

# Introduction to the Entry Level Mathematics (ELM)

The purpose of the Entry Level Mathematics is to determine whether you are prepared to undertake college-level work in mathematics. The results of the test will be used to place you in the appropriate course.

The ELM is composed of 50 multiple-choice questions; 45 of the questions actually count toward your score. You have 90 minutes to complete the test. The following areas are covered (but the questions will **not be labeled or in this order**):

Contents of the Entry Level Mathematics (ELM)	
Number Sense and Data	35% or about 15–20 Questions
Algebra	35% or about 15–20 Questions
Geometry	30% or about 13–17 Questions
<b>Total</b>	50 Questions

Total time for the ELM is 90 minutes or 1 hour and 30 minutes.

The number of questions and the types of questions may be adjusted slightly in later tests. Also note that the five “trial” questions could be scattered anywhere on the exam.

## Scoring the ELM

Remember, the ELM is a placement test. It uses “cut scores” to place you in appropriate mathematics classes. If you score at or above the cut score of 50 on the ELM, which is scored from 0–80, then you will be placed in regular, college-level math classes. If you score below 50, you will need to take remedial coursework in math. The subscores given for Number Sense and Data, Algebra, and Geometry will provide guidance in determining your remediation coursework.

## Basic Skills and Concepts You Should Be Familiar With

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The following list will give you an indication of the basic skills and topics you should be familiar with to pass the ELM. You may wish to use this official list of ELM Topics as a checklist when you begin your review.

### CSU ELM Topics

#### NUMBER SENSE AND DATA (approximately 35%)

- Carry out basic arithmetic calculations
- Understand and use percent in context
- Compare and order rational numbers expressed as fractions and/or decimals
- Solve problems involving fractions and/or decimals in context
- Interpret and use ratio and proportion in context
- Use estimation appropriately
- Evaluate the reasonableness of a solution to a problem
- Evaluate and estimate square roots
- Represent and understand data presented graphically (including pie charts, bar and line graphs, histograms, and other formats for presenting data visually used in print and electronic media)
- Calculate and understand the arithmetic mean
- Calculate and understand the median
- Make estimates and predictions based on data
- Distinguish between reasonable and unreasonable claims based on data

#### ALGEBRA (approximately 35%)

- Evaluate and interpret algebraic expressions
- Simplify algebraic expressions
- Express relationships among quantities using variables
- Use properties of exponents
- Perform polynomial arithmetic (add, subtract, multiply, divide, and factor)
- Perform arithmetic operations involving rational expressions
- Solve linear equations (with both numerical and literal coefficients)
- Solve systems of linear equations in two unknowns
- Solve linear inequalities
- Solve problems in context that are modeled by linear equations
- Solve quadratic and rational equations (with both numerical and literal coefficients; real solutions only)
- Solve problems in context that are modeled by quadratic equations
- Solve equations involving absolute value (in one variable)
- Solve inequalities involving absolute value (in one variable)
- Find and use slopes and intercepts of lines
- Use constant and average rates to solve problems in context (using appropriate units)

**GEOMETRY (approximately 30%)**

- Find the perimeter, area, or volume of geometric figures (including triangles, quadrilaterals, rectangular parallelepipeds, circles, cylinders, and combinations of these figures)
- Calculate the ratio of corresponding geometric measurements of similar figures (e.g., if the perimeters are in a 3:2 ratio, the areas are in a 9:4 ratio)
- Use the Pythagorean Theorem
- Use properties of congruent or similar geometric objects
- Solve geometric problems using the properties of basic geometric figures (including triangles, quadrilaterals, polygons, and circles)
- Determine angles in the plane (using properties of intersecting lines, parallel lines, and perpendicular lines)
- Identify and plot points on the number line
- Identify and plot points in the coordinate plane
- Plot points on the graph of a function determined by an algebraic expression
- Graph linear functions in one variable
- Graph quadratic functions in one variable
- Relate basic information about a function to features of its graph (e.g., linearity, positivity or negativity, increasing or decreasing)
- Find the length or midpoint of a line segment in the coordinate plane

## Strategies and Samples

Carefully review the following strategies and sample problems. They will help give you insight into how to approach many different question types.

### Circle or Underline

**Take advantage of being allowed to mark on the test booklet by always underlining or circling what you are looking for. This will ensure that you are answering the right question.**

### Samples

1. If  $x + 6 = 9$ , then  $3x + 1 =$

- A. 3
- B. 9
- C. 10
- D. 34
- E. 46

You should first circle or underline  $3x + 1$  because this is what you are solving for. Solving for  $x$  leaves  $x = 3$ , then substituting into  $3x + 1$  gives  $3(3) + 1$ , 10. The most common mistake is to solve for  $x$ , which is 3, and *mistakenly choose A* as your answer. But remember, you are solving for  $3x + 1$ , not just  $x$ . You should also notice that most of the other choices would all be possible answers if you made common or simple mistakes. *Make sure that you are answering the right question.* The correct answer is C.

2. Together, a hat and coat cost \$125. The coat costs \$25 more than the hat. What is the cost of the coat?

- A. \$25
- B. \$50
- C. \$75
- D. \$100
- E. \$125

The key words here are cost of the coat, so circle those words. To solve algebraically,

$$\begin{aligned} x &= \text{hat} \\ x + \$25 &= \text{coat (cost \$25 more than the hat)} \end{aligned}$$

Together they cost \$125.

$$\begin{aligned} (x + 25) + x &= 125 \\ 2x + 25 &= 125 \\ 2x &= 100 \\ x &= 50 \end{aligned}$$

But this is the cost of the hat. Notice that \$50 is one of the answer choices, B. Since  $x = 50$ , then  $x + 25 = 75$ . Therefore, the coat costs \$75, which is Choice C. *Always answer the question that is being asked.* Circling the key word or words will help you do that. The correct answer is C.

3. Which of the following is between  $\frac{1}{4}$  and 0.375?

- A. 0.0094
- B. 0.291
- C. 0.38
- D. 0.4
- E. 0.51

First underline the word “between.” Next simplify  $\frac{1}{4}$  to .25. A quick glance at the choices is valuable because it tips you off that you are working in decimals. Simply check which decimal is between .250 and .375. The correct answer is B—.291 is between .250 and .375. Notice that changing .25 to .250 makes the problem even easier. (Adding or eliminating zeros to the far right of a decimal doesn’t change the value of the number.)

## Pull Out Information

**Pulling information out of the word problem structure can often give you a better look at what you are working with; therefore, you gain additional insight into the problem.**

### Samples

1. Phil works for a furniture store  $\frac{3}{4}$  of the year and for a pool supply store for  $\frac{1}{6}$  of the same year. He takes a vacation for the remainder of the year. How much more time does he spend working for the furniture store than for the pool supply store?
- A. 1 month
  - B. 2 months
  - C. 7 months
  - D. 9 months
  - E. 11 months

First circle or underline “How much more time.” Next, pull out the information regarding Phil’s work for the year:

furniture store  $\frac{3}{4}$  of a year is  $\frac{3}{4} \times 12$ , or 9 months

pool supply store  $\frac{1}{6}$  of a year, or  $\frac{2}{12}$ , which is 2 months

The difference is 9 months minus 2 months, which is 7 months. The correct answer is C. Notice that taking a quick look at the answer choices is helpful, letting you know that the answer is in months.

2. If the ratio of boys to girls in a drama class is 2 to 1, which of the following is a possible number of students in the class?
- A. 10
  - B. 16
  - C. 19
  - D. 25
  - E. 30

First underline or circle “possible number of students.” Then, pulling out information gives you the following.

$$b:g = 2:1$$

Because the ratio of boys to girls is 2:1, the possible total number of students in the class must be a multiple of  $2 + 1$  (boys plus girls), or 3. The multiples of 3 are 3, 6, 9, 12, 15 and so on. Only Choice E, 30, is a multiple of 3.

## Work Forward

If you quickly see the method to solve the problem, then do the work. Work forward.

### Samples

1.  $\frac{10^{-3}}{10^{-6}} =$

- A.  $10^{-3}$
- B.  $10^{-2}$
- C.  $10^2$
- D.  $10^3$
- E.  $10^4$

This is a straightforward mechanical problem. You must know the rules for dividing numbers of the same base with exponents. When you divide numbers with exponents and the bases of the numbers are the same, then you keep the same base and subtract the exponents. For example,  $x^a$  divided by  $x^b$  is  $x^{a-b}$ .

In this case,  $= \frac{10^{-3}}{10^{-6}} = 10^{-3} \div 10^{-3}(-6) = 10^{-3+6} = 10^3$

The correct answer is D.

2. If  $|x| = 6$ , what is the value of  $x$ ?

- A.  $-6$  or  $0$
- B.  $-6$  or  $6$
- C.  $0$  or  $6$
- D.  $0$  or  $12$
- E.  $12$  or  $-12$

You can work this problem forward, using the definition of absolute value. If you know that absolute value refers to actual distance on a number line, and not direction, then it is evident that  $x$  can be  $-6$  or  $6$ . The correct answer is B.

You can also work this problem by plugging in the answers, but you still need to know how to work with absolute values.

3. If  $x = 3$  and  $y = 5$ , then  $6x^2 - 4y =$

- A. 16
- B. 34
- C. 54
- D. 60
- E. 64

To evaluate an expression, simply plug in the given numbers or values. These types of problems are usually easy to solve as long as you are careful in your calculations and understand the order of operations. Plugging in the values given for  $x$  and  $y$ :

$$\begin{aligned} 6x^2 - 4y &= \\ 6(3)^2 - 4(5) &= \\ 6(9) - 4(5) &= \\ 54 - 20 &= 34 \end{aligned}$$

The correct answer is B.

Remember, the order of operations is

**P**arentheses

**E**xponents

**M**ultiplication or **D**ivision

**A**ddition or **S**ubtraction

A good tool for remembering the order of operations is **PEMDAS**.

## Work Backward

In some instances, it will be easier to work from the answers. Do not disregard this method because it will at least eliminate some of the choices and could give you the correct answer.

### Samples

1. Which of the following points is the  $y$ -intercept of the line  $3x + 2y = 6$ ?

- A. (2, 0)
- B. (2, 1)
- C. (0, 2)
- D. (0, 3)
- E. (3, 0)

Probably the fastest method to answer this question is working from the answers. First, note that you are looking for the  $y$ -intercept. The  $y$ -intercept is where the line crosses the  $y$ -axis, so  $x$  must be 0. Eliminate choices A, B, and E because the  $x$  coordinate is not 0. Next, plug in choices C and D to see which is true for the equation.

Choice C (0, 2):

$$\begin{aligned} 3x + 2y &= 6 \\ 3(0) + 2(2) &? 6 \\ 0 + 4 &\neq 6 \end{aligned}$$

So you can eliminate C. The correct answer is D.

Choice D (0, 3):

$$\begin{aligned} 3x + 2y &= 6 \\ 3(0) + 2(3) &? 6 \\ 0 + 6 &= 6 \end{aligned}$$

Another method for solving this problem is to change the equation of the line to slope-intercept form,  $y = mx + b$ , where  $b$  is the  $y$ -intercept.

$$3x + 2y = 6$$

Subtract  $3x$  from each side.

$$\begin{array}{r} 3x + 2y = 6 \\ -3x \phantom{=} \phantom{=} \\ \hline 2y = -3x + 6 \end{array}$$

Divide both sides by 2.

$$\begin{aligned} \frac{2y}{2} &= \frac{-3x}{2} + \frac{6}{2} \\ y &= \left(\frac{-3}{2}\right)x + 3 \end{aligned}$$

So the  $y$ -intercept is 3.

- 2.** Mr. Tuchman can paint 30 surfboards in an hour. Mr. Christianson can paint 60 surfboards in an hour. If they are both painting surfboards, how long does it take them to paint a total of 45 surfboards?
- A. 30 minutes
  - B. 45 minutes
  - C. 60 minutes
  - D. 90 minutes
  - E. 120 minutes

You are looking for *how long it takes to paint 45 surfboards if both men are painting*. You can work this problem from the answers. First, try Choice A, 30 minutes. If Mr. Tuchman can paint 30 surfboards in an hour, then he can paint 15 surfboards in half an hour. If Mr. Christianson can paint 60 surfboards in an hour, then he can paint 30 surfboards in half an hour.

So, together they can paint 45 surfboards in 30 minutes. The correct answer is A.

You can also work this problem algebraically. If  $t$  is the number of hours, and Mr. Tuchman paints at a rate of 30 surfboards an hour, this can be expressed as  $30t$ . If Mr. Christianson paints at a rate of 60 surfboards an hour, then this can be expressed as  $60t$ . They both have to work to paint a total of 45 surfboards, so you can set up the equation  $30t + 60t = 45$ . Now, solve as follows:

$$\begin{aligned}30t + 60t &= 45 \\90t &= 45\end{aligned}$$

Dividing by 90 gives

$$t = \frac{45}{90}$$

So,  $t = \frac{1}{2}$  hour, or 30 minutes. The correct answer is A.

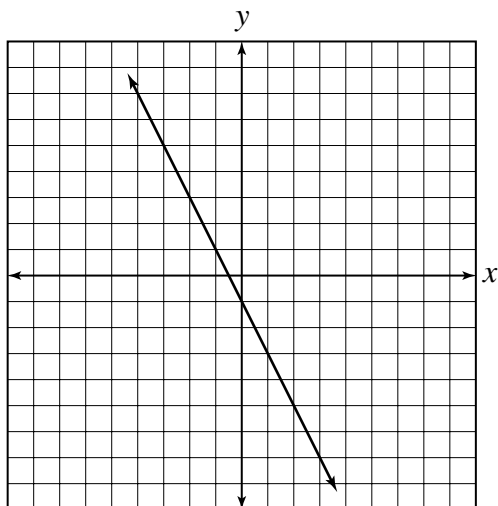
- 3.** The square root of 90 is between
- A. 9 and 10
  - B. 10 and 11
  - C. 11 and 12
  - D. 12 and 13
  - E. 13 and 14

This problem is most easily answered by working backward from the answers. Start with Choice A, 9 and 10. If you square 9, that is,  $9 \times 9$ , you get 81, which is below 90. Next, try squaring the second number, 10, and you get 100. Since 90 is between 81 and 100, the square root of 90 is between 9 and 10. The correct answer is A.

You can work this problem forward by approximating the square root of 90. First, find the closest perfect square number below 90. That is 81. Next, find the closest perfect square number above 90, which is 100.

Since  $\sqrt{90}$  is between  $\sqrt{81}$  and  $\sqrt{100}$ , it falls somewhere between 9 and 10.

$$\sqrt{81} < \sqrt{90} < \sqrt{100}$$



4. Which of the following is the equation of the line shown in the preceding graph?

- A.  $y = 2x + 1$
- B.  $y = 2x + 2$
- C.  $y = -2x - 1$
- D.  $y = -2x - 2$
- E.  $y = -2x - 3$

A quick look at the choices lets you eliminate some by looking for the  $y$ -intercept first. Since the line crosses the  $y$ -axis at  $-1$ , the  $y$ -intercept on the graph is  $-1$ . You can eliminate choices A, B, D, and E. The correct answer is C.

In this question, you don't even need to deal with the slopes. But if you are working with the slopes, you can eliminate choices A and B immediately because they are positive. Since the line goes down to the right, the slope is negative.

## Substitute Simple Numbers

Substituting numbers for variables can often be an aid to understanding a problem. Remember to substitute simple numbers, since you have to do the work.

### Sample

1. If  $x > 1$ , which of the following decreases as  $x$  increases?

- A.  $x + x^2$
- B.  $2x^2 - x$
- C.  $2x - x^2$
- D.  $x^3 - 3x$
- E.  $x^4 - 4x$

This problem is most easily solved by taking each situation and substituting in simple numbers. However, in the first situation, A,  $x + x^2$ , you should recognize that this expression will increase as  $x$  increases. So you can eliminate A.

Next, Choice B. Trying  $x = 2$  in the expression  $2x^2 - x$  gives  $2(2)^2 - 2$ , or  $2(4) - 2 = 6$ .

Now trying  $x = 3$  in the expression gives  $2(3)^2 - 3 = 2(9) - 3 = 18 - 3 = 15$ .

This expression also increases as  $x$  increases. Therefore, eliminate B.

Next, Choice C.

Trying  $x = 2$  gives  $2(2) - (2)^2 = 4 - 4 = 0$ .

Now trying  $x = 3$  gives  $2(3) - (3)^2 = 6 - 9 = -3$ .

So the correct answer is C.

Be sure to make logical substitutions. Use a positive number, a negative number, or zero when applicable to get the full picture.

## Be Reasonable

Sometimes you will immediately recognize a simple method to solve a problem. If this is not the case, try a reasonable approach and then check the answers to see which one is most reasonable.

### Samples

1. Barney can mow the lawn in 5 hours, and Fred can mow the lawn in 4 hours. How long will it take them to mow the lawn together?
- A. 5 hours
  - B.  $4\frac{1}{2}$  hours
  - C. 4 hours
  - D.  $2\frac{2}{9}$  hours
  - E. 1 hour

Suppose that you are unfamiliar with the type of equation for this problem. Try the “reasonable” method. Since Fred can mow the lawn in 4 hours by himself, it will take less than 4 hours if Barney helps him. Therefore, choices A, B, and C are ridiculous. Taking this method a little further, suppose that Barney could also mow the lawn in 4 hours. Then, together it would take Barney and Fred 2 hours. But since Barney is a little slower than this, the total time should be a little more than 2 hours. The correct answer is D,  $2\frac{2}{9}$  hours.

Using the equation for this problem would give the following calculations:

$$\frac{1}{5} + \frac{1}{4} = \frac{1}{x}$$

In 1 hour, Barney could do  $\frac{1}{5}$  of the job, and in 1 hour, Fred could do  $\frac{1}{4}$  of the job. Unknown  $\frac{1}{x}$  is that part of the job they could do together in 1 hour. Now solving, you calculate as follows:

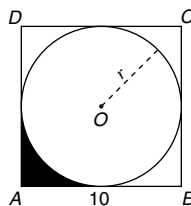
$$\frac{4}{20} + \frac{5}{20} = \frac{1}{x}$$

Cross multiplying gives

$$9x = 20$$

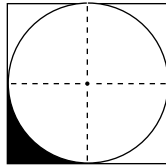
Therefore,

$$x = \frac{20}{9} \text{ or } 2\frac{2}{9}$$



2. Circle  $O$  is inscribed in square  $ABCD$  as shown above. The area of the shaded region is approximately
- A. 10
  - B. 25
  - C. 30
  - D. 50
  - E. 75

Using a reasonable approach, you would first find the area of the square:  $10 \times 10 = 100$ . Then divide the square into four equal sections as follows:



Since a quarter of the square is 25, then the shaded region must be much less than 25. The only possible answer is Choice A, 10.

Another approach to this problem would be to first find the area of the square:  $10 \times 10 = 100$ . Then subtract the approximate area of the circle:

$$A = \pi(r^2) \cong 3(5^2) = 3(25) = 75.$$

Therefore, the total area inside the square but outside the circle is approximately 25. One quarter of that area is shaded. Therefore,  $\frac{25}{4}$  is approximately the shaded area. The closest answer is A, 10.

## Sketch a Diagram

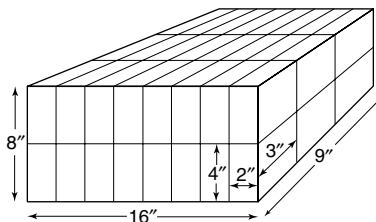
Sketching diagrams or simple pictures can also be very helpful in problem solving because the diagram may tip off either a simple solution or a method for solving the problem.

## Samples

1. What is the maximum number of milk cartons, each 2" wide by 3" long by 4" tall, that can be fit into a cardboard box with inside dimensions of 16" wide by 9" long by 8" tall?
- A. 12  
 B. 18  
 C. 20  
 D. 24  
 E. 48

Drawing a diagram, as shown below, may be helpful in envisioning the process of fitting the cartons into the box. Notice that eight cartons will fit across the box, three cartons deep, and two "stacks" high:

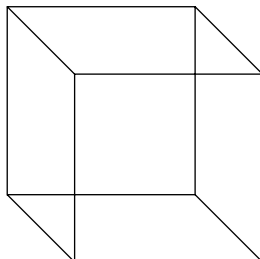
$$8 \times 3 \times 2 = 48 \text{ cartons}$$



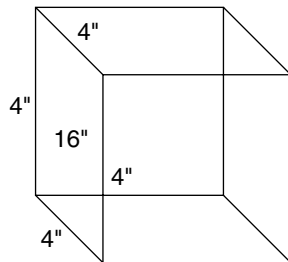
The correct answer is E.

2. If the area of one face of a cube is 16 square inches, what is the volume of the cube in cubic inches?
- A. 8  
 B. 12  
 C. 24  
 D. 64  
 E. 96

First underline or circle the word "volume." Now draw a cube.



Next, label one face of the cube. This helps you determine that each edge of the cube is 4 inches because the face of a cube is square and all edges are equal.



The formula for the volume of a cube is length times width times height, or  $V = lwh$ .

So the volume is  $4 \times 4 \times 4 = 64$ , which is Choice D.

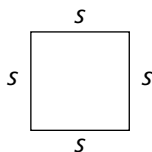
Keep in mind that in a cube all edges are the same length and all six sides have the same area.

**3.** If all sides of a square are doubled, the area of that square:

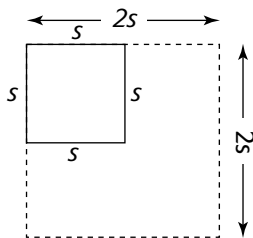
- A. is doubled.
- B. is tripled.
- C. is multiplied by 4.
- D. remains the same.
- E. is multiplied by 8

One way to solve this problem is to draw a square and then double all its sides. Then compare the two areas.

Your first diagram



Doubling every side

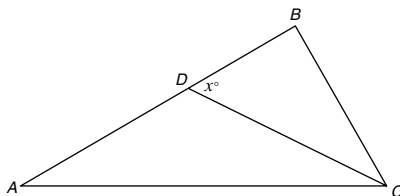


You can see that the total area of the new square will now be four times the original square. The correct answer is C.

## Mark in Diagrams

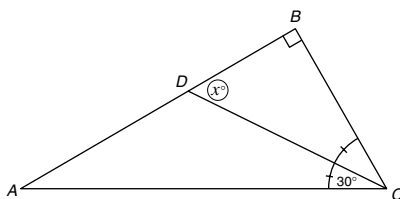
Marking in or labeling diagrams as you read the questions can save you valuable time. Marking can also give you insight into how to solve a problem because you will have the complete picture clearly in front of you.

### Samples



1. In the triangle above,  $CD$  is an angle bisector, angle  $ACD$  is  $30^\circ$ , and angle  $ABC$  is a right angle. What is the measurement of angle  $x$  in degrees?
- A.  $30^\circ$
  - B.  $45^\circ$
  - C.  $60^\circ$
  - D.  $75^\circ$
  - E.  $80^\circ$

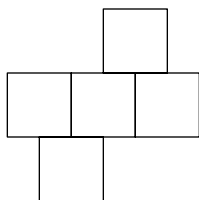
You should have read the problem and marked as follows: In the triangle above,  $CD$  is an angle bisector (*stop and mark in the drawing*), angle  $ACD$  is  $30^\circ$  (*stop and mark in the drawing*), and angle  $ABC$  is a right angle (*stop and mark in the drawing*). What is the measurement of angle  $x$  in degrees? (*Stop and mark in or circle what you are looking for in the drawing.*)



Now, with the drawing marked in, it is evident that, since angle  $ACD$  is  $30^\circ$ , then angle  $BCD$  is also  $30^\circ$  because they are formed by an angle bisector (divides an angle into two equal parts). Since angle  $ABC$  is  $90^\circ$  (right angle) and angle  $BCD$  is  $30^\circ$ , then angle  $x$  is  $60^\circ$  because there are  $180^\circ$  in a triangle.

$$180 - (90 + 30) = 60$$

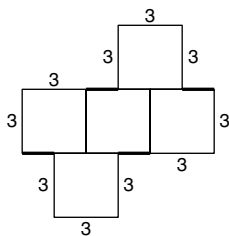
The correct answer is C. Always mark in diagrams as you read descriptions and information about them. This includes what you are looking for.



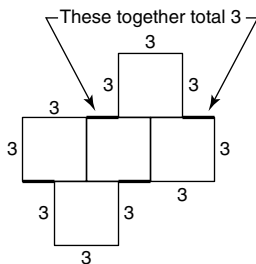
2. If each square in the figure above has a side of length 3, what is the perimeter?

- A. 12
- B. 14
- C. 21
- D. 30
- E. 36

Mark the known facts.

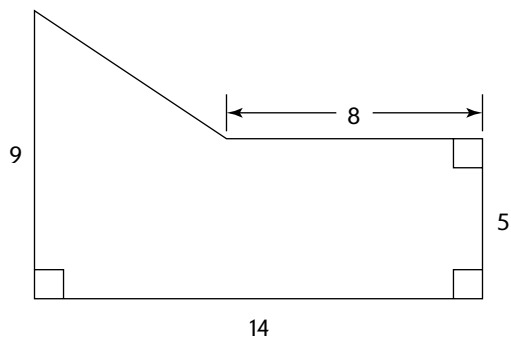


We now have a calculation for the perimeter: 30 plus the darkened part. Now look carefully at the top two darkened parts. They will add up to 3. (Notice how the top square may slide over to illustrate that fact.)



The same is true for the bottom darkened parts. They will add up to 3.

Thus, the total perimeter is  $30 + 6 = 36$ , Choice E.



3. What is the area of the preceding figure in square units?

- A. 60
- B. 70
- C. 82
- D. 94
- E. 104

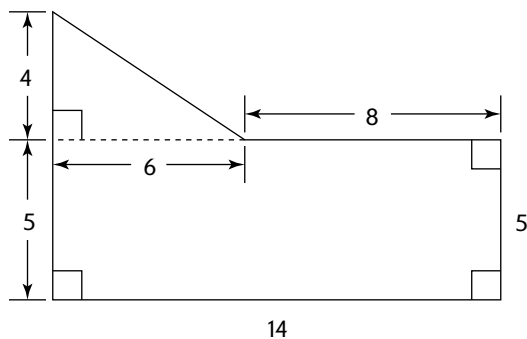
Underline or circle the words area of the figure. Since this is an irregular figure, use common shapes to work it out.

Start by finding the area of the rectangle, which is length times width.

$$14 \times 5 = 70$$

So, the area of the rectangle is 70 square units. At this point, you can eliminate choices A and B. The area must be greater than 70.

Now, you need to find the dimensions of the triangle. By using the dimensions given and subtracting from the length, you get the following dimensions:



You only need the base and height of the triangle to find the area. The height is 4 and the base is 6.

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(6)(4)$$

$$A = \frac{1}{2}(24) = 12$$

Now add the areas of the rectangle and triangle,  $70 + 12 = 82$ . The correct answer is C.

## Approximate

If it appears that extensive calculations are going to be necessary to solve a problem, check to see how far apart the choices are and then approximate. The reason for checking the answers first is to give you a guide to how freely you can approximate.

## Sample

1. Which is the best estimate of  $931 \times 311$ ?

- A. 2,700
- B. 27,000
- C. 270,000
- D. 2,700,000
- E. 27,000,000

First, check the answer choices to see how far apart they are. This gives you an indication of how close your approximation should be. In this case, the choices are far apart, so your approximation does not need to be too accurate. As a matter of fact, these choices differ only by the number of zeros after 27. So be careful that you have the correct number of zeros in your estimate.

Round each number to the nearest hundred.

$$\begin{array}{r} 932 \times 311 \\ \downarrow \quad \downarrow \\ 900 \times 300 = 270,000 \end{array}$$

So the best estimate is 270,000. The correct answer is C.