

APPENDIX E CONFIDENCE INTERVALS FOR THE MEAN 576

APPENDIX F MAP PROJECTION COORDINATE SYSTEMS 582

F.1 Introduction / 582

F.2 Mathematics of the Lambert Conformal Conic Map Projection / 583

F.2.1 Zone Constants / 584

F.2.2 Direct Problem / 585

F.2.3 Inverse Problem / 585

F.3 Mathematics of the Transverse Mercator / 586

F.3.1 Zone Constants / 587

F.3.2 Direct Problem / 588

F.3.3 Inverse Problem / 588

F.4 Reduction of Observations / 590

F.4.1 Reduction of Distances / 590

F.4.2 Reduction of Geodetic Azimuths / 593

APPENDIX G COMPANION CD 595

G.1 Introduction / 595

G.2 File Formats and Memory Matters / 596

G.3 Software / 596

G.3.1 ADJUST / 596

G.3.2 STATS / 597

G.3.3 MATRIX / 598

G.3.4 Mathcad Worksheets / 598

G.4 Using the Software as an Instructional
Aid / 599

BIBLIOGRAPHY 600

INDEX 603

